Technical University of Ilmenau

Department of Automation Engineering

AT.325.EN

System Identification

Date: Friday, August 10th, 2018

Time: 10:00 am

Location: K-Hs 1

Duration: 2 hours

Version: AT.325.EN.2018.a.SAMPLE

Instructor: Prof. Yuri A.W. Shardt

Student's Name: ________________SAMPLE FINAL__________________________

Student's ID Number: ____

Degree Programme: ____

Student's Signature: ____

General Instructions

1) Address all inquiries to the supervisor. Do not communicate with other candidates.
2) Please write your ID number on each page.
3) Please write all answers in the allocated space.
4) Should you need additional paper, please ask the invigilators for it. Please place your ID number on all pages.
5) Should you feel ill, please inform the supervisor **before the start of the examination** and leave the examination room immediately.
6) Please do not cheat.
7) A regular calculator is permitted. All other electronic devices (including cell phones, smart watches, and computers) are strictly forbidden.
8) The examination is open book, that means that you are permitted to use your own course notes, a dictionary, and a copy of the textbook.
9) The date and time when you can review your examination will be posted on the departmental website: http://tu-ilmenau.de/en/dept-automation/.

Total Pages: 10 (including this cover page)

Department of Automation Engineering

AT.325.EN System Identification

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Final Mark: ____/155 Marks

Question 1 (30 marks): True or False

Please write for each of the following statements "true" or "false" with justifications. Please only use the words "true" or "false". (One point for true/false and 2 points for the justification.)

a) The national animal of Canada is the beaver.
b) If H_1 is rejected when performing a statistical test, this implies that H_0 must be true.
c) As the number of parameters in a model increase, the value of $R^2 \to 0$.
d) In linear regression, if the confidence interval for a parameter covers zero, then it should be excluded from the model.
e) Adding replicates to a model can improve the regression results.
f) If we are performing a 3^3 experiment with two replicates, then a total of 54 experiments will be performed.
g) If the complete defining relation is I = ABCG = ABDHE, then we have a Resolution V design.
h) Performing a central composite design experiment allows the determination of curvature in the model.
i) If the residuals have a pattern, the it can be concluded that the model is poor and should be revised.
j) The response surface methodology seeks to determine the saddle points of a system.

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Question 2 (40 marks): Design of Experiments I

Consider the following experiment for optimising the production of rubber in a factory. Two factors were considered: x_2, which represents temperature in °C and has been scaled as $x_2 = (T - 250)/30$ and x_1, which represents reaction time in hours and has been scaled as $x_1 = (t - 12)/8$. Based on the questions and data below, perform a detailed analysis of the given problem to determine what the optimal conditions are.

The model fit can be written as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2$$

and the data are given in Table 1.

Table 1: Data for the rubber question

y	x_1	x_2
83.8	-1	0
81.7	1	0
82.4	0	0
82.9	0	0
84.7	0	-1
75.9	0	1
81.2	0	0
81.3	-1.4142	-1.4142
83.1	-1.4142	1.41421
85.3	1.41421	-1.4142
72.7	1.41421	1.41421
82	0	0

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0.16346	0	0	0	-0.0481	-0.0481
0	0.1	0	0	0	0
0	0	0.1	0	0	0
0	0	0	0.0625	0	0
-0.0481	0	0	0	0.27885	-0.2212
-0.0481	0	0	0	-0.2212	0.27885

Figure 1: $(\mathcal{A}^T\mathcal{A})^{-1}$ matrix

β	82.0	-1.115	-2.407	-1.800	0.861	-1.589
$\pm\delta$	1.38	1.076	1.076	0.851	1.797	1.797

Figure 2: Parameter estimates and their confidence intervals

tinv(α, 0.05)	2.446912
TSS	144.4367
SSR	132.8342
F-critical, model	4.387374

Figure 3: Additional useful information about the model

a) What type of design is this?

b) Are there any replicates?

c) Is this an orthogonal design?

d) Can the *F*-test for the parameters be used?

e) Based on the provided data, determine which parameters are significant. Would you need to refit the data in order to obtain new parameter estimates?

f) Determine R^2.

g) Analysis the residual plots shown in Figure 4 and Figure 5.

h) Would you conclude that the complete model is good?

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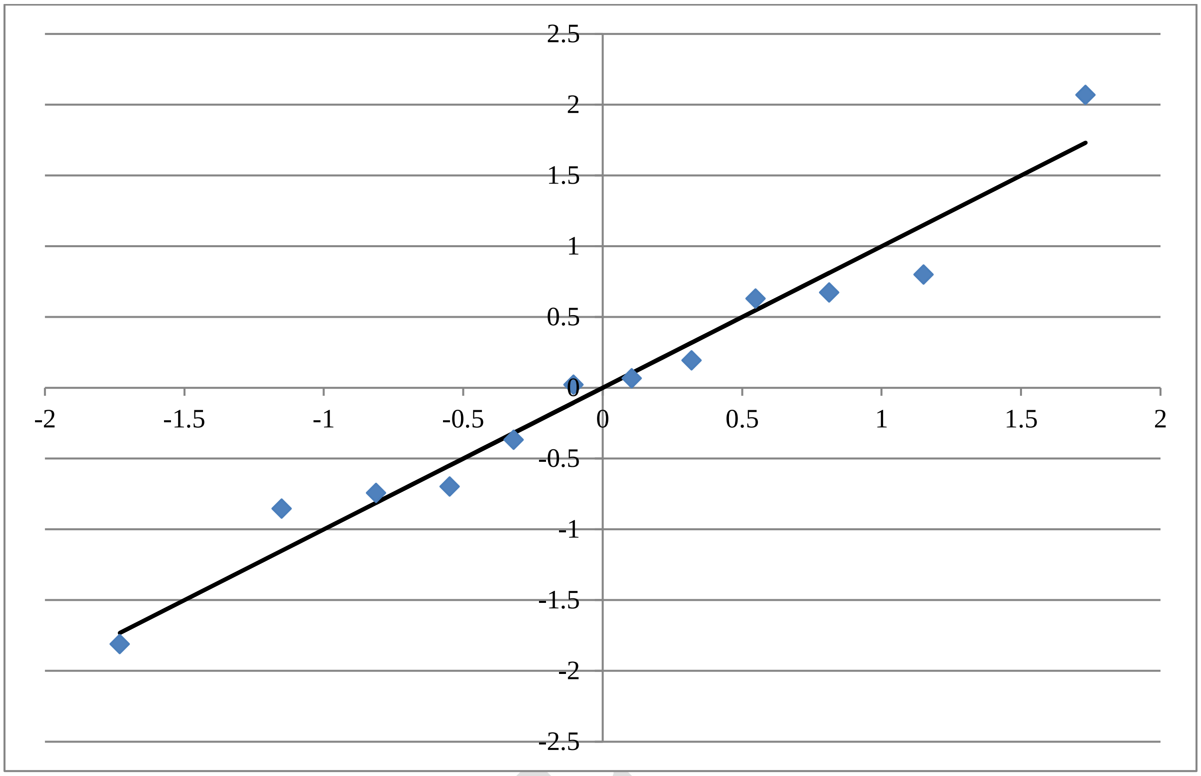

Figure 4: Normplot of the residuals

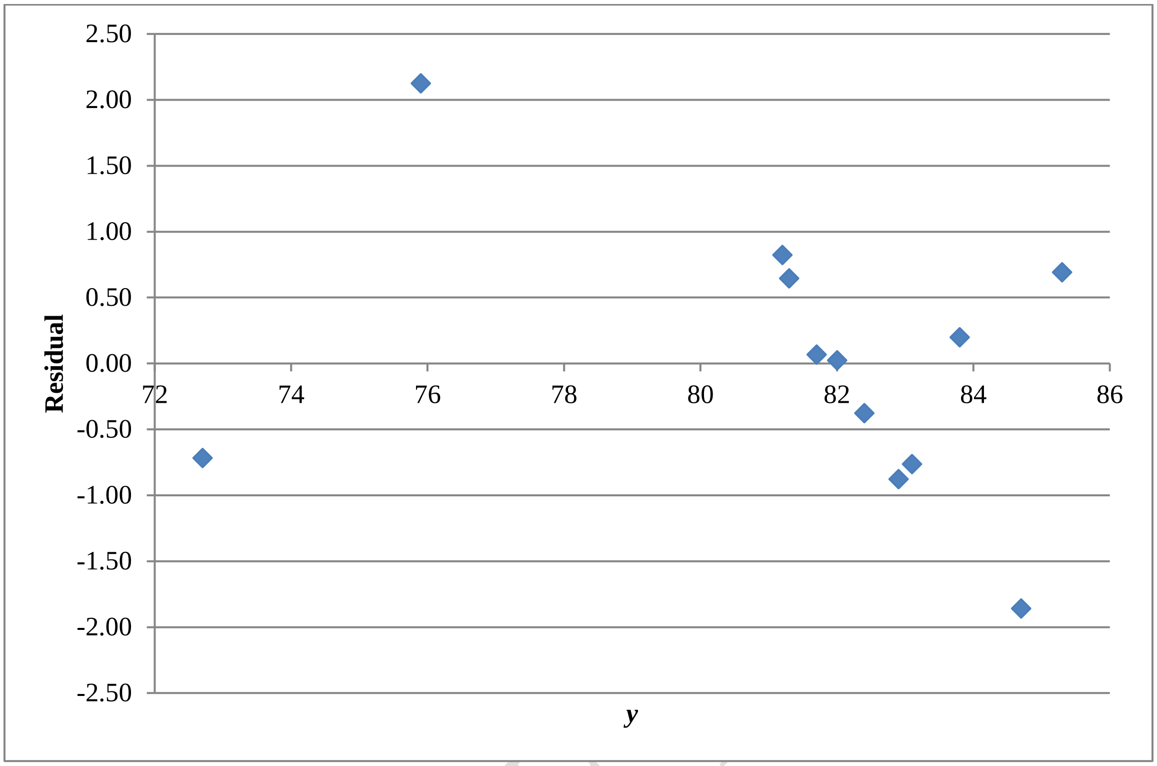

Figure 5: Residuals as a function of $\hat{y}$

Question 3 (20 marks): Design of Experiments II

You have to run a fractional factorial experiment where there are 6 (A, B, C, D, E, and F) factors each at two different levels. You have decided that a 2^{6-2} fractional factorial experiment will be performed with the following two generators:

$$I = \text{ACD}$$

$$I = \text{CDF}$$

Answer the following questions:

a) What is the complete defining relation?
b) What is the confounding pattern for all the first-order interactions?
c) What is the resolution of this experiment?
d) If a resolution of IV was desired, how could the above generators be changed to achieve this? Give the complete defining relation to prove that the design is indeed resolution IV.

Question 4 (20 marks): Design of Experiments III

You ran a 4^2 full factorial experiment. Not wanting to learn the material in §4.7 in the textbook regarding the development of an orthogonal basis for such an experiment, you decided to analysis this experiment as a 2-level factorial experiment. Answer the following questions:

a) What model can you fit with the original 4^2 experiment?

b) Clearly explain how you could analysis this experiment as a 2-level factorial experiment.

c) Can you fit the original model using this type of analysis?

Question 5 (15 marks): The Theory of Statistics

Consider the following probability density function

$$f\left(x\right)=\begin{cases}0.5\sin\left(x\right) & 0\le x\le\pi\\ 0 & \text{otherwise}\end{cases}$$

and answer the following questions:

a) Show that *f*(*x*) is indeed a probability density function.

b) Compute the mean value of this distribution.

c) If a parameter ψ is well-described by this distribution, compute the 95% confidence interval for ψ given $\hat{\psi}=6$ and $\hat{\sigma}_{\hat{\psi}}=1.0$. Could the true value of ψ be 5.5?

d) After sampling 200 times a distribution claimed to be equal to the one given above, the mean value was obtained to be 1.1 with a variance of 0.5. Formally test at the 95% level (α = 0.05) whether or not this result equals the true value obtained in b). Note that you should clearly state what the null and alternative hypotheses are and what test needs to be performed. Useful information: Z0.95 = 1.64 and Z0.975 = 1.96 (all left probabilities).

e) Compute P(0.25 < x < 0.5) for this probability density function.

Question 6 (25 marks): Regression Analysis

Consider the problem of fitting the Antoine equation

$$P^{vap}=\exp\left(\theta_0-\frac{\theta_1}{\theta_2+T}\right)$$

where P^{vap} is the vapour pressure of the compound corresponding to the given temperature, *T*, and *A*, *B*, and *C* are parameters to be determined. Consider the attached Excel spreadsheet for the problem and answer the following questions:

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a) Is the model linear or nonlinear? How do you know this?

b) Compute the derivatives required for forming the $\mathcal{J}$ matrix.

c) Are the parameter estimates significant?

d) What is the standard deviation of the model?

e) Compute R^2.

f) Compute the F-statistic. Is the F-test significant?

g) Consider the residuals shown in Figure 7. Assess the validity of the assumptions on the residuals.

h) Based on the above analysis, would you consider the model to be good?

i) Linearise the model and set up the linear, regression matrices?

j) Given the residuals for the linearised model shown in Figure 8, what can you conclude about the linearised model?

T (°C)	Pvap (mm Hg)	∂P/∂θ0	∂P/∂θ1	∂P/∂θ2	ŷ	residuals	r2	1 V
-4.4	5	4.996	-0.023	0.336	5.00	-0.0043	0.0000	1
6.4	10	9.970	-0.044	0.607	9.97	-0.0301	0.0009	1
18.4	20	19.951	-0.084	1.095	19.95	-0.0492	0.0024	1
31.8	40	40.010	-0.160	1.966	40.01	0.0097	0.0001	1
40.3	60	59.925	-0.232	2.754	59.92	-0.0752	0.0057	1
51.9	100	99.816	-0.370	4.202	99.82	-0.1843	0.0340	1
69.5	200	200.104	-0.696	7.424	200.10	0.1044	0.0109	1
89.5	400	400.431	-1.302	12.985	400.43	0.4307	0.1855	1
110.6	760	759.642	-2.312	21.572	759.64	-0.3585	0.1285	1
136.5	1502	1502.068	-4.238	36.649	1502.07	0.0680	0.0046	1
						sum	0.372569572	
						tinv(0.05)	2.364624251	
						TSS	2090188.1	
						SSR	2090187.727	
						F-critical	4.346831402	

Information Matrix		
0.0131	8.49317	0.44604
8.49317	5517.98	290.436
0.44604	290.436	15.3255

Parameter		Interval
θ0	15.9631	0.06244
θ1	3065.53	40.5235
θ2	217.959	2.13562

m	10
n	3

Figure 6: Excel data for Question 3 for the nonlinear model

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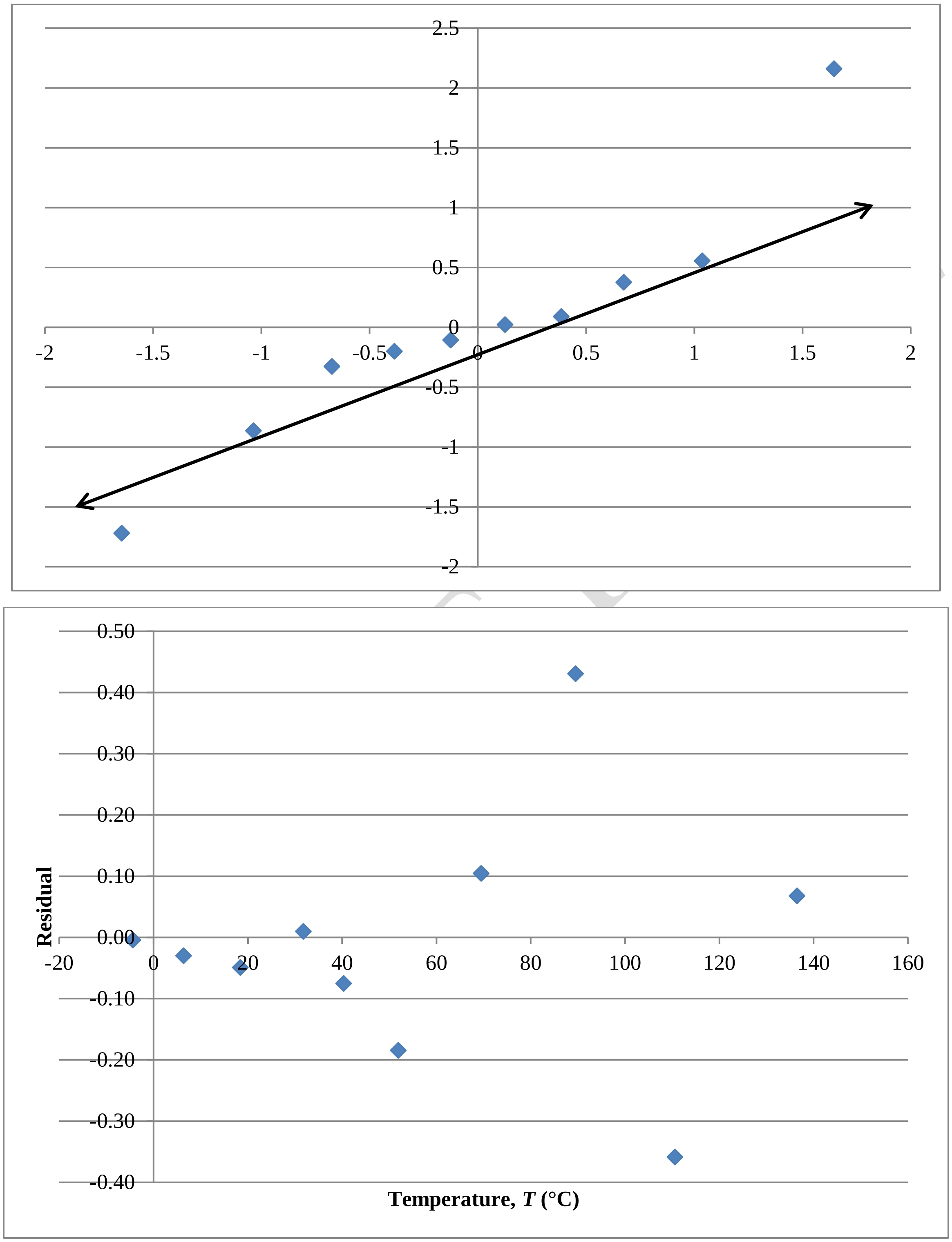

Figure 7: (top) Normplot of the residuals and (bottom) residuals as a function of temperature for the nonlinear model

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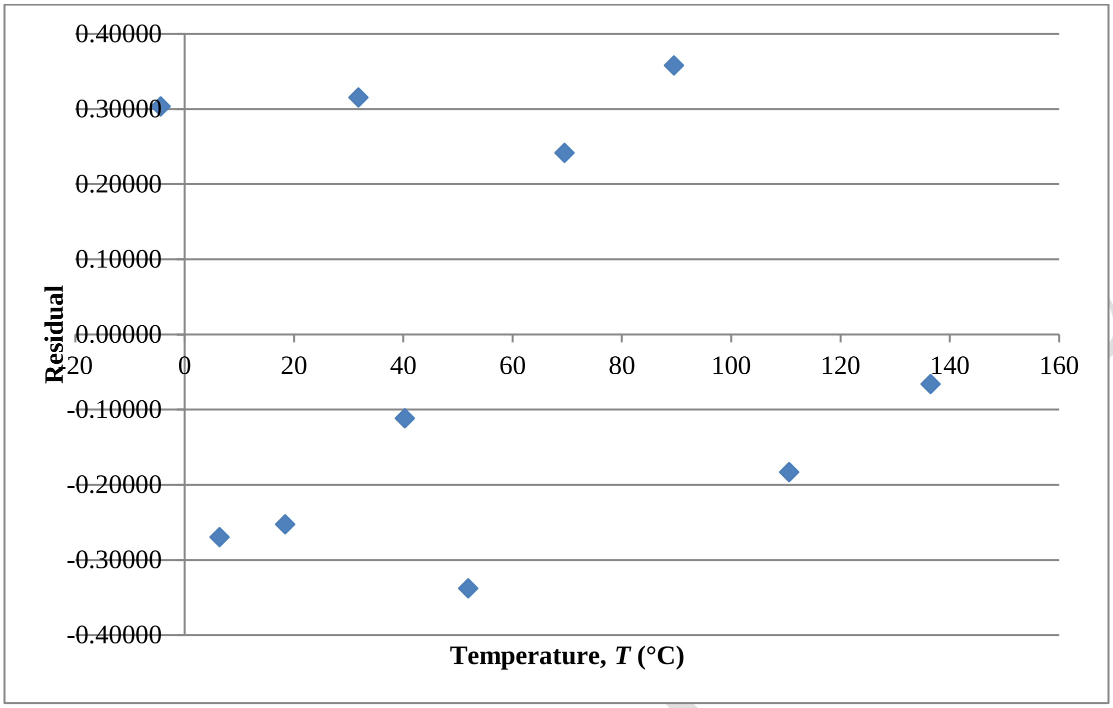

Figure 8: Residuals as a function of the temperature for the linearised model