Cycles, Convexity, and Searching in Graphs

Summary We want to make progress on several joint research projects which were initiated during a three months visit of **Professor Dr. Dieter Rautenbach (TU Ilmenau)** and **Lucia Draque Penso, Ph.D. (TU Ilmenau)** as guest researchers at the UFRJ in Rio de Janeiro at the beginning of 2009, and which already culminated in several joint research manuscripts.

The main topics of our joint research are:

The cycle and path structure of graphs which belong to restricted graph classes. We want to extend some of the fundamental results concerning the well-known circulant graphs to related graph classes. Circulant graphs have many special properties and are often used as interconnection, local area or large area networks for being essencial in achieving high algorithmic efficiency in many applications. Some examples of usage may be encountered in VLSI design, data alignment, parallel architectures and distributed computing. For instance time complexity of several problems can be tremendously reduced due to its particular topology, which allows a sense of direction: how to recognize if two different paths lead to the same vertex without complete topological information and relying only on a local edge labeling. Having recently shown how to extend some core characteristics of circulant graphs to a more general class named distance graphs, we want to go further and investigate other such properties with an algorithmic impact for both distance graphs and a number of related classes.

Convexity notions in graphs. The geometric notion of convexity plays a central role in optimization. Many of the classical results concerning convex sets can be extended to combinatorial structures such as graphs. We want to pursue some research questions which we identified during the above-mentioned research visit. Some of those regard the geodetic (minimum path) and the monophonic (induced path) convexities, while others relate to the Steiner convexity, with interesting applications in computational biology. In particular, we want to study partition problems related to suitably defined convex sets in graphs.

Searching problems in graphs. Searching related problems in graphs are imperative in network problems as well as in parallel and distributed computing. Our focus here is particularly directed to optimal solutions for the following two problems: rendezvous and blackhole search. We are interested in both deterministic and randomized efficient strategies. We consider the rendezvous between two or more agents exploring environments represented by graphs from a specific class. A rendezvous is a meeting (in a vertex or in an edge) between two or more agents starting (simultaneously or not) in different unknown places of a network graph and having no pre-planned meeting point. The graph topology may be unknown, completely known or partially known. A partially known topology may be one where some properties are known to exist, such as degree regularity, acyclicity or even the presence of a consistent edge labeling. The question is how can two or more autonomous exploring agents find one another without or with few communication, with limited memory, and over long distances, if they start exploring at different locations in a given environment. Furthermore, we are interested in the blackhole search problem, in which agents have the mission of finding one or more malicious vertices in contexts represented by diverse types of graphs. Again, the graph topology may be unknown, completely known or partially known.

Participants

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