What the hell do they mean by 'locomotion'?

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Abstract

In current mechanics literature one very frequently encounters the concepts *locomotion* and *locomotion system*. Unfortunately, strong definitions of these seemingly intuitively clear notions are generally missing. The following paper tries to fill this gap.

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1 Introduction

Throwing a glance over the current Applied Mechanics literature one recognizes a frequent use of the concepts *locomotion* and *locomotion system*, see the References (by far not exhausting!). No standard definition of these notions is presented anywhere, explanations if given at all, are manifold, vague, and may provoke criticisms, some examples are quoted below. Obviously, *locomotion* is handled as being intuitively clear - 'Fortbewegung' in its German equivalent. But is it indeed? Just think of a moving planet, a running cat or sportsman, a falling cat or a somersaulting athlete, is it *loco*motion what they are exercising? In connection with current research done at the Department of Mechanics at Ilmenau University of Technology we had a sporadic series of discussions about these items during the last years. Summing up, the results thereby obtained are not fully convincing and the participants could not bear them all with the same ease.

The present paper is a try to give some definitions of basic notions which in turn could bring some transparency to the domain of live and technical locomotion systems. Some detailed representations may be felt as overdone, they are a tribute to forthcoming tutorial work.

2 Some excerpts from literature

The following is a loose collection of explanations concerning 'locomotion' found in literature. Quotations are given in italics, some critical remarks within braces.

(1) *The term "locomotion" refers to autonomous movement from place to place.* [19] {What is 'place'?}

(2) *By locomotion it is understood that the forces causing the motion of the object originate within the object itself.* [23] {Motion=translation and rotation?}

(3) *Hyper-redundant robot locomotion is the process of generating net displacements ... via internal mechanism deformations.* [10] {What is 'net displacement'?}

(4) *Locomotion: Autonomous, internally driven change of location ... during which base of support and centre of mass of the body are displaced.* [7] {Counterexample: Worm in plane could shift support and center while keeping one further point fixed: locomotion? Both support and center to be displaced?}

(5) *Lokomotion im Zeitintervall T heißt die Bewegung des Körpers genau dann, wenn die Verschiebung $\overrightarrow{U}(\overrightarrow{\xi})$ aller Punkte des Körpers im Moment $t = T$ von Null verschieden ist ...* [1] {Counterexample: Snake in plane, straight configuration at $t = 0 \rightarrow$ elliptic configuration at $t = T$ with each point displaced but center of mass kept fixed: locomotion?}

(6) *Als Lokomotion bezeichnet man die, durch eine stetige Veränderung der Lage des Massenmittelpunktes gekennzeichnete Ortsveränderung eines natürlichen bzw. technischen Systems, einschließlich seiner Kontaktflächen zum umgebenden Medium.* [13] {see (4)}

(7) *Locomotion is defined as the act of moving from one place to another. ... fundamentally involve interaction with their environment: locomotion is achieved by pushing or sliding or rolling or a combination of all of these.* [10], [17] {'Place' in which space?}

(8) *Undulatory locomotion is the process of generating net displacements of a robotic mechanism via a coupling of internal deformations to an interaction between the robot and its environment.* [19],[20] { What is 'net displacement'?}

(9) *This general method of locomotion (i.e., generating net motions by cycling certain control variables) ...* [15] {What are 'net motions'?}

3 Bodies, configuration, motion, shape

Clearly, doing Applied Mechanics means to observe and describe macroscopic *bodies* and their behavior in space-time. Mostly, atomic structure and substructures and corresponding phenomena are outside the mainstream interests of mechanical science. Disregarding gases, a *body* is commonly understood as a compact collection of matter that can be observed in the euclidean point space $\mathbb{R}^3$. *What* matter is, *what* its internal structure is, *how* its particles are arranged in space - these are, as a rule, no questions in mechanics, there suffices the working hypothesis, that the particles somehow are spread over a finite space region and that they can be observed (so that they are individuals, opposite to what is fundamental in quantum theory!).

The space region filled up with that matter may - under influence of neighbored matter - change, but it always remains a (3-dimensional) region: a cuboid cannot be turned into a (2-dimensional) patch of a surface. Tearing a body to pieces (i.e., transition of a region into several disjoint regions) as well as the occurrence of holes will not play any role in the present context. Penetration of bodies is excluded anyway, since otherwise particles would coincide and thereby lose their individuality.

In order to capture these facts through a suitable mathematical model we adopt a definition from [18] in a slightly modified version.

Definition 3.1. *A* ***body*** *is a set $\mathcal{B}$ of particles equipped with a set $\mathbf{\Phi}$ of maps $\varphi \mid \mathcal{B} \to \mathbb{R}^3$ and a positive measure $\mu \mid \sigma(\mathcal{B}) \to \mathbb{R}^+$ which have the following properties:*
(i) every $\varphi \in \mathbf{\Phi}$ is injective, so the inverse map $\varphi^{-1} \mid \varphi(\mathcal{B}) \to \mathcal{B}$ exists;
(ii) for every $\varphi \in \mathbf{\Phi}$ the image $\varphi(\mathcal{B}) =: B \subset \mathbb{R}^3$ is a compact set;
(iii) for each pair of maps $\varphi, \psi \in \mathbf{\Phi}$ the composition

$$\psi \circ \varphi^{-1} =: f \mid \varphi(\mathcal{B}) \to \psi(\mathcal{B})$$

is the restriction to $\varphi(\mathcal{B})$ of a smooth map of $\mathbb{R}^3$ into itself;[1]
(iv) with any diffeomorphism H of $\mathbb{R}^3$ onto itself and $\varphi \in \mathbf{\Phi}$ it holds

$$H \circ \varphi =: \psi \in \mathbf{\Phi}.$$

(v) Every $\varphi \in \mathbf{\Phi}$ is measurable with respect to the $\sigma-$algebra $\sigma(\mathcal{B})$ on which μ is defined and the $\sigma-$algebra $\beta(B)$ of the Borel sets of $B = \varphi(\mathcal{B})$; the induced measure on B is $m_\varphi = \mu \circ \varphi^{-1}$.

[1](1.iii) implies that $f^{-1} = \varphi \circ \psi^{-1}$ exists and is smooth as well. So, disregarding what happens on boundaries, f is in fact a *diffeomorphism* of some class $C^k,\ k \geq 1$.

Some comments on the foregoing definition are supported by Figure 1.

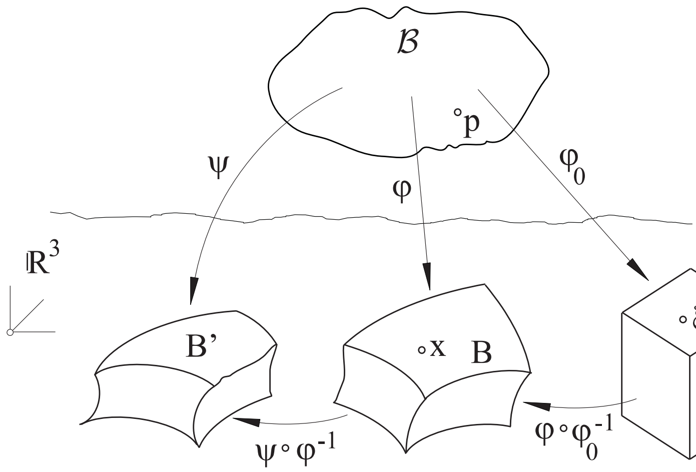

Figure 1: Set of particles, and configurations.

$\mathcal{B}$ is kept as an abstract set whose elements *are called* particles. Every map $\varphi \in \mathbf{\Phi}$ describes an embedding of the particles in $\mathbb{R}^3$: each particle is mapped onto one and only one space point. These space points, then, are called *material points*. Every material point is - on account of injectivity (i) - the image of only one particle, this implies the individual observability of the particles. In the present context it is sufficient to imagine the compact set B as a finite union of regions, surfaces, arcs, and isolated points (a compound of $3-, 2-, 1-$, and $0-$dimensional parts, connected or not). So in fact, Definition 3.1 covers the notion of a *system of bodies* as well. All its 'lower-dimensional parts' serve, adapted to concrete problems, as useful approximations of bodies with some negligible dimensions. The set $B = \varphi(\mathcal{B})$ is called a *configuration* of the body. The diffeomorphic map $f := \psi \circ \varphi^{-1} \mid B \rightarrow B' = \psi(\mathcal{B})$ describes a *change of configuration* which, following (iv), could be the restriction of *any* diffeomorphism in $\mathbb{R}^3$.

The measure μ is the *mass distribution* of the body, the measure $m_\varphi = \mu \circ \varphi^{-1}$ is the corresponding mass distribution in configuration B. $\mu(\mathcal{B}) =: M$ is the *total mass* of the body, it does not change with configuration: $m_\varphi(B) = (\mu \circ \varphi^{-1})(B) = \mu(\mathcal{B}) = M$ (conservation of mass) neither does the mass of any *part of the body*, i.e., of any set $b \in \beta(B)$.

Remark 3.2. *In [18] μ is supposed to be continuous with respect to the Lebesgue measure on $\mathbb{R}^3$. Consequently, every Lebesgue null set of B gets zero mass. In the present setting, however, any lower dimensional part of B is to represent a plate, a wire, or a masspoint, and thus it should carry a positive mass. That is why this continuity has been dropped.*

As a rule, it is useful to choose <u>one</u> particular map $\varphi_0 \in \mathbf{\Phi}$ and to define a *reference configuration* by $B_0 := \varphi_0(\mathcal{B})$. In a particular mechanical problem, it is not necessary (but often taken for granted) that the body actually appears in this configuration B_0, but it may be comfortable to describe any (actual) configuration relative to the reference configuration B_0. A *spatial reference frame* can be introduced by gluing a 3-frame $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ at a point $O \in B_0$ (or at least O fixed to B_0). Preferably, $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is chosen as an *orthonormal* triplet of vectors, then $\{O, (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)\}$ is in fact a cartesian coordinate system fixed in $\mathbb{R}^3$ (inertial reference frame). If the respective cartesian coordinates in space are denoted by $\xi^i,\ i = 1, 2, 3$, then to each particle $p \in \mathcal{B}$ the map φ_0 uniquely assigns a triplet of numbers $\xi = (\xi^1, \xi^2, \xi^3)^T \in B_0$. They become *body-fixed coordinates* by being used as 'names' of the particles, i.e., once and for all marking the material points of the body in any configuration.

Also every map $\varphi \in \mathbf{\Phi}$ causes an assignment $p \mapsto x = (x^1, x^2, x^3)^T \in B = \varphi(\mathcal{B})$ (*space coordinates* of the material point p in configuration B of the body). Then, the description of configuration B relative to the reference configuration B_0: $\xi \mapsto p = \varphi_0^{-1}(\xi) \mapsto x = \varphi(p)$ is given by

$$h := \varphi \circ \varphi_0^{-1} : \xi \mapsto x = h(\xi).$$

In view of (1.iii) in Definition 3.1, h is diffeomorphic and it describes a *coordinate transformation* (body-fixed to actual coordinates).

A family of maps of $\mathbf{\Phi}$, whose members depend on a real parameter t='*time*', where t is from an interval (t_0, t_1), is called a *motion*, i.e., a temporal sequence of configurations (t running from t_0 to t_1) of the body, $B_0 \to B_t =: h_t(B_0)$, if the family is denoted by $\{\varphi_t \mid t \in (t_0, t_1)\}$ and $h_t := \varphi_t \circ \varphi_0^{-1}$. Using the more familiar notation $h_t(\xi) =: h(\xi, t)$, then, as noted above, at any fixed time $h(\cdot, t)$ is the restriction of a diffeomorphism, so its inverse, $h^{-1}(\cdot, t)$ exists and is smooth, too. The time-dependent coordinate transformation

$$\xi \to x = h(\xi, t), \text{ for short } \ \xi \to x(\xi, t)$$

describes the positions in space at time t of the material points ξ, this is known as the *Lagrangean representation* of a motion. As to the dependence on t, a piecewise smoothness $h(\xi, \cdot) \in D^2([t_0, t_1], \mathbb{R}^3)$ is required in most cases.

Until now, the diffeomorphisms h which describe changes of configurations may be largely arbitrary: two configurations B and $B' = h(B)$ of a body may visually be quite different: being very distant from each other, or either exhibiting external and internal *deformations* with respect to the other. It is important to clarify this issue despite its seeming evidence.

Definition 3.3. *Let B and $B' = h(B)$ be two configurations of a body. If there exists a direct congruent transformation c of $\mathbb{R}^3$ such that $c(B) = B'$, then the body is said to have the same* ***shape*** *in either configuration.*

In a nutshell: The configuration B can be made coincide with B' by suitable translation and rotation in $\mathbb{R}^3$ iff B and B' have the same shape. Or: *Shape* *means configuration up to arbitrary direct congruent transformation;* let $\mathbb{C}$, $\mathbb{S}$, and $SE(\mathbb{R}^3)$ be the set of configurations, the set of shapes, and the special euclidean group, respectively, then $\mathbb{S}$ is the quotient set $\mathbb{S} = \mathbb{C}/\ SE(\mathbb{R}^3)$.

To complete the picture a word on bodies in mutual contact seems reasonable. Let $\{B_1, B_2\}$ be a system of two separate bodies in some configuration. In view of Definition 1 this should be a two-component closed set (individual material points!). In view to systems of bodies, where separate bodies may roll upon each other or are connected by joints, we relax the individuality for boundary points.

Definition 3.4. *Two separate bodies B_1 and B_2 are said to be in **contact** iff subsets of their boundaries mutually coincide, i.e., $B_1 \cap B_2 \subset \partial B_1 \cap \partial B_2 \neq \emptyset$.*

Note that a system of $n > 1$ bodies may change its shape also in case of non-deforming (rigid) bodies. To describe a configuration of a system of bodies two ingredients are required: (i) to describe the shape of the system, and (ii) to tell how the bodies are placed in space (both in relation to the reference configuration, say). Symbolically:

$$configuration = (position\ in\ space,\ shape).$$

This is seemingly a lucid scheme but in fact it is by no means self-evident how to give it a strong and handy analytical form. Difficulties arise in particular in the context of motion where shape and position depend on time t and may undergo certain kinematic and dynamic coupling.

One way out of this dilemma opens by taking the system's mass distribution into consideration. Choose any $\xi_0 \in B_0$, and determine $\{\mathbf{e}_1^0, \mathbf{e}_2^0, \mathbf{e}_3^0\}$ as spanning the principal axes of inertia of B_0 at ξ_0 (eigenvectors of the inertia tensor at ξ_0). Then, in configuration $B = h(B_0)$, the material point ξ_0 is at $x_0 = h(\xi_0)$, and now choose the orthonormal vector triplet $\{\mathbf{E}_1^0, \mathbf{E}_2^0, \mathbf{E}_3^0\}$ as to span the principal axes of inertia of B at x_0 (apart from configurations with some symmetry both triplets are unique). Finally, $\{\xi_0; \mathbf{e}_1^0, \mathbf{e}_2^0, \mathbf{e}_3^0\} \to \{x_0; \mathbf{E}_1^0, \mathbf{E}_2^0, \mathbf{E}_3^0\}$ defines a congruent transformation that is uniquely determined by h and can be seen as the change of the system's position in space.

A slight modification is as follows. In the reference configuration B_0 with mass distribution $m_0 := \mu \circ \varphi_0^{-1}$ determine $\xi^* := \frac{1}{M}\int_{B_0} \xi dm_0(\xi)$, the center of mass, and the principal axes of inertia of B_0 at ξ^*, spanned by $\{\mathbf{e}_1^*, \mathbf{e}_2^*, \mathbf{e}_3^*\}$. Note that the frame $\mathcal{F}_0^* = \{\xi^*; \mathbf{e}_1^*, \mathbf{e}_2^*, \mathbf{e}_3^*\}$ represents a cartesian coordinate system. The same construction in configuration $B = h(B_0)$ with mass distribution $m := \mu \circ \varphi^{-1} = \mu \circ (\varphi_0^{-1} \circ \varphi_0 \circ \varphi^{-1}) = m_0 \circ h^{-1}$, center of mass at $x^* := \frac{1}{M}\int_{B_0} h(\xi) dm(\xi)$ (note that in general $x^* \neq h(\xi^*)$!) generates a frame $\mathcal{F}^* = \{x^*; \mathbf{E}_1^*, \mathbf{E}_2^*, \mathbf{E}_3^*\}$. Both frames are (apart from cases of symmetry) *uniquely* attached to B_0 and B, respectively, though $\xi^* \notin B_0$ or $x^* \notin B$ might happen (for instance if a configuration is like a donut with constant mass density). For short, every $\mathcal{F}^*$ is called the *principal frame* of the respective configuration.

Again, $\mathcal{F}_0^* \to \mathcal{F}^*$ defines a congruent transformation (a *direct* one simply by suitable enumeration of the vectors) that now can be seen as the *change of the system's position in space*.

The foregoing considerations point out the fact that there are several options in choosing a reasonable frame which indicates the position in space. In view of *one single* configuration this choice is without any restriction. In view of a moving system of bodies there must be a *unique rule* how to adjoin the frame to each configuration, $h(B_0, t) =: B_t \mapsto \mathcal{F}_t$. This is guaranteed in either of the examples given above. In any case, *shape at time t* then means *configuration with respect to this frame* $\mathcal{F}_t$. As any two frames (at vertices ξ_0 or ξ^*, say) are connected by a congruent transformation, the shape at time t is by definition independent of the frame used.

In practice, the effective choice of an adjoined frame is strongly determined by both the *kind of system* under consideration and by the *aim of observations* and investigations to be done. Two examples are sketched in the following Figure: (a) Snake in $\mathbb{R}^3$: ξ_0 at head, $\{x_0; \mathbf{E}_1^0, \mathbf{E}_2^0, \mathbf{E}_3^0\}$ actual Frenet frame of backbone curve at head (so this has nothing to do with principal axes of inertia); (b) Athlete, somersaulting: actual frame $\mathcal{F}^* = \{x^*; \mathbf{E}_1^*, \mathbf{E}_2^*, \mathbf{E}_3^*\}$.

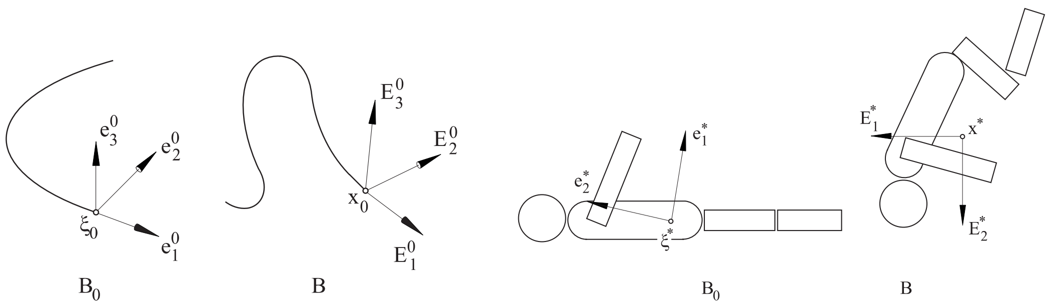

Figure 2: Frames used for position in space. (a) Snake, (b) athlete.

Now $t \mapsto \mathcal{F}_t$ describes the journey of the body (system of bodies) through space, accompanied by a temporal change of shape. Example (b) above most impressively demonstrates how different this journey appears to an observer sitting on B_0, when using one frame or another: here $t \mapsto x^*(t)$ is simply the parabola of free fall under gravitation whereas, if using ξ_0 at a foot, say, then $t \mapsto x_0(t)$ describes a complicated curve in 3−space. It is the business of the investigator to decide for the preference of either description or for yet another one.

Let us close this Section by a short remark on *rigid bodies*.

Definition 3.5. *A* ***rigid body*** *is a body as described above but undergoing the restriction that for any pair of maps* $\varphi, \psi \in \mathbf{\Phi}$ *the change of configurations* $h = \psi \circ \varphi^{-1}$ *be a direct congruent transformation.*

The definition claims more than just the preservation of the 'outer form': *each part* of the body keeps its shape. Of course, a system of rigid bodies may change its shape if any two components of the system are allowed to change their relative position to each other.

4 Autonomous motion systems, locomotion

By definition a rigid body has a fixed shape, a rigid-body configuration is basically nothing else but position in 3−space, and this can be described by means of any body-fixed frame (cf. examples above). Six parameters are needed to fix a 3−frame: the free rigid body has *degree-of-freedom* (DOF) equal to 6.

Arguing in a Newton-Euler setting, a rigid body needs *external forces* (influences of bodies in its environment such as gravity, push or pull, friction) for acceleration, i.e., to start or to change motion. This is the basic outcome of of the principles of linear and angular momentum: a single rigid body cannot propel itself, *it cannot perform an autonomous motion* (neither translation nor rotation).

For a system composed of several rigid bodies this feature totally changes! Each pair of bodies belonging to the system may *interact* through forces (which are classified as *internal forces* of the system if they are exclusively due to internal causes). In technical systems these forces frequently result from devices like rotatory joints, linear or rotatory motors, piezo-elements, springs or dampers, in live systems these forces are primarily caused by muscles or hydraulic elements. Such devices are often modeled massless ("ghost"-components of the system). Now, following Newton's third law of action-reaction, these internal forces pairwise cancel, thus they cannot cause the center of mass, x^*, to accelerate but they can well influence the *rotation* of a frame at x^*!

Striking example: *Spacecraft in orbit*. Taking care while entering the orbit, the principal frame undergoes pure translation along the orbit (vectors of frame $\mathcal{F}^*$ remains fixed in space). Seen from this frame the system 'spacecraft' is free of external forces (gravity and centrifugal force compensate). Driving certain rotors (by *internal* forces) - and thereby changing shape! - the principal frame, on account of conservation of angular momentum, gets a rotation (goal: to approach a desired orientation in space).[2] Thus, the system has driven itself (without any support by external forces): a change of shape has caused a change of position in space.

Every such change of shape - angular or linear displacement - or the corresponding internal forces can be understood as the output of an appropriate actuator that is part of the system. An actuator output of this kind is called an *internal drive*.

[2]This is due to the Principle of angular momentum: time-rate of angular momentum equals resultant moment of *external* forces. Then, in absence of external forces, conservation of the system's angular momentum follows. See [11], [12], [21].

On the other hand it is clear by the principle of linear momentum that an *external driving force* (i.e., an accelerating action of the system's environment) is necessary to cause a motion of x^*. If emphasis is on purely autonomous motion of the system then every external force under whose sole action the system could start a motion from rest has to be excluded from consideration. What remains as feasible kinds of external force are *impressed forces* ('eingepraegte Kraefte' after Hamel) which arise from non-zero relative velocities (such as friction and Lorentz forces) and *workless constraint forces* (reactions to scleronomic constraints): *non-driving external forces.*

Striking example: *Walking man.* Vertical gravity is of no interest for horizontal motion. Drive is achieved by changing certain joint angles - change of shape! - via *internal* muscular forces. The external force necessary for forward motion is the workless reaction to the constraint 'no sliding of feet'.

Mutatis mutandis, the same considerations can be applied to systems containing one or more deformable bodies.

Based on the foregoing considerations a definition can be attempted.

Definition 4.1. *Consider a mechanical system that exhibits the following features.*
(i) The system may or may not have contact to a (material) environment;
(ii) there are internal drives;
(iii) there exists at least one particular internal drive such that, in presence of only non-driving external forces, and starting from rest, the principal frame $\mathcal{F}_t^*$ *does not remain fixed in space.*
Then the system is called an ***autonomous motion system****.*

Examples: Spacecraft on orbit (see above); multiple pendulum with active rotatory joints (i.e., equipped with motors) and fixed pivot; locomotive on rail; falling cat.

Counterexample: Flying projectile, sailplane.

Remark 4.2. *Note the different meanings of* 'autonomous' *in the theories of differential equations and motion systems: an autonomous differential equation is one whose right-hand-side does not explicitly depend on time t whereas in general an autonomous motion system is governed by a heteronomic differential equation whose t−dependence is caused by the outputs of the internal actuators. Furthermore, there is some contrast to the very meaning of physical autonomy, since by Definition 4.1* autonomous motion *is independent of whether the actuators' control signals and power supply come from inside or outside the system.*

Definition 4.3. *An autonomous motion system is called a* ***locomotion system*** *if there exist a particular internal drive and a time interval* (t_0, t_0+T)*, such that in presence of only non-driving external forces*

$$x^*(t_0+T) \neq x^*(t_0) \ \wedge \ x(\xi, t_0+T) \neq x(\xi, t_0) \ \ for\ every\ \xi \in B_0,$$

i.e., neither the center of mass nor any material point remain fixed or run a cycle in space on that time interval.
Every motion of this kind is called ***locomotion****.*

Examples: Locomotive on rail; two rigid bodies $1, 2$, coupled by an actuator with output $t \mapsto dist_{1,2}(t)$, both bodies contacting their environment with anisotropic friction; automobile on ground; snake on plane ground.

Counterexamples: Any autonomous motion system with no contact to a material environment; the above mentioned pendulum; continuum analogues: elephant trunk considered separate is rather for *manipulation*; snake in plane with head kept fixed.

By definition there is no frame describing motion in space of the *locomotion system* which performs pure rotation under that particular drive. In other words: a locomotion system can perform (if suitably driven) a kind of motion that is also *intuitively* classified as *loco*-motion. But the system is by no means restricted to this kind of motion: the above mentioned snake in plane could pass from a straight-line configuration to an elliptic one while keeping $x^*(t)$ or even $\mathcal{F}_t^*$ fixed! An earthworm, certainly a locomotion system, does not achieve locomotion during free fall. *The heart of locomotion is the conversion of internal drive into motion by interaction with the environment through non-driving external forces.*

Position in space described by $\mathcal{F}^*$, say, can be seen as partitioned

$$\mathcal{F}^* = \{x^*; \mathbf{E}_1^*, \mathbf{E}_2^*, \mathbf{E}_3^*\} =: (location;\ orientation).$$

So, following Definition 4.3, *locomotion* relies basically on temporal change of *loc*ation whereas a change of orientation is of minor interest. Of course, there might be situations with primary or additional need for observation of how orientation behaves in time (adjustment of an antenna or of some tool in surgery, e.g.). To this end the concept locomotion system could be appropriately augmented.

It is indeed obvious that the overwhelming majority of both live and technical systems is characterized by the fact that those particular internal drives which make them *locomotion systems* are periodic (or at least reciprocating) in time. Examples: knee-joint torque or angle of a running man; wing-stroke of a bird; alternating contraction-expansion of a creeping worm; relative motion piston-cylinder of a steam-roller.

The following definition is adapted from [19] and [20].

Definition 4.4. *Every locomotion that is based on a periodic internal drive is called an* ***undulatory locomotion****.*

In practice, both analytical description and computational handling of configurations $(\mathcal{F}_t^*, shape)$ require the introduction of coordinates. In $3-$space these are 6 coordinates to capture $\mathcal{F}_t^*$ (3 - possibly cartesian - coordinates for x^*, and 3 - possibly angles - α, β, γ for the frame vectors), and some further coordinates (finitely many for a system of rigid bodies, more if there are deformable bodies) for shape. Abbreviating these coordinates by X and q, respectively, then there is, at least locally, a one-to-one correspondence

$$configuration \longleftrightarrow (X, q) =: (\textit{position variables}, \textit{shape variables}).$$

Now in considering (undulatory) locomotion in the sense of Definitions 4.3 and 4.4 one focuses on the motion $t \mapsto x^*(t)$ while a rotation of $\mathcal{F}_t^*$ enjoys minor interest. In short, during a particular undulatory locomotion a *periodic* function $t \mapsto q(t)$ generates a, say, *monotonic* function $t \mapsto X(t)$ through assistance of the external body 'environment'.

Any locomotion $t \mapsto (X(t), q(t))$ can be seen as a curve $\mathbf{l}$ in configuration space. Kelly and Murray described undulatory locomotion by means of fiber bundle concepts [14] taking the shape space as basis and position space as fiber. Simplifying, this description can be nicely visualized by depicting the configuration $(X, q)-$space as a $3-$space with horizontal $q-$plane and a vertical $X-$axis. A $T-$periodic function $t \mapsto q(t)$ appears as a cycle $\mathbf{c}$ in the $q-$plane. To each $q(t)$ a position $X(t)$ is uniquely attached (via influence of environment). Now starting at $t = t_0$ from configuration (X_0, q_0) then at time $t_1 = t_0 + T$ the curve $\mathbf{l}$ has been run through to configuration (X_1, q_1) where $q_1 = q(t_0 + T) = q_0$ (one cycle run) whereas in case of locomotion $X_1 = X(t_0 + T) \neq X_0$. So the curve $\mathbf{l}$ is like a helix having $\mathbf{c}$ as its projection into the $q-$plane, see following Figure. Note that the 'helix' is determined by the kind of interaction system-environment.

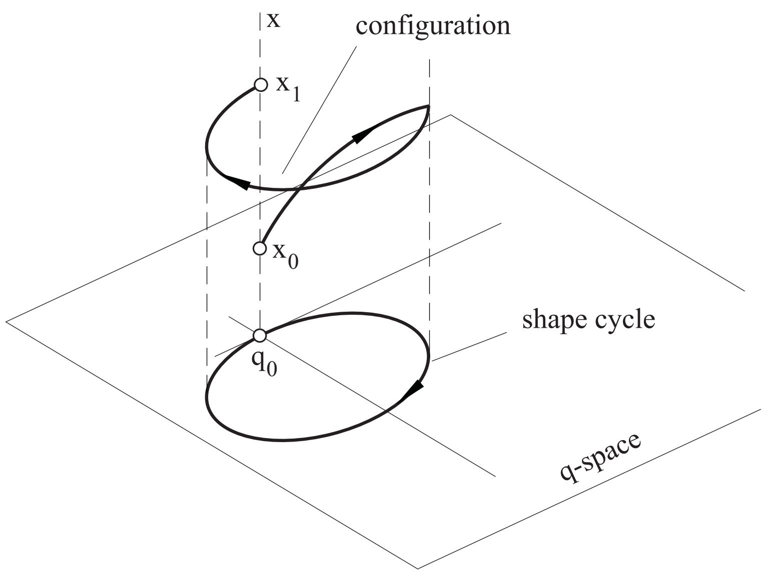

Figure 3: Undulatory locomotion visualized (one period).

It is important to realize the evident fact that, given a locomotion system, the environmental correspondence *periodic* $q(\cdot) \rightarrow$ *monotonic* $X(\cdot)$ *cannot* be

described in the form $X = f(q)$ where f is some function. If this was the true interdependence of position and shape then this would mean that the environment gave rise to a holonomic (geometric) constraint. Rather the fundamental locomotive coupling $q \to X$ could originate
(i) in the system's kinematics: from nonholonomic constraints (as in case of a motorcar with its steered and rolling wheels) or from structural switches (a climber with alternating feet and hands fixed - similar to an inch-worm; or a walking man with alternating left and right foot fixed) or
(ii) in the system's dynamics: from peculiarities of the differential equations of motion (e.g., generated by anisotropic, i.e., orientation dependent, friction forces).

Investigations of locomotion in case (i) can often be accomplished within kinematics. This kinematical theory may then be supplemented by dynamical considerations. It can also serve as a basis for treating case (ii) which anyway requires the use of dynamics from the very beginning.

Conclusion

In the foregoing Sections we have suggested a definition for the concept *locomotion* which is more stringent than what is usually presented in literature. Emphasis is, first, on change of position in 3-space, thereby covering the intuitive picture of locomotion in common mind, but a slightly enlarged definition would admit to take also orientation into account thus aiming at 'change of place in $SE(\mathbb{R}^3)$'. Second, the definition restricts locomotion to autonomous systems. So the classification now appears as being clarified: an earthworm and an autonomously moving endoscope are locomotion systems, my TV receiver, thrown through the window and falling down is not. Nevertheless, some things still remain uncertain: consider, e.g., a rocket in free space. Indeed one would like to see it as a locomotion system, for it has certain (thermal, chemical, or nuclear) actuators on board which are to start and change the rocket motion. But their outputs can be classified as *internal* drives only for the system 'rocket trunk plus ejected gas' whereas, considering the isolated rocket trunk as the proper motion system the thrust appears as an *external driving force*. Which (fictitious) material environment does this force result from? So what is the dilemma? Items for further discussion! Why not?

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