Elgersburg School 2010
Differential-Algebraic Equations - Control and Numerics
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DAEs - Control and Numerics
Exercise Sheet 2 - Numerical solutions

Exercise 5 (Classical solvers and higher index - Matlab experiment)
Consider the "pure" DAE

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}$$

with $u(t) = e^t$ (or any other smooth function). Do numerical simulations by applying implicit Euler, Gauss ($s = 2$) and Radau ($s = 2$) solvers to the above DAE. Compare the results with the exact solutions.

Exercise 6 (Time varying DAEs with classical solvers - Matlab experiment)
Consider the time-varying DAE

$$\begin{bmatrix} -t & t^2 \\ -1 & t \end{bmatrix} \begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}, \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Apply again implicit Euler, Gauss ($s = 2$) and Radau ($s = 2$) to numerically solve this DAE. Does any of the solver "recognize" the non-uniqueness of the solution?

Exercise 7 (Parameter depended solutions)
Find the solutions of the DAE

$$\begin{bmatrix} 0 & 0 \\ 1 & \eta t \end{bmatrix} \begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} -1 & -\eta t \\ 0 & -(1+\eta) \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

depending on the parameter $\eta \in \mathbb{R}$. What happens when you apply an implicit Euler solver to this problem?

Exercise 8 (Normal form and derivative array)
Find the normal form of the DAE

$$\begin{bmatrix} 0 & t \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}.$$

What is the derivative array and the corresponding condensed form of this DAE?