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Compressive Sensing based High-Resolution DoA Estimation by Beamspace Covariance Gradient Descent

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***Abstract*—In this paper, we present a high-resolution direction of arrival (DoA) estimation scheme using compressive measurements for mmWave band communications. We first propose an off-grid refinement algorithm to refine an initial on-grid DoA estimation through a gradient descent search in the beamspace covariance (BSC) domain. Then we integrate the proposed refinement algorithm into the covariance orthogonal matching pursuit (COMP) algorithm, such that an on-grid detected source is firstly refined and then off-grid canceled during the successive iterations. Numerical results show that the proposed method has a lower complexity than the state-of-the-art off-grid compressive DoA estimation method in case of OFDM signals, while it can reliably estimate the DoAs with high accuracy.**

***Index Terms*—compressive sensing, DoA estimation, beamspace covariance gradient descent, off-grid cancellation.**

I. Introduction

The millimeter wave (mmWave) band provides a wide bandwidth for high data throughput and has been adopted by 5G new radio (NR) cellular communication systems. To overcome the high path-loss through mmWave band propagation while at the same time being cost and power efficient, large antenna arrays based on analog beamforming with phase shifters have become a popular architecture not only on the base station (BS) side but also on the user equipment (UE) side. Antenna arrays bring the opportunity to perform direction of arrival (DoA) sensing. On one hand, DoA sensing improves the spatial channel estimates, which further helps to improve the multiple-input-multiple-output (MIMO) communication efficiency [1]. On the other hand, DoA sensing provides additional perception information to a mobile device, which is useful for integrated sensing and communication (ISAC) applications [2] . However, due to RF chain sharing among multiple antennas, the baseband processor does not directly observe the channel state information (CSI) in the antenna element space, but rather the projected version in the beamspace. This makes accurate DoA estimation for mmWave communications challenging [3].

By exploring the sparsity of the mmWave propagation channel, compressive sensing (CS) techniques can be explored to estimate the DoAs using a small measurement overhead. CS based DoA estimation for OFDM systems has been proposed by [4], which shows that the spatial channel transfer function in the antenna element space can be recovered through compressive measurements using the orthogonal matching pursuit (OMP) algorithm. The heuristic nature of the OMP algortihm leads to a reduced complexity compared to gridless methods [5]. However, the estimates are on-grid which limits the DoA estimation accuracy. To improve the estimation accuracy, a two-step estimation algorithm called gradient descent orthogonal matching pursuit (GOMP) has been proposed by [6]. The algorithm first conducts initial on-grid DoA estimations using a conventional OMP algorithm, then it introduces a gradient descent based off-grid refinement to further improve the estimation accuracy for all the on-grid detected sources. The algorithm achieves a better resolution than on-grid DoA estimation techniques. However, since the refinement is applied in the channel snapshots domain, the optimal gradient has to be computed for all snapshots jointly, which introduces a high complexity when the number of snapshots is increased. Compressive DoA estimation based on the so-called covariance orthogonal matching pursuit (COMP) algorithm has been proposed by [7], where DoA estimation is conducted in the beamspace covariance (BSC) domain. Since the compressive recovery is done after BSC combining over different snapshots, the complexity is much lower than the recovery in the channel snapshots domain. However, the estimation by COMP is still on-grid, whose resolution is bounded by the grid density.

To address the aforementioned issues, we first propose a novel off-grid refinement algorithm called beamspace covariance gradient descent (BSC-GD), to refine the on-grid estimated DoA for a single source. The refinement is based on the gradient descent technique using a quadratic cost function in the BSC domain. Then, instead of treating on-grid estimation and off-grid refinement as two independent steps to estimate the DoAs as in [6], we propose to integrate the developed BSC-GD algorithm into the successive procedure of the COMP algorithm. This means that an on-grid detected DoA source is off-grid refined, then canceled in the BSC domain, before the next source is on-grid detected. This method mitigates the power spreading issue for the off-grid DoAs. Numerical results show that the proposed method achieves a lower complexity than GOMP, and a higher accuracy than GOMP and COMP.

Notation: Upper-case and lower-case bold-faced letters denote matrices and vectors, respectively. The Frobenius norm, transpose, conjugate, Hermitian transpose, pseudo-inverse of a matrix are denoted by $\|.\|_F$, $\{\cdot\}^T$, $\{\cdot\}^*$, $\{\cdot\}^H$, $\{\cdot\}^\dagger$, respectively. The $\text{diag}\{\cdot\}$, $\text{vec}\{\cdot\}$ denote a diagonal matrix construction and the vectorization operators, respectively.

II. Problem Formulation

We consider a typical mmWave MIMO communication system with a base station (BS) and a user equipment (UE) each is equipped with T and N antennas, respectively. The antennas on both sides are assumed to form uniform linear arrays (ULAs) which use phase shifters for transmit and receive beamforming. The BS transmits a measurement burst of M OFDM symbols each containing K pilot reference sub-carriers, with the same frequency domain allocation. We assume that the transmit beam at the BS is fixed during the measurement burst, whereas the UE uses M different measurement beams to receive the M OFDM symbols where $M \ll N$. The switching time from one measurement beam to the next is assumed to be shorter than the cyclic prefix (CP) duration. We further assume that all M OFDM symbols are transmitted within the channel coherence time so that the channel could be assumed to be constant during the measurement. On the receiver side, after applying the FFT operation for the m^{th} received OFDM symbol, the frequency domain received signal on k^{th} sub-carrier is formulated as:

$$y_m(k) = \mathbf{w}_m^H \mathbf{H}(k) \mathbf{f} s_m(k) + \mathbf{w}_m^H \mathbf{z}_m(k) \tag{1}$$

where $\mathbf{z}_m(k) \in \mathbb{C}^{N\times 1}$ is a zero mean complex Gaussian noise vector with variance σ^2, $s_m(k)$ is the pilot reference signal on the k^{th} sub-carrier of the m^{th} OFDM symbol and $|s_m(k)| = 1$. Moreover, $\mathbf{f} \in \mathbb{C}^{T\times 1}$ is the beamforming vector at the BS which is fixed over the measurement burst while $\mathbf{w}_m^H \in \mathbb{C}^{1\times N}$ is the receive beamforming row vector at the UE for the m^{th} OFDM symbol. Since the beams are formed by using analog phase shifters, $\mathbf{f}$ and $\mathbf{w}_m^H$ contain only unit modulus entries. $\mathbf{H}(k) \in \mathbb{C}^{N\times T}$ is the sampled frequency domain channel transfer function (CTF) on the k^{th} sub-carrier. A typical mmWave massive MIMO channel is assumed with L scatterers. The l^{th} scatterer accounts for one time delay τ_l, one complex gain α_l, and one pair of spatial frequencies $(\mu_{T,l}, \mu_{R,l})$, corresponding to a direction of departure (DoD) and a DoA. Then $\mathbf{H}(k)$ is modeled as [8]:

$$\mathbf{H}(k) = \sum_{l=0}^{L-1} \alpha_l e^{-j2\pi k \Delta f \tau_l} \mathbf{a}_R(\mu_{R,l}) \mathbf{a}_T^T(\mu_{T,l}) \tag{2}$$

where Δf is the sub-carrier spacing, $\mathbf{a}_R(\mu_{R,l}) = [1, e^{j\mu_{R,l}}, \cdots, e^{j(N-1)\mu_{R,l}}]^T \in \mathbb{C}^{N\times 1}$ and $\mathbf{a}_T^T(\mu_{T,l}) = [1, e^{j\mu_{T,l}}, \cdots, e^{j(T-1)\mu_{T,l}}]^T \in \mathbb{C}^{T\times 1}$ are the array steering vectors at the UE and the BS corresponding to the l^{th} scatterer. Since the transmit beamforming vector $\mathbf{f}$ is fixed for all m OFDM symbols, (1) can be written as:

$$y_m(k) = \mathbf{w}_m^H \mathbf{h}(k) s_m(k) + \mathbf{w}_m^H \mathbf{z}_m(k) \tag{3}$$

where

$$\begin{aligned} \mathbf{h}(k) &= \sum_{l=0}^{L-1} \alpha_l e^{-j2\pi k \Delta f \tau_l} \mathbf{a}_R(\mu_{R,l}) \mathbf{a}_T^T(\mu_{T,l}) \mathbf{f} \\ &= \sum_{l=0}^{L-1} x_l(k) \mathbf{a}_R(\mu_{R,l}) \end{aligned} \tag{4}$$

and $x_l(k) \triangleq \alpha_l e^{-j2\pi k \Delta f \tau_l} \mathbf{a}_T^T(\mu_{T,l}) \mathbf{f}$ is the complex valued gain of the l^{th} incoming DoA source on the k^{th} sub-carrier. Since we only focus on DoA estimation, in the rest of the paper, for simplicity, we use the notation $\mathbf{a}(\mu_l)$ instead of $\mathbf{a}_R(\mu_{R,l})$ to represent the steering vector corresponding to the spatial frequency of the l^{th} incoming DoA source. We then apply the descrambling operation by multiplying with $s_m^*(k)$ on the k^{th} sub-carrier and stack the descrambled signal for all M measurements together. We also stack the steering vectors of all L sources into a steering matrix $\mathbf{A} = [\mathbf{a}(\mu_0), \ldots, \mathbf{a}(\mu_{L-1})] \in \mathbb{C}^{N\times L}$ and stack the complex valued gains of all L sources on the k^{th} sub-carrier into a gain vector $\mathbf{x} = [x_0(k), \ldots, x_{L-1}(k)]^T \in \mathbb{C}^{L\times 1}$. Then we get the measurement vector in the beamspace as

$$\mathbf{y}(k) = \begin{bmatrix} \mathbf{w}_1^H \\ \vdots \\ \mathbf{w}_M^H \end{bmatrix} [\mathbf{a}(\mu_0), \ldots, \mathbf{a}(\mu_{L-1})] \begin{bmatrix} x_0(k) \\ \vdots \\ x_{L-1}(k) \end{bmatrix} + \tilde{\mathbf{z}}(k) \tag{5}$$

which could be written in the compact form:

$$\mathbf{y}(k) = \mathbf{W}\mathbf{A}\mathbf{x}(k) + \tilde{\mathbf{z}}(k), \ \ k \in 0, \ldots, K-1 \tag{6}$$

where $\mathbf{y}(k) \in \mathbb{C}^{M\times 1}$ is the stacked vector on the k^{th} sub-carrier for all M measurements whose m^{th} element is $[\mathbf{y}(k)]_m = y_m(k) s_m^*(k)$. The term $\tilde{\mathbf{z}}(k) \in \mathbb{C}^{M\times 1}$ is the modulated noise vector whose m^{th} element is $[\tilde{\mathbf{z}}(k)]_m = \mathbf{w}_m^H \mathbf{z}_m(k) s_m^*(k)$. The matrix $\mathbf{W} \in \mathbb{C}^{M\times N}$ is the analog codebook for the UE to receive the pilot OFDM symbols, which is also denoted as the projection matrix. For the measured signal, a wideband beamspace covariance matrix can be derived as:

$$\mathbf{R}_{\mathbf{y}} = \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{y}(k) \mathbf{y}^H(k) \tag{7}$$

Using (6), we rewrite (7) as:

$$\mathbf{R}_{\mathbf{y}} = \mathbf{W}\mathbf{A}\mathbf{R}_{\mathbf{x}}\mathbf{A}^H\mathbf{W}^H + \frac{\sigma^2}{K}\mathbf{W}\mathbf{W}^H \tag{8}$$

where $\mathbf{R}_{\mathbf{x}} \triangleq \frac{1}{K}\sum_{k=0}^{K-1} \mathbf{x}(k)\mathbf{x}^H(k)$.

III. The Proposed Approach

In this section, we first present a DoA refinement for a given coarse DoA estimate of a single source. The refinement is applied in the BSC domain by the gradient descent search. We then propose an improved version of the COMP algorithm called gradient descent covariance orthogonal matching pursuit (GCOMP). GCOMP integrates the proposed refinement algorithm into the successive procedure of COMP, which allows off-grid DoA estimation for multiple sources.

A. DoA Refinement by BSC Gradient Descent

For the single source case, equation (8) reduces to:

$$\mathbf{R}_{\mathbf{y}} = \mathbf{W}\mathbf{a}(\mu)\sigma_{\mathbf{x}}^2\mathbf{a}^H(\mu)\mathbf{W}^H + \frac{\sigma^2}{K}\mathbf{W}\mathbf{W}^H \tag{9}$$

where $\sigma_{\mathbf{x}}^2 \in \mathbb{R}$ and $\mathbf{a}(\mu) \in \mathbb{C}^{N\times 1}$. In the BSC domain, the objective function for DoA estimation can be formulated in matrix form as:

$$\underset{\sigma_{\mathbf{x}}^2, \mu}{\arg\min} \left\| \mathbf{R}_{\mathbf{y}} - \mathbf{W}\mathbf{a}(\mu)\sigma_{\mathbf{x}}^2\mathbf{a}^H(\mu)\mathbf{W}^H \right\|_F \tag{10}$$

Note that the minimization problem in (10) is non-convex due to joint optimization over μ and $\sigma_{\mathbf{x}}^2$. Similar to the approach in [6], we use an iterative alternating optimization procedure between μ and $\sigma_{\mathbf{x}}^2$ where a step is taken to optimize one variable while assuming that the other is fixed.

In the iteration $(i+1)$, the first step is a local refinement to compute $\mu^{(i+1)}$ with a fixed $\sigma_{\mathbf{x}}^{2\,(i)}$. The cost function in (10) becomes a function of μ only and the objective is to compute a change in μ which is denoted hereforth as $\delta \in \mathbb{R}$ which minimizes the cost function. Using the first-order Taylor approximation of $\mathbf{a}(\mu) \in \mathbb{C}^{N\times 1}$ as:

$$\mathbf{a}(\mu^{(i+1)}) \approx \mathbf{a}(\mu^{(i)}) + \mathbf{g}(\mu^{(i)})\delta, \quad \delta = \mu^{(i+1)} - \mu^{(i)} \quad (11)$$

where $\mathbf{g}(\mu^{(i)})$ is the gradient of vector $\mathbf{a}$ with respect to μ at $\mu = \mu^{(i)}$ and is given as :

$$\mathbf{g}(\mu^{(i)}) = j\mathrm{diag}\left\{0, \cdots, N-1\right\}\mathbf{a}(\mu^{(i)}), \quad (12)$$

For simplicity of notation, we replace $\mathbf{g}(\mu^{(i)})$, $\mathbf{a}(\mu^{(i)})$, and $\mathbf{a}(\mu^{(i+1)})$ by $\mathbf{g}_i$, $\mathbf{a}_i$, and $\mathbf{a}_{i+1}$, respectively. The cost function in (10) can be approximated as $f(\delta)$ which is given as:

$$\begin{aligned} f(\delta) &= \left\|\mathbf{R_y} - \mathbf{W}\mathbf{a}(\mu)\sigma_{\mathbf{x}}^2\mathbf{a}^H(\mu)\mathbf{W}^H\right\|_F \\ &\approx \left\|\mathbf{R_y} - \mathbf{W}(\mathbf{a}_i + \mathbf{g}_i\delta)\sigma_{\mathbf{x}}^2(\mathbf{a}_i + \mathbf{g}_i\delta)^H\mathbf{W}^H\right\|_F \\ &= \|\mathbf{R_y} - \sigma_{\mathbf{x}}^2\mathbf{W}\mathbf{a}_i\mathbf{a}_i^H\mathbf{W}^H - \sigma_{\mathbf{x}}^2\mathbf{W}(\mathbf{a}_i\mathbf{g}_i^H + \mathbf{g}_i\mathbf{a}_i^H)\mathbf{W}^H\delta \\ &- \sigma_{\mathbf{x}}^2\mathbf{W}\mathbf{g}_i\mathbf{g}_i^H\mathbf{W}^H\delta^2\|_F = \left\|\mathbf{u}_0 + \mathbf{u}_1\delta + \mathbf{u}_2\delta^2\right\|^2 \end{aligned} \quad (13)$$

where $\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2$ are given by:

$$\begin{aligned} \mathbf{u}_0 &= \mathrm{vec}\left(\mathbf{R_y} - \sigma_{\mathbf{x}}^2\mathbf{W}\mathbf{a}_i\mathbf{a}_i^H\mathbf{W}^H\right) && \in \mathbb{C}^{M^2\times 1} \\ \mathbf{u}_1 &= -\sigma_{\mathbf{x}}^2\mathrm{vec}\left(\mathbf{W}(\mathbf{a}_i\mathbf{g}_i^H + \mathbf{g}_i\mathbf{a}_i^H)\mathbf{W}^H\right) && \in \mathbb{C}^{M^2\times 1} \\ \mathbf{u}_2 &= -\sigma_{\mathbf{x}}^2\mathrm{vec}\left(\mathbf{W}\mathbf{g}_i\mathbf{g}_i^H\mathbf{W}^H\right) && \in \mathbb{C}^{M^2\times 1} \end{aligned} \quad (14)$$

From (13), $\hat{\delta}$ which minimizes $f(\delta)$ is obtained by solving the equation $\frac{\partial f}{\partial \delta} = 0$. Therefore, we have:

$$\frac{\partial f}{\partial \delta} = 2\left[w_0 + w_1\delta + w_2\delta^2 + w_3\delta^3\right] = 0 \quad (15)$$

where w_0, w_1, w_2, w_3 are given by:

$$\begin{aligned} w_0 &= \Re\left\{\mathbf{u}_1^H\mathbf{u}_0\right\} \quad, w_1 = \left\{2\Re\{\mathbf{u}_0^H\mathbf{u}_2\} + \|\mathbf{u}_1\|^2\right\} \\ w_2 &= 3\Re\left\{\mathbf{u}_1^H\mathbf{u}_2\right\} \quad, w_3 = 2\left\|\mathbf{u}_2\right\|^2 \end{aligned} \quad (16)$$

Equation (15) is a third-order polynomial with 3 roots $\delta_1, \delta_2, \delta_3$ and $\hat{\delta}$ is selected as the real root which minimizes the approximate cost function $f(\delta)$ as follows:

$$\hat{\delta} = \min_{\delta\in\{\delta_1,\delta_2,\delta_3\}} \|\mathbf{u}_0 + \mathbf{u}_1\delta + \mathbf{u}_2\delta^2\|^2 \quad (17)$$

The second step of the iteration $(i+1)$ is to take the least squares (LS) estimate of $\sigma_{\mathbf{x}}^{2\,(i+1)}$ from (9) by fixing $\mu^{(i+1)}$

$$\sigma_{\mathbf{x}}^{2\,(i+1)} = \left\{\mathbf{W}\mathbf{a}_{i+1}\right\}^\dagger \mathbf{R_y}\left\{\mathbf{a}_{i+1}^H\mathbf{W}^H\right\}^\dagger \in \mathbb{R} \quad (18)$$

This alternating procedure is performed iteratively and it terminates if the maximum number of iterations $I_{\max}$ is reached or the cost function in (10) has reached a local minimum. The proposed algorithm is summarized in Algorithm 1.

B. The GCOMP Algorithm

According to the compressive sensing framework, the wideband beamspace covariance matrix in (8) can also be written using the dictionary as in the following:

$$\mathbf{R_y} = \mathbf{W}\mathring{\mathbf{A}}\mathring{\mathbf{R}}_{\mathbf{x}}\mathring{\mathbf{A}}^H\mathbf{W}^H + \frac{\sigma^2}{K}\mathbf{W}\mathbf{W}^H \quad (19)$$

where $\mathring{\mathbf{A}} = [\mathbf{a}(\mathring{\mu}_1), \ldots, \mathbf{a}(\mathring{\mu}_P)] \in \mathbb{C}^{N\times P}$ is the dictionary matrix of P steering vectors with pre-defined spatial frequencies, $\mathbf{a}(\mathring{\mu}_i)$ denotes the i^{th} steering vector with spatial frequency $\mathring{\mu}_i$, and $\mathring{\mathbf{R}}_{\mathbf{x}}$ is a sparse $P\times P$ Hermitian matrix.

By assuming that $\mathring{\mathbf{R}}_{\mathbf{x}}$ is sparse, the original COMP algorithm in [7] iteratively detects the index of the grid-point which corresponds to the strongest DoA source, cancels its contribution from $\mathbf{R_y}$, and then detects the next strongest DoA source. When a DoA source is off-grid, the power of the source leaks into multiple grid points, so that the sparsity of $\mathring{\mathbf{R}}_{\mathbf{x}}$ is reduced. When the cancellation is applied only on the pre-defined grid, the contribution of a detected off-grid DoA source to the $\mathbf{R_y}$ is not totally eliminated, which impacts the detection performance for the next DoA source. To mitigate this, the developed BSC-GD algorithm is further integrated into the COMP procedure, such that instead of subtracting the on-grid contributions of the detected sources, we subtract the refined off-grid contributions from $\mathbf{R_y}$, before a further DoA source is detected. The proposed algorithm is summarized in Algorithm 2. This method clearly differs from the two-step approach by the GOMP algorithm in [6], which computes the on-grid estimates of all sources in a first step, and then applies the refinement for all on-grid estimates jointly in a second step.

Algorithm 1 Proposed BSC-GD Algorithm

Input: $\mathbf{R_y}$, $\mathbf{W}$, $\mu^{(0)}$
Compute $\sigma_{\mathbf{x}}^{2\,(0)} = \{\mathbf{W}\mathbf{a}_0)\}^\dagger \mathbf{R_y}\left\{\mathbf{a}_0^H\mathbf{W}^H\right\}^\dagger$
Compute $\epsilon^{(0)} = \|\mathbf{R_y} - \mathbf{W}\mathbf{a}_0\sigma_{\mathbf{x}}^{2\,(0)}\mathbf{a}_0^H\mathbf{W}^H\|_F$
for $i = 0$ to $I_{\max}$ **do**
  For given $\sigma_{\mathbf{x}}^{2\,(i)}, \mu^{(i)}$, find roots of (15)
  Compute $\hat{\delta}$ as in (17)
  Compute $\mu^{(i+1)} = \hat{\delta} + \mu^{(i)}$
  Compute $\sigma_{\mathbf{x}}^{2\,(i+1)}$ as in (18)
  $\epsilon^{(i+1)} = \|\mathbf{R_y} - \mathbf{W}\mathbf{a}_{i+1}\sigma_{\mathbf{x}}^{2\,(i+1)}\mathbf{a}_{i+1}^H\mathbf{W}^H\|_F$
  if $\epsilon^{(i+1)} \geq \epsilon^{(i)}$
    return $\hat{\mu} = \mu^{(i)}$ and terminate
  end if
  $\hat{\mu} = \mu^{(i+1)}$
end for
Output: The estimated parameter $\hat{\mu}$

Algorithm 2 Proposed GCOMP Algorithm

Input: $\mathbf{y}(k)$, $\mathbf{W}$, $\mathring{\mathbf{A}}$, L, K
Compute $\mathbf{R_y} = \frac{1}{K}\sum_{k=0}^{K-1}\mathbf{y}(k)\mathbf{y}^H(k)$, $\mathbf{V} = \mathbf{R_y}$, $\mathbf{A} = \emptyset$
for $l = 0$ to $L-1$ **do**
  $\mathring{\mu}_l = \arg\max_{\mathring{\mu}_i\in\{\mathring{\mu}_1,\ldots,\mathring{\mu}_P\}}(\mathbf{W}\mathbf{a}(\mathring{\mu}_i))^H\mathbf{V}\mathbf{W}\mathbf{a}(\mathring{\mu}_i)$
  $\mu_l = \text{BSC\_GD}(\mathbf{V}, \mathbf{W}, \mathring{\mu}_l)$
  $\mathbf{A} = [\mathbf{A}, \mathbf{a}(\mu_l)]$
  $\mathbf{R_x} = \{\mathbf{W}\mathbf{A}\}^\dagger\mathbf{R_y}\left(\{\mathbf{W}\mathbf{A}\}^\dagger\right)^H$
  $\mathbf{V} = \mathbf{R_y} - (\mathbf{W}\mathbf{A})\mathbf{R_x}(\mathbf{W}\mathbf{A})^H$
end for
Output: The estimated parameter $\hat{\boldsymbol{\mu}} = [\mu_0, \ldots, \mu_{L-1}]$

IV. Numerical Results

A. Computational Complexity

We compare the computational complexity of our proposed algorithm with GOMP which treats each sub-carrier as one snapshot and computes the gradients in the channel snapshots domain instead of the BSC domain. We use the methods introduced in [9] for matrix operations to compute the total number of floating point operations (FLOPS) for both algorithms. Table I compares the FLOPS of GCOMP and GOMP with $I_{\text{max}} = 5$ and $L = 1$. We see that GCOMP has a much lower complexity than GOMP. This is mainly because the complexity of the pseudo-inverse step in GOMP is directly proportional to $\mathcal{O}(K^2)$, whereas the GCOMP algorithm computes the pseudo-inverse with complexity proportional to $\mathcal{O}(M^2)$. Since, $K \gg M$, it is clear that GCOMP scales much better than GOMP with the number of sub-carriers. Note that $\mathbf{R}_{\mathbf{y}}$ in the GCOMP is computed according to (7) only once and requires $M^2(K+1)$ FLOPS.

B. Performance Evaluation

We compare our proposed GCOMP algorithm with GOMP from [6] and COMP from [7]. The same sensing matrix is used for all algorithms and it has been designed to minimize the mutual coherence (MC) following the schemes introduced in [6]. For the dictionary design, we configure $P = N$. The simulations are conducted using OFDM signals with the central carrier frequency of 28 GHz and 120 kHz sub-carrier spacing. The DoAs of the configured sources are randomly selected within the range from $[-\frac{\pi}{2}, \ldots, \frac{\pi}{2}]$. We evaluate the mean-squared error (MSE) of the estimated spatial frequency vector for the DoA sources by averaging over 2000 Monte Carlo runs per signal to noise ratio (SNR) point. The SNR is swept from -10 dB to 20 dB with steps of 5 dB.

In the first scenario, only a single DoA source is configured. We further configure $M = 4$, $N = 8$ and we vary the number of sub-carriers to be $K = 64$ and $K = 128$, respectively. The algorithm performance in this scenario is shown by Fig. 1. The simulation figure shows that both GOMP and GCOMP achieve a similar accuracy, although the proposed GCOMP enjoys a much smaller computational complexity due to the BSC domain processing. Meanwhile, both GOMP and GCOMP significantly outperform COMP, which is due to the gain of the gradient descent based local refinement. Furthermore, the figure shows that the performance of the local refinement improves with an increased number of sub-carriers, which is also expected. In the second scenario, multiple DoA sources are configured. Hereby, two setups are evaluated: In the first setup we configure $L = 2$, $M = 4$, $N = 8$ while in the second setup we configure $L = 3$, $M = 8$, $N = 16$. The number of sub-carriers is fixed to be $K = 64$ in both setups. The algorithm performance in this scenario is shown in Fig. 2. We can see that in both setups GOMP only achieves a marginal improvement over COMP. That is due to the fact that GOMP treats the on-grid estimation for all DoA sources and the off-grid refinement for all DoA sources as two independent steps. In a multi-source scenario, when an early on-grid detected DoA source is off-grid, its leaked power will impact the on-grid detection of the next DoA source. As a result, the on-grid estimation step does not provide reliable initial estimates for all DoA sources, which may lead to a failure of the later local refinement. On the other hand, we see that in both setups, the proposed GCOMP significantly outperforms both GOMP and COMP. That is because GCOMP integrates the BSC-GD into the successive procedure of the COMP, such that an early detected DoA source is off-grid refined and then canceled before the next DoA source is detected. In this way, the grid mismatch issue is mitigated.

K	$M=4,\ N=8$		$M=8,\ N=16$	
	GOMP	**GCOMP**	**GOMP**	**GCOMP**
32	69448	9145	186768	59645
64	219528	9657	532176	61693
128	765448	10681	1714512	65789

TABLE I: FLOPS comparison with $I_{\text{max}} = 5$ and $L = 1$

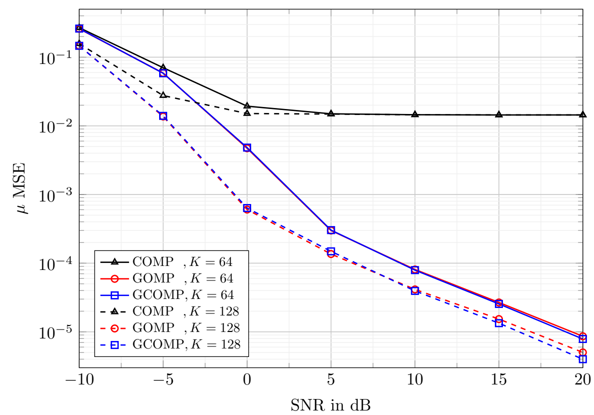

Fig. 1: MSE of the spatial frequency μ for single source with $M = 4$, $N = 8$.

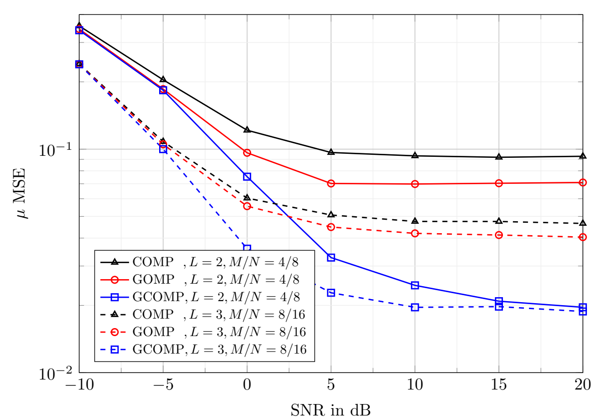

Fig. 2: MSE of the spatial frequency μ for multiple sources with $K = 64$.

V. Conclusions

We present a compressive sensing based high resolution DoA estimation scheme using BSC-GD. Compared with an existing gradient descent based DoA refinement algorithm in the channel snapshots domain, the proposed refinement algorithm operates in the BSC domain so that the complexity does not scale with the number of snapshots. The proposed BSC-GD refinement is further integrated into a known COMP algorithm, such that an on-grid detected source is first refined and then off-grid canceled before the next source is detected. Such an approach can eliminate the power spreading issue when an off-grid source is on-grid canceled. Numerical results show the effectiveness of the proposed method.

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