

## Calculation of specular and semi-specular luminaires using backward ray-tracing method.

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### 1. Introduction

In order to calculate luminaires of real shapes, in which the optical elements are constructed of materials of diversified photometric characteristics, computer calculation methods need to be applied. The algorithm for carrying out the calculation may be designed on the basis of the methods which make it possible to perform the calculation without having to make simplifying assumptions which should be taken into account when using the commonly known analytical method of computing luminaires [1, 2]. One of the computer methods is the ray tracing method employed in its various forms (light ray method, backward ray method). In this paper the backward ray tracing method [3, 4] was used in computing the luminaires. The method is applied in computer graphics to create image visualisation. As it has been proved in many studies, the method may be applied to carry out various computations in the lighting engineering [6, 7, 8, 9].

### 2. Backward ray-tracing method

The basis for calculation of luminaire is determining the luminous intensity  $I_{\gamma,C}$  of the luminaire on given planes C and angles  $\gamma$ . Having the values of the luminous intensity in respective directions, it is possible to determine other quantities characteristic of the luminaire, such as the luminous flux  $\Phi_{lum}$  or the light output ratio  $\eta_{lum}$ . The luminous intensity of the luminaire  $I_{\gamma,C}$  can be computed by means of the inverse square law, which describes the following relation for the point source:

$$I_{\gamma,C} = E_i \cdot r^2 \quad (1)$$

where:

$E_i$  – illuminance  $E_i$  in a given point “i” at surface  $S_i$  in a distance  $r$  from light center of luminaire

Illuminance  $E_i$  in a given point “i” is determined by means of the definition expressed by the equation (2) [12] where the integral is replaced by the luminance summation  $L_{S_j}$  on the surfaces  $S_j$ , where the luminance is connected with the elementary beam of radiation which propagates at a solid angle  $d\omega$  and incident on the surface normal  $S_i$  at the angle of  $\theta$ :

$$E_i = \int_{\omega} L_{S_j} \cdot \cos\theta \, d\omega = \sum_{S_{j1}}^{S_{jk}} L_{S_j} \cdot \cos\theta \cdot \Delta\omega \quad (2)$$

In order to determine the luminance  $L_{S_j}$  on the surfaces  $S_j$  of the system it is possible to apply the ray tracing method. Its assumption is that the radiation propagates along the linear paths called rays, and the traced rays are subject to repeated reflections between the surfaces of given shapes and given photometric properties [3, 4]. The process connected with the propagation of radiation is described by the laws of geometrical optics.

In the backward ray tracing method the rays are traced starting from the point of view of a conventional observer placed in the given environment, and at the same time consideration is given to any possible reflections or refractions resulting from the intersection of rays with surfaces which are part of a given system (Fig. 1). New, secondary rays form in the reflection points. Each of the newly formed rays is traced in the same way until it is sufficiently absorbed or encounters an object on its way. In each hitting point a direct component is calculated, which results from illuminating the given point by a light source. With subsequent reflections in the subsequent nodes the value resulting from the determined direct component of a given node and from the sum of values determined in the remaining nodes is calculated. The process is ended after summing up the values of all the nodes and thus we obtain the value of the luminance for the area around the point which was hit by the original ray.

The assumptions for the proposed method are based on the assumed model of the system (Fig. 1) and the equation (3) which makes it possible to determine the luminance  $L_r$  in the direction “v” of the surface element  $S_i$  [6].

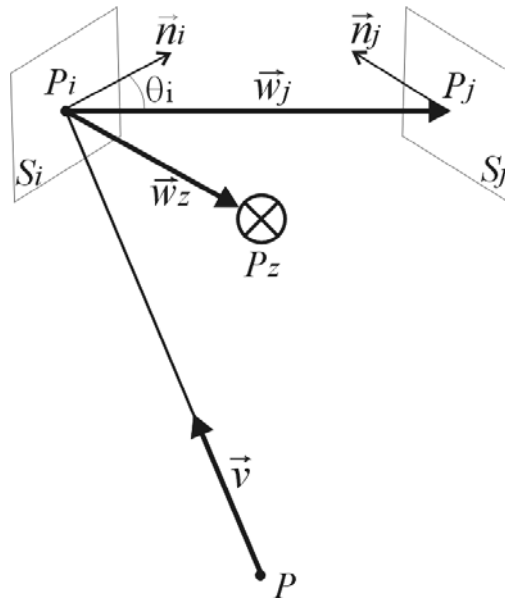


Fig. 1. The geometrical interpretation of the designations in the assumed model. The observer is placed in point P, and the light source in point  $P_z$ . The surfaces found in the given system are denoted as  $S_i$ ,  $S_j$ .

$$L_r(P_i, -\vec{v}) = \sum_z q(P_i, -\vec{v}, \vec{w}_z) \cdot g(P_i, P_z) \cdot L(P_z, -\vec{w}_z) \cdot |\cos \theta_z| \cdot \omega_z + \frac{\pi}{N} \cdot \sum_{j=0}^{N-1} q(P_i, -\vec{v}, \vec{w}_j) \cdot g(P_i, P_j) \cdot L(P_j, -\vec{w}_j) \quad (3)$$

where:  $q$  – the luminance coefficient,  $g$  – the factor dependant on the system geometry, it assumes the value 0 when points  $P_i$  and  $P_j$  are mutually invisible or the value 1 when there is no obstacle on the line connecting the two points.

In the equation (3) two parts can be differentiated: the part responsible for determining the direct component and the part responsible for determining the indirect component. Both components may be determined for materials which are characterised by specular reflection, diffuse reflection or mixed reflection (the combination of ideal reflections: the specular and diffuse reflections), where the photometric properties of the materials are described by means of the luminance coefficient  $q$  (Fig. 2).

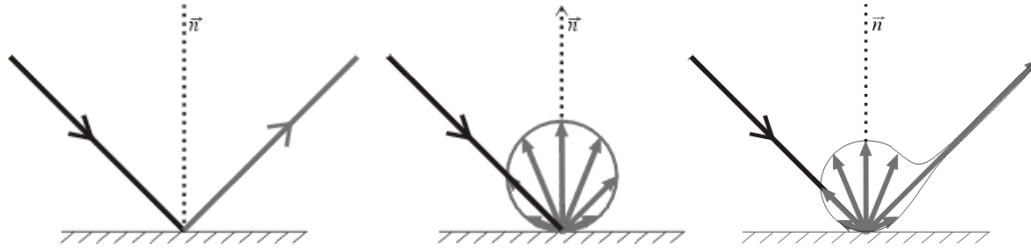


Fig.2. Specular, diffuse and mixed reflection.

### 3. Calculation of diffuse luminaires

The actual surfaces of the optical elements of luminaires (reflectors, globes, arc tube bodies and lamp bulbs) are replaced by a set of small, flat surfaces [10] (Fig. 3, Fig. 4 and Fig. 5). The shapes of the reflectors, globes and other elements are created by means of software of the CAD type. Out of available spatial models of objects the face objects are used (flat surface – polygon).

A detailed description of the method for computing diffusing luminaires has already been included in the published papers [7, 8, 9].

The analytical method of computing the diffusing luminaires [1, 2] applied for the simplest cases (spherical globe) provides precise and reliable results. Therefore the first verification of correctness of the applied computer method was performed for the ideal, diffusing spherical globe and for the open globe with a handle (Fig. 3).

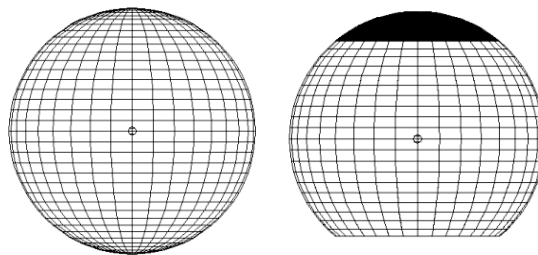


Fig. 3. The model of an ideal spherical globe of 30 cm diameter and the open globe with a handle. The assumed transmittance of the luminous flux for the globe was  $\tau=0.6$ , the reflectance  $\rho=0.3$ , and the luminous flux of the light source  $\Phi_0$  being 1350 lm.

The luminance of the globe surface and the luminous intensity curve were calculated, and then, on its basis, (by means of partial flux method) the luminous flux and the light output ratio of the luminaire. The obtained results (Table 1) were compared with the calculation results obtained by means of the analytical method.

Just ideal convergence was found between the computations effected by means of the ray tracing method and the computations performed by means of the analytical method which is a reference for the analysed cases. As the complexity of the shape of the computed object does not put a constraint on the computer method, it can be presumed that a similar precision of calculations might be obtained for systems of more complex shapes.

Table 1. Luminance of the globe  $L$  and luminous flux of the luminaire  $\Phi_{lum}$  calculated of the analytical and backward ray-tracing method

		Analytical method	Backward ray-tracing method
$L$ [ $cd/m^2$ ]	Ideal spherical globe	<b>1 303</b> [ $cd/m^2$ ]	<b>1 306</b> [ $cd/m^2$ ]
	Open globe with a handle	<b>1 184</b> [ $cd/m^2$ ]	<b>1 187</b> [ $cd/m^2$ ]
$\Phi_{lum}$ [lm]	Ideal spherical globe	<b>1 157</b> [lm]	<b>1 166</b> [lm]
	Open globe with a handle	<b>1 011</b> [lm]	<b>997</b> [lm]

The next example for which the calculations were carried out regards a luminaire with an open, diffusing globe (Fig. 4). In the luminaire some objects were placed, which with regard to their shapes and photometric properties corresponded to the lampholder and the fitting elements. The light source was a 100 W standard incandescent lamp, which was presented as a sphere of 2 cm diameter, and an actual distribution of luminous intensity of a standard incandescent lamp with a clear bulb was assigned to it.

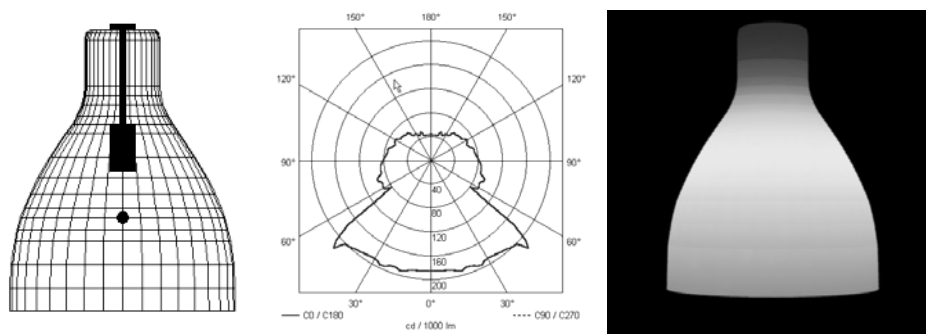


Fig. 4. The luminaire model (left) with an open, diffusing globe (the luminous flux transmittance coefficient  $\tau=0.6$ , the reflectance  $\rho=0.3$ ). The objects filled black are the handle and lampholder and the light source (a 100 W standard incandescent lamp with a clear bulb). The luminous intensity curve (the centre) and the visualisation of the luminance distribution on the surface of the luminaire (right).

Next example: luminaire with louvre for four compact fluorescent lamps, 36W power (Fig. 5).

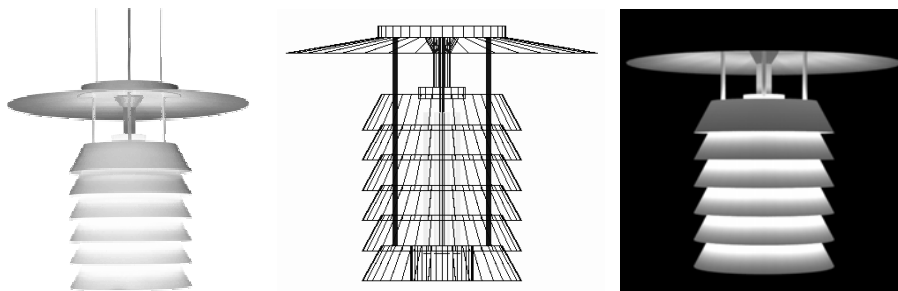


Fig.5. Photo (left) and model of the luminaire (centre) with louvre and the luminance distribution on the surface of the luminaire (right).

#### 4. Calculation of specular reflectors

Assuming there occurs the specular reflection i.e. the reflection without diffusion in accordance with the laws of the geometrical optics [2, 12], the traced rays are reflected at the point of intersection with subsequent surfaces until they hit the light source or encounter any object on their way. The value of luminance of the surface  $S_i$  which was hit by the original ray is computed based on the overall luminance of the light source  $P_z$  and the specular reflectance of the surfaces from  $S_i$  to  $S_j$  (Fig. 6).

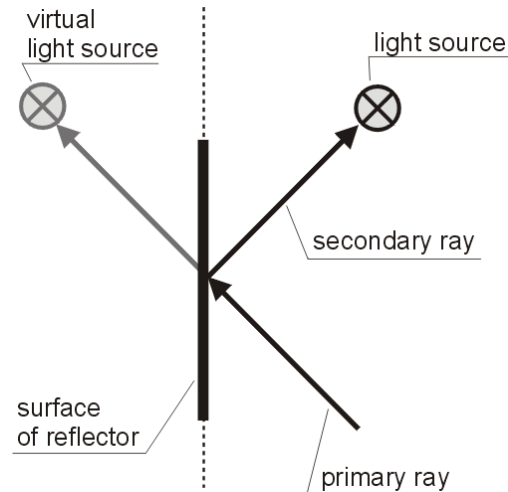


Fig.6. Virtual light source created for surface characterized by specular reflection.

Hitting the surface which is characterised by specular reflection results in sending a secondary ray from the hitting point. The ray is sent in the direction for which the angle between the surface normal is equal to the angle, at which the original ray hit the given surface. However, sending a big number of rays towards the elementary surface does not guarantee that any of the secondary rays reflected by the surface will hit the light source. It is also assumed that the actual overall surface of the light source is subject to discretisation, and only one ray is sent towards each elementary (discrete) flat surface. Therefore, in order to find the hitting point on the surface of the reflector in which an image of the light source will form, the so called virtual light source [5] is introduced, which is formed on the opposite side of the elementary surface. The traced ray is sent towards the virtual source and it intersects the surface of the reflector in the place where the image of the actual light source will form (Fig. 6).

The verification of precision of the suggested method was conducted by means of carrying out calculations for a paraboloidal reflector [11] (Fig. 7) with the light source in a ball shape placed in the focus of parabola, and two diameters of the light sources were given:  $d_1=1\text{cm}$  and  $d_2=4\text{ cm}$ . The value of the flux of the light source is 1000 lumens.

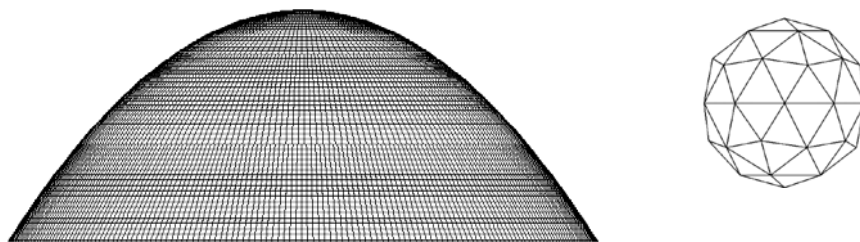


Fig. 7. The model of a paraboloidal reflector (left), whose actual surface was replaced by 13230 flat surfaces. The reflector height 17.2 cm, the diameter of the opening 43.9 cm, the focal length 7 cm. The model of the light source (right), whose surface was divided into 144 discrete flat surfaces. Proportions between the objects have not been kept.

The luminous intensity curve was calculated for several distances “r” from the light centre of the luminaire (Fig. 8, Fig. 9). The far-field photometry is respectively. Distance  $r_f$  where results of inverse-square law are accurate within one percent is:  $r_f=21\text{m}$  for the light source of the diameter  $d_1=1\text{ cm}$  and  $r_f=5\text{m}$  for the light source of the diameter  $d_2=4\text{cm}$  [1, 2].

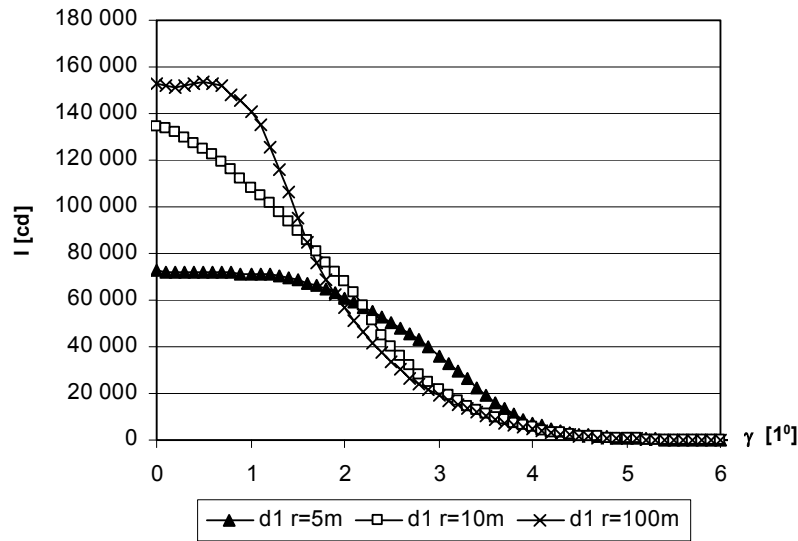


Fig.8. Luminous intensity curve calculated for the light source of the diameter  $d_1=1\text{ cm}$  and three distances  $r$ : 5m, 10m and 100m.

In the case of a light source of a relatively small diameter  $d_1=1\text{cm}$  it is possible to obtain a large concentration of the light beam and a great value of the maximum luminous intensity  $I_{\max}$  and the shape of the luminous intensity curve to a large extent depends on the calculating distance  $r$  (8).

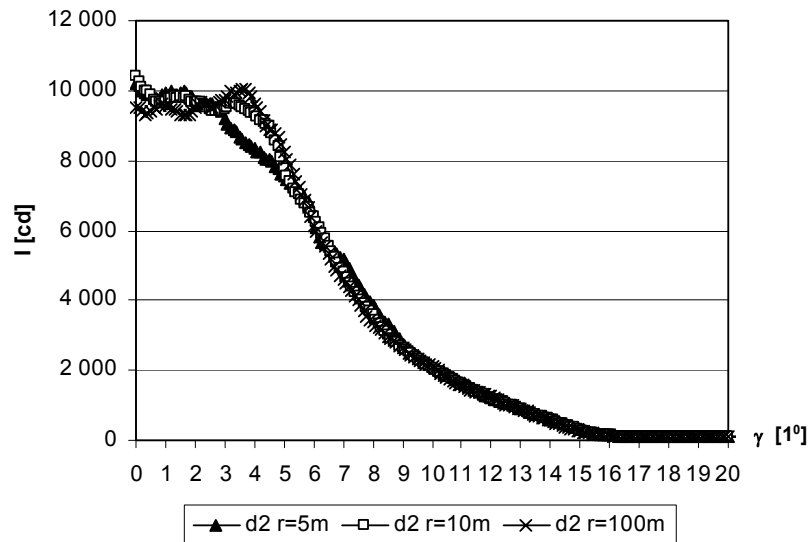


Fig.9. Luminous intensity curve calculated for the light source of the diameter  $d_1=4\text{ cm}$  and three distances  $r$ : 5m, 10m and 100m.

In the case of a light source of a relatively large diameter  $d_2=4\text{cm}$  it is not possible to obtain a large concentration of the light beam or a great value of the maximum luminous intensity  $I_{\max}$  and the shape of the luminous intensity curve hardly changes with different distances  $r$  (Fig. 9).

On the basis of the obtained luminous intensity curves, the luminaire flux  $\Phi_{\text{lum}}$  the maximum luminous intensity  $I_{\max}$ , the smallest  $\delta_{\min}$  and the largest  $\delta_{\max}$  beam divergence angle were calculated. The obtained results were compared with the calculation results

obtained by means of the analytical method (Table 2).

Table 2. Luminous flux of the luminaire  $\Phi_{lum}$ , the maximum luminous intensity  $I_{max}$ , the smallest  $\delta_{min}$  and the largest  $\delta_{max}$  beam divergence angle calculated for paraboloidal reflector and spherical light source of the diameter  $d_1=1\text{cm}$  i  $d_2=4\text{cm}$ .

	$d_1=1\text{cm}$			$d_2=4\text{cm}$		
	Analytical method	Backward ray-tracing method	Error	Analytical method	Backward ray-tracing method	Error
$\Phi_{lum}$	<b>929 [lm]</b>	<b>950 [lm]</b>	<b>2 %</b>	<b>929 [lm]</b>	<b>917 [lm]</b>	<b>-1 %</b>
$I_{max}$	<b>145 121 [cd]</b>	<b>~152 000 [cd]</b>	<b>5 %</b>	<b>9 072 [cd]</b>	<b>~9 600 [cd]</b>	<b>6 %</b>
$\delta_{min}$	<b>2.4 [°]</b>	<b>~1.8 [°]</b>	-	<b>9.4 [°]</b>	<b>~8.2 [°]</b>	-
$\delta_{max}$	<b>8.1 [°]</b>	<b>~9.8 [°]</b>	-	<b>29.7 [°]</b>	<b>~32.2 [°]</b>	-

Considerable convergence was found between the computations effected by means of the ray tracing method and the computations performed by means of the analytical method which is a reference for the analysed cases. Similarly as in the case of computing the diffusing luminaires, the complexity of the shape of the computed object does not put a constraint on the computer method, it can be presumed that a similar precision of calculations might be obtained for systems of more complex shapes.

## 5. Calculation of semi-specular reflectors

Computation of luminaires is connected with having to account for the interreflections. The luminous flux interreflections may be accounted for in the proposed method only for the materials which reflect the light in an ideally specular and ideally diffuse way. Therefore the calculations for the reflectors made from the materials characterised by mixed reflection properties may only be done under condition that ideal reflections are combined: specular and diffuse (Fig. 2).

The calculations were performed for the computer model of an actual reflector of a road lighting luminaire (Fig. 10) designed for application with a compact, 150 W high pressure sodium lamp (Fig. 11).

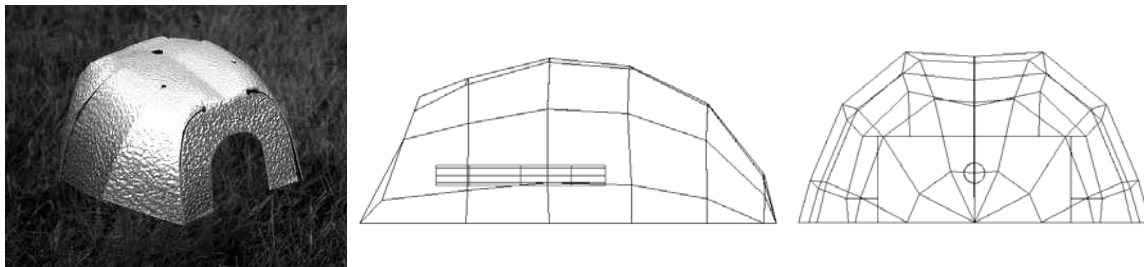


Fig.10. Photo and CAD model of the reflector.

The reflector is made of Alanod Dessin Stucco G aluminium sheet. On the basis of the catalogue data from the sheet producer, the value of the overall reflection coefficient of the luminous flux was assumed at  $\rho=0.86$ . The computer model of the reflector was constructed by means of replacing its actual surface with a set of flat surfaces (Fig. 10). Similarly the high pressure sodium lamp model was created. In order to reconstruct, as

accurately as possible, the true photometric properties of the lamp, the computer model consists of arc tube, bulb and cap (Fig. 11).

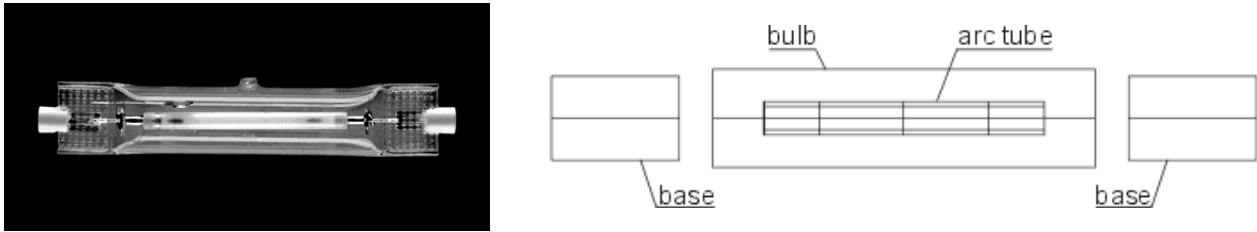


Fig.11. Photo and CAD model of sodium lamp.

The lamp arc tube was divided into four parts, so as to take into account the non-uniform distribution of luminance on the surface of the actual arc tube. The surface luminance of the two terminal parts is 20% smaller than the luminance of the two middle parts of the arc tube [9]. The surface of the bulb which surrounds the arc tube has the glass photometric properties of the refractive index  $n=1.52$ . The luminance of the arc tube surface was selected in such a way that the luminous flux of the lamp with all its elements (arc tube, bulb, cap) has the value of 15000 lumens. By means of the ray tracing method, the luminous intensity curves were calculated for the above described lamp, and the results were correlated with the catalogue data (Fig. 12) [9].

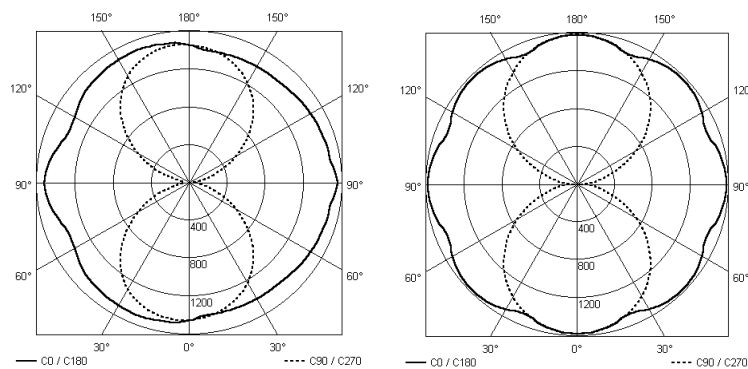


Fig.12. Luminous intensity curve [cd] of sodium lamp: catalogue data (left) and calculations performed using backward ray-tracing method for model of lamp from Fig.11 (right).

The calculations and measurements of the luminous intensity curves of the luminaire with the reflector and the high pressure sodium lamp were performed (Fig. 13).

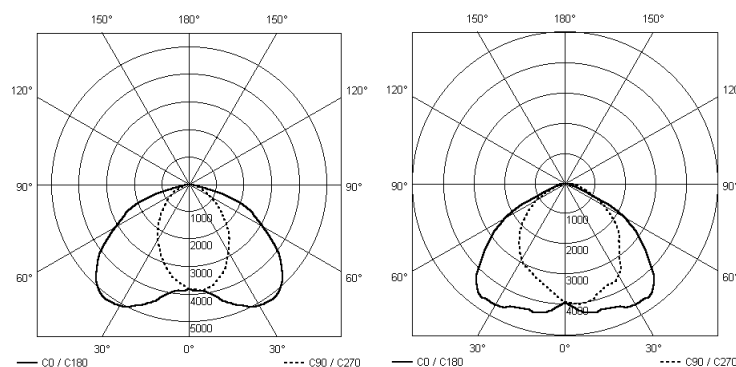


Fig.13. Luminaire with reflector and sodium lamp: luminous intensity curve [cd] measured in lighting laboratory (left) and calculated using backward ray-tracing method (right).



On the basis of the obtained luminous intensity curves the luminous flux and the light output ratio of the luminaire were computed (Table 3). The calculation results were verified against the results of measurements which were conducted in the photometric laboratory using the real reflector and the real high pressure sodium lamp (Table 3). The luminous intensity curves were measured by means of a moving detector photometer, and the luminous flux and the light output ratio of the luminaire were measured by means of the Ulbricht sphere.

Table 3. Luminous flux  $\Phi_{lum}$  and the output ratio  $\eta_{lum}$  of the luminaire, received from measurements and calculations performed using backward ray-tracing method.

	Measurements	Calculations backward ray-tracing method	Error
$\Phi_{lum}$	<b>12 435 [lm]</b>	<b>12 137 [lm]</b>	<b>-2 %</b>
$\eta_{lum}$	<b>0.83</b>	<b>0.81</b>	

Comparing the luminous intensity curves obtained through the measurements with those calculated by means of the ray tracing method, it should be stated that there is a considerable similarity in their shapes. The maximum luminous intensity in the plane C0-C180 is for the angle  $\gamma=35^\circ$ . The relative differences in percentage terms between the values of the luminous intensity for particular angles may exceed 100%, but the biggest of them occur in the case of angles over  $60^\circ$ , i.e. where the absolute values of the luminous intensity are relatively small (Fig. 14).

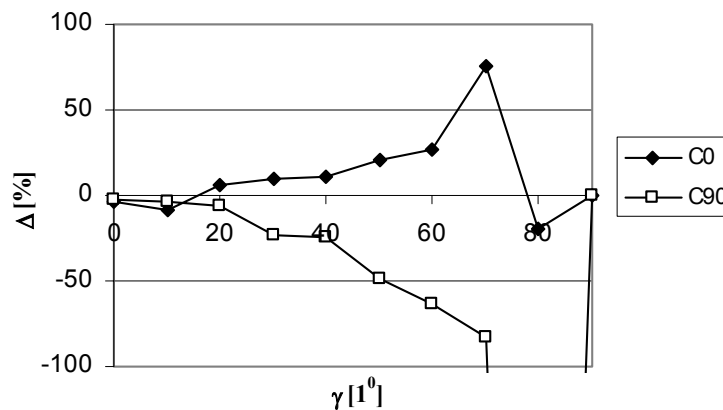


Fig.14. Relative proportional difference curve  $\Delta$  between measured and calculated value of the luminous intensity of luminaire for particular angles  $\gamma$  in two planes C0 and C90.

The discrepancies between the calculation results and the measurement results come from two sources. Firstly, taking measurements involves occurrence of a number of errors. These include e.g. an error in the system scaling connected with the model accuracy and imprecise fixing of the reflector and lamp in the casing of the prototype luminaire. Secondly, the accuracy of calculations depends to a large extent on the way of representing the actual shape of the reflector and on including the photometric properties of the applied materials in the calculations. The reflector shape was represented in an approximate way. A more precise representation would have required a replacement of the actual shape of the reflector with a bigger number of flat surfaces. Moreover, measurements of the photometric properties of the reflector material were not carried out.

These were assumed on the basis of the data provided by the aluminium sheet producer.

## 6. Summation

The presented method makes it possible to calculate luminaires which are constructed of materials having various photometric properties. Accurate results of calculations are possible to obtain for any complicated shapes of optical elements of luminaires (globe, reflector). An additional advantage of the proposed method is the possibility of applying it for educational purposes to teach students how to design and compute luminaires. The available didactic materials enable them to conduct the computations independently after becoming acquainted with the course which is generally accessible on the Internet [10].

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