

## LUMINOUS FLUXMETER SYSTEM WITH A CYLINDRICAL INTEGRATOR: GENERALIZATION AND ANALYSIS OF THE MODEL ERROR

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### Abstract:

A closed cylindrically shaped integrating space (integrator) (ICS-C), as a part of the system for electrical light sources (ELs) luminous flux measurement, is considered in this paper. In that sense, the error introduced by the ICS-C model, when such an integrator is used instead of the Ulbricht's sphere in the integrating fluxmeter system, is theoretically evaluated.

Mathematical model for the indirect illuminance determination in the vicinity of the point  $P_1$  ( $P_1$  - the point in the centre of one basis where the indirect illuminance is predicted to be measured), is realized in two steps:

- First, the luminous flux distribution is determined using the system of interreflection equations of luminous flux (SIE-LF) for the  $N$ -surface model of the uniformly and Lambertian painted cylinder (two basis and  $(N-2)$  ring shaped lateral surfaces).
- Afterwards, using the method of equivalent luminous exitances, the existing system is transformed into the plane of the opposite basis as concentric radiated rings of calculated exitances. Based on these exitances, the indirect illuminance in the centre of the first basis, i.e. at point  $P_1$ , is easily determined.

Based on the obtained expression, the error introduced by the ICS-C model, when it is used instead of the ICS-S, can be formulated and analyzed. Analyzing the error of the ICS-C model, the geometry of the ICS-C model can be designed.

### 1. INTRODUCTION

The idea to use some other form of the integrator, instead of integrating sphere, in the integrating fluxmeter system is not a new one (i.e. cube [1]), but has not been yet widely applied.

Due to a rather expensive production of the integrating sphere, some other possibilities of using a different ICS for the same purpose were analyzed at the Laboratory of Electrical Installations and Illuminating Engineering at the Faculty of Electronic Engineering in Niš. Some of them are: Parallelepiped with square bases (ICS-PSS), cylinder with flat or spherical sector bases, or a combination of a flat and spherical sector bases (ICS-CFF, ICS-CSS, ICS-CFS, ICS-CSF). A number of these ICSs are realized, and using a laboratory ICS-PSS model, corresponding measurements were done. In the frame of the experiment, the integrating factor is firstly determined using the standard (etalon) ELS of OSRAM type.

Theoretical analyses, which preceded the construction of the Laboratory ICS model, are based on deduced expressions for the indirect illuminance in the measuring point  $P_1$  vicinity. The point  $P_1$  is always adopted to be in the centre of one of the bases. In order to deduce the expression for the indirect illuminance at point  $P_1$ , two-surface model is used (one basis and the remaining surface of the ICS), or a three-surface model (two bases and lateral surface) and a corresponding SIE-SF. For the purpose of estimation of the luminous flux distribution on the ICS surfaces, luminous intensity distributions of ELs, which are the subject of the measurement and are used for the ICS illumination, are assumed to be

known. For that purpose, two types of ELSs were used. The ELS is placed arbitrarily on the parallelepiped or cylinder axis. Afterwards, using the method of equivalent exitances, the expression for indirect illuminance at point  $P_1$  is deduced. This expression is modelled in such a way so it can be compared to the corresponding one for the ICS-S, i.e. the expression that presents the error of the used ICS model regarding the ICS-S is evaluated. Regarding the previous papers, in this one, the expressions for error of the ICS-C model (which is considered a multi-surface model of two bases and two-three ring shaped lateral surfaces), were generalised. This was done in order to investigate the accuracy of the indirect illuminance calculation at point  $P_1$ , since the accuracy increases with the increment of the constitutive surface number.

The paper is organized in a way that in the second Section is given a theoretical background for solving the problem of  $N$ -surface ICS-C model with flat bases. The ICS-C is illuminated using an ELS placed arbitrarily on the cylinder axis. Two types of ELSs, previous papers [3, 4] denoted as Type-A ELS (isotropic light source) and Type-D ELS (FL-fluorescent lamp), were adopted.

In the first part of this Section, the SIE-LF is formed, and all parameters necessary for its solving, including direct luminous fluxes on each cylinder surface, were determined. Afterwards, in the second part of the Section, the method of equivalent exitances was applied, and the expression for the indirect illuminance at point  $P_1$ , was determined.

In the next, third Section, a number of numerical experiments is presented, based on which certain conclusions can be made.

A number of numerical experiments relates to the analysis of the multi-surface model, i.e. to the dependence of the indirect illuminance at point  $P_1$  on the constitutive surface number. Remaining numerical experiments analyse the error of the ICS-C model versus the cylinder geometry and the ELS position. These experiments allow the design of the ICS-C.

Then follows the fourth Section – Conclusion, and afterwards Appendix and the list of references.

## 2. THEORETICAL BACKGROUND

### 2.1 GEOMETRY DESCRIPTION AND SIE-LF

A system for luminous flux measurement with cylindrically shaped ICS with flat circle bases, illustrated in *Fig. 1*, is considered. Generally, cylindrically shaped ICS can be considered as a  $N$ -surface model with flat circle bases of radius  $R_c$ , surfaces  $S_{c1} = S_{cN} = \pi R_c^2$ , and lateral surface of length  $H_c$  which consists of  $N-2$  ring shaped surfaces  $S_{ck} = 2\pi R_c \Delta H_k$ ,  $k = 2, 3, \dots, N-1$ . Optical centre of the ELS is placed on the cylinder axis at distance  $h$  from the measuring point  $P_1$ . Interior surface is uniformly coloured and its reflectance is  $\rho_c = \rho_{ck}$ ,  $k = 1, 2, \dots, N$ . Total surface of the closed space is  $S_c = S_{c1} + \dots + S_{cN}$ .

In the case of a closed space, SIE-LF that presents the flux conservation law is of the following form

$$\Phi_{0k} = \sum_{i=1}^N (\delta_{ik} - \rho_i f_{i,k}) \Phi_i, \quad k = 1, 2, \dots, N, \quad (1)$$

where:  $\delta_{ik}$  - Kronecher symbol,  $\Phi_i = \Phi_{0i} + \Phi_{ind i}$  - total luminous flux of the  $i$ -th surface ( $\Phi_{0i}$  - direct component and  $\Phi_{ind i}$  - indirect component),  $f_{i,k}$  - form factor (interreflection coefficient) given by the double surface Fredholm's integral:

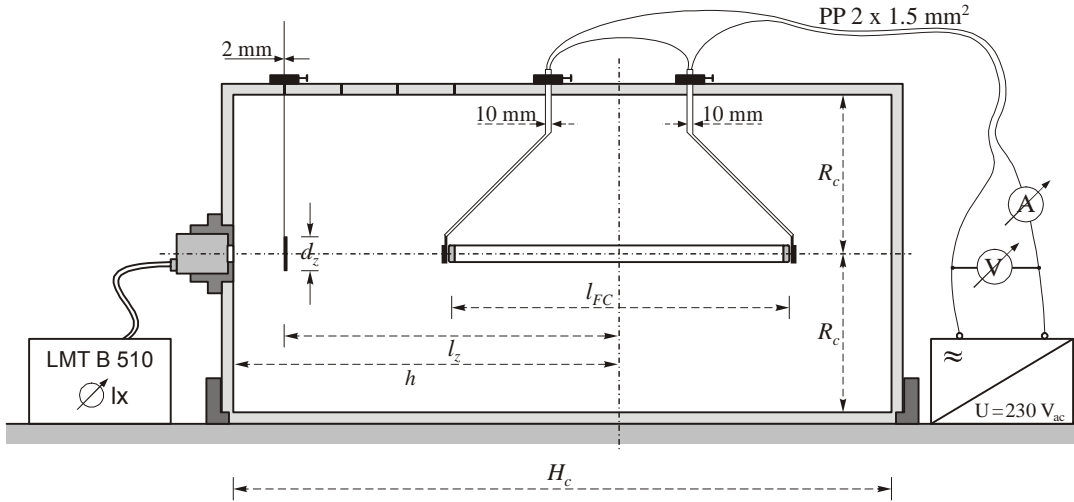


Fig. 1: Schematic illustration of the integrating fluxmeter system (IFS) with a cylindrically shaped integrator with flat bases (ICS-C).

$$f_{i,k} = \frac{1}{S_i} \iint_{S_i S_k} \frac{\cos \gamma_i \cdot \cos \gamma_k}{\pi r_{ik}^2} dS_i dS_k, \quad i, k = 1, 2, \dots, N. \quad (2a)$$

For any kind of closed space consisting of  $N$  Lambertian surfaces, general relations are valid

$$S_i f_{i,k} = S_k f_{k,i}, \quad i, k = 1, 2, \dots, N, \quad (2b)$$

$$\sum_{k=1}^N f_{i,k} = 1, \quad i = 1, 2, \dots, N, \quad (2c)$$

where (2b) represents the reciprocity law, and (2c) represents the consequence of the luminous flux conservation.

## 2.2 DETERMINATION OF THE DIRECT FLUXES ON THE ICS-C SURFACES

In order to determine the total luminous flux distribution on the interior ICS-C surfaces, based on SIE-LF, it is necessary to know the direct luminous flux distribution.

Two types of ELSs, whose schematic illustrations are given in Fig. 2, are used for the direct luminous flux determination. Rotationally symmetric luminous intensity distributions of these ELSs, can be approximately analytically presented in the following form:

$$\text{Type A ELS – Isotropic light source} \quad \Rightarrow \quad I(\gamma, \varphi) = I_0, \quad \gamma \in [0, \pi]; \quad (3a)$$

$$\text{Type D ELS – Fluorescent lamp} \quad \Rightarrow \quad I(\gamma, \varphi) = I_0 \sin \gamma, \quad \gamma \in [0, \pi]. \quad (3b)$$

In accordance with the luminous intensity distributions (3a and 3b), sketches of polar diagrams are also shown in Fig. 2.

Under an assumption that the luminous intensity distribution,  $I(\hat{r})$ , is known, total radiated luminous flux of the ELS is

$$\Phi_{ELS} = \Phi_{0c} = \int_{\Omega=4\pi \text{ Sr}} I(\hat{r}) d\Omega, \quad (4)$$

where  $I(\hat{r})$  is given by (3a, b), and  $\hat{r}$  denotes the direction of the space angle  $d\Omega$  axis, from the ELS location point, i.e. the direction defined by the sphere angles  $\gamma$  and  $\varphi$ .

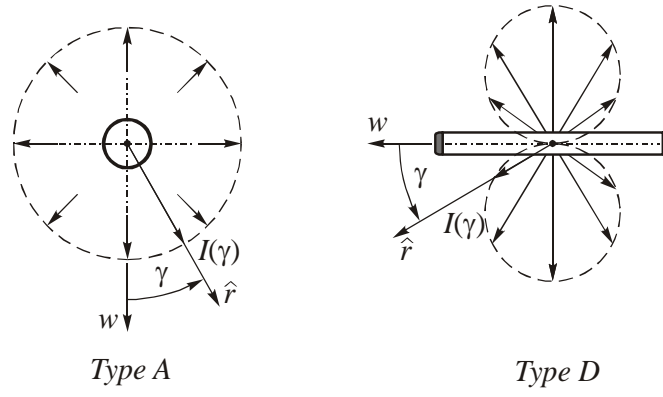


Fig. 2: Schematic illustration of two ELS types and sketches of their polar diagrams.

If  $I(\hat{r}) = I(\gamma, \varphi)$ , according to (3a, b), is substituted in (4), then the following is obtained for the total ELS luminous flux: for Type A ELS -  $\Phi_{0c} = 4\pi I_0$ ; and for Type D ELS -  $\Phi_{0c} = \pi^2 I_0$ .

Direct luminous flux distribution on the ICS-C surfaces can be obtained using the following expressions:

For the Type A ELS  $\Phi_{0k} = \frac{\Phi_{0c}}{2} (\cos \gamma_{k-1} - \cos \gamma_k)$ ,  $k = 1, 2, \dots, N$  (6a)

For the Type D ELS  $\Phi_{0k} = \frac{\Phi_{0c}}{\pi} (\gamma_k - \gamma_{k-1} - \sin \gamma_k \cos \gamma_k + \sin \gamma_{k-1} \cos \gamma_{k-1})$ ,  $k = 1, 2, \dots, N$  (6b)

where:

$$\gamma_k = \begin{cases} \arctg \frac{R_c}{h - (k-1)\Delta H}, & (h - (k-1)\Delta H) \geq 0 \\ \frac{\pi}{2} + \arctg \frac{R_c}{(k-1)\Delta H - h}, & (h - (k-1)\Delta H) < 0 \end{cases},$$

$$\cos \gamma_k = \frac{h - (k-1)\Delta H}{\sqrt{R_c^2 + (h - (k-1)\Delta H)^2}}, \quad \sin \gamma_k = \frac{R_c}{\sqrt{R_c^2 + (h - (k-1)\Delta H)^2}}, \quad k = 2, 3, \dots, N-1,$$

$$\sin \gamma_0 = \sin \gamma_N = 0, \quad \cos \gamma_0 = 1, \quad \cos \gamma_N = -1, \quad \Delta H = H_c / (N-2).$$

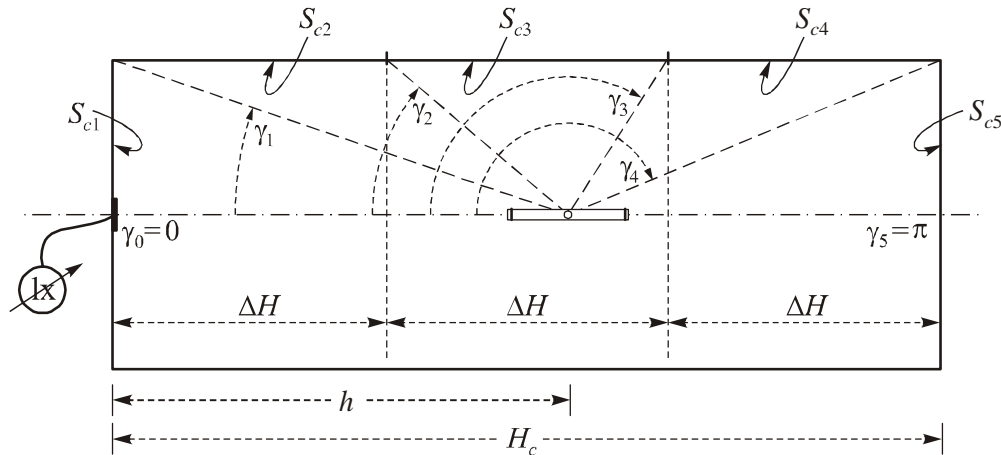
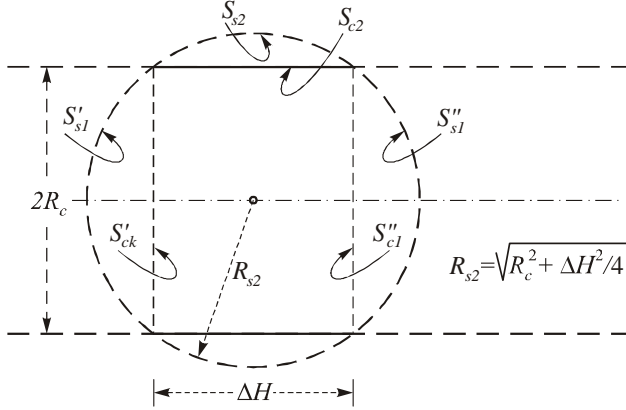


Fig. 3: Auxiliary schematic illustration of the five-surface ICS-C model for the direct luminous flux determination.

### 2.3 CALCULATION OF THE FORM FACTORS $f_{ik}$

In order to determine form factors  $f_{i,k}$ ,  $i, k = 1, 2, \dots, 5$ , equivalent spheres circumscribed around corresponding cylinder parts and relation equations (2b) and (2c), which apply to a closed space consisting of  $N$  ideally diffused surfaces, are used in this paper along with the definition expression (2a).

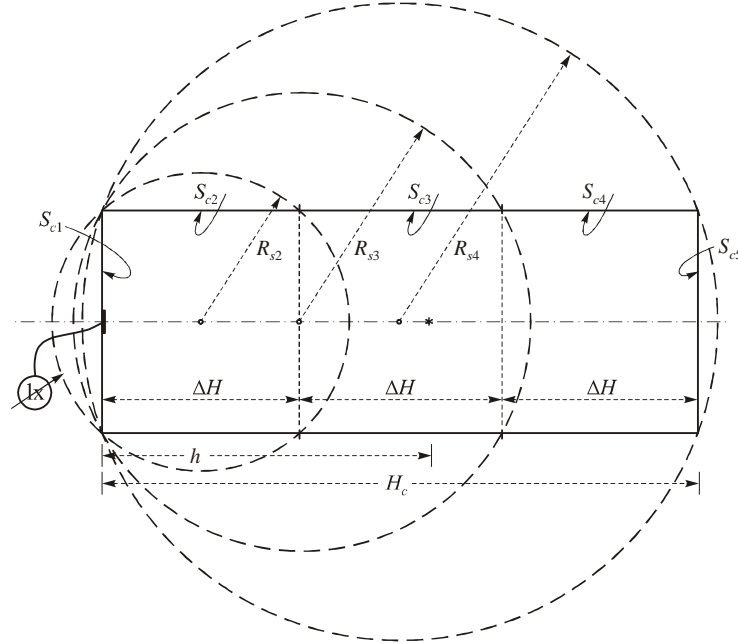
Form factors for a  $N$  - surface ICS model,  $N = 3, 4, 5$ , can be presented in a matrix form:



$$f_{i,k} = \begin{bmatrix} f_{1,1} & f_{2,1} & \cdots & f_{5,1} \\ f_{1,2} & f_{2,2} & \cdots & f_{5,2} \\ \vdots & \vdots & \ddots & \vdots \\ f_{1,5} & f_{2,5} & \cdots & f_{5,5} \end{bmatrix} \quad (7)$$

*Fig. 4: Auxiliary schematic illustration for determination of form factor  $f_{k,k}$ ,  $k = 2, 3, \dots, N-1$ .*

Form factors  $f_{1,1}$  and  $f_{N,N}$  are determined based on a definition expression and are equal to zero, i.e.  $f_{1,1} = f_{N,N} = 0$ . Considering the fact that the ring shaped surfaces are of the same length  $\Delta H = H_c / (N-2)$ , the form factors along the main diagonal are also the same. They



*Fig. 5: Auxiliary schematic illustration for determination of form factor  $f_{1,k}$ ,  $k = 2, 3, \dots, N-1$*

are evaluated using the method of equivalent sphere circumscribed around the first ring of surface  $S_{c2}$ , illustration in *Fig. 4*. Value of form factor  $f_{k,k}$  versus the cylinder parameter  $A_c$ ,  $A_c = 2R_c / H_c$  and number  $N$ , is

$$f_{2,2} = f_{k,k} = 1 - \frac{\sqrt{1 + A_c^2(N-2)^2} - 1}{A_c(N-2)}, \quad k = 2, 3, \dots, N-1. \quad (8)$$

The illustration given in *Fig. 5* and relations (2b, c) are used to determine form factors of the first column  $f_{1,k}$ ,  $k = 2, 3, \dots, N-1$ , and  $f_{1,N}$ . For that purpose, the auxiliary illustration in *Fig. 5* is used, i.e.  $N-2$  spheres of corresponding radius  $R_{sk}$  are circumscribed around the basis,  $S_{c1}$ , and  $k$  ring shaped lateral surfaces. After the form factors  $f_{1,k}$  are obtained, the remaining form factors in (7) are determined using relations (2b, c).

The expressions used for determination of form factors for three-, four- and five-surface ICS-C models are explicitly given in the following sections.

### 2.3.1 Three-surface model

Three-surface ICS model consists of a measuring basis of surface  $S_{c1} = \pi R_c^2$ , a cylinder lateral surface  $S_{c2} = 2\pi R_c H_c$  and an opposite basis  $S_{c3} = \pi R_c^2$ . Applying the previously described procedure of form factors determination, following values for form factors  $f_{i,k}$ ,  $i, k = 1, 2, 3$ , are obtained:

$$\begin{aligned} f_{1,1} = f_{3,3} = 0, \quad f_{1,2} = f_{3,2} = 2 \frac{\sqrt{A_c^2 + 1} - 1}{A_c^2}, \quad f_{1,3} = f_{3,1} = \frac{(\sqrt{A_c^2 + 1} - 1)^2}{A_c^2}, \\ f_{2,1} = f_{2,3} = \frac{\sqrt{A_c^2 + 1} - 1}{2A_c}, \quad f_{2,2} = 1 - \frac{\sqrt{A_c^2 + 1} - 1}{A_c}, \quad A_c = 2R_c / H_c \end{aligned} \quad (9)$$

### 2.3.2 Four-surface model

Four-surface ICS model consists of a measuring basis of surface  $S_{c1} = \pi R_c^2$ , two ring shaped parts of cylinder lateral surface, each of height  $\Delta H = H_c / 2$  and surface  $S_{c2} = S_{c3} = 2\pi R_c \Delta H$ , and an opposite basis  $S_{c4} = \pi R_c^2$ . In this case, form factors  $f_{i,k}$ ,  $i, k = 1, 2, 3, 4$  have the following values:

$$\begin{aligned} f_{1,1} = f_{4,4} = 0, \quad f_{1,2} = f_{4,3} = \frac{\sqrt{(2A_c)^2 + 1} - 1}{2A_c^2}, \quad f_{1,3} = f_{4,2} = \frac{4\sqrt{A_c^2 + 1} - \sqrt{(2A_c)^2 + 1} - 3}{2A_c^2} \\ f_{1,4} = f_{4,1} = \frac{(\sqrt{A_c^2 + 1} - 1)^2}{A_c^2}, \quad f_{2,1} = f_{3,4} = \frac{\sqrt{(2A_c)^2 + 1} - 1}{4A_c}, \quad f_{2,2} = f_{3,3} = 1 - \frac{\sqrt{(2A_c)^2 + 1} - 1}{2A_c} \\ f_{2,3} = f_{3,2} = \frac{\sqrt{(2A_c)^2 + 1} - 2\sqrt{A_c^2 + 1} + 1}{2A_c}, \quad f_{2,4} = f_{3,1} = \frac{4\sqrt{A_c^2 + 1} - \sqrt{(2A_c)^2 + 1} - 3}{4A_c}. \end{aligned} \quad (10)$$

### 2.3.3 Five-surface model

Five-surface ICS model consists of a measuring basis of surface  $S_{c1} = \pi R_c^2$ , three ring shaped parts of cylinder lateral surface, each of height  $\Delta H = H_c / 3$  and surface  $S_{c2} = S_{c3} = S_{c4} = 2\pi R_c \Delta H$ , and an opposite basis  $S_{c5} = \pi R_c^2$ . In this case, form factors  $f_{i,k}$ ,  $i, k = 1, 2, 3, 4, 5$ , have the following values:

$$\begin{aligned}
 f_{1,1} = f_{5,5} &= 0, \quad f_{1,2} = f_{5,4} = 2 \frac{\sqrt{(3A_c)^2 + 1} - 1}{(3A_c)^2}, \quad f_{1,3} = f_{5,3} = \frac{4\sqrt{(3A_c)^2 + 4} - 2\sqrt{(3A_c)^2 + 1} - 6}{(3A_c)^2} \\
 f_{1,4} = f_{5,2} &= \frac{18\sqrt{A_c^2 + 1} - 4\sqrt{(3A_c)^2 + 4} - 10}{(3A_c)^2}, \quad f_{1,5} = f_{5,1} = \frac{(\sqrt{A_c^2 + 1} - 1)^2}{A_c^2}, \\
 f_{2,1} = f_{4,5} &= \frac{\sqrt{(3A_c)^2 + 1} - 1}{6A_c}, \quad f_{2,2} = f_{3,3} = f_{4,4} = 1 - \frac{\sqrt{(3A_c)^2 + 1} - 1}{3A_c}, \\
 f_{2,3} = f_{3,4} = f_{3,2} = f_{4,3} &= \frac{\sqrt{(3A_c)^2 + 1} - \sqrt{(3A_c)^2 + 4} + 1}{3A_c}, \\
 f_{2,4} = f_{4,2} &= \frac{4\sqrt{(3A_c)^2 + 4} - \sqrt{(3A_c)^2 + 1} - 9\sqrt{A_c^2 + 1} + 2}{6A_c}, \\
 f_{2,5} = f_{4,1} &= \frac{9\sqrt{A_c^2 + 1} - 2\sqrt{3A_c^2 + 4} - 5}{6A_c}, \\
 f_{3,1} = f_{3,5} &= \frac{2\sqrt{(3A_c)^2 + 4} - \sqrt{(3A_c)^2 + 1} - 3}{6A_c}. \tag{11}
 \end{aligned}$$

### 2.4 DETERMINATION OF THE INDIRECT ILLUMINANCE IN THE VICINITY OF THE MEASURING BASIS CENTRE - $E_{ind P_1}$

The method of equivalent exitances is used for determination of the measuring point  $P_1$  illuminance. For that purpose, it is necessary to determine equivalent exitances for each of  $N-2$  lateral surfaces, as well as equivalent exitance of the basis opposite to the measuring point  $P_1$ . Exitance of each  $N$  ICS surfaces is determined based on a definition expression:

$$M_k = \rho_c \frac{\Phi_k}{S_{ck}}, \quad k = 1, 2, \dots, N, \quad N = 2, 3, 4, 5, \tag{12}$$

where:  $\Phi_k$  - total flux of the  $k$ -th surface that is determined solving the SIE-LF,  $\rho_c$  - reflectance of the ICS interior surface, and  $S_{ck}$  - the surface of the  $k$ -th cylinder part.

In order to determine the indirect illuminance of the measuring point  $P_1$  vicinity, the existing system is transformed, based on the method of equivalent exitances, into the plane of the other basis as concentric radiated rings of calculated exitances. A schematic illustration for appliance of the method of equivalent exitances is given in Fig. 6. For a  $N$ -surface model with the measuring point  $P_1$  in the centre of a flat surface  $S_{c1}$ , the indirect illuminance is obtained as a sum of illuminances of each  $N-1$  concentric surfaces:

$$E_{ind P_1} = \sum_{k=2}^N M_k (\sin^2 \beta_{k-1} - \sin^2 \beta_k), \quad (13)$$

where:  $\sin \beta_k = \frac{R_c}{\sqrt{R_c^2 + H_{ck}^2}}$ ,  $k = 1, 2, \dots, N-1$ ,  $\sin \beta_N = 0$ ,  $H_{ck} = (k-1)\Delta H$ .

A general canonical form, can be given for expression (13)

$$E_{ind P_1} = C_c (1 + \varepsilon_c) \Phi_{0c} \quad (14)$$

where:  $C_c = \frac{1}{S_{eq}} \frac{\rho_c}{1 - \rho_c}$  - integration factor of the cylinder,  $S_{eq}$  - equivalent cylinder surface,  $\varepsilon_c$  - error of the model compared to the ICS-S, and  $\Phi_{0c}$  - total luminous flux of the (ELS).

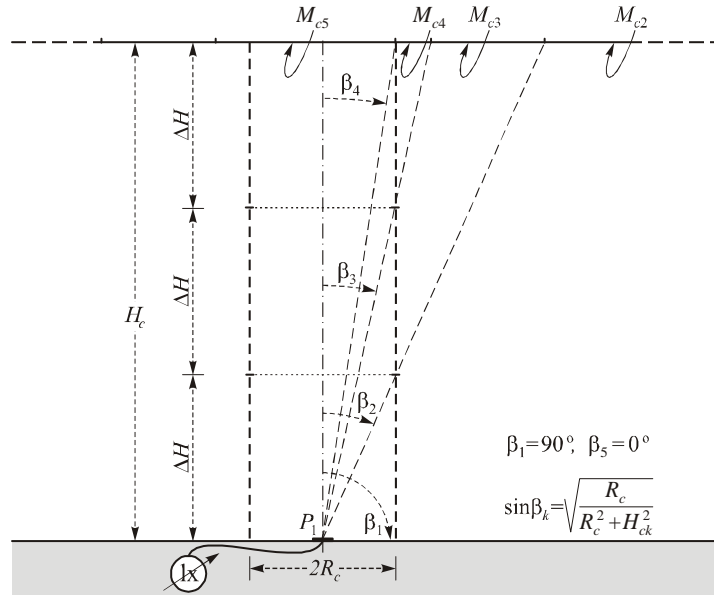


Fig. 6: Auxiliary schematic illustration for appliance of the method of equivalent exitances.

For a three-surface model, the elements in the expression (14) have the following values:

$$S_{eq} = S_{c2} \left\{ \frac{4}{4 + A_c^2} \left[ 1 - \frac{A_c}{2} \frac{2C}{1-F} \right] \right\}^{-1},$$

$C = \rho_c f_c / (1 + \rho_c f_c)$ ,  $F = C(F_c / f_c - 1)$ ,  $F_c = f_{1,3}$ ,  $f_c = f_{2,1}$  according to (9);

$\varepsilon_c = \varepsilon_{c11} \varepsilon_{\Phi 1} + \varepsilon_{c13} \varepsilon_{\Phi 3}$  - error of the model compared to the ICS-S,

where  $\varepsilon_{c11}$  and  $\varepsilon_{c13}$  are errors which are functions of geometry and reflectance of the ICS-C, given by expressions,

$$\varepsilon_{c11} = \frac{\left[ -1 + \frac{A_c}{2} \left( \frac{2F}{1+F} \right) \right] \frac{1-C}{1-F}}{1 - \left( 1 - \frac{2}{A_c} \right) \frac{2C}{1-F}} (1 - \rho_c), \quad \varepsilon_{c13} = \frac{\left[ -1 + \frac{A_c}{2} \left( \frac{2}{1+F} \right) \right] \frac{1-C}{1-F}}{1 - \left( 1 - \frac{2}{A_c} \right) \frac{2C}{1-F}} (1 - \rho_c),$$

$\varepsilon_{\Phi 1} = \Phi_{01} / \Phi_{0c}$  and  $\varepsilon_{\Phi 3} = \Phi_{03} / \Phi_{0c}$  are errors introduced by the direct luminous flux distribution, according to (6a, b).



### 3. NUMERICAL RESULTS

Adopting a real ICS-C geometry, i.e. radius  $R_c = 1\text{m}$  and reflectance  $\rho_c = 0.8$ , indirect illuminances,  $E_{ind P_1}$ , are given in table for three-, four- and five-surface ICS-C model, for different geometry ratio  $A_c = 2R_c / H_c$ , and different distances of the ELS from the measuring point  $P_1$  expressed by the ratio  $h / H_c$ , for  $\Phi_{oc} = 1000\text{lm}$  being the total luminous flux of ELS.

Based on the results shown in Table 1 and Table 2, one can conclude that for certain ICS geometries expressed by  $A_c = 2R_c / H_c$ , and for certain positions of the luminaire expressed by ratio  $h / H_c$ , the value of the indirect illuminance at the measuring point varies in a small range for different number of consisting surfaces,  $N = 3, 4, 5$ . This indicates that it is enough to analyze the error using a three-surface model in order to begin the design of the ICS.

Table 1: Indirect illuminance at point  $P_1$  for a ELS of Type A

$E_{ind P_1}$ [lx] – ELS Type A		$A_c$			
		0.5	1	1.5	2
$h / H_c = 0.25$	$N=3$	127.006	207.322	264.787	309.595
	$N=4$	148.833	210.794	263.455	307.941
	$N=5$	151.353	210.842	263.442	307.840
$h / H_c = 0.5$	$N=3$	129.178	213.069	272.624	318.310
	$N=4$	129.178	213.069	272.624	318.310
	$N=5$	126.364	214.845	273.522	318.661
$h / H_c = 0.75$	$N=3$	128.738	215.939	279.364	327.025
	$N=4$	106.910	212.466	280.696	328.679
	$N=5$	104.431	213.433	281.609	329.233

Table 1: Indirect illuminance at point  $P_1$  for a ELS of Type D

$E_{ind P_1}$ [lx] – ELS Type D		$A_c$			
		0.5	1	1.5	2
$h / H_c = 0.25$	$N=3$	128.730	209.518	265.413	308.326
	$N=4$	154.839	214.200	263.500	305.866
	$N=5$	157.755	214.202	263.561	305.773
$h / H_c = 0.5$	$N=3$	130.456	215.440	274.070	318.310
	$N=4$	130.456	215.440	274.070	318.310
	$N=5$	126.595	218.239	275.576	318.920
$h / H_c = 0.75$	$N=3$	129.933	217.795	281.169	328.294
	$N=4$	103.824	213.113	283.082	330.754
	$N=5$	101.281	214.473	284.422	331.599

Errors of the three-surface model  $\varepsilon_c$  versus the ELS position  $h$ , ( $h / H_c$ ), for different forms of ICS-C geometry expressed by ratio  $A_c = 2R_c / H_c$ , i.e. for  $A_c$  as a parameter, are given in Fig. 7a and Fig. 7b.

From the presented results it can be concluded that choosing the right cylinder geometry and ELS position, model error can be minimized in a desired range. Also, one can conclude that the error can be annulled for a certain ratio  $A_c = 2R_c / H_c$ , i.e. ICS-C with flat circle bases is, regarding the error, equivalent to the sphere in these cases.

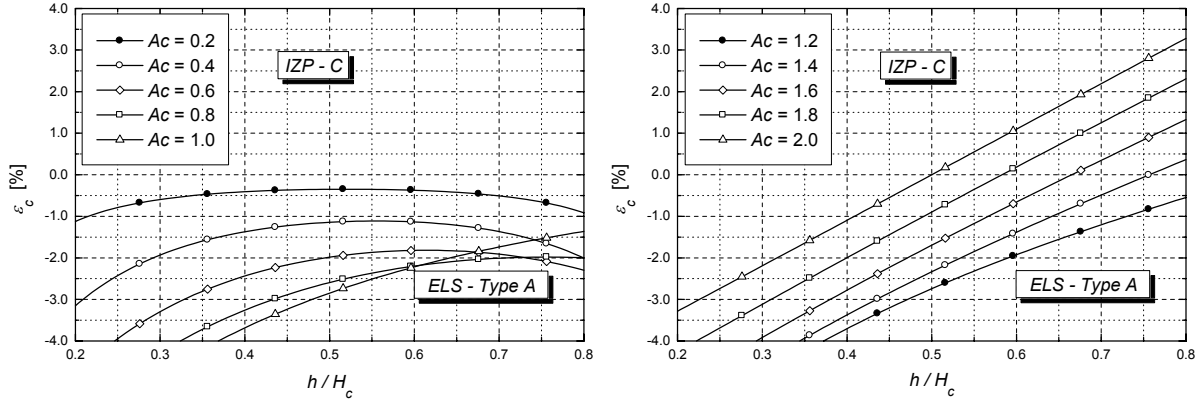


Fig. 7a: Model error versus the position of the optical centre of Type A ELS, for different ICS-C geometry values expressed by parameter  $A_c$ .

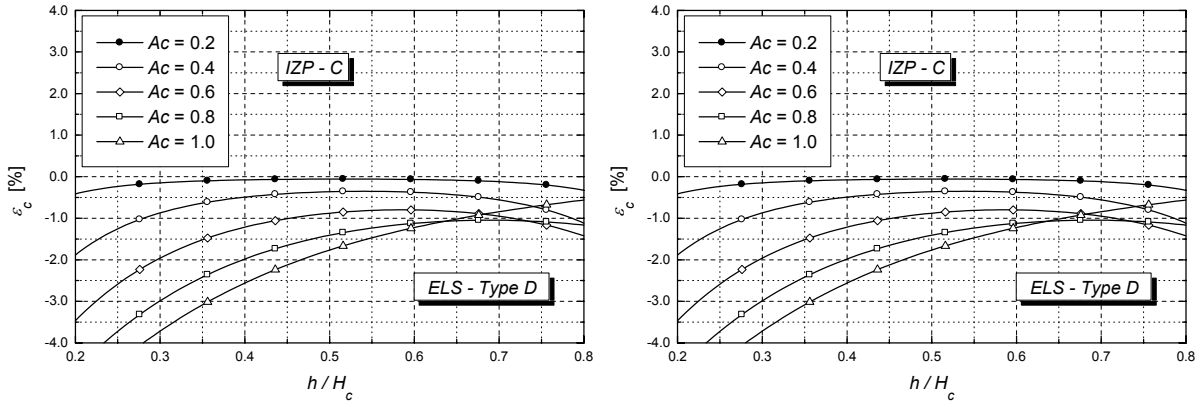


Fig. 7b: Model error versus the position of the optical centre of Type D ELS, for different ICS-C geometry values expressed by parameter  $A_c$ .

#### 4. CONCLUSION

A possibility of construction of a system for luminous flux measurement, which instead of the sphere uses an ICS cylindrically shaped with flat circle bases, is theoretically presented in this paper.

Based on the mathematical model, using the SIE-LF and the method of equivalent exitances, the indirect illuminance at measuring point  $P_1$ , when the ICS-C is considered as multi-surface model,  $N = 3, 4, 5$ , where  $N$  presents the number of constitutive ICS surfaces, is determined.

Analysing the results for the indirect illuminance at point  $P_1$  for different ICS models expressed by a number of constitutive surfaces  $N$ ,  $N = 3, 4, 5$ , it can be concluded that its value varies in a small range for certain ICS geometries, expressed through  $A_c = 2R_c / H_c$ , for example for  $A_c \geq 1$ , and for specific positions of the luminaire, expressed by ratio  $h / H_c$ , for example for  $h / H_c \geq 0.5$ .

Real construction feasibility of the system is presented through analysis of the general error expression for the three-surface model, which occurs as a consequence of using the ICS cylinder instead of the integrating sphere.

Analyzing the general expression for the model error based on the numerical experiments it can be concluded that the error could be minimized choosing the ICS-C geometry. Choosing the ICS-C geometry and the ELS position, i.e. for  $A_c \geq 1.2$  and  $h / H_c \geq 0.5$ , this

error can be even annulled. However, such an oblate form of the ICS cylinder is not suitable for all types of ELSs, for example for Type D ELSs.

Numerical experiments presented in this paper allow, with the model error given in advance, reliable design of the ICS-C in a system for luminous flux measurement for one group of ELS. For the proposed system, the methodology of the luminous flux measurement of ELS is the same as the one in the case of a system that uses the integrating sphere.

The ICS-C surfaces should be uniformly and ideally diffusely painted in order to achieve the highest possible value for the reflectance  $\rho_c > 0.8$ , since the minimized error is even more decreased in that way.

Form factors between the ring shaped lateral surfaces, as well as between the basis surfaces and ring shaped surfaces, necessary for calculation of the indirect illuminance are obtained. Solutions for these form factors are not given in [5], Fig. 9.30 - 9.33.

## 5. APPENDIX

### TWO-SURFACE ICS-C MODEL

ICS-C, as a two-surface model, consists of basis with surface  $S_{c1}$ ,  $S_{c1} = \pi R_c^2$ , with measuring point  $P_1$  in its centre, and surface  $S_2$  that presents the sum of lateral surface  $S_{c2}$ ,  $S_{c2} = 2\pi R_c H_c$  and the surface of the opposite basis  $S_{c3} = S_{c1} = \pi R_c^2$ , i.e.  $S_2 = S_{c2} + S_{c3}$ . The parameter of the ICS-C geometry is the ratio of the diameter and length of the cylinder, i.e.  $A_c = 2R_c / H_c$ , where:  $R_c$ -cylinder radius, and  $H_c$ -cylinder length. Reflectance of all surfaces is  $\rho_c$ .

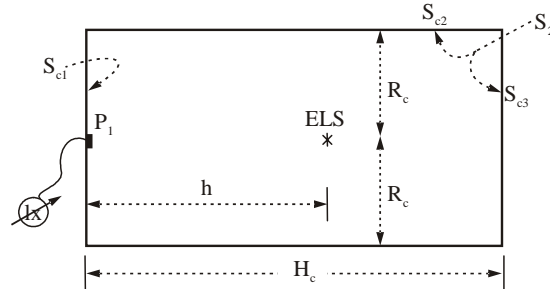


Fig. 8: Schematic illustration of the ICS-C two-surface model.

Using the definition expression (2a), relation equations (2b, c) and the method of equivalent sphere, the following is obtained for form factors  $f_{i,k}$ ,  $i, k = 1, 2$

$$f_{1,1} = 0, \quad f_{1,2} = 1, \quad f_{2,1} = \frac{A_c}{A_c + 4}, \quad f_{2,2} = \frac{4}{A_c + 4}.$$

The indirect illuminance in the vicinity of the measuring point  $P_1$  ( $P_1$  - basis centre), can be expressed in a canonical form as:

$$E_{ind P_1} = C_c (1 + \varepsilon_c) \Phi_{0c} = \frac{1}{S_{eq}} \frac{\rho_c}{1 - \rho_c} (1 + \varepsilon_c) \Phi_{0c},$$

where

$$S_{eq} = S_2 / (1 - C), \quad C = \rho_c f_{21} / (1 + \rho_c f_{21});$$

$\varepsilon_c = -(1 - \rho_c) \frac{\Phi_{01}}{\Phi_{0c}}$  - model error compared to the ICS-S; and  $\Phi_{0c}$  - radiated ELS's luminous flux.

Two-surface model does not offer enough information for optimal design of the ICS-C, because, it can be concluded from the model error that: model error is decreased if the ICS-C length is increased and if the ICS-C is uniformly coloured with greatest possible value of the reflectance. By increasing the length, the distance of the ELS from the measuring basis can be increased, so the direct luminous flux  $\Phi_{01}$  is smaller. For real parameters of ICS-C construction, the model error can be estimated, i.e.: if the reflectance is  $\rho_c = 0.8$ ,  $h/H_c = 0.5$ , and  $A_c = 1$ , i.e.  $\Phi_{01}/\Phi_0 = 1/5$  which corresponds to a isotropic ELS, than the model error, compared to the ICS-S, is around 4%,  $\varepsilon_c \cong 4\%$ .

*Acknowledgement* – Authors thank to Prof. Dr.-Ing. habil. Dietrich Gall and Dr.-Ing. Cornelia Vandahl, as well as the LiTG (Deutsche Lichttechnische Gesellschaft) without whose support it would not be possible to present this manuscript at the Lux Junior 2005, Dörfeld, Germany. Authors also thank the MSEP of Republic of Serbia.

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