

Fourth International
Conference on
Combinatorics,
Graph Theory,
and Applications

Elgersburg

March 21–25, 2011

Dear Participants,

welcome to the Fourth International Conference on Combinatorics, Graph Theory, and Applications in Elgersburg.

This conference brings together mathematicians of distinct parts of the world interested in graph theory, combinatorics and their interactions. It succeeds the conferences of 1996, 2000, and 2009 which also took place in Elgersburg.

The scientific programme consists of seven lectures of invited speakers and of many contributed talks. We are proud to welcome as our invited speakers:

Adrian Bondy, Lyon and Paris, France
Maria Chudnovsky, New York, USA
Bojan Mohar, Burnaby, Canada and Ljubljana, Slovenia
André Raspaud, Bordeaux, France
Akira Saito, Tokyo, Japan
Paul Seymour, Princeton, USA
Carsten Thomassen, Lyngby, Denmark

This booklet contains the schedule, the abstracts for the lectures and talks as were sent to us by the authors as well as a list of the participants and their email addresses.

We wish you a pleasant and successful stay in Elgersburg. Enjoy the conference!

Jochen Harant
Arnfried Kemnitz
Dieter Rautenbach
Ingo Schiermeyer
Michael Stiebitz

Decomposing Infinite Matroids into Their 3-Connected Minors

ELAD AIGNER-HOREV
(Hamburg University, Department of Mathematics)

Joint work with Reinhard Diestel, Luke Postle, and Julian Pott

Recently, Diestel et al. solved a long-standing open problem of Rado by establishing the axioms of infinite matroids with duality. One may then be prompted to ask whether there is a theory of infinite matroids; we are in pursuit of this question.

In this talk, we report on a recent result of ours. We prove that every (infinite) connected matroid M admits a unique decomposition graph-theoretical tree whose vertices correspond to minors of M that are 3-connected, circuits, or cocircuits and whose edges correspond to 2-separations of M . In addition, we show that the decomposition of M determines the decomposition of its dual in a natural manner.

Such a decomposition is well-known for finite matroids as this was established by Cunningham and Edmonds and by Seymour. Our approach is inspired by that of Cunningham and Edmonds, but necessarily different. The proofs establishing this decomposition for finite matroids do not seem to extend to infinite matroids. In particular, in the finite case the approach is to repeatedly "cut" the matroid along 2-separations of a certain type; such a process ends due to the matroid being finite. Such an approach cannot be employed in the infinite case.

Finding an Induced Subdivision of a Digraph

JØRGEN BANG-JENSEN
(IMADA University of Southern Denmark, Odense)

We consider the following problem for oriented graphs and digraphs: Given an oriented graph (digraph) H ; does it contain an induced subdivision of a prescribed oriented graph (digraph) D ? That is, can we find $|V(D)|$ distinct vertices $\{h_v | v \in V(D)\}$ in H such that for every arc uv of D there is an induced path P_{uv} from h_u to h_v in H and furthermore there is no arc in H between distinct paths $P_{uv}, P_{u'v'}$ (i.e., no arc with precisely one end on P_{uv} and the other on $P_{u'v'}$)? The complexity of this problem depends on whether H is an oriented graph or contains 2-cycles. We give a number of examples of polynomial instances as well as several NP-completeness proofs.

Rainbow Cycles in Cube Graphs

JENS-P. BODE

(Technische Universität Braunschweig)

Joint work with A. Kemnitz and S. Struckmann

A graph G is called rainbow with respect to an edge coloring if no two edges of G have the same color. Given a host graph H and a guest graph $G \subseteq H$, an edge coloring of H is called G -anti-Ramsey if no subgraph of H isomorphic to G is rainbow. The anti-Ramsey number $f(H, G)$ is the maximum number of colors for which there is a G -anti-Ramsey edge coloring of H . We consider cube graphs Q_n as host graphs and even cycles C_k as guest graphs.

Induced Decompositions of Graphs

J. ADRIAN BONDY

(Université Lyon 1 and Université Paris 6)

We consider those graphs G which admit decompositions into copies of a fixed graph F , each copy being an induced subgraph of G . We are interested in finding the extremal graphs with this property, that is, those graphs G on n vertices with the maximum possible number of edges. We report on joint work with Jayme Szwarcfiter concerning the cases where F is a complete r -partite graph, a cycle, a star, or a graph on at most four vertices. We also discuss recent contributions to the topic by Nathann Cohen and Zsolt Tuza.

Minimum Feedback Vertex Set in Graphs

MIECZYSLAW BOROWIECKI

(University of Zielona Góra, Department of Discrete Mathematics
and Theoretical Computer Science)

A *feedback vertex set* (*decycling set*) of a graph is a subset of its vertices whose removal induces an acyclic subgraph. The problem of finding a *minimum feedback* vertex set in G and its cardinality $\tau_F(G)$ has been widely studied since it has interesting applications. It is easy to see that $\tau_F(G)$ is the minimum cardinality of a transversal of cycles. Also several papers address the problem of finding the maximum number $a(G)$ of vertices of G that induce a forest; this is an equivalent problem since

$$a(G) = |V(G)| - \tau_F(G).$$

More generally, in 1973 Hedetniemi has been proved the following generalization of Gallai's Theorem:

For a graph G of order n ,

$$\alpha_{\mathcal{P}}(G) + \tau_{\mathcal{P}}(G) = n,$$

where \mathcal{P} is an induced hereditary property of graphs, $\alpha_{\mathcal{P}}(G)$ is the maximum cardinality of \mathcal{P} -independent set in G , $\tau_{\mathcal{P}}(G)$ is the minimum cardinality of a vertex \mathcal{P} -cover (a transversal of non \mathcal{P} -independent sets) of G .

In a talk some results concerning $\tau_{\mathcal{P}}(G)$ for selected properties \mathcal{P} and classes of graphs will be given.

The k -Connected Components of a Graph

JOHANNES CARMESIN

(Universität Hamburg)

Joint work with R. Diestel, F. Hundertmark, M. Stein

A vertex-set is called k -inseparable if it contains at least $k + 1$ vertices and no separation of order at most k separates the set. For every graph G , we build an $\text{Aut}(G)$ -invariant tree decomposition such that for all k any two maximal k -inseparable sets are separated by a separation of order at most k corresponding to an edge of the tree.

Tournament Heroes

MARIA CHUDNOVSKY
(Columbia University, New York)

The chromatic number of a tournament T is the smallest number of transitive tournaments that partition $V(T)$. Let us say that a tournament S is a hero if for every tournament T not containing S , the chromatic number of T is at most a constant $c(S)$. Recently, in joint work with Eli Berger, Krzysztof Choromanski, Jacob Fox, Martin Loebl, Alex Scott, Paul Seymour and Stephan Thomasse, we proved a theorem that gives a complete description of all heroes. This talk will describe the result, and survey some of the proof ideas.

Traceability of Oriented Graphs with Independence Number Two

MARIETJIE FRICK
(University of South Africa)

Joint work with Susan van Aardt, Alewyn Burger, Jean Dunbar, John Harris and Joy Singleton

An oriented graph is k -traceable if each of its subdigraphs of order k is traceable. The Traceability Conjecture (TC) states that for $k \geq 2$ every k -traceable oriented graph of order at least $2k - 1$ is traceable.

It follows from the well-known Chen-Manalastas Theorem that every nontraceable oriented graph with independence number $\alpha = 2$ has at least three strong components. Using the strong component structure of nontraceable k -traceable oriented graphs with $\alpha = 2$, we obtain results towards proving the TC for oriented graphs with $\alpha = 2$. The importance of these results lies in the fact that for $k \geq 2$ every k -traceable oriented graph of order greater than $6k - 20$ has $\alpha = 2$.

Extending Bondy's Theorem — Further Results

ANIKA FRICKE

(Institute of Mathematics and Applied Computer Science,
University of Hildesheim)

Joint work with J.-P. Bode and A. Kemnitz

In 1989 Bondy proved the following theorem:

Let G be a k -connected graph of order $n \geq 3$. If the degree sum of every $k + 1$ pairwise nonadjacent vertices is at least $\frac{1}{2}((k + 1)(n - 1) + 1)$ then G is hamiltonian.

We prove that it is possible to allow some independent $(k + 1)$ -sets violating the degree sum condition of Bondy's Theorem and still implying hamiltonicity of the graph. We give some upper bounds for such exceptional $(k + 1)$ -sets improving previous results.

Taming the Apex Set

JAN-OLIVER FRÖHLICH

(Universität Hamburg)

Joint work with Theodor Müller

A well known result from the Graph Minor papers of Robertson and Seymour roughly states that any graph not containing some fixed graph as a minor has a near embedding into a surface in which the forbidden minor cannot be embedded. Diestel, Kawarabayashi, Müller and Wollan recently proved that we can also get a 'rich' near embedding with a list of several additional structural properties. This talk is about how to modify a rich near embedding such that the vertices of the so-called apex set attach 'nicely' to the embedded part of the graph.

Preserving High Connectivity

FRANK GÖRING
(TU Chemnitz)

Some known results are obtained by the trick of constructing an auxiliary graph with high connectivity. Examples of such results are given in the introduction. Motivated by these examples the talk presents a generalized tool to preserve high connectivity of a graph under altering it at many locations simultaneously.

Cube Graphs with Saturated Edge Rainbow Colorings

HEIKO HARBORTH
(TU Braunschweig, Germany)

Joint work with Arnfried Kemnitz

An edge rainbow coloring of a graph G uses pairwise different colors for the edges of G . Let $f(Q_n, Q_k)$ denote the minimum number of colors for the edges of the cube graph Q_n such that no subgraph Q_k has a rainbow coloring, however, each recoloring of an edge increasing the total number of colors induces a rainbow coloring of a Q_k . — Some exact values of $f(Q_n, Q_k)$ are determined.

2-Colorings in k -Regular k -Uniform Hypergraphs

MICHAEL A. HENNING
(University of Johannesburg)

Joint work with Anders Yeo (Royal Holloway, University of London)

A hypergraph is 2-colorable if there is a 2-coloring of the vertices with no monochromatic hyperedge. Let \mathcal{H}_k denote the class of all k -uniform k -regular hypergraphs. The Lovász Local Lemma, devised by Erdős and Lovász in 1975 to tackle the problem of hypergraph 2-colorings, implies that every hypergraph $H \in \mathcal{H}_k$ is 2-colorable, provided $k \geq 9$. Alon and Bregman [Graphs Combin. 4 (1988) 303–306] proved the slightly stronger result that every hypergraph $H \in \mathcal{H}_k$ is 2-colorable, provided $k \geq 8$. It is implicitly known in the literature that the Alon-Bregman result is true for all $k \geq 4$, as remarked by Vishwanathan [J. Combin. Theory Ser. A 101 (2003), 168–172] even though we have not seen it explicitly proved. For completeness, we provide a short proof of this result using results of Thomassen [J. Amer. Math. Soc. 5 (1992), 217–229] and Seymour [Quart. J. Math. Oxford Ser. 25 (1974), 303–312]. As remarked by Alon and Bregman the result is not true when $k = 3$, as may be seen by considering the Fano plane.

Our main result in this talk is a strengthening of the above results. For this purpose, we define a set X of vertices in a hypergraph H to be a free set in H if we can 2-color $V(H) \setminus X$ such that every edge in H receives at least one vertex of each color. Equivalently, X is a free set in H if it is the complement of two disjoint transversals in H . For every $k \geq 13$, we prove that every hypergraph $H \in \mathcal{H}_k$ of order n has a free set of size at least $n/14$. For any $\epsilon > 0$ where $0 < \epsilon < 1$ and for sufficiently large k , we prove that every hypergraph $H \in \mathcal{H}_k$ of order n has a free set of size at least $c_k \times n$, where $c_k = 1 - (\ln(k) + (\ln(k))^2)/((1 - \epsilon)k)$, and so $c_k \rightarrow 1$ as $k \rightarrow \infty$. As an application, we show that the total restrained domination number of a graph on n vertices with sufficiently large minimum degree k is at most $\frac{1}{2}(1 - c_k)n$, which significantly improves the best known bound of $\frac{1}{2}n + 1$ for $k \geq 3$.

We conjecture that for $k \geq 4$, every hypergraph $H \in \mathcal{H}_k$ has a free set of size $k - 3$ in H and we show that the bound $k - 3$ cannot be improved for any $k \geq 4$. Our proofs use results from areas such as transversal in hypergraphs and disjoint cycles in digraphs, as well as probabilistic arguments.

Rank Number of a Prism

MIRKO HORŇÁK

(Institute of Mathematics, P.J. Šafárik University, Košice, Slovakia)

Joint work with Juan Ortiz, Andrew Zemke, Hala King, Darren Narayan

Let k be a positive integer. A k -ranking of a graph G is such a mapping $\varphi : V(G) \rightarrow \{1, \dots, k\}$ that any path joining vertices x, y with $x \neq y$ and $\varphi(x) = \varphi(y)$ has an internal vertex z satisfying $\varphi(z) > \varphi(x)$. The rank number of G is the minimum k admitting a k -ranking of G .

Theorem. *The rank number of an n -sided prism, $n \geq 4$, is equal to $\lfloor \log_2(n-1) \rfloor + \lfloor \log_2(n-1-2^{\lfloor \log_2(n-2) \rfloor - 1}) \rfloor + 3$.*

Nonrepetitive Vertex Colourings of Graphs

STANISLAV JENDROL'

(Institut of Mathematics, Pavol Jozef Šafárik University in Košice, Slovakia)

Joint work with Jochen Harant (Institute für Mathematik, TU Ilmenau, Germany)

Let ϕ be a colouring of the vertices of a simple graph G of maximum degree d . A simple path $P = (v_1, \dots, v_{2r})$ in G is *repetitive* if $\phi(v_i) = \phi(v_{i+r})$ for all $i = 1, \dots, r$.

1. The *Thue chromatic number* $\pi(G)$ is the minimum number of colours in such a vertex colouring of G that no path in G is repetitive. We show that

$$\pi(G) \leq \left\lceil 12.92 (d-1)^2 \right\rceil.$$

2. The *facial Thue chromatic number* $\pi_f(G)$ of a 2-connected plane graph G is the minimum number of colours in a vertex colouring of G that no facial path (i.e. path of vertices on the boundary walk of a face) of G is repetitive. We prove that for any $d \geq 3$

(i) $\pi_f(G) \leq \min \left\{ 29(d-2); 39\sqrt{d-2}; 47\sqrt[3]{d-2}; 120 \ln d \right\}$ and

(ii) $\pi_f(G) \leq 16$ if G is hamiltonian.

Fractional Total Colorings of Graphs with Large Girth

FRANTIŠEK KARDOŠ

(P. J. Šafárik University, Košice, Slovakia)

Joint work with Daniel Král' and Jean-Sébastien Sereni

Reed conjectured that for every $\varepsilon > 0$ and every integer Δ , there exists g such that the fractional total chromatic number of every graph with maximum degree Δ and girth at least g is at most $\Delta + 1 + \varepsilon$. The conjecture was proven to be true when $\Delta = 3$ or Δ is even. We settle the conjecture by proving it for the remaining cases.

Maximum Weight of a Connected Graph of Given Order and Size

MARIA KOCH

(TU Bergakademie Freiberg)

Joint work with Ingo Schiermeyer

The weight of an edge $e = xy$ of a graph G is $w(e) := \deg_G(x) + \deg_G(y)$ and the weight of G is $w(G) := \min(w(e) : e \in E(G))$. For a positive integer n , $m \in \{0, \dots, \binom{n}{2}\}$ and a graph property \mathcal{P} let

$$w(n, m, \mathcal{P}) := \max(w(G) : |V(G)| = n, |E(G)| = m, G \in \mathcal{P}).$$

At Czechoslovak Symposium on Combinatorics, Graphs and Complexity in 1990 Erdős posed the problem of determining $w(n, m, \mathcal{I})$ for the most general property \mathcal{I} of all graphs. The problem has been solved first partially by Invančo and Jendrol' and then completely by Jendrol' and Schiermeyer.

$$G \in \mathcal{C} \leftrightarrow G \text{ is connected.}$$

This talk will present partial results for $w(n, m, \mathcal{C})$.

On b -Colorings of Graphs with Independence Number 2

ANJA KOHL
(TU Bergakademie Freiberg)

A b -coloring of a graph G by k colors is a proper vertex coloring such that each color class contains a color-dominating vertex, that is, a vertex having neighbors in all other $k - 1$ color classes. The b -chromatic number $\chi_b(G)$ is the maximum integer k for which G has a b -coloring by k colors. A graph G is called b -continuous if G admits a b -coloring by k colors for all integers k satisfying $\chi(G) \leq k \leq \chi_b(G)$.

In this talk we present bounds on $\chi_b(G)$ for graphs G with $\alpha(G) = 2$ and we show that these graphs are b -continuous. After this we consider the r th power C_n^r of a cycle of order n . Effantin and Kheddouci established in 2003 the b -chromatic number $\chi_b(C_n^r)$ for all values of n and r , except for $2r + 3 \leq n \leq 3r$. For these cases, where, incidentally, C_n^r has independence number 2, we determine the exact value or new bounds on $\chi_b(C_n^r)$.

Nonseparating K_4 -Subdivisions in Graphs of Minimum Degree at least 4

MATTHIAS KRIESELL
(IMADA, University of Southern Denmark)

We first outline a proof that for every vertex x of a 4-connected graph G there exists a subgraph H in G isomorphic to a subdivision of the complete graph K_4 on four vertices such that $G - V(H)$ is connected and contains x . This implies an affirmative answer to a question of W. KÜHNEL whether every 4-connected graph G contains a subdivision H of K_4 as a subgraph such that $G - V(H)$ is connected.

The motor for our induction is a result of FONTET and MARTINOV stating that every 4-connected graph can be reduced to a smaller one by contracting a single edge, unless the graph is the square of a cycle or the line graph of a cubic graph. It turns out that this is the only ingredience of the proof where 4-connectedness is used. We then generalize our result to connected graphs of minimum degree at least 4, by developing the respective motor, *a structure theorem for the class of simple connected graphs of minimum degree at least 4*:

A simple connected graph G of minimum degree 4 can not be reduced to a smaller such graph by deleting a single edge or contracting a single edge and simplifying if and only if it is the square of a cycle or the edge disjoint union of copies of certain *bricks* as follows: Each brick is isomorphic to K_3 , K_5 , $K_{2,2,2}$, K_5^- , $K_{2,2,2}^-$, or one the four graphs K_5^∇ , $K_{2,2,2}^\nabla$, $K_5^{\triangleright\triangleleft}$, $K_{2,2,2}^{\triangleright\triangleleft}$ obtained from K_5 and $K_{2,2,2}$ by deleting the edges of a triangle, or replacing a vertex x by two new vertices and adding four edges to the endpoints of two disjoint edges of its former neighborhood, respectively. Bricks isomorphic to K_5 or $K_{2,2,2}$ share exactly one vertex with the other bricks of the decomposition, vertices of degree 4 in any other brick are not contained in any further brick of the decomposition, and the vertices of a brick isomorphic to K_3 must have degree 4 in G and have pairwise no common neighbors outside that brick.

Hamilton Paths and Cycles in Distance Graphs

CHRISTIAN LÖWENSTEIN

(Department of Mathematics, University of Johannesburg)

Joint work with Dieter Rautenbach, Friedrich Regen, and Roman Soták

For a positive integer $n \in \mathbb{N}$ and a finite set of positive integers $D \subseteq \mathbb{N}$, the finite, undirected, and simple *distance graph* G_n^D has vertex set $V(G_n^D) = \{0, 1, \dots, n-1\}$, and two vertices u and v of G_n^D are adjacent exactly if $|u - v| \in D$.

We consider the following two questions:

- Let $D \subset \mathbb{N}$ be a finite set such that $|D| \geq 2$ and $\gcd(D) = 1$. Is there some $n_D \in \mathbb{N}$ such that for every $n \geq n_D$, G_n^D has a Hamiltonian cycle?
- Let $D \subset \mathbb{N}$ be a finite set such that $|D| \geq 2$ and $\gcd(D) = 1$. Is there some $n_D \in \mathbb{N}$ such that for every $n \geq n_D$, G_n^D has a Hamiltonian path with endvertices 0 and $n - 1$?

On Factorial Properties of Graphs

VADIM V. LOZIN

(DIMAP and Mathematics Institute, The University of Warwick, UK)

Joint work with Colin Mayhill and Viktor Zamaraev

For a graph property X , let X_n be the number of graphs with vertex set $\{1, \dots, n\}$ having property X , also known as the speed of X . A property X is called factorial if X is hereditary (i.e. closed under taking induced subgraphs) and $n^{c_1 n} \leq X_n \leq n^{c_2 n}$ for some constants c_1 and c_2 . Hereditary properties with the speed slower than factorial are surprisingly well defined [2]. The situation with factorial properties is more complicated and less explored, although this family includes many properties of theoretical or practical importance, such as interval graphs, permutation graphs, line graphs, forests, threshold graphs, all classes of graphs of bounded vertex degree, of bounded clique-width [1], all proper minor-closed graphs classes (including planar graphs) [3]. To simplify the study of factorial properties, we propose the following conjecture: the speed of a hereditary property X is factorial if and only if the fastest of the following three properties is factorial: bipartite graphs in X , co-bipartite graphs in X and split graphs in X . We verify the conjecture for hereditary properties defined by forbidden induced subgraphs with at most 4 vertices. We also introduce a structural tool allowing to reveal variety of factorial properties of graphs and illustrate it with a number of examples.

References

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- [3] S. NORINE, P. SEYMOUR, R. THOMAS and P. WOLLAN, Proper minor-closed families are small, *J. Combinatorial Theory B* 96 (2006) 754–757.

The Complexity of Finding Disjoint Cycles and Dicycles in a Digraph with Cycle Transversal Number 1

ALESSANDRO MADDALONI

(Department of Mathematics and Computer Science,
University of Southern Denmark, Odense)

Joint work with Jørgen Bang-Jensen, Matthias Kriesell and Sven Simonsen

In this and the companion talk by S.Simonsen we determine the complexity of the following problem:

Given a digraph D , decide if there is a cycle B in D and a cycle C in its underlying undirected graph $UG(D)$ such that $V(B) \cap V(C) = \emptyset$.

It was proved in [J.Bang-Jensen, M.Kriesell, On the problem of finding disjoint cycles and dicycles in a digraph, *Combinatorica*, to appear] that one can decide the problem (and find the cycles) in polynomial time if D is strongly connected.

When D is not necessarily strongly connected it is possible to give a complete characterization for the complexity of the problem in terms of the cycle transversal number $\tau(D)$ of D , i.e. the smallest cardinality of a set T of vertices in D such that $D - T$ is acyclic.

We show that the general problem is \mathcal{NP} -complete. A reduction from 3-SAT is presented, this reduction produces a graph D with $\tau(D) = 1$.

Focusing on the class of graphs with $\tau(D) = 1$, we will see that the parameter who makes the problem hard is the number of transversal vertices: it will be shown how to decide the problem in polynomial time (and find the cycles if they exist) when the number of transversal vertices is bounded.

Generalized Colorings of Distance Graphs

MASSIMILIANO MARANGIO

(Computational Mathematics, Technische Universität Braunschweig)

The *integer distance graph* $G(D)$ has vertex set \mathbb{Z} and two vertices u and v are adjacent if and only if $|u - v|$ is an element of the distance set $D \subseteq \mathbb{N}$.

An *additive hereditary property* of graphs is a class of simple graphs which is closed under unions, subgraphs and isomorphism.

Let \mathcal{P} and \mathcal{Q} be additive hereditary properties of graphs. A \mathcal{P} -*vertex coloring* (\mathcal{P} -*edge coloring*) of a graph G is a coloring of the vertices $V(G)$ (edges $E(G)$) of G such that for each color i the vertices (edges) colored by i induce a subgraph of property \mathcal{P} . A $(\mathcal{P}, \mathcal{Q})$ -*total coloring* of a graph G is a coloring of the vertices $V(G)$ and edges $E(G)$ of G such that for each color i the vertices colored by i induce a subgraph of property \mathcal{P} , the edges colored by i induce a subgraph of property \mathcal{Q} , and incident vertices and edges obtain different colors.

In this talk we present general basic results on such generalized colorings of integer distance graphs.

Stars in Minimum Rainbow Subgraphs

STEPHAN MATOS CAMACHO

(TU Bergakademie Freiberg)

Joint work with Ingo Schiermeyer and Maria Koch

Let G be an edge-coloured graph. Then a minimum rainbow subgraph $F \subseteq G$ is a subgraph of smallest order containing exactly one edge of each colour in G . Since this problem is NP-hard, and even APX-hard, our goal is to find polynomial time algorithms with a small approximation ratio.

In our talk we will discuss a Greedy heuristic using stars for approximation. We will evaluate the performance on bipartite and general graphs.

Recent Advances in the Degree/Diameter Problem

MIRKA MILLER

(School of Electrical Engineering and Computer Science, University of Newcastle,
Australia

Department of Mathematics, University of West Bohemia, Pilsen, Czech Republic

Department of Computer Science, King's College London, UK

Department of Mathematics, ITB Bandung, Indonesia)

A well-known fundamental problem in extremal graph theory is the *degree/diameter problem*, which is to determine the largest (in terms of the number of vertices) graphs or digraphs or mixed graphs of given maximum degree, respectively, maximum outdegree, respectively, mixed degree; and given diameter. General upper bounds, called Moore bounds, exist for the largest possible order of such graphs, digraphs and mixed graphs of given maximum degree d (respectively, maximum out-degree d , resp., maximum mixed degree d) and diameter k .

The Moore bound for a directed graph of maximum out-degree d and diameter k is

$$M_{d,k} = 1 + d + d^2 + \dots + d^k.$$

It is known that digraphs of order $M_{d,k}$ (Moore digraphs) do not exist for $d > 1$ and $k > 1$. Similarly, the Moore bound for an undirected graph of degree d and diameter k is

$$M_{d,k}^* = 1 + d + d(d-1) + \dots + d(d-1)^{k-1}.$$

Undirected Moore graphs for $d > 2$ and $k > 1$ exist only when $k = 2$ and $d = 3, 7$ and possibly 57.

Mixed (or partially directed) Moore graphs of diameter $k = 2$ were first studied by Bosák. The Moore bound for mixed graphs is

$$M_{d,z,k} = 1 + d + zd + r(d-1) + \dots + zd^{k-1} + r(d-1)^{k-1}$$

where $d = z + r$.

In recent years, there have been many interesting new results in all the three versions of the degree/diameter problem, resulting in improvements in both the lower bounds and the upper bounds on the largest possible number of vertices.

In this talk we present a short overview of the current state of the degree/diameter problem, for undirected, directed and mixed graphs, and we outline several related open problems.

Immersions in Graphs and Digraphs

BOJAN MOHAR

(Simon Fraser University, Burnaby and IMFM, Ljubljana)

An immersion of a graph H into a graph G is a one-to-one mapping $f : V(H) \rightarrow V(G)$ and a collection of edge-disjoint paths, one for each edge of H , such that the path P_{uv} corresponding to edge uv has endpoints $f(u)$ and $f(v)$. As a central result, it will be shown that every simple graph with average degree $\Omega(t)$ immerses the complete graph K_t . Moreover, if G is dense enough, then there is an immersion of K_t in which each path P_{uv} is of length precisely 2.

Most of the results presented in the talk are joint work with Matt DeVos, Zdenek Dvorak, Jacob Fox, Jessica McDonald, and Diego Scheide.

Degree Conditions for H -Linked Digraphs

FLORIAN PFENDER

(Universität Rostock)

Joint work with Michael Ferrara and Michael Jacobson

Given a (multi)digraph H , a digraph D is H -linked if every injective function $\iota : V(H) \rightarrow V(D)$ can be extended to an H -subdivision. In this paper, we give sharp degree conditions that assure a sufficiently large digraph D is H -linked for arbitrary H . The notion of an H -linked digraph extends the classes of m -linked, m -ordered and strongly m -connected digraphs.

First, we give sharp minimum semi-degree conditions for H -linkedness, extending results of Kühn and Osthus on m -linked and m -ordered digraphs. It is known that the minimum degree threshold for an undirected graph to be H -linked depends on a partition of the (undirected) graph H into three parts. Here, we show that the corresponding semi-degree threshold for H -linked digraphs depends on a partition of H into as many as nine parts. We also determine sharp Ore-Type degree-sum conditions assuring that a digraph D is H -linked for general H . As a corollary, we obtain (previously undetermined) sharp degree-sum conditions for m -linked and m -ordered digraphs.

Partitions of Graphs with Bounded Maximum Average Degree

ANDRÉ RASPAUD

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A graph G is called *improperly* (d_1, \dots, d_k) -colorable, or just (d_1, \dots, d_k) -colorable, if the vertex set of G can be partitioned into subsets V_1, \dots, V_k such that the graph $G[V_i]$ induced by the vertices of V_i has maximum degree at most d_i for all $1 \leq i \leq k$. This notion generalizes those of proper k -coloring (when $d_1 = \dots = d_k = 0$) and d -improper k -coloring (when $d_1 = \dots = d_k = d \geq 1$).

Proper and d -improper colorings have been widely studied. As shown by Appel and Haken [1, 2], every planar graph is 4-colorable, i.e. $(0, 0, 0, 0)$ -colorable. Eaton and Hull [3] and independently Škrekovski [5] proved that every planar graph is 2-improperly 3-colorable (in fact, 2-improper choosable), i.e. $(2, 2, 2)$ -colorable.

This latter result was extended by Havet and Sereni to not necessarily planar sparse graphs as follows:

Theorem. [4] *For every $k \geq 0$, every graph G with $\text{mad}(G) < \frac{4k+4}{k+2}$ is k -improperly 2-colorable (in fact k -improperly 2-choosable), i.e. (k, k) -colorable.*

We recall that $\text{mad}(G) = \max \left\{ \frac{2|E(H)|}{|V(H)|}, H \subseteq G \right\}$ is the maximum average degree of a graph G .

In this talk we will present obtained results concerning the (k, j) -colorability for some values of k and j ($k \neq j$) for graphs with a given maximum average degree.

References

- [1] K. Appel and W. Haken. Every planar map is four colorable. Part I. Discharging. *Illinois J. Math.*, 21:429–490, 1977.
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Closure, Clique Covering and Degree Conditions for Hamilton-connectedness in Claw-free Graphs

ZDENĚK RYJÁČEK

(University of West Bohemia, Plzeň)

Joint work with Roman Kužel, Jakub Teska and Petr Vrána

We strengthen the closure concept for Hamilton-connectedness in claw-free graphs, introduced recently by Z.R. and Petr Vrána, such that the strong closure G^M of a claw-free graph G is a line graph of a (multi)graph containing at most two triangles or at most one double edge.

Using the strong closure, we show that 3-connected claw-free graphs that can be covered by “few” cliques are Hamilton-connected (more precisely, a 3-connected claw-free graph G is Hamilton-connected if $\vartheta(G) \leq 5$, or $\vartheta(G) \leq 6$ and $\delta(G) \geq 4$, or $\vartheta(G) \leq 5$ and $\delta(G) \geq 6$, where $\vartheta(G)$ denotes the clique covering number of G).

Finally, by reconsidering the relation between degree conditions and clique coverings in the case of the strong closure G^M , we show that every 3-connected claw-free graph G of minimum degree $\delta(G) \geq 24$ and minimum degree sum $\sigma_8(G) \geq n + 50$ (or, as a corollary, of order $n \geq 142$ and minimum degree $\delta(G) \geq \frac{n+50}{8}$) is Hamilton-connected.

We also show that our results are asymptotically sharp.

Forbidden Subgraphs and 2-Factors

AKIRA SAITO

(Nihon University, Japan)

Joint work with R.E.L. Aldred (Otago University, New Zealand), Jun Fujisawa (Kochi University, Japan) and Bruhn Fujimoto (University of Paris VI, France)

For a connected graph H , a graph G is said to be H -free if G does not contain an induced subgraph isomorphic to H . More generally, for a set of connected graphs \mathcal{H} , G is said to be \mathcal{H} -free if G is H -free for each $H \in \mathcal{H}$. The set \mathcal{H} is often referred as *forbidden subgraphs*. If $|\mathcal{H}| = 2$ (resp. $|\mathcal{H}| = 3$), we call \mathcal{H} a *forbidden pair* (resp. *forbidden triple*). In this talk, we investigate the relationship between forbidden subgraphs and the existence of a 2-factor. Faudree and Gould have determined all the forbidden pairs that force the existence of a hamiltonian cycle in a 2-connected graph of sufficiently large order. Later, Faudree, Faudree and Ryjáček have extended the research and characterized the set of forbidden pairs that imply the existence of a 2-factor, again in a 2-connected graph of sufficiently large order. First, we consider the same problem, not in the class of 2-connected graphs but in the class of connected graphs. We determine the set of forbidden pairs, and observe that it is considerably smaller than the one characterized by Faudree, Faudree and Ryjáček. Then we extend the result to forbidden triples. We will encounter several strange graphs in the triples, which do not appear in the pairs. If time permits, we also investigate the effect of minimum degree that affect the forbidden pairs.

On Finite Convexity Spaces Induced by Sets of Paths in Graphs

PHILIPP MATTHIAS SCHÄFER
(Universität Ulm)

Joint work with Mitre Costa Dourado, Dieter Rautenbach

A finite convexity space is a pair (V, \mathcal{C}) consisting of a finite set V and a set \mathcal{C} of subsets of V such that $\emptyset, V \in \mathcal{C}$ and \mathcal{C} is closed under intersection. A graph G with vertex set V and a set \mathcal{P} of paths of G naturally define a convexity space $(V, \mathcal{C}(\mathcal{P}))$ where $\mathcal{C}(\mathcal{P})$ contains all subsets C of V such that whenever C contains the endvertices of some path P in \mathcal{P} , then C contains all vertices of P .

We prove that for a finite convexity space (V, \mathcal{C}) and a graph G with vertex set V , there is a set \mathcal{P} of paths of G with $\mathcal{C} = \mathcal{C}(\mathcal{P})$ if and only if

- every set S which is not convex with respect to \mathcal{C} contains two distinct vertices whose convex hull with respect to \mathcal{C} is not contained in S and
- for every two elements x and z of V and every element y of the convex hull of $\{x, z\}$ with respect to \mathcal{C} distinct from x and z , the subgraph of G induced by the convex hull of $\{x, z\}$ with respect to \mathcal{C} contains a path between x and z with y as an internal vertex.

Furthermore, we prove that the corresponding algorithmic problem can be solved efficiently.

Disjoint Paths in Tournaments

PAUL SEYMOUR
(Princeton)

Joint work with Maria Chudnovsky and Alex Scott

Given k pairs of vertices of a tournament, how can we test in polynomial time (for fixed k) whether there are k directed paths joining the pairs, pairwise vertex-disjoint? For $k = 2$, there was an algorithm by Bang-Jensen and Thomassen; but recently we found an algorithm for all fixed k .

It turns out the same algorithm also answers the following more general question — given k pairs in a tournament, as before, and k integers, test whether the paths exist, pairwise vertex-disjoint, such that each path has length at most the corresponding integer.

The Complexity of Finding Disjoint Cycles and Dicycles in a Digraph with Cycle Transversal Number 2

SVEN SIMONSEN

(Department of Mathematics and Computer Science,
University of Southern Denmark, Odense)

Joint work with Jørgen Bang-Jensen, Matthias Kriesell and Alessandro Maddaloni

In this and the companion talk by A. Maddaloni we determine the complexity of the following problem:

Given a digraph D , decide if there is a cycle B in D and a cycle C in its underlying undirected graph $UG(D)$ such that $V(B) \cap V(C) = \emptyset$.

It was proved in [J.Bang-Jensen, M.Kriesell, On the problem of finding disjoint cycles and dicycles in a digraph, *Combinatorica*, to appear] that one can decide the problem (and find the cycles) in polynomial time if D is strongly connected.

When D is not necessarily strongly connected it is possible to give a complete characterization for the complexity of the problem in terms of the cycle transversal number $\tau(D)$ of D , i.e. the smallest cardinality of a set T of vertices in D such that $D - T$ is acyclic.

If $\tau(D) \geq 3$ then the desired cycles always exist.

For $\tau(D) = 1$ the problem is NP-complete as covered in the talk by A. Maddaloni.

If $\tau(D) = 2$ then we give an overview of how one can decide the existence of B, C in polynomial time (and find the cycles if they exist). Our approach is based on the classification for strongly connected digraphs given by J. Bang-Jensen and M. Kriesell and handles general digraphs based on the classification of their unique nontrivial strongly connected component.

Generalized Fractional and Circular Colorings

ROMAN SOTÁK

(Institute of Mathematics, P. J. Šafárik University, Košice, Slovakia)

An additive hereditary property of graphs is a class of simple graphs which is closed under unions, subgraphs and isomorphism. Let \mathcal{P} be a graph property and r, s be positive integers, $r \geq s$. A *strong \mathcal{P} -circular (r, s) -colouring* of a graph G is an assignment $f : V(G) \rightarrow \mathbf{Z}_r$, such that the edges $uv \in E(G)$ satisfying $|f(u) - f(v)| < s$ or $|f(u) - f(v)| > r - s$, induce a subgraph of G with the property \mathcal{P} . We present some basic results on strong \mathcal{P} -circular (r, s) -colourings. We introduce the strong circular \mathcal{P} -chromatic number of a graph and we determine the strong \mathcal{P} -circular chromatic number of complete graphs for additive and hereditary graph properties.

Let \mathcal{P} and \mathcal{Q} be additive hereditary properties of graphs. For positive integers r, s a *(weak) $(\mathcal{P}, \mathcal{Q})$ -total fractional/circular (r, s) -coloring* of a simple graph G is a coloring of the vertices $V(G)$ and edges $E(G)$ of G by arbitrary/consecutive s -element subsets of \mathbf{Z}_r such that for each color i the vertices colored by sets containing i induce a subgraph of property \mathcal{P} , the edges colored by sets containing i induce a subgraph of property \mathcal{Q} and incident vertices and edges obtain disjoint sets. We present general basic results on $(\mathcal{P}, \mathcal{Q})$ -total fractional/circular (r, s) -colorings. For specific properties we determine the $(\mathcal{P}, \mathcal{Q})$ -total fractional and circular chromatic numbers of complete graphs.

The Weak 3-Flow Conjecture

CARSTEN THOMASSEN

(Technical University of Denmark)

Tutte's 3-flow conjecture says that every 4-edge-connected graph has an orientation of its edges such that every vertex is balanced modulo 3. Jaeger suggested in 1988 to replace 4 by a larger constant. In this talk we prove this weaker statement, known as the weak 3-flow conjecture, and discuss some of its consequences.

The Traceability Conjecture for Oriented Graphs Holds for $k < 8$

SUSAN VAN AARDT
(University of South Africa)

Joint work with Marietjie Frick, Joy Singleton, Alewyn Burger, Jean Dunbar, John Harris

An oriented graph is traceable if it contains a path that visits every vertex, and it is k -traceable if each of its subdigraphs of order k is traceable. The Traceability Conjecture (TC) states that for $k \geq 2$ every k -traceable oriented graph of order at least $2k - 1$ is traceable.

We have observed that every nontraceable k -traceable oriented graph contains a hypotraceable oriented graph of order at least $k + 1$ as induced subdigraph. This, together with the fact that there are no hypotraceable oriented graphs of order $n \leq 7$, enabled us to prove that the TC holds for $k \leq 6$.

For $k = 7$ we need a different approach since we know that there exist hypotraceable oriented graphs of every order $n \geq 8$ except for $n = 9$. In this case we use the following result: if every k -traceable oriented graph of order n_1 and order n_2 is traceable and D is a k -traceable oriented graph of order $n_1 + n_2 - 1$ or $n_1 + n_2 - 2$ containing a vertex of sufficiently small in- and out-degree, then D is traceable. By an iterative application of this result we show that the TC holds for $k = 7$.

On Arbitrarily Partitionable Graphs

MARIUSZ WOŹNIAK

(AGH University of Science and Technology, Cracow, Poland)

A graph G of order n is said to be *arbitrarily partitionable* or *arbitrarily vertex decomposable* if for each sequence (n_1, \dots, n_k) of positive integers such that $n_1 + \dots + n_k = n$ there exists a partition (V_1, \dots, V_k) of the vertex set of G such that for each $i \in \{1, \dots, k\}$, V_i induces a connected subgraph of G on n_i vertices.

There is an interesting motivation for investigation of avd graphs. Consider a network connecting different computing resources; such a network is modeled by a graph. Suppose there are k different users, where the i -th one needs n_i resources in our graph G . The subgraph induced by the set of resources attributed to each user should be connected and each resource should be attributed to one user. So we have the problem of seeking a *realization of the sequence* $\tau = (n_1, \dots, n_k)$ in G .

This problem can also be considered as a natural analogy of the similar notion in which vertices are replaced by edges.

In the talk, a complete characterization of some families of arbitrarily vertex decomposable graphs will be given. We shall consider also the ‘on-line’ and recursive versions of the problem.

Acute Triangulations of Surfaces

CAROL T. ZAMFIRESCU
(TU Dortmund, Germany)

A *triangulation* of a two-dimensional space means a collection of (full) triangles covering the space such that the intersection of any two triangles is either empty, or a vertex, or an edge. A triangulation is called *geodesic*, if all of its triangles are geodesic, meaning that their edges are *segments*, i.e. shortest paths between the corresponding vertices. In this talk we shall always work with geodesic triangulations. (Colin de Verdière showed how to transform a triangulation of a compact surface of non-positive curvature into a geodesic triangulation.)

We have a *non-obtuse* or an *acute* triangulation, if all angles within the geodesic triangles are not larger than, respectively smaller than, $\pi/2$. A *balanced* triangulation is an acute triangulation with all angles measuring more than $\pi/6$.

In this talk we will present recent results concerning non-obtuse, acute and balanced triangulations of minimal size, investigating polygons, the *Platonic surfaces* (i.e. the boundaries of the Platonic solids), and so-called *double polygons*: two congruent planar convex bodies can be identified along their boundaries in accordance with the congruence, creating a (degenerate) convex surface. Regarding triangulations of double polygons, results are spread so thinly that we can present an exhaustive list. Note that in general, one cannot simply triangulate — acutely for instance — a polygon, and then apply this same triangulation to the copy, as easily a situation might occur where two triangles have two edges in common, which contradicts the definition of a triangulation! Even if it is possible to simply copy the triangulation, this is often not desirable, as better configurations (i.e. triangulations of smaller size) might exist.

We end the talk by exhibiting several interesting open problems.

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Schedule

Elgersburg 2011

March 21–25

	Monday	Tuesday		Wednesday	Thursday	Friday	
		9:00– 9:20	M. Chudnovsky		M.A. Henning	J.A. Bondy	A. Raspaud
		9:25– 9:45			S. Jendrol'		
		9:55–10:15	P. Seymour		M. Horňák	M. Borowiecki	M. Miller
		10:20–10:40			M. Woźniak/ R. Kalinowski	P.M. Schäfer	J. Carmesin
		10:40–11:10	Coffee break		Coffee break	Coffee break	Coffee break
		11:10–11:30	S. van Aardt		B. Mohar	J. Bang-Jensen	J.-O. Fröhlich
		11:35–11:55	M. Frick			H. Harborth	C. Zamfirescu
		12:00–14:00	Lunch break		Lunch break	Lunch break	Lunch
			Room A	Room B			
14:00–14:05	Opening	14:00–14:20	M. Marangio	F. Pfender	Excursion	Z. Ryjáček	
14:05–14:50	C. Thomassen	14:30–14:50	C. Löwenstein	A. Maddaloni		F. Kardoš	
14:55–15:40	A. Saito	15:00–15:20	A. Fricke	S. Simonsen		M. Kriesell	
15:40–16:10	Coffee break	15:30–16:00	Coffee break			Coffee break	
16:10–16:30	V. Lozin	16:00–16:20	J.-P. Bode	F. Göring		M. Koch	
16:40–17:00	R. Soták	16:30–16:50	S. Matos Camacho	E. Aigner-Horev		A. Kohl	
		17:00–18:00	Problem Session				
18:00	Dinner	18:00	Dinner				Dinner
		19:00				Conference Dinner	