Tight Lower Bounds for Greedy Routing in Higher-Dimensional Small- World Grids

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Vortrag

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We consider Kleinberg's celebrated small world graph model (Kleinberg, 2000), in which a D-dimensional grid {0,□,n-1}^D is augmented with a constant number of additional unidirectional edges leaving each node. These long range edges are determined at random according to a probability distribution (the augmenting distribution), which is the same for each node. Kleinberg suggested using the inverse D-th power distribution, in which node v is the long range contact of node u with a probability proportional to 1/d(u,v)^d, where d(u,v) is the Manhattan distance between u and v . He showed that such an augmenting distribution allows to route a message efficiently in the resulting random graph: The greedy algorithm, where in each intermediate node the message travels over a link that brings the message closest to the target w.r.t. the Manhattan distance, finds a path of expected length O ((log n)^2) between any two nodes. We prove that greedy routing does not perform asymptotically better for any uniform and isotropic augmenting distribution, i.e., the probability that node u has a particular long range contact v is independent of the labels of u and v and only a function of d(u,v). In particular, we show that for such graphs the expected greedy routing time between two arbitrary nodes s and t is Omega(log(d(s,t))^2). This lower bound proves and strengthens a conjecture by Aspnes, Diamadi, and Shah (2000). In order to obtain the result, we introduce a novel proof technique: We define a so-called budget game, in which a token travels over a game board, from one end to the other, while the player manages a ``probability budget''. In each round, the player ``bets'' part of her remaining probability budget on step sizes. A step size is chosen at random according to a probability distribution of the player's bet. The token then makes progress as determined by the chosen step size, while some of the player's bet is removed from her probability budget. We prove a tight lower bound for such a budget game, and then obtain a lower bound for greedy routing in the D-dimensional grid by a reduction.