Last name, first name: … **Major: …** **Mat.-Nr.: …**

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Mobile Communications

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Total time: 120 min Total points: 40+2 Bonus

NB: Those tasks highlighted **bold-faced** can be solved independently from the previous ones. Label the axis of all your graphs properly.

1. Consider a propagation model where the received power is proportional to d^{-n}, where d is the distance between transmitter and receiver and n is the path loss exponent. The received power has been measured to be $P_{\mathrm{R},1} = -80$ dBm at a distance $d_1 = 50$ m and $P_{\mathrm{R},2} = -90$ dBm at a distance $d_2 = 100$ m. (7 pt)

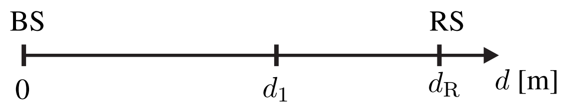
   (a) What is the path loss exponent n? Hint: $10 \cdot \log_{10}(2) \approx 3$.

   (b) Compute the power received at a distance of $d_3 = 500$ m.

   (c) Compute the maximum distance $d_{\max}$ so that the received power level maintains above the detection threshold of -166.7 dBm (Hint: $-166.7 \approx -500/3$, $10^{2.3} \approx 200$, $10^{2.6} \approx 400$).

   (d) Assume the path loss is $n = 3$ and the maximum distance $d_{\max} = 600$ m. We would like to double the cell radius to 1200 m without changing the transmit power or the noise power. Explain how this can be achieved by deploying multiple antennas at the receiver: What is the appropriate receive filtering technique and how many antenna elements are needed?

   We now consider a base station (BS) with a transmit power of $P_{\mathrm{T},1} = 42$ dBm and a relay station (RS) with a transmit power of $P_{\mathrm{T},2} = 33$ dBm. The distance between base station and relay station is $d_\mathrm{R} = 600$ m. Our goal is to assign users to the base station or the relay station depending on the received power level.



   (e) Compute the distance d_1 where the power received from the base station is equal to the power received from the relay station. As before, assume a path loss exponent of $n = 3$. Hint: $10^{0.3} \approx 2$.

2. Let the random variable X by Gaussian distributed with zero mean and variance one, i.e., $X \sim \mathcal{N}(0, 1)$. Then a new random variable Y is given by $Y = g(X)$, where $g(x) = x^2 \ \forall x$. Derive the probability density function $f_Y(y)$. (5 pt)

3. Consider an L-path model with $L = 2$ paths that have the following parameters: $c_1 = 2$, $\tau_1 = 100$ ns, $f_{\mathrm{D},1} = -10$ Hz, $c_2 = 1$, $\tau_2 = 200$ ns, $f_{\mathrm{D},2} = 20$ Hz. (14 pt)

   (a) Provide the explicit expressions for $h(\tau, t)$, $H(f, t)$, $H(f, f_\mathrm{D})$, and $h(\tau, f_\mathrm{D})$.

   (b) Give the explicit expression for the power-delay profile $\varphi_h(\tau)$ and sketch it.

   (c) Compute the average delay $\bar{\tau}$, the squared average $\overline{\tau^2}$, and the RMS delay spread τ_{rms} (Hint: no calculator needed).

(d) Provide the explicit expression for the spaced-frequency correlation function $\varphi_h(\Delta f)$.

(e) Give the formula for the scattering function $\Phi_h(\tau, f_\mathrm{D})$ and sketch it (pseudo-3D or view from top).

Next, consider $L = 3$ paths that have the following (transmit) directions of departure $\theta_{\mathrm{T},\ell}$ and (receive) directions of arrival $\theta_{\mathrm{R},\ell}$: $\theta_{\mathrm{T},1} = -10°$, $\theta_{\mathrm{T},2} = 40°$, $\theta_{\mathrm{T},3} = 30°$, $\theta_{\mathrm{R},1} = 70°$, $\theta_{\mathrm{R},2} = -60°$, $\theta_{\mathrm{R},3} = 0°$. Moreover, the amplitudes of the three paths are $c_1 = 1$, $c_2 = 3$, $c_3 = 2$.

(f) Sketch the angular power spectrum at the transmitter $\varphi_{h,\mathrm{T}}(\theta_\mathrm{T})$.

(g) Sketch the angular power spectrum at the receiver $\varphi_{h,\mathrm{R}}(\theta_\mathrm{R})$.

(h) Sketch the 2-D angular power spectrum $\varphi_h(\theta_\mathrm{R}, \theta_\mathrm{T})$ (pseudo-3D or view from top).

(i) Sketch the 2-D angular power spectrum $\varphi_{h,\mathrm{kron}}(\theta_\mathrm{T}, \theta_\mathrm{R})$ that is obtained by computing a Kronecker approximation to the channel (pseudo-3D or view from top).

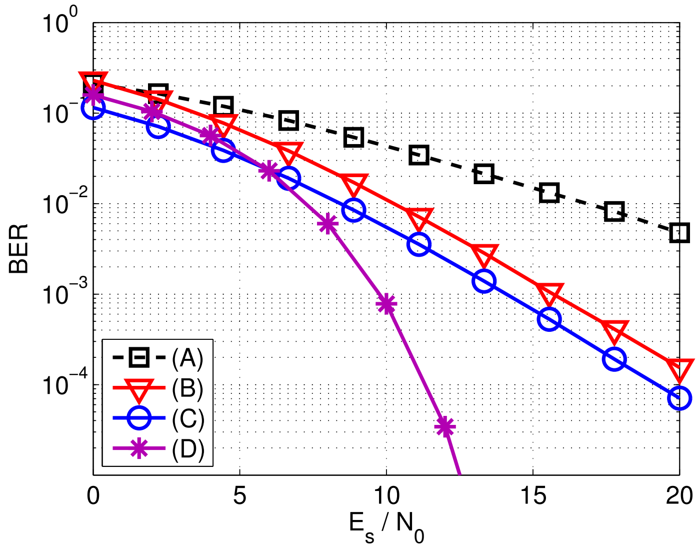
4. The following graph shows the bit error rate performance of an uncoded QPSK transmission under different conditions: (8 pt)

(i) a pure AWGN channel.

(ii) a single-input single-output (SISO) frequency-flat fading Rayleigh channel;

(iii) a multiple-input single-output (MISO) frequency-flat fading Rayleigh channel ($\boldsymbol{H}_w$) with $M_\mathrm{T} = 2$ transmit antennas using Alamouti Space-Time Coding at the transmitter;

(iv) a single-input multiple-output (SIMO) frequency-flat fading Rayleigh channel ($\boldsymbol{H}_w$) with $M_\mathrm{R} = 2$ receive antennas using Maximum Ratio Combining (MRC) at the receiver;



(a) Identify the curves (A), (B), (C), and (D) with the propagation conditions (i), (ii), (iii), and (iv). Which curve belongs to which scenario?

(b) Explain in your own words: Why and how does diversity improve the reliability of a transmission in a fading noisy environment?

(c) Explain in your own words: Why and how does array gain improve the reliability of a transmission in a fading noisy environment?

(d) For the curves (A), (B), and (C), what is the diversity order?

(e) For the curves (A), (B), and (C), what is the transmit and receive array gain?

(f) How do the diversity order and the array gain change for (iii) if the transmitter possesses channel knowledge and therefore changes its transmission scheme from Alamouti Space-Time Coding to transmit MRC?

(g) How do the diversity order and the array gain change for (iii) if a second receive antenna is added (i.e., a 2×2 MIMO system is considered), where the receiver performs MRC (the transmitter still performs Alamouti Space-Time Coding)?

5. Consider the following channel realizations of a 2×2 MIMO system: (6+2 pt)

$$\boldsymbol{H}^{(\mathrm{a})} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \boldsymbol{H}^{(\mathrm{b})} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The corresponding singular values are given by

$$\sigma_1^{(\mathrm{a})} = 2,\ \sigma_2^{(\mathrm{a})} = 0 \text{ for } \boldsymbol{H}^{(\mathrm{a})}$$
$$\sigma_1^{(\mathrm{b})} = \sqrt{2},\ \sigma_2^{(\mathrm{b})} = \sqrt{2} \text{ for } \boldsymbol{H}^{(\mathrm{b})}$$

(a) For $P_{\mathrm{T}} = 1$, $\sigma_{\mathrm{n}}^2 = 1$ (SNR = 0 dB), compute the open-loop MIMO capacity of $\boldsymbol{H}^{(\mathrm{a})}$ and $\boldsymbol{H}^{(\mathrm{b})}$.

(b) What is the optimal power distribution γ_1^{opt}, γ_2^{opt} for SNR $\to \infty$ obtained via water pouring for $\boldsymbol{H}^{(\mathrm{a})}$ and $\boldsymbol{H}^{(\mathrm{b})}$? You can leave the result as $\log_2(X)$ once you found the (scalar) X.

(c) Describe how the water pouring solution changes for lower SNRs for $\boldsymbol{H}^{(\mathrm{a})}$ and $\boldsymbol{H}^{(\mathrm{b})}$.

(d) What is the spatial multiplexing gain (i.e., the number of streams that can be multiplexed over the channel) in the system if the optimal (capacity-achieving) prefiltering scheme is applied for $\boldsymbol{H}^{(\mathrm{a})}$ and $\boldsymbol{H}^{(\mathrm{b})}$? Why?

(e) (Bonus +2 pt): Compute the closed-loop MIMO capacity for $P_{\mathrm{T}} = 1$, $\sigma_{\mathrm{n}}^2 = 1$ (SNR = 0 dB) for $\boldsymbol{H}^{(\mathrm{a})}$ and $\boldsymbol{H}^{(\mathrm{b})}$.