Blind Estimation of SIMO Channels Using A Tensor-Based Subspace Method

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***Abstract* — In this paper, we introduce a tensor-based subspace method for solving the blind channel estimation problem in a single-input multiple-output (SIMO) system. Since the measurement data is multidimensional, previously proposed blind channel estimation methods require stacking the multiple dimensions into one highly structured vector and estimate the signal subspace via a singular value decomposition (SVD) of the correlation matrix of the measurement data. In contrast to this, we define a 3-way measurement tensor of the received signals and obtain the signal subspace via a multidimensional extension known as Higher-Order SVD (HOSVD). This allows us to exploit the structure inherent in the measurement data and leads to improved estimates of the signal subspace. Numerical simulations demonstrate that the proposed method outperforms previously proposed subspace based blind channel estimation methods in terms of the channel estimates accuracy. Furthermore, we show that the accuracy of the estimations is significantly improved by employing overlapping observed data windows at the receiver.**

***Index Terms*—Blind channel estimation, signal subspace, Higher-Order SVD, tensor.**

I. Introduction

In general, second-order or higher-order statistics of the measurement data are utilized for blind channel estimation [1], [2], [3], [4], [5]. The method proposed in [4] is viewed as a pioneering work which provides a subspace method for the blind estimation of single-input multiple-output (SIMO) channels. This subspace-based blind channel estimation algorithm performs a singular value decomposition (SVD) of the correlation matrix of the measurement data to separate the observed space into two orthogonal subspaces, namely the signal subspace and the noise subspace. Then, the channel can be estimated up to a scalar factor by exploiting the orthogonality property between the signal and noise subspaces. The concept behind the subspace method is quite useful, since it is easily extendible to the blind estimation of a multiple-input multiple-output (MIMO) channel and space-time OFDM channels [6], [7].

However, in the existing subspace-based approaches to blind channel estimation, the measurement data is stored in one highly structured vector by a stacking operation. As a result, the structure inherent in the measurement data is not considered in the subspace estimation step. A more natural approach to store and exploit the inherent structure of the measurement data is given by tensors. Tensor-based signal processing has become increasingly popular in many different areas of signal processing. This is due to the fact that it offers several fundamental advantages compared to matrix-based techniques. First of all, multilinear decompositions are essentially unique without additional constraints and allow to separate more components compared to the bilinear (matrix) approach, which renders them attractive for component separation tasks [8]. Moreover, since the structure of the data is preserved, structured denoising can be applied, which leads to an improved tensor-based signal subspace estimate enhancing any subspace-based parameter estimation scheme [9].

In this paper, we extend the matrix-based blind channel estimation technique of [4] to the tensor case. We use a 3-way tensor to model the measurement data. Moreover, the truncated Higher-Order SVD (HOSVD) is used to obtain the tensor-based signal subspace estimate. This tensor-based signal subspace estimate yields a higher accuracy of the subspace [9], [10] compared to the matrix-based signal subspace estimate, thereby leading to an improved estimate of the channels. As we show in our simulations, the proposed tensor-based subspace method outperforms the matrix-based subspace method in terms of the channel estimation accuracy. Furthermore, we introduce a smoothing window to observe the received data symbols with smoothing parameter η. The parameter η denotes the number of new measurements in the next observed data window as depicted in Figure 6. It is found that the estimation accuracy is increased with decreasing η.

The paper is organized as follows. The notation is introduced in Section II. The system description and the general signal model are presented in Section III. In Section IV, a short introduction of the matrix-based subspace method for blind estimation of SIMO channels is given. The proposed tensor-based method follows in Section V. In Section VI and VII, we show simulation results and a conclusion, respectively.

II. Notation

The following notation is used throughout the paper to distinguish between scalars, vectors, matrices, and tensors: Scalars are represented as italic letters (a, $b \dots A$, $B \dots$), column vectors are written as lower-case bold-faced letters ($\boldsymbol{a}$, $\boldsymbol{b}$, $\dots$), matrices are denoted as upper-case bold-faced letters ($\boldsymbol{A}$, $\boldsymbol{B}$, $\dots$), tensors are indicated by bold-faced calligraphic letters ($\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{B}}$, $\dots$). We use $\boldsymbol{A}(:, i)$ to denote the ith column of the matrix $\boldsymbol{A}$. The superscripts $^{\mathrm{T}}$, $^{\mathrm{H}}$, $^{+}$ are for transposition, Hermitian transposition, and Moore-Penrose pseudo-inverse, respectively.

An R-dimensional tensor $\boldsymbol{\mathcal{A}} \in \mathbb{C}^{M_1 \times M_2 \times \dots \times M_R}$ is an R-way array which has the size M_r along mode r. The tensor operation we use are consistent with [11].

- **The r-mode vectors of $\boldsymbol{\mathcal{A}}$** are obtained by varying the r-th index, while keeping all other indices fixed.
- **The r-mode unfolding of $\boldsymbol{\mathcal{A}}$** is obtained by collecting all r-mode vectors into a matrix and represented by $[\boldsymbol{\mathcal{A}}]_{(r)} \in \mathbb{C}^{M_r \times M_{r+1} \cdot \ldots \cdot M_R \cdot M_1 \cdot \ldots \cdot M_{r-1}}$. The ordering of the columns in $[\boldsymbol{\mathcal{A}}]_{(r)}$ is chosen in accordance with [11].
- **The r-rank of $\boldsymbol{\mathcal{A}}$** is defined as the rank of $[\boldsymbol{\mathcal{A}}]_{(r)}$. Note that in general, all the r-ranks of a tensor $\boldsymbol{\mathcal{A}}$ can be different.

- **The r-mode product of tensor $\boldsymbol{\mathcal{A}}$** and a matrix $\boldsymbol{U}_r \in \mathbb{C}^{J_r \times M_r}$ is denoted as $\boldsymbol{\mathcal{B}} = \boldsymbol{\mathcal{A}} \times_r \boldsymbol{U}_r$. It is visualized by multiplying all r-mode vectors of $\boldsymbol{\mathcal{A}}$ from left-hand side by the matrix $\boldsymbol{U}_r$, i.e., $[\boldsymbol{\mathcal{B}}]_{(r)} = \boldsymbol{U}_r [\boldsymbol{\mathcal{A}}]_{(r)}$.
- **The HOSVD of $\boldsymbol{\mathcal{A}}$** is given by

$$\boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{S}} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2 \ldots \times_R \boldsymbol{U}_R,$$

where $\boldsymbol{\mathcal{S}} \in \mathbb{C}^{M_1 \times M_2 \times \ldots \times M_R}$ is the core tensor which satisfies the all-orthogonality conditions [11] and $\boldsymbol{U}_r \in \mathbb{C}^{M_r \times M_r}$ are the unitary matrices of r-mode singular vectors for $r = 1, \ldots, R$.

III. System Model

We consider a SIMO system where the receiver is equipped with M_{R} receive antennas. The channel between each transmit and receive antenna pair is modeled as an FIR filter with $L+1$ taps. Let $s(k)$ denote the symbol emitted over the transmit antenna at time kT. Here T indicates the symbol duration. The discrete-time signal experiences the unknown channel which is assumed to be time-invariant during the observation interval. Then, the received signal vector at time kT is denoted as

$$\boldsymbol{y}(k) = \sum_{\ell=0}^{L} \boldsymbol{h}_\ell s(k-\ell) + \boldsymbol{n}(k) \in \mathbb{C}^{M_{\mathrm{R}}}. \tag{1}$$

Here, $\boldsymbol{h}_\ell \in \mathbb{C}^{M_{\mathrm{R}}}$ contains the coefficients of the channel impulse responses corresponding to the ℓth channel tap. The elements of the noise vector $\boldsymbol{n}(k)$ are circularly symmetric complex Gaussian distributed with variance σ^2 and assumed mutually uncorrelated in space and time.

IV. Matrix-based Subspace Method for SIMO Channels

In practice, the measurement data is observed during consecutive data windows over all receive antennas. The matrix-based subspace method for the blind estimation of SIMO channels (e.g., [4]) stacks the dimensions of M_{R} receive antennas and data window length into one highly structured vector. Then, the signal subspace is estimated via an SVD of the correlation matrix of the measurement data.

A. Matrix-Based Measurement Data Model

We use W to indicate the length of the observed data window. Stacking M_{R} observation vectors into one $M_{\mathrm{R}} \cdot W \times 1$ vector, the measurement data with respect to the nth observed data window is given by

$$\boldsymbol{y}_n = \boldsymbol{H}_T \boldsymbol{s}_n + \boldsymbol{n}, \tag{2}$$

where $\boldsymbol{y}_n = \left[\boldsymbol{y}_n^{(1)^{\mathrm{T}}}, \boldsymbol{y}_n^{(2)^{\mathrm{T}}}, \ldots, \boldsymbol{y}_n^{(M_{\mathrm{R}})^{\mathrm{T}}}\right]^{\mathrm{T}}$. The vector $\boldsymbol{s}_n = [s(nW), s(nW-1), \ldots, s(nW-W-L+1)]^{\mathrm{T}}$ denotes the input data sequence with dimension $(W+L) \times 1$. The matrix $\boldsymbol{H}_T \in \mathbb{C}^{M_{\mathrm{R}} \cdot W \times (W+L)}$ is the filtering matrix and structured as

$$\boldsymbol{H}_T = \left[\boldsymbol{H}_T^{(1)^{\mathrm{T}}},\ \boldsymbol{H}_T^{(2)^{\mathrm{T}}}, \ldots, \boldsymbol{H}_T^{(M_{\mathrm{R}})^{\mathrm{T}}}\right]^{\mathrm{T}}, \tag{3}$$

where $\boldsymbol{H}_T^{(j)} \in \mathbb{C}^{W \times (W+L)}$ indicates a banded Toeplitz matrix associated to the jth receive antenna's impulse response $\boldsymbol{h}^{(j)}$. The vector $\boldsymbol{h}^{(j)}$ is defined as

$$\begin{aligned} \boldsymbol{h}^{(j)} &\overset{\text{def}}{=} \left[h_0^{(j)},\ h_1^{(j)}, \ldots, h_L^{(j)}\right]^{\mathrm{T}} \\ &\overset{\text{def}}{=} \left[h^{(j)}(t_0),\ h^{(j)}(t_0+T), \ldots, h^{(j)}(t_0+LT)\right]^{\mathrm{T}}. \end{aligned} \tag{4}$$

Then, we have

$$\boldsymbol{H}_T^{(j)} = \begin{bmatrix} h_0^{(j)} & \cdots & h_L^{(j)} & 0 & \cdots & \cdots & 0 \\ 0 & h_0^{(j)} & \cdots & h_L^{(j)} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & h_0^{(j)} & \cdots & h_L^{(j)} \end{bmatrix}. \tag{5}$$

B. Signal Subspace Estimation

Observing N consecutive data windows at the receiver, the space-time correlation matrix $\boldsymbol{R}_{yy} \in \mathbb{C}^{M_{\mathrm{R}} \cdot W \times M_{\mathrm{R}} \cdot W}$ of the measurement data can be estimated as

$$\begin{aligned} \boldsymbol{R}_{yy} &= \mathrm{E}\left\{\boldsymbol{y}_n \boldsymbol{y}_n^{\mathrm{H}}\right\} = \boldsymbol{H}_T \boldsymbol{R}_{ss} \boldsymbol{H}_T^{\mathrm{H}} + \sigma^2 \boldsymbol{I}_{M_{\mathrm{R}} \cdot W} \\ &\approx \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{y}_n \boldsymbol{y}_n^{\mathrm{H}} = \hat{\boldsymbol{R}}_{yy}. \end{aligned} \tag{6}$$

Here, the matrix $\boldsymbol{R}_{ss} = \mathrm{E}\left\{\boldsymbol{s}_n \boldsymbol{s}_n^{\mathrm{H}}\right\}$ denotes the correlation matrix of the transmit data with dimension $(W+L) \times (W+L)$. Calculating an SVD of the estimated correlation matrix $\hat{\boldsymbol{R}}_{yy}$, the first $(W+L)$ dominant left singular vectors span the estimated signal subspace represented as $\hat{\boldsymbol{U}}_s \in \mathbb{C}^{M_{\mathrm{R}} \cdot W \times (W+L)}$.

The following assumptions are the necessary conditions for the channel identifiability.

1) The correlation matrix $\boldsymbol{R}_{ss}$ is full-rank but otherwise unknown, which requires $N \geq (W+L)$.
2) The matrix $\boldsymbol{H}_T$ has a full column rank.
3) The observed data window length is greater than the channel order L (i.e., $W > L$).
4) The number of channel taps $L+1$ has been correctly estimated before.
5) The noise samples are uncorrelated with the input data.

C. Signal Subspace Based Parameter Estimation

Since the column space of $\hat{\boldsymbol{U}}_s$ is the linear space spanned by the columns of the filtering matrix $\boldsymbol{H}_T$, the unknown SIMO channel coefficients incorporated in the filtering matrix $\boldsymbol{H}_T$ can be identified up to a scalar factor by maximizing the following quadratic form

$$q(\boldsymbol{H}) \overset{\text{def}}{=} \sum_{i=1}^{W+L} \left\| \hat{\boldsymbol{U}}_s(:,i)^{\mathrm{H}} \boldsymbol{H}_T \right\|_2^2, \tag{7}$$

where the matrix $\boldsymbol{H} \in \mathbb{C}^{(L+1) \times M_{\mathrm{R}}}$ is a combined channel matrix for all M_{R} subchannels denoted as $\boldsymbol{H} = \left[\boldsymbol{h}^{(1)},\ \boldsymbol{h}^{(2)}, \ldots, \boldsymbol{h}^{(M_{\mathrm{R}})}\right]$. In order to specify the quadratic dependence of $q(\boldsymbol{H})$ on the matrix $\boldsymbol{H}$ rather than on the associated filtering matrix $\boldsymbol{H}_T$ found in (7), the commutativity $\hat{\boldsymbol{U}}(:,i)^{\mathrm{T}} \boldsymbol{H}_T = \mathrm{vec}(\boldsymbol{H})^{\mathrm{T}} \boldsymbol{G}_i$ can be applied. The matrix $\boldsymbol{G}_i \in \mathbb{C}^{M_{\mathrm{R}}(L+1) \times (L+W)}$ is the filtering matrix associated to the vector $\hat{\boldsymbol{U}}(:,i) \in \mathbb{C}^{M_{\mathrm{R}} \cdot W}$ and has the structure $\boldsymbol{G}_i = \left[\boldsymbol{G}_i^{(1)^{\mathrm{T}}}, \boldsymbol{G}_i^{(2)^{\mathrm{T}}}, \ldots, \boldsymbol{G}_i^{(M_{\mathrm{R}})^{\mathrm{T}}}\right]^{\mathrm{T}}$. Each $\boldsymbol{G}_i^{(j)} \in \mathbb{C}^{(L+1) \times (L+W)}$ is a banded Toeplitz matrix corresponding to the vector $\hat{\boldsymbol{U}}^{(j)}(:,i) \in \mathbb{C}^{W}$, where the vector $\hat{\boldsymbol{U}}^{(j)}(:,i)$ is obtained by splitting the vector $\hat{\boldsymbol{U}}(:,i)$ into M_{R} subvectors (i.e., $\hat{\boldsymbol{U}}(:,i) = \left[\hat{\boldsymbol{U}}^{(1)^{\mathrm{T}}}(:,i),\ \hat{\boldsymbol{U}}^{(2)^{\mathrm{T}}}(:,i), \ldots, \hat{\boldsymbol{U}}^{(M_{\mathrm{R}})^{\mathrm{T}}}(:,i)\right]^{\mathrm{T}}$). Then, the maximization problem in equation (7) can be transformed to the maximization of the following quadratic form

$$\begin{aligned} q(\boldsymbol{H}) &\overset{\text{def}}{=} \mathrm{vec}(\boldsymbol{H})^{\mathrm{H}} \cdot \left(\sum_{i=1}^{W+L} \boldsymbol{G}_i \boldsymbol{G}_i^{\mathrm{H}}\right) \cdot \mathrm{vec}(\boldsymbol{H}) \\ &\overset{\text{def}}{=} \mathrm{vec}(\boldsymbol{H})^{\mathrm{H}} \cdot \boldsymbol{G} \cdot \mathrm{vec}(\boldsymbol{H}). \end{aligned} \tag{8}$$

The solution of the maximization problem in (8) is the eigenvector associated to the largest eigenvalue of the matrix $\boldsymbol{G}$ [4].

V. Proposed Tensor-Based Subspace Method for SIMO Channels

In this section, we propose a tensor-based subspace method for blind estimation of SIMO channels. Instead of the stacking operation employed in the definition of $\boldsymbol{y}_n$ in equation (2), we introduce a 3-way tensor to model the measurement data. In this case, an enhanced estimate of the signal subspace is achieved by computing an HOSVD based signal subspace estimate.

A. Tensor-Based Measurement Data Model

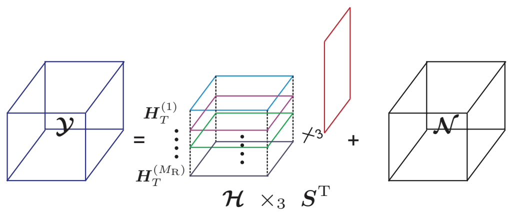


Fig. 1. Block diagram of the tensor based data model in equation (9).

The received signal is modeled by a 3-way tensor $\boldsymbol{\mathcal{Y}} \in \mathbb{C}^{M_\mathrm{R} \times W \times N}$, where the three dimensions of the tensor $\boldsymbol{\mathcal{Y}}$ represent receive antennas, observed data window length, and the number of data windows, respectively. The corresponding input output data model can be expressed as

$$\boldsymbol{\mathcal{Y}} = \boldsymbol{\mathcal{H}} \times_3 \boldsymbol{S}^\mathrm{T} + \boldsymbol{\mathcal{N}}. \tag{9}$$

The matrix $\boldsymbol{S} = [\boldsymbol{s}_1,\ \boldsymbol{s}_2, \ldots, \boldsymbol{s}_N]$ has the dimension $(W+L) \times N$ and contains input data sequences associated to N sequentially observed data windows at the receiver. Each input data sequence $\boldsymbol{s}_n = [s(nW), s(nW-1), \ldots, s(nW-W-L+1)]^\mathrm{T}$ has the dimension $(W+L) \times 1$ for $n = 1, 2, \ldots, N$. The filtering tensor $\boldsymbol{\mathcal{H}} \in \mathbb{C}^{M_\mathrm{R} \times W \times (W+L)}$ is constructed by aligning the banded Toeplitz matrices of the M_R subchannels along the first dimension as shown in Figure 1. The tensor $\boldsymbol{\mathcal{N}}$ contains noise samples and has the same size as the tensor $\boldsymbol{\mathcal{Y}}$.

The channel identification of the tensor-based subspace method requires the following necessary conditions which are consistent with the channel identifiablity conditions of the matrix-based subspace method.

1) The matrix $\boldsymbol{S}$ of the transmit signal has the rank $W+L$, which requires $N \geq (W+L)$.
2) the 3-mode unfolding of the filtering tensor $\boldsymbol{\mathcal{H}}$ has a full row rank $W+L$.
3) The observed data window length is greater than the channel order L (i.e., $W > L$).
4) The number of channel taps $L+1$ has been correctly estimated before.
5) The noise samples are uncorrelated with the input data.

B. Signal Subspace Estimation

Instead of computing SVD of the estimated space-time correlation matrix $\hat{\boldsymbol{R}}_{yy}$ found in (6), we employ a truncated HOSVD of the measurement tensor $\boldsymbol{\mathcal{Y}}$ to obtain an enhanced estimate of the signal subspace. We have [9]

$$\boldsymbol{\mathcal{Y}} \approx \boldsymbol{\mathcal{S}}^{[s]} \times_1 \boldsymbol{U}_1^{[s]} \times_2 \boldsymbol{U}_2^{[s]} \times_3 \boldsymbol{U}_3^{[s]}, \tag{10}$$

where $\boldsymbol{\mathcal{S}}^{[s]} \in \mathbb{C}^{r_1 \times r_2 \times r_3}$, $\boldsymbol{U}_1^{[s]} \in \mathbb{C}^{M_\mathrm{R} \times r_1}$, $\boldsymbol{U}_2^{[s]} \in \mathbb{C}^{W \times r_2}$, and $\boldsymbol{U}_3^{[s]} \in \mathbb{C}^{N \times r_3}$. Here, r_n $(n = 1, 2, 3)$ denotes the n-rank of the noiseless tensor $\widetilde{\boldsymbol{\mathcal{Y}}}$ (i.e., $\widetilde{\boldsymbol{\mathcal{Y}}} = \boldsymbol{\mathcal{H}} \times_3 \boldsymbol{S}^\mathrm{T}$). In our application, we have $r_1 = \min(M_\mathrm{R},\ L+1)$, $r_2 = \min(W,\ N \cdot M_\mathrm{R})$, and $r_3 = \min(N,\ W+L)$. Due to the assumption of $N \geq W+L$, we can conclude that r_2 is the same as the observed data window length W and r_3 is equal to $W+L$.

From equation (10), we define the estimated signal subspace tensor $\hat{\boldsymbol{\mathcal{U}}}^{[s]} \in \mathbb{C}^{M_\mathrm{R} \times W \times r_3}$ as [9]

$$\hat{\boldsymbol{\mathcal{U}}}^{[s]} = \boldsymbol{\mathcal{S}}^{[s]} \times_1 \boldsymbol{U}_1^{[s]} \times_2 \boldsymbol{U}_2^{[s]}. \tag{11}$$

We observe that the columns of $\left[\hat{\boldsymbol{\mathcal{U}}}^{[s]}\right]_{(3)}^\mathrm{T} \in \mathbb{C}^{M_\mathrm{R} \cdot W \times r_3}$ span the estimated signal subspace. Compared to the estimated signal subspace $\hat{\boldsymbol{U}}_s$ from the matrix case, $\left[\hat{\boldsymbol{\mathcal{U}}}^{[s]}\right]_{(3)}^\mathrm{T}$ provides a more accurate estimate under the conditions that the measurement tensor $\widetilde{\boldsymbol{\mathcal{Y}}}$ is rank-deficient in the first or second mode [9], [10] (i.e., $M_\mathrm{R} > r_1$ or $W > r_2$). Otherwise, both the tensor-based and the matrix-based signal subspace estimation yield exactly the same accuracy. Since r_2 is equal to W for our model, $\left[\hat{\boldsymbol{\mathcal{U}}}^{[s]}\right]_{(3)}^\mathrm{T}$ can achieve better estimate under the condition $M_\mathrm{R} > L+1$.

C. Signal Subspace Based Parameter Estimation

Since the column spaces of $\left[\hat{\boldsymbol{\mathcal{U}}}^{[s]}\right]_{(3)}^\mathrm{T}$ and $[\boldsymbol{\mathcal{H}}]_{(3)}^\mathrm{T}$ approximately coincide, the unknown SIMO channel coefficients incorporated in the filtering tensor $\boldsymbol{\mathcal{H}}$ can be identified up to a scalar factor by solving the maximization of the following quadratic form

$$q(\boldsymbol{H}) \stackrel{\mathrm{def}}{=} \sum_{i=1}^{W+L} \left\| \hat{\boldsymbol{U}}_{s_T}(:, i)^\mathrm{H} \, [\boldsymbol{\mathcal{H}}]_{(3)}^\mathrm{T} \right\|_2^2. \tag{12}$$

Here, we use $\hat{\boldsymbol{U}}_{s_T}$ to indicate the estimated signal subspace of the tensor case for notational simplicity (i.e., $\hat{\boldsymbol{U}}_{s_T} = \left[\hat{\boldsymbol{\mathcal{U}}}^{[s]}\right]_{(3)}^\mathrm{T}$). The channel parameter estimation scheme obeys the exact same procedure as the scheme mentioned in Section IV.C, except for replacing $\hat{\boldsymbol{U}}_s(:, i)$ and $\boldsymbol{H}_T$ by $\hat{\boldsymbol{U}}_{s_T}(:, i)$ and $[\boldsymbol{\mathcal{H}}]_{(3)}^\mathrm{T}$, respectively.

D. Oversampled Antenna Array

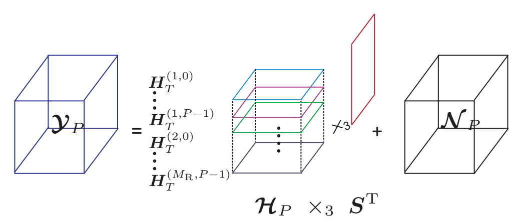


Fig. 2. Block diagram of the tensor based data model with oversampling.

As mentioned above, the performance benefit of the tensor-based subspace method is achieved under the condition $M_\mathrm{R} > L+1$. To maintain the performance benefit of the tensor model for the case $M_\mathrm{R} \leq L+1$, we introduce an oversampling of the received signals by a factor $P = T/\Delta$. Then, for the received signal of the jth receive antenna $\boldsymbol{y}_n^{(j)}$ in the nth observed data window, a set of P sequences are constructed according to $\boldsymbol{y}_n^{(j,m)} = \left[y_n^{(j)}(k + \frac{m}{P}), y_n^{(j)}(k + 1 + \frac{m}{P}), \ldots, y_n^{(j)}(k + W - 1 + \frac{m}{P})\right]^\mathrm{T}$ for $m = 0, 1, \ldots, P-1$. Notice that the vector $\boldsymbol{y}_n^{(j,0)}$ corresponds to the vector $\boldsymbol{y}_n^{(j)}$. Each sequence $\boldsymbol{y}_n^{(j,m)}$ depends on the oversampled discrete-time impulse responses $\boldsymbol{h}^{(j,m)}$. We have

$$\begin{aligned} \boldsymbol{h}^{(j,m)} &\stackrel{\mathrm{def}}{=} \left[h_0^{(j,m)},\ h_1^{(j,m)}, \ldots, h_L^{(j,m)}\right] \\ &\stackrel{\mathrm{def}}{=} \left[h^{(j)}(t_0 + m\Delta), h^{(j)}(t_0 + m\Delta + T), \right. \\ &\qquad \left. \ldots, h^{(j)}(t_0 + m\Delta + LT)\right], \end{aligned} \tag{13}$$

where the vector $\boldsymbol{h}^{(j,0)}$ is equal to the vector $\boldsymbol{h}^{(j)}$ in equation (4).

The filtering matrix in this case is constructed as

$$\boldsymbol{H}_{T_P} = \begin{bmatrix} \boldsymbol{H}_T^{(1,0)} \\ \boldsymbol{H}_T^{(1,1)} \\ \vdots \\ \boldsymbol{H}_T^{(1,P-1)} \\ \boldsymbol{H}_T^{(2,0)} \\ \vdots \\ \boldsymbol{H}_T^{(M_{\mathrm{R}},P-1)} \end{bmatrix}, \tag{14}$$

where the matrix $\boldsymbol{H}_T^{(j,m)}$ is a banded Toeplitz matrix associated to the discrete-time impulse responses $\boldsymbol{h}^{(j,m)}$ and constructed as in equation (5). Here, m changes from 0 to $P-1$ for each j.

We still can use a 3-way tensor $\boldsymbol{\mathcal{Y}}_p \in \mathbb{C}^{M_{\mathrm{R}}\cdot P\times W\times N}$ to model the oversampled received signals. Compared to the tensor $\boldsymbol{\mathcal{Y}}$, only the first dimension changes due to the oversampling. We describe the corresponding input output data model as

$$\boldsymbol{\mathcal{Y}}_P = \boldsymbol{\mathcal{H}}_P \times_3 \boldsymbol{S}^{\mathrm{T}} + \boldsymbol{\mathcal{N}}_P. \tag{15}$$

The filtering tensor $\boldsymbol{\mathcal{H}}_P$ has the dimension $M_{\mathrm{R}}\cdot P \times W \times (W+L)$ and is organized by stacking the slices of the block matrices defined in equation (14) along the first dimension of the tensor $\boldsymbol{\mathcal{H}}_P$ as shown in Figure 2. The noise tensor $\boldsymbol{\mathcal{N}}_P$ has the same size as the tensor $\boldsymbol{\mathcal{Y}}_P$. Notice that the noise samples are temporally correlated due to the oversampling.

By computing the truncated HOSVD of the measurement tensor $\boldsymbol{\mathcal{Y}}_P$, the signal subspace tensor $\hat{\boldsymbol{\mathcal{U}}}_P^{[s]} \in \mathbb{C}^{M_{\mathrm{R}}\cdot P\times W\times r_{3_P}}$ can be estimated as

$$\hat{\boldsymbol{\mathcal{U}}}_P^{[s]} = \boldsymbol{\mathcal{S}}_P^{[s]} \times_1 \boldsymbol{U}_{1_P}^{[s]} \times_2 \boldsymbol{U}_{2_P}^{[s]}, \tag{16}$$

where $\boldsymbol{\mathcal{S}}_P^{[s]} \in \mathbb{C}^{r_{1_P}\times r_{2_P}\times r_{3_P}}$, $\boldsymbol{U}_1^{[s]} \in \mathbb{C}^{M_{\mathrm{R}}\cdot P\times r_{1_P}}$ and $\boldsymbol{U}_2^{[s]} \in \mathbb{C}^{W\times r_{2_P}}$. The terms r_{n_P} $(n=1,2,3)$ denote the n-rank of the noiseless tensor $\widetilde{\boldsymbol{\mathcal{Y}}}_P$ (i.e., $\widetilde{\boldsymbol{\mathcal{Y}}}_P = \boldsymbol{\mathcal{H}}_P \times_3 \boldsymbol{S}^{\mathrm{T}}$). It is found that $r_{1_P} = \min(M_{\mathrm{R}}\cdot P,\ L+1)$. The parameter r_{2_P} is equal to W and r_{3_P} is the same as $W+L$. In this case, the condition for achieving more accurate signal subspace estimate of the tensor case is loosened to $M_{\mathrm{R}}\cdot P > L+1$.

VI. Simulation Results

In this section, we demonstrate the performance improvement of the tensor-based subspace method for the blind estimation of SIMO channels. The comparisons between the matrix-based subspace method and the tensor-based subspace method are shown in terms of the root mean square error (RMSE) of the estimated normalized channels. This RMSE is defined as

$$\mathrm{RMSE} = \frac{1}{P}\sqrt{\mathrm{E}\left\{\left\|\hat{\boldsymbol{H}}a - \boldsymbol{H}\right\|_F^2\right\}}. \tag{17}$$

Here, $a = \frac{\mathrm{vec}(\hat{\boldsymbol{H}})^{\mathrm{H}}\mathrm{vec}(\boldsymbol{H})}{\left|\mathrm{vec}(\hat{\boldsymbol{H}})^{\mathrm{H}}\mathrm{vec}(\boldsymbol{H})\right|}$ is a scalar factor due to the fact that the unknown SIMO channels are only estimated up to a multiplication by a scalar. The channel matrix $\boldsymbol{H}$ is normalized to unit Frobenius norm. The RMSE is averaged over 500 channel realizations.

Monte Carlo simulations have been conducted where the successive symbols are generated statistically independent and emitted in 4-QAM format. The signal to noise ratio (SNR) is defined as $10\log_{10}\frac{\mathrm{E}\{\|\boldsymbol{s}(k)\|_2^2\}}{\mathrm{E}\{\|\boldsymbol{n}(k)\|_2^2\}}$. To simulate a multipath environment, we adopt a commonly used model [1] to construct an $(L+1)$-ray multipath continuous-time channel $h^{(j)}(t)$ between the jth receive antenna and the transmit antenna using a raised cosine pulse shaping filter $g_c(t-\gamma_\ell,\beta)$. We have

$$h^{(j)}(t) = \sum_{\ell=0}^{L} \alpha_\ell^{(j)} g_c(t-\gamma_\ell^{(j)},\beta), \tag{18}$$

where the roll-off factor β is set to 0.5 in the simulations and $\alpha_\ell^{(j)}$ are zero mean, i.i.d., unit variance complex Gaussian random variables. The term $\gamma_\ell^{(j)}$ indicates the delay of the ℓth path. The discrete-time channel is obtained by sampling $h^{(j)}(t)$ at a rate of T/P. We observe the measurement data with a smoothing window. The length of the observed data window is set to $W=10$.

A. Smoothing Window with $\eta = W$

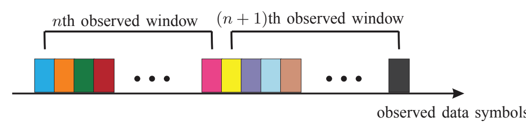
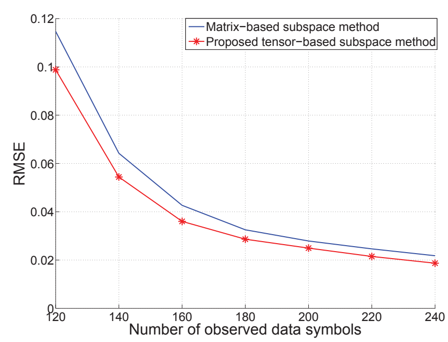
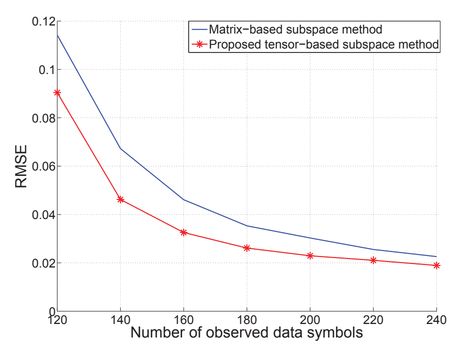


Fig. 3. The smoothing window with the smoothing paramter $\eta = W$.

The smoothing parameter is denoted by η which denotes the number of the new measurements in the next observed data window. First, we consider the case $\eta = W$, where the adjacent observed data windows do not overlap with each other as shown in Figure 3. Under this condition, Figures 4(a) and 4(b) show the comparions between the proposed tensor method and the matrix-based subspace method. Under the condition $M_{\mathrm{R}} > L+1$, an enhanced estimate has been achieved by the proposed tensor-based subspace method, especially for a small number of observed data symbols. Much larger differences between M_{R} and $L+1$ lead to a more significant improvement.



(a) for $M_{\mathrm{R}} = 4$ and $r_1 = L+1 = 3$



(b) for $M_{\mathrm{R}} = 6$ and $r_1 = L+1 = 3$

Fig. 4. RMSE for blind estimation of SIMO channels at SNR = 20 dB

Figure 5 shows the case $M_{\mathrm{R}} < L+1$, where the both methods achieve the same performance. In order to maintain the benefit of the tensor method, we use oversampling at the receiver. For a fair comparison, the oversampling is utilized for both the matrix-based method and the proposed tensor-based method. There is a significant performance improvement achieved by the tensor-based method, since the condition for achieving an improved estimate is loosened to $M_{\mathrm{R}} \cdot P > L+1$. Notice that the additive noise is not necessarily uncorrelated due to the oversampling.

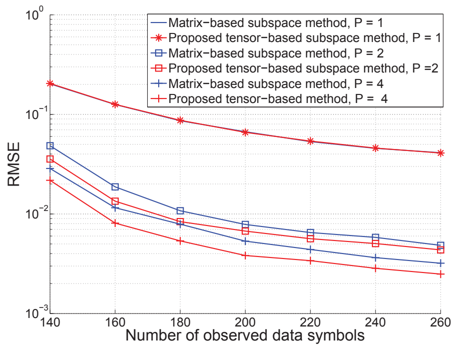


Fig. 5. RMSE for blind estimation of SIMO channels for $M_{\mathrm{R}} = 4$, $L+1 = 5$ at SNR $= 20$dB.

B. Smoothing Window with $1 \leq \eta < W$

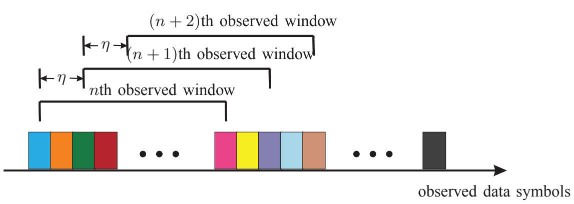


Fig. 6. The smoothing window with the smoothing paramter $1 \leq \eta < W$.

Next, we investigate the overlap of the observed data windows as shown in Figure 6. We evaluate the RMSE performance of the matrix-based subspace method and the proposed tensor-based method with different smoothing parameters η. Due to the assumption $N \geq W + L$, the cases with various η have different requirements for the minimum number of observed data symbols. It is observed in Figure 7 that the accuracy of the estimate improves with the decrease of the parameter η. But notice that, with the same number of observed data symbols, the smaller parameter η leads to a larger number of observed data windows which results in an increased computation time. The proposed tensor-based method always outperforms the matrix-based subspace method for any η.

VII. Conclusions

In this paper, we propose a new tensor-based subspace method for the blind estimation of SIMO channels. Compared to the matrix-based subspace methods, the proposed method models the measurement data via a 3-way tensor which allows us to exploit the structure inherent in the data. The HOSVD is employed for the tensor method to obtain the estimate of the signal subspace. When the measurement tensor has a low rank in the first mode (i.e., $M_{\mathrm{R}} > r_1 = L+1$), the proposed tensor-based method can achieve an improved estimate compared to the matrix-based subspace method.

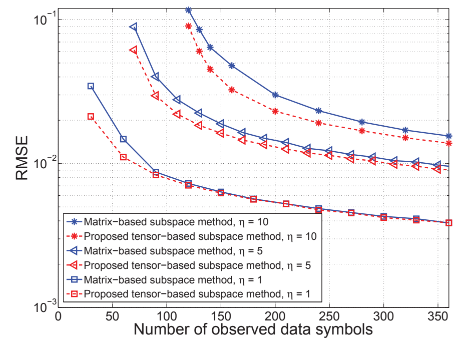


Fig. 7. RMSE for blind estimation of SIMO channels with varied smoothing parameter η. $M_{\mathrm{R}} = 6$, $r_1 = L+1 = 3$, and SNR $= 20$dB.

Once the measurement tensor has a full rank in the first mode (i.e., $L+1 \geq r_1 = M_{\mathrm{R}}$), the proposed tensor method has exactly the same performance as the matrix-based subspace method. However, we can ensure the performance improvement of the proposed tensor method for the full rank case by introducing oversampling at the receiver. Then, the condition for the improved estimate is loosened to $M_{\mathrm{R}} \cdot P > r_{1_P} = L+1$, where P indicates the oversampling factor.

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References

[1] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *25th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, 1991, pp. 856–860.

[2] L. Tong, G. Xu, B. Hassibi, and T. Kailath, "Blind channel identification based on second-order statistics: A frequency-domain approach," *IEEE Trans. Inform. Theory*, vol. 41, pp. 329–333, 1995.

[3] Y. Li and Z. Ding, "Blind channel identification based on second order cyclostationary statistics," in *Int. Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Minneapolis, Minnesota, 1993, pp. 81–84.

[4] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. on Signal Processing*, vol. 43(2), pp. 516–525, 1995.

[5] B. Chen and P. Petropulu, "Frequency domain blind MIMO system identification based on second and high order statistics," *IEEE Trans. on Signal Processing*, vol. 49, pp. 1677–1688, 2001.

[6] N. Nefedov, "On subspace channel estimation in multipath SIMO and MIMO channels," in *11th International Conference on Telecommunications (ICT)*, Fortaleza, Brazil, 2004, pp. 532–540.

[7] S. Zhou, B. Muquet, and G. Giannakis, "Subspace-based (semi)blind channel estimation for block precoded space-time OFDM," *IEEE Trans. on Signal Processing*, vol. 50, pp. 1215–1228, 2002.

[8] T. G. Kolda and B. W. Bader, "Tensor decompositions and applications," *SIAM Review*, vol. 51, no. 3, Sep 2009.

[9] M. Haardt, F. Roemer, and G. Del Galdo, "Higher-order SVD based subspace estimation to improve the parameter estimation accuracy in multi-dimensional harmonic retrieval problems," *IEEE Trans. on Signal Processing*, vol. 56, pp. 3198–3213, 2008.

[10] F. Roemer, H. Becker, M. Haardt, and M. Weis, "Analytical performance evaluation for HOSVD-based parameter estimation schemes," in *IEEE Int. Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP 2009)*, Aruba, Dutch Antilles, 2009, pp. 77–80.

[11] L. de Lathauwer, B. de Moor, and J. Vanderwalle, "A multilinear singular value decomposition," *SIAM J. Matrix Anal. Appl.*, vol. 21, pp. 1253–1278, 2000.