DISTRIBUTED BEAMFORMING FOR TWO-WAY RELAYING NETWORKS WITH INDIVIDUAL POWER CONSTRAINTS

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ABSTRACT

In this paper we study the sum rate maximization problem in a multi-pair two-way relaying network with multiple single antenna amplify-and-forward relays where each relay has its own transmit power constraint. The optimization problem is non-convex and in general NP-hard. First, we propose a monotonic optimization based algorithm. Due to its high computational complexity, this algorithm can only be used as a benchmark. Afterwards, inspired by the polynomial time difference of convex functions (POTDC) method, we develop a sub-optimal solution which has lower complexity but comparable performance. To further reduce the computational complexity, we propose two other algorithms, i.e., the total SINR eigen-beamformer and an interference neutralization based design which are the low SNR and high SNR approximations of the original optimization problem, respectively. Simulation results show that all the proposed suboptimal methods only suffer small losses compared to the global optimal solution especially when there is a sufficient number of relays in the network.

Index Terms— Two-way relaying, amplify and forward, sum rate maximization, monotonic optimization.

1. INTRODUCTION

Relay networks are important for future mobile networks since they can improve the network performance by extending the coverage and increasing the network capacity. In contrast to one-way relaying, two-way relaying techniques can compensate the spectral efficiency loss due to the half-duplex constraint of the relay and, therefore, use the radio resources in a more efficient manner. Multi-pair two-way amplify-and-forward (AF) relay networks have been investigated in [1], [2], and [3]. Reference [1] deals with the adaptive power allocation problem while assuming only orthogonal transmission in the network, i.e., no inter-pair interference is created during the data transmission. References [2] and [3] propose beamforming techniques for networks with inter-pair interference. Moreover, the optimum beamforming design for maximizing the sum rate of this system is developed in [3]. However, a sum power constraint is assumed in [3]. Thereby, this motivates us to extend it to the case where each relay has its own transmit power constraint because this case has not been dealt with prior to our work.

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In this paper, we consider the problem of maximizing the sum rate of a multi-pair two-way AF relaying network where each relay has its individual power constraint. This optimization problem is non-convex and in general NP-hard. Since the maximization problem fulfills the monotonic optimization framework, we first propose an (ϵ, η)-global optimal solution which is obtained using the polyblock approximation algorithm. However, due to its high computational complexity, this algorithm is only suitable for benchmarking. Afterwards, inspired by the polynomial time difference of convex functions (POTDC) method in [4], we develop a sub-optimal algorithm which converges faster than the polyblock algorithm and has a comparable performance. To further reduce the computational complexity, we propose the total SINR eigen-beamformer and the interference neutralization based design which are the low SNR and the high SNR approximation of the original optimization problem, respectively. Simulation results have demonstrated that especially when there are enough relays in the network all the proposed suboptimal algorithms have close to optimum performance. Moreover, the interference neutralization based design yields the lowest computational complexity.

Notation: Uppercase and lower case bold letters denote matrices and vectors, respectively. The expectation operator, trace of a matrix, transpose, conjugate, and Hermitian transpose are denoted by $\mathbb{E}\{\cdot\}$, Tr$\{\cdot\}$, $\{\cdot\}^{\mathrm{T}}$, $\{\cdot\}^*$, and $\{\cdot\}^{\mathrm{H}}$, respectively. The $m \times m$ identity matrix is $\boldsymbol{I}_m$. The Euclidean norm of a vector is denoted by $\|\cdot\|$ and $\succeq$ is the generalized inequality. The Hadamard (element-wise) product is denoted by $\odot$ and $\mathrm{diag}\{\boldsymbol{v}\}$ creates a diagonal matrix by aligning the elements of the vector $\boldsymbol{v}$ onto its diagonal entries. The natural logarithmic function is denoted as $\log(\cdot)$.

2. SYSTEM MODEL

The scenario under investigation is shown in Fig. 1. K pairs of single-antenna users would like to communicate with each other via the help of N single-antenna relays. We assume perfect synchronization and the channel is frequency flat and quasi-static block fading. The channel vector from the $(2k-1)$th user (on the left-hand side of Fig. 1) to the relays is denoted as $\boldsymbol{f}_{2k-1} = [f_{2k-1,1}, f_{2k-1,2}, \ldots, f_{2k-1,N}]^{\mathrm{T}} \in \mathbb{C}^N$, while the channel from the $2k$th user (on the right-hand side of Fig. 1) to the relay is denoted as $\boldsymbol{g}_{2k} = [g_{2k,1}, g_{2k,2}, \ldots, g_{2k,N}]^{\mathrm{T}} \in \mathbb{C}^N$, for $k \in \{1, 2, \cdots, K\}$. For notational simplicity, we assume an ideal time-division duplex (TDD) system, i.e., the channels are *reciprocal*. The transmission takes two time slots. In the first time slot, the

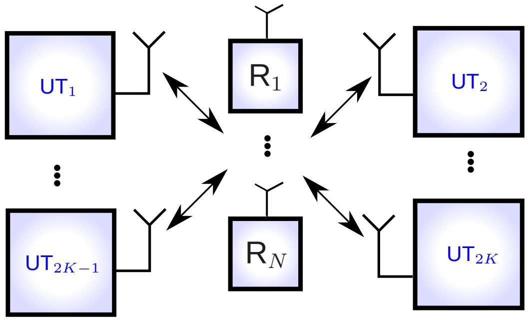

Fig. 1. Multi-pair two-way relaying with multiple single-antenna amplify and forward relays.

signal received at all relays can be combined in a vector as

$$\boldsymbol{r} = \sum_{k=1}^{K} (\boldsymbol{f}_{2k-1} s_{2k-1} + \boldsymbol{g}_{2k} s_{2k}) + \boldsymbol{n}_{\mathrm{R}} \in \mathbb{C}^N \tag{1}$$

where s_{2k-1} and s_{2k} are i.i.d. symbols with zero mean and unit power. The vector $\boldsymbol{n}_{\mathrm{R}}$ contains the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise and $\mathbb{E}\{\boldsymbol{n}_{\mathrm{R}}\boldsymbol{n}_{\mathrm{R}}^{\mathrm{H}}\} = \sigma_{\mathrm{r}}^2 \boldsymbol{I}_N$.

Afterwards, the AF relays broadcast the weighted signal as

$$\bar{\boldsymbol{r}} = \boldsymbol{W} \cdot \boldsymbol{r} \tag{2}$$

where $\boldsymbol{W} = \mathrm{diag}\{\boldsymbol{w}^*\}$ and $\boldsymbol{w} = [w_1, w_2, \ldots, w_N]^{\mathrm{T}}$ is the vector which consists of the N complex weights of all the relays.

In the second time slot, the received signal at the $(2k-1)$th user (on the left-hand side of Fig. 1) is expressed as [2]

$$\begin{aligned} y_{2k-1} &= \underbrace{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1} \boldsymbol{g}_{2k} s_{2k}}_{\text{desired signal}} + \underbrace{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1} \boldsymbol{f}_{2k-1} s_{2k-1}}_{\text{self-interfernce}} \\ &+ \underbrace{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1} \sum_{\substack{\ell \neq k \\ \ell=1}}^{K} (\boldsymbol{f}_{2\ell-1} s_{2\ell-1} + \boldsymbol{g}_{2\ell} s_{2\ell})}_{\text{inter-pair interference}} \\ &+ \underbrace{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1} \boldsymbol{n}_{\mathrm{R}} + n_{2k-1}}_{\text{effective noise}} \end{aligned} \tag{3}$$

where $\boldsymbol{F}_{2k-1} = \mathrm{diag}\{\boldsymbol{f}_{2k-1}\}$ and n_{2k-1} is the ZMCSCG noise with variance $\sigma_{\mathrm{u}}^2, \forall k$. Similar expressions can be obtained for the $2k$th user.

We assume that perfect channel knowledge can be obtained such that the self-interference terms can be canceled and perfect synchronization is available. Let $P_{\mathrm{R},i}$ be the transmit power constraint of the ith relay in the network. Our goal is to find the optimal weighting vector $\boldsymbol{w}_{\mathrm{opt}}$ such that the sum rate of the system is maximized subject to these individual power constraint.

3. SUM RATE MAXIMIZATION

The optimization problem can be formulated as

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \frac{1}{2} \sum_{m=1}^{2K} \log_2(1 + \mathrm{SINR}_m) \\ \text{subject to} \quad & \mathbb{E}\{\|\bar{r}_i\|^2\} \leq P_{\mathrm{R},i}, \forall i \in \{1, 2, \cdots, N\} \end{aligned} \tag{4}$$

where the factor $1/2$ is due to the two channel uses (half duplex). When $m = 2k-1$, from the expression (3), the SINR of the mth UT is described as [3]

$$\mathrm{SINR}_{2k-1} = \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{B}_{2k-1} \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} (\boldsymbol{D}_{2k-1} + \boldsymbol{E}_{2k-1}) \boldsymbol{w} + \sigma_{2k-1}^2} \tag{5}$$

where $\boldsymbol{D}_{2k-1} = \sum_{\substack{\ell \neq k \\ \ell=1}} (\tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{o})} \tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{o})\mathrm{H}} + \tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{e})} \tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{e})\mathrm{H}})$ and $\boldsymbol{B}_{2k-1} = \boldsymbol{h}_{2k-1} \boldsymbol{h}_{2k-1}^{\mathrm{H}}$ are $N \times N$ positive semidefinite Hermitian matrices. The matrices $\boldsymbol{D}_{2k-1}$ and $\boldsymbol{B}_{2k-1}$ are related to the interference power and the desired signal power, respectively, ($\boldsymbol{h}_{2k-1} = \boldsymbol{f}_{2k-1} \odot \boldsymbol{g}_{2k}$, $\tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{o})} = \boldsymbol{f}_{2k-1} \odot \boldsymbol{f}_{2\ell-1}$ and $\tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{e})} = \boldsymbol{f}_{2k-1} \odot \boldsymbol{g}_{2\ell}$). The term which is related to the forwarded noise from the relay is denoted by an $N \times N$ full rank diagonal matrix $\boldsymbol{E}_{2k-1} = \sigma_{\mathrm{R}}^2 \boldsymbol{F}_{2k-1} \boldsymbol{F}_{2k-1}^{\mathrm{H}}$. Similar SINR expressions can be obtained when $m = 2k$. Furthermore, the ith relay's transmit power is given by $\mathbb{E}\{\|\bar{r}_i\|^2\} = \boldsymbol{w}^{\mathrm{H}} \boldsymbol{\Upsilon}_i \boldsymbol{w}$ with $\boldsymbol{\Upsilon}_i = \Gamma_{i,i} \boldsymbol{e}_i \boldsymbol{e}_i^{\mathrm{H}}$. The vector $\boldsymbol{e}_i$ is the ith column of an identity matrix. The scalar $\Gamma_{i,i}$ is the (i,i)th element of the following diagonal matrix

$$\boldsymbol{\Gamma} = \sum_{k=1}^{K} (\boldsymbol{F}_{2k-1} \boldsymbol{F}_{2k-1}^{\mathrm{H}} + \boldsymbol{G}_{2k} \boldsymbol{G}_{2k}^{\mathrm{H}}) + \sigma_{\mathrm{R}}^2 \boldsymbol{I}_N. \tag{6}$$

Given the above definitions, problem (4) can be rewritten as

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \prod_{m=1}^{2K} \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2} \\ \text{subject to} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{\Upsilon}_i \boldsymbol{w} \leq P_{\mathrm{R},i}, \forall i \end{aligned} \tag{7}$$

or equivalently

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \sum_{m=1}^{2K} \left(\log(\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2) - \log(\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2) \right) \\ \text{subject to} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{\Upsilon}_i \boldsymbol{w} \leq P_{\mathrm{R},i}, \forall i \end{aligned} \tag{8}$$

where $\boldsymbol{C}_m = \boldsymbol{D}_m + \boldsymbol{E}_m$ and $\boldsymbol{A}_m = \boldsymbol{B}_m + \boldsymbol{C}_m$ are positive definite. Note that for simplicity the scalar $\frac{1}{2}$ is dropped and the natural logarithm is used instead. The formulations (7) or (8) are still nonconvex.

3.1. Generalized Polyblock Algorithm

In [3] we have proven that the sum rate maximization problem in such a relay network with a total power constraint satisfies the monotonic optimization framework. Similarly, problem (7) is also a monotonic optimization problem which can be solved using a unified algorithm, which is called the polyblock approximation approach [5]. In the following we prove that the problem (7) is a monotonic optimization problem and then adapt the polyblock algorithm to solve it.

Problem (7) is equivalent to the following problem

$$\max_{\boldsymbol{y}} \{\Phi(\boldsymbol{y}) | \boldsymbol{y} \in \mathbb{D}\} \tag{9}$$

where $\Phi(\boldsymbol{y}) = \prod_{m=1}^{2K} y_m$ and $\mathbb{D} = \mathbb{G} \cap \mathbb{L}$. The sets $\mathbb{G} = \{\boldsymbol{y} \in \mathbb{R}_+^{2K} | y_m \leq \max_{\boldsymbol{w}} \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2}\}$ and $\mathbb{L} = \{\boldsymbol{y} \in \mathbb{R}_+^{2K} | y_m \geq \min_{\boldsymbol{w}} \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2}\}$ are a normal set and a reverse normal set, respectively [5]. The domain of $\boldsymbol{w}$ is defined as $\{\boldsymbol{w} \in \mathbb{C}^N | \boldsymbol{w}^{\mathrm{H}} \boldsymbol{\Upsilon}_i \boldsymbol{w} \leq$

$P_{\mathrm{R},i}, \forall i\}$. Moreover, the function $\Phi(\boldsymbol{y})$ is an increasing function since $\Phi(\bar{\boldsymbol{y}}) \geq \Phi(\tilde{\boldsymbol{y}})$, $\forall \bar{\boldsymbol{y}} \succeq \tilde{\boldsymbol{y}}$. Thereby, problem (7) is the maximization of an increasing function over an intersection of normal and reverse normal sets. As shown in [5], such a formulation is a monotonic optimization problem. The definitions of the increasing function, the normal set, and the reverse normal set are the same as in [5].

Now let us briefly review the polyblock algorithm in [3]. A polyblock $\mathbb{P}$ with vertex set $\mathbb{T} \subset \mathbb{R}_+^{2K}$ is defined as the finite union of all the boxes $[\mathbf{0}, \boldsymbol{z}]$, $\boldsymbol{z} \in \mathbb{T}$. It is dominated by its proper vertices. A vertex $\boldsymbol{z}$ is proper if there is no $\bar{\boldsymbol{z}} \neq \boldsymbol{z}$ and $\bar{\boldsymbol{z}} \succeq \boldsymbol{z}$ for $\bar{\boldsymbol{z}} \in \mathbb{T}$. The global maximum of a monotonic optimization problem, if it exists, is obtained on $\partial^+\mathbb{D}$, i.e., the upper boundary of $\mathbb{D}$. The main idea of the polyblock approximation algorithm for solving (9) is to approximate $\partial^+\mathbb{D}$ by polyblocks, i.e., to construct a nested sequence of polyblocks which approximate $\mathbb{D}$ from above, that is,

$$\mathbb{P}_1 \supset \mathbb{P}_2 \supset \cdots \supset \mathbb{D} \text{ s.t. } \max_{\boldsymbol{y} \in \mathbb{P}_k} \Phi(\boldsymbol{y}) \rightarrow \max_{\boldsymbol{y} \in \mathbb{D}} \Phi(\boldsymbol{y}) \qquad (10)$$

when $k \rightarrow \infty$ and $\boldsymbol{y}_k \succeq \boldsymbol{y}_\ell$ for all $\ell \geq k$ [5].

Following the same procedure as in [3], the (ϵ, η)-optimal solution of problem (7) is obtained. Note that the major difference between the problem in [3] and our problem is the calculation of $\alpha_k \in (0, 1]$ at the kth step. The scalar α_k determines the unique intersection between the ray through $\mathbf{0}$ and $\bar{\boldsymbol{y}}_k$ and the upper boundary $\partial^+\mathbb{D}$ where $\bar{\boldsymbol{y}}_k$ is the vertex in $\mathbb{T}_k$ which maximizes the function $\Phi(\boldsymbol{y})$. Instead of solving an unconstrained max-min problem as in [3], we need to solve the following constrained problem

$$\begin{aligned} \alpha_k = \max_{\boldsymbol{w}} \min_{m} \quad & \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2}{\bar{y}_{k,m} \boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w} + \sigma_{\mathrm{u}}^2} \\ \text{subject to} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{\Upsilon}_i \boldsymbol{w} \leq P_{\mathrm{R},i}, \forall i \end{aligned} \qquad (11)$$

Similar as in [3], problem (11) is solved using semidefinite relaxation together with the bisection search (the concept of this method is elaborated in Section 3.3).

3.2. POTDC Inspired Approach

As illustrated in [3], the computational complexity of the polyblock algorithm can be non-polynomial time in the worst case. Thus, it is worth to look for a polynomial time solution. In this section, we develop a suboptimal but polynomial time solution.

Let us first define $\boldsymbol{X} = \boldsymbol{w}\boldsymbol{w}^{\mathrm{H}}$. Then problem (8) can be reformulated as

$$\begin{aligned} \min_{\boldsymbol{X}, \alpha_m, \beta_m, \forall m} \quad & -\sum_{m=1}^{2K} \log(\alpha_m) - \left(-\sum_{m=1}^{2K} \log(\beta_m) \right) \\ \text{subject to} \quad & \mathrm{Tr}\{\boldsymbol{\Upsilon}_i \boldsymbol{X}\} \leq P_{\mathrm{R},i}, \forall i, \\ & \mathrm{Tr}\{\boldsymbol{A}_m \boldsymbol{X}\} + \sigma_{\mathrm{u}}^2 = \alpha_m, \\ & \mathrm{Tr}\{\boldsymbol{C}_m \boldsymbol{X}\} + \sigma_{\mathrm{u}}^2 = \beta_m, \forall m, \\ & \boldsymbol{X} \succeq 0, \quad \mathrm{rank}\{\boldsymbol{X}\} = 1. \end{aligned} \qquad (12)$$

If we further drop the non-convex rank-1 constraint, such a method is called semidefinite relaxation (SDR) [6].

The objective function of problem (12) is the difference of convex functions (D.C.) and therefore is non-convex and in general NP-hard. Inspired by the POTDC algorithm in [4], we replace the concave part of the objective function in (12) by its linear approximation, i.e., $\log(\beta_m)$ is replaced by its first order Taylor polynomial $\log(\beta_{m,0}) + \frac{\beta_m - \beta_{m,0}}{\beta_{m,0}}, \forall m$. After the substitutions, the cost function in (12) becomes convex. Finally, we obtain the following problem:

$$\begin{aligned} \min_{\boldsymbol{X}, \alpha_m, \beta_m, t_m \forall m} \quad & -\sum_{m=1}^{2K} \log(\alpha_m) + \sum_{m=1}^{2K} t_m \\ \text{subject to} \quad & \mathrm{Tr}\{\boldsymbol{\Upsilon}_i \boldsymbol{X}\} \leq P_{\mathrm{R},i}, \forall i, \boldsymbol{X} \succeq 0 \\ & \mathrm{Tr}\{\boldsymbol{A}_m \boldsymbol{X}\} + \sigma_{\mathrm{u}}^2 = \alpha_m, \\ & \mathrm{Tr}\{\boldsymbol{C}_m \boldsymbol{X}\} + \sigma_{\mathrm{u}}^2 = \beta_m \\ & \log(\beta_{m,0}) + \frac{1}{\beta_{m,0}} (\beta_m - \beta_{m,0}) \leq t_m. \end{aligned} \qquad (13)$$

Problem (13) is a convex semidefinite programming (SDP) problem and can be solved using the standard interior-point algorithm [7].

Clearly, the first order Taylor polynomial approximation in problem (13) is the true Taylor expansion of $\log(\beta_m)$ in (12) only if $\beta_{m,0}$ equals to the optimal $\beta_{m,\mathrm{opt}}$. Thus, similarly as in [4], we propose an iterative algorithm as described in Table 1 for obtaining the optimal $\boldsymbol{X}_{\mathrm{opt}}$ of problem (13). The proposed algorithm has preserved the convergence properties from the original POTDC. That is, the optimal values obtained over the iterations are non-decreasing. Furthermore, the proposed algorithm provides a polynomial-time solution since it solves a sequence of convex problems.

Table 1. Iterative algorithm for obtaining $\boldsymbol{X}_{\mathrm{opt}}$ inspired by POTDC

Initialization step: set initial values $\beta_{m,0}, \forall m$
maximum iteration number $N_{\max}$ and the tolerance factor ϵ.

Main step:
for $p = 1$ to $N_{\max}$ **do**
  Solve (13) finding optimal value $f^{\star(p)}$ and $\beta_m^{(p)}$.
  $\beta_{m,0}^{(p+1)} = \beta_m^{(p)}, m = 1, \cdots, 2K$
  if $\left| f^{\star(p)} - f^{\star(p-1)} \right| \leq \epsilon$ **then**
    break
  end if
end for

In the end, to obtain $\boldsymbol{w}_{\mathrm{opt}}$ we need to extract a rank-1 solution from $\boldsymbol{X}_{\mathrm{opt}}$. In our work, the randomization technique described in [6] is applied.

3.3. Total SINR Eigen-Beamformer

Although the POTDC inspired algorithm has a comparable performance and guaranteed polynomial time solution compared to the polyblock algorithm, it requires iterations and therefore is still computationally inefficient. To further reduce the computational complexity, we propose a low SNR approximation of problem (4), i.e., the total SINR eigen-beamformer (denoted as ToT in the simulation results). As stated in [3], the total SINR eigen-beamformer aims at maximizing the ratio between the sum of the received signal powers of all the UTs and the sum of interference plus noise power of all the UTs. This beamformer design can be applied to our problem but a closed-form solution as in [3] cannot be obtained due to the individual relay power constraints. In the following we apply the concept of the total SINR eigen-beamformer and develop the solution to it.

Let us define $\boldsymbol{S}_{\mathrm{tot}} = \sum_{m=1}^{2K} \boldsymbol{B}_m$ and $\boldsymbol{U}_{\mathrm{tot}} = \sum_{m=1}^{2K} \boldsymbol{C}_m$. Thus, $\boldsymbol{w}^{\mathrm{H}} \boldsymbol{S}_{\mathrm{tot}} \boldsymbol{w}$ and $\boldsymbol{w}^{\mathrm{H}} \boldsymbol{U}_{\mathrm{tot}} \boldsymbol{w}$ are the sum of the signal power and the sum of the interference power plus the forwarded noise power from all the relays, respectively. Then our proposed total SINR eigen-beamformer solves the following problem

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \frac{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{S}_{\mathrm{tot}}\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{U}_{\mathrm{tot}}\boldsymbol{w}+2K\sigma_{\mathrm{u}}^{2}} \\ \text{subject to} \quad & \boldsymbol{w}^{\mathrm{H}}\boldsymbol{\Upsilon}_i\boldsymbol{w} \le P_{\mathrm{R},i}, \forall i \end{aligned} \tag{14}$$

Although problem (14) is in general non-convex and NP-hard, it is well studied in the literature, e.g., [8], [9]. In our work, we use the SDR together with a bisection search which is similar to [8]. In the following we briefly introduce this algorithm. Applying the SDR method, problem (14) is reformulated as

$$\begin{aligned} \max_{\boldsymbol{X},t} \quad & -t \\ \text{subject to} \quad & \mathrm{Tr}\{\boldsymbol{\Upsilon}_i\boldsymbol{X}\} \le P_{\mathrm{R},i}, \forall i, \boldsymbol{X}\succeq 0 \\ & \mathrm{Tr}\{(t\boldsymbol{U}_{\mathrm{tot}}-\boldsymbol{S}_{\mathrm{tot}})\boldsymbol{X}\} \le -2Kt\sigma_{\mathrm{u}}^2, \forall i \end{aligned} \tag{15}$$

For a fixed t, problem (15) is a feasibility check problem. Thereby, the optimal $\boldsymbol{X}_{\mathrm{opt}}$ can be obtained via a bisection search over an interval $[t_{\min}, t_{\max}]$. In our case, we select $t_{\min}=0$ and $t_{\max}=\mathcal{P}((\boldsymbol{U}_{\mathrm{tot}}+2K\sigma_{\mathrm{u}}^2\boldsymbol{\Gamma}/(\sum_i^N P_{\mathrm{R},i}))^{-1}\boldsymbol{S}_{\mathrm{tot}})$ where $\mathcal{P}(\cdot)$ is the dominant eigenvalue of a square matrix. After obtaining $\boldsymbol{X}_{\mathrm{opt}}$, the optimal beamforming vector $\boldsymbol{w}_{\mathrm{opt}}$ is found using the randomization techniques described in [6].

Next we prove that problem (14) is the low SNR approximation of the original problem (4). Applying the the Taylor expansion of the logarithmic function $\log(1+x)$, we have $\forall m$

$$\log\left(1+\frac{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{B}_m\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}(\boldsymbol{D}_m+\boldsymbol{E}_m)\boldsymbol{w}+\sigma_{\mathrm{u}}^2}\right) \approx \frac{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{B}_m\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}(\boldsymbol{D}_m+\boldsymbol{E}_m)\boldsymbol{w}+\sigma_{\mathrm{u}}^2}.$$

Using the fact that in the low SNR regime ($\sigma_{\mathrm{r}}^2 \to +\infty$) we have $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{D}_m\boldsymbol{w} \ll \boldsymbol{w}^{\mathrm{H}}\boldsymbol{E}_m\boldsymbol{w} \approx \sigma_{\mathrm{r}}^2, \forall m$, we can rewrite the objective function in (4) as

$$\begin{aligned} & \frac{1}{2K}\sum_{m=1}^{2K}\log\left(1+\frac{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{B}_m\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}(\boldsymbol{D}_m+\boldsymbol{E}_m)\boldsymbol{w}+\sigma_{\mathrm{u}}^2}\right) \\ \approx & \frac{1}{2K}\sum_{m=1}^{2K}\frac{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{B}_m\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}(\boldsymbol{D}_m+\boldsymbol{E}_m)\boldsymbol{w}+\sigma_{\mathrm{u}}^2} \approx \frac{\sum_{m=1}^{2K}\boldsymbol{w}^{\mathrm{H}}\boldsymbol{B}_m\boldsymbol{w}}{2K(\sigma_{\mathrm{r}}^2+\sigma_{\mathrm{u}}^2)} \\ \approx & \frac{\sum_{m=1}^{2K}\boldsymbol{w}^{\mathrm{H}}\boldsymbol{B}_m\boldsymbol{w}}{\sum_{m=1}^{2K}(\boldsymbol{w}^{\mathrm{H}}(\boldsymbol{D}_m+\boldsymbol{E}_m)\boldsymbol{w}+\sigma_{\mathrm{u}}^2)} = \frac{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{S}_{\mathrm{tot}}\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{U}_{\mathrm{tot}}\boldsymbol{w}+2K\sigma_{\mathrm{u}}^2}. \end{aligned}$$

3.4. Interference Neutralization Based Design

In this section, we propose a high SNR approximation of the original problem (4). The proposed algorithm is based on the interference neutralization which is a technique that tunes the interfering signals such that they neutralize each other at the receiver [10]. Mathematically, interference neutralization for our scenario requires that

$$\begin{cases} (\boldsymbol{f}_{2k-1}\odot\boldsymbol{f}_{2\ell-1})^{\mathrm{H}}\boldsymbol{w}\cdot s_{2\ell-1}=0 & \forall \ell,k,\ell\neq k \\ (\boldsymbol{f}_{2k-1}\odot\boldsymbol{g}_{2\ell})^{\mathrm{H}}\boldsymbol{w}\cdot s_{2\ell}=0 & \forall \ell,k,\ell\neq k \\ (\boldsymbol{g}_{2k}\odot\boldsymbol{f}_{2\ell-1})^{\mathrm{H}}\boldsymbol{w}\cdot s_{2\ell-1}=0 & \forall \ell,k,\ell\neq k \\ (\boldsymbol{g}_{2k}\odot\boldsymbol{g}_{2\ell})^{\mathrm{H}}\boldsymbol{w}\cdot s_{2\ell}=0 & \forall \ell,k,\ell\neq k \end{cases} \tag{16}$$

or equivalently

$$\underbrace{\begin{bmatrix} (\boldsymbol{f}_1\odot\boldsymbol{f}_3)^{\mathrm{H}} \\ (\boldsymbol{f}_1\odot\boldsymbol{g}_4)^{\mathrm{H}} \\ (\boldsymbol{g}_2\odot\boldsymbol{f}_3)^{\mathrm{H}} \\ (\boldsymbol{g}_2\odot\boldsymbol{g}_4)^{\mathrm{H}} \\ \vdots \\ (\boldsymbol{f}_{2K-3}\odot\boldsymbol{f}_{2K-1})^{\mathrm{H}} \\ (\boldsymbol{f}_{2K-3}\odot\boldsymbol{g}_{2K})^{\mathrm{H}} \\ (\boldsymbol{g}_{2K-2}\odot\boldsymbol{f}_{2K-1})^{\mathrm{H}} \\ (\boldsymbol{g}_{2K-2}\odot\boldsymbol{g}_{2K})^{\mathrm{H}} \end{bmatrix}}_{\boldsymbol{H}^{(\mathrm{e})}} \cdot\boldsymbol{w}=\boldsymbol{0} \tag{17}$$

Utilizing the commutative property of the Hadamard product, duplicat rows in $\boldsymbol{H}^{(\mathrm{e})}$ are removed. The matrix $\boldsymbol{H}^{(\mathrm{e})}$ is found to have a dimension of $2K(K-1)\times N$. Equation (17) is solvable only if the null space of $\boldsymbol{H}^{(\mathrm{e})}$ is not empty, i.e., $N>2K(K-1)$. Define the SVD of $\boldsymbol{H}^{(\mathrm{e})}=\boldsymbol{U}\boldsymbol{\Sigma}[\boldsymbol{V}_{\mathrm{s}} \quad \boldsymbol{V}_{\mathrm{n}}]^{\mathrm{H}}$, where $\boldsymbol{V}_{\mathrm{n}}$ contains the last $(N-2K(K-1))$ right singular vectors and thus forms an orthonormal basis for the null subspace of $\boldsymbol{H}^{(\mathrm{e})}$. Without loss of generality, we define the interference neutralization based beamformer (denoted as IntNeu in the simulation results) as $\boldsymbol{w}=\boldsymbol{V}_{\mathrm{n}}\bar{\boldsymbol{w}}$, where $\bar{\boldsymbol{w}}\in\mathbb{C}^{N-2K^2+2K}$ has a smaller dimension than $\boldsymbol{w}\in\mathbb{C}^N$. In other words, searching over $\bar{\boldsymbol{w}}$ yields a lower computational complexity. Furthermore, observing that we have $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{E}_m\boldsymbol{w}\to\sigma_{\mathrm{r}}^2, \forall m$ also in the high SNR regime ($\sigma_{\mathrm{r}}^2\to 0$), the cost function in (8) is then reformulated as

$$\sum_{m=1}^{2K}\log(\bar{\boldsymbol{w}}^{\mathrm{H}}\boldsymbol{V}_{\mathrm{n}}^{\mathrm{H}}\boldsymbol{A}_m\boldsymbol{V}_{\mathrm{n}}\bar{\boldsymbol{w}}+\sigma_{\mathrm{u}}^2)-\sum_{m=1}^{2K}\log(\sigma_{\mathrm{r}}^2+\sigma_{\mathrm{u}}^2) \tag{18}$$

Replacing the cost function in (8) by (18) and dropping the constant terms, we obtain the following problem

$$\begin{aligned} \max_{\bar{\boldsymbol{w}}} \quad & \sum_{m=1}^{2K}\log(\bar{\boldsymbol{w}}^{\mathrm{H}}\bar{\boldsymbol{A}}_m\bar{\boldsymbol{w}}+\sigma_{\mathrm{u}}^2) \\ \text{subject to} \quad & \bar{\boldsymbol{w}}^{\mathrm{H}}\bar{\boldsymbol{\Upsilon}}_i\bar{\boldsymbol{w}} \le P_{\mathrm{R},i}, \forall i \end{aligned} \tag{19}$$

where $\bar{\boldsymbol{A}}_m=\boldsymbol{V}_{\mathrm{n}}^{\mathrm{H}}\boldsymbol{A}_m\boldsymbol{V}_{\mathrm{n}}, \forall m$ and $\bar{\boldsymbol{\Upsilon}}_i=\boldsymbol{V}_{\mathrm{n}}^{\mathrm{H}}\Gamma_{ii}\boldsymbol{e}_i\boldsymbol{e}_i^{\mathrm{H}}\boldsymbol{V}_{\mathrm{n}}$. Again applying the SDR, we have the following convex SDP problem

$$\begin{aligned} \min_{\bar{\boldsymbol{X}},\bar{\alpha}_m,\forall m} \quad & -\sum_{m=1}^{2K}\log(\bar{\alpha}_m) \\ \text{subject to} \quad & \mathrm{Tr}\{\bar{\boldsymbol{\Upsilon}}_i\bar{\boldsymbol{X}}\} \le P_{\mathrm{R},i}, \forall i, \boldsymbol{X}\succeq 0 \\ & \mathrm{Tr}\{\bar{\boldsymbol{A}}_m\bar{\boldsymbol{X}}\}+\sigma_{\mathrm{u}}^2=\bar{\alpha}_m. \end{aligned} \tag{20}$$

where $\bar{\boldsymbol{X}}=\bar{\boldsymbol{w}}\bar{\boldsymbol{w}}^{\mathrm{H}}$. After obtaining the optimal $\bar{\boldsymbol{X}}_{\mathrm{opt}}$, the rank-1 extraction of $\boldsymbol{V}_{\mathrm{n}}\bar{\boldsymbol{X}}\boldsymbol{V}_{\mathrm{n}}^{\mathrm{H}}$, which is computed using the randomization technique, yields the final $\boldsymbol{w}_{\mathrm{opt}}$.

4. SIMULATION RESULTS

In this section, the performance of the proposed algorithms is evaluated via Monte-Carlo simulations. The simulated flat fading channels are spatially uncorrelated Rayleigh fading channels. Each relay has an identical transmit power constraint of $1/N$. The noise variances at all nodes are the same, i.e., $\sigma_{\mathrm{r}}^2=\sigma_{\mathrm{u}}^2$ and the SNR is defined as $\mathrm{SNR}=1/\sigma_m^2$. There are $K=2$ pairs of users in the network. All the simulation results are obtained by averaging over 1000 channel realizations. "Polyblock", "POTDC", "ToT", and "IntNeu" denote the algorithms in Sections 3.1, 3.2, 3.3, and 3.4, respectively. For the polyblock algorithm, the POTDC algorithm, and the ToT algorithm, the stopping criterion is set to be a tolerance factor of 10^{-4}.

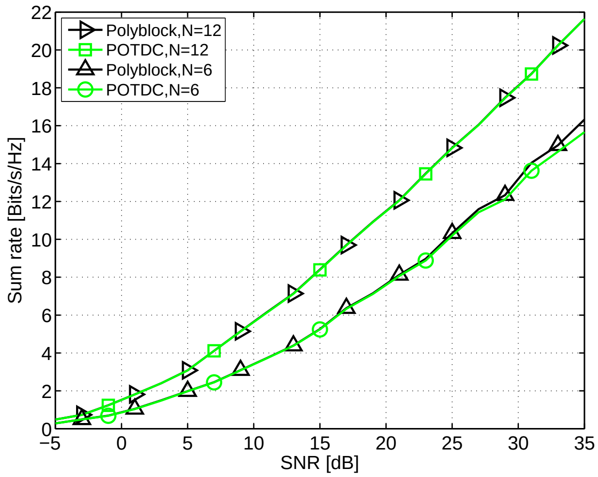

Fig. 2. Sum rate comparison of the polyblock algorithm and the POTDC algorithm.

Fig. 2 shows the comparison of different algorithms with $N = 6$ relays and $N = 12$ relays in the network. Clearly, the POTDC algorithm has close to optimal performance especially in the low SNR regime and when there is a sufficient number of relays in the network (e.g., $N = 12$). Thus, the POTDC algorithm can also be used as a benchmark for the other sub-optimal algorithms since it has a lower computational complexity but a comparable performance when compared to the global optimal solution.

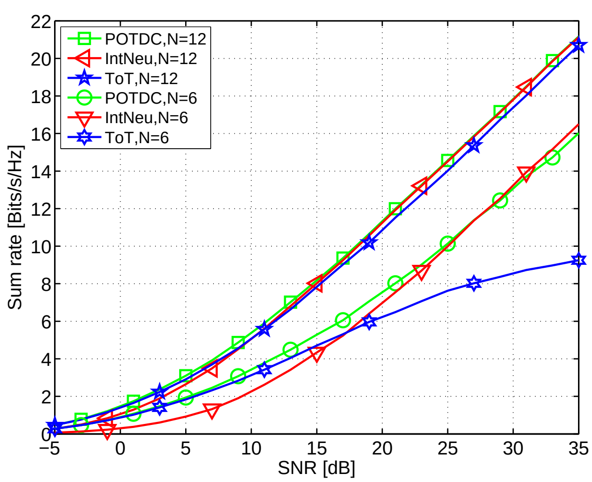

Fig. 3. Sum rate comparison of the POTDC algorithm and the total SINR eigen-beamfomer (low SNR approximation) and the interference neutralization based design (high SNR approximation).

Fig. 3 demonstrates the comparison of different suboptimal algorithms. As depicted in the figure, the total SINR eigen-beamformer (ToT) and the interference neutralization based design (IntNeu) show a low SNR performance and a high SNR performance of the global optimum solution, respectively. Moreover, when there are enough relays in the network, both the total SINR eigen-beamformer and the interference neutralization based design are very close to the optimum solution but have much lower computational complexity.

5. CONCLUSION

In this paper, we have investigated the sum rate maximization problem in two-way AF relaying networks where each relay has its own transmit power constraint. The optimization problem fits into the monotonic optimization framework and thus can be solved using the generalized polyblock approximation algorithm. Since the D.C. formulation of the original problem has a similar structure as in [4], we develop a generalized POTDC algorithm which is inspired by the POTDC algorithm in [4]. To further reduce the computational complexity, we propose the total SINR eigen-beamformer which maximizes the total SINR of the network. The total SINR eigen-beamformer is a low SNR approximation of the original problem and it only suffers a little loss compared to the polyblock algorithm and the generalized POTDC algorithm when there is a sufficient number of relays in the network. The same performance can be observed for the proposed interference neutralization based design which provides a high SNR approximation of the optimum solution. Moreover, the interference neutralization based design has the lowest computational complexity among all the proposed algorithms.

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