# FULL DUPLEX WIRELESS COMMUNICATIONS WITH PARTIAL INTERFERENCE CANCELLATION

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## ABSTRACT

In this paper we study a full duplex (FD) multiple-antenna point-to-point communication system with a limited RF self-interference cancellation capability. Thereby, transmitter side digital signal processing techniques can be developed to guarantee the reliability of the aforementioned FD system. For this purpose, we propose a common system model for a FD system with partial self-interference cancellation ability. Using the proposed system model, we develop linear transmit strategies which maximize the system sum rate for the MIMO and the MISO setup. The numerical results show that a significant gain over the half duplex (HD) operation mode can be obtained and the magnitude of the gain depends on the self-interference cancellation ability and the system setup.

***Index Terms—*** full duplex, MIMO

## 1. INTRODUCTION

Full duplex (FD) wireless systems have the potential to double the system spectral efficiency compared to half duplex (HD) systems [1]. The main difficulty in implementing a FD system is that the strong loop-back self-interference exceeds the limited dynamic range at the receiver. This phenomenon is critical since it saturates the receiver which will not only prevent the correct reception of the desired signal but may also damage the device. Recently, several approaches have been proposed to implement SISO FD transceivers via self-interference cancellation. Most of them involve advanced concepts in both RF transceiver architecture and digital signal processing at the receiver. The simplest approach is to use directional transmit (Tx) and receive (Rx) antennas to decouple the Tx and Rx signals [2]. However, this approach is only suitable when the data source and the data sink are sufficiently separated spatially (probably different devices). In [3], an antenna cancellation approach was proposed, which requires two Tx antennas. By proper position adjustment, the signals of both Tx antennas overlap destructively at the Rx antenna, which leads to a certain degree of self-interference cancellation. This approach can be regarded as a static beamforming approach and has the drawback that it is only suitable for narrow band transmissions and requires accurate manual tuning of antenna positions. In [1] and [4], more advanced approaches are proposed, which can cope with larger bandwidths. In [1] a balun is used at the Tx antenna input to feed an inverted and amplitude- and phase- adjusted version of the RF Tx signal to the output of the Rx antenna to cancel the self-interference. In [4], an auxiliary Tx path is used to feed a cancellation signal to the Rx input for RF self-interference cancellation, where the cancellation signal is a preprocessed version of the own Tx signal to match the actual self-interference signal.

Using the above schemes, a considerable part of the self-interference can be canceled/avoided in the RF domain. However, the self-interference cancellation ability of these schemes has not yet been verified in real-world applications and thus their stability is unknown. Nor the extension of the above schemes for MIMO systems is clear. Actually, it is possible that after the RF cancellation, the receiver might be still saturated. This situation is what we mean by limited/partial self-interference cancellation ability. Thus, in this paper we propose transmitter side precoding techniques to cope with such a situation so that both the reliability and the performance of full duplex systems are improved. In other words, if the interference cancellation ability provided by RF cancellation techniques is sufficient, the proposed precoding techniques focus only on the performance of the system. If the interference cancellation ability is not enough, the proposed precoding techniques will first help to reduce the self-interference by either turning down the transmit power or allocating a certain amount of power into the null space of the self-interference channel. Combining the proposed transmit strategies, the RF techniques, and the subtraction of the residual interference in the digital baseband of the

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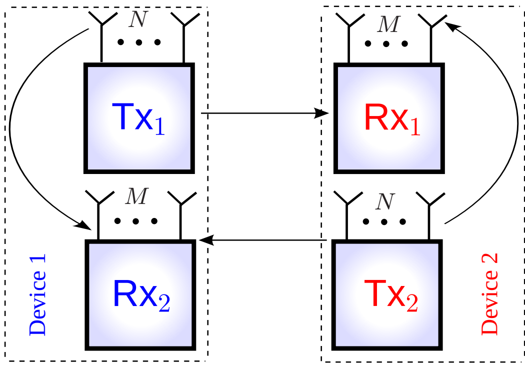



**Fig. 1**. A full duplex point-to-point system.

receiver, high performance FD systems are then built. To design the precoding techniques, a common signal model is required, which does not depend on a specific RF model but also takes into account the ability of the digital subtraction of the residual interference at the receiver. Based on the proposed system model, transmit strategies can be developed such that the system performance is optimized. Afterwards, the theoretical upper bounds of a FD system with constrained hardware abilities are found. In summary, the contributions of this work are:

- We have proposed a common system model for a FD wireless system with limited RF self-interference cancellation ability.
- Based on the proposed system model, we study the achievable sum rate of the FD system with single or multiple antenna transceivers. Optimal linear transmit strategies are developed and the possibilities of analytic solutions are discussed for the MIMO and the MISO setup, respectively.

**Notation:** Upper case and lower case bold letters denote matrices and vectors, respectively. The expectation operator, trace of a matrix, transpose, conjugate, and Hermitian transpose are denoted by $\mathbb{E}\{\cdot\}$, $\mathrm{Tr}\{\cdot\}$, $\{\cdot\}^{\mathrm{T}}$, $\{\cdot\}^{*}$, and $\{\cdot\}^{\mathrm{H}}$, respectively. The Euclidean norm of a vector is denoted by $\|\cdot\|$.

## 2. SYSTEM MODEL

Two transceivers with identical hardware configurations communicate with each other as depicted in Fig. 1. Each transceiver has $N$ Tx antennas and $M$ Rx antennas. The transmitter and the receiver at the same transceiver are indexed by $\{i, j\} \in \{1, 2\}$ and $i \neq j$. We assume perfect synchronization. The channel is frequency flat and quasi-static block fading. The desired channel from the $i$th transmitter to the $i$th receiver is denoted as $\boldsymbol{H}_{ii} \in \mathbb{C}^{M\times N}$ while the self-interference channel from the $i$th transmitter to the $j$th receiver is $\boldsymbol{H}_{ji} \in \mathbb{C}^{M\times N}$. All the channels have full rank, i.e., $\mathrm{rank}(\boldsymbol{H}_{ii}) = \mathrm{rank}(\boldsymbol{H}_{ji}) = \min(M, N)$.

Let the $i$th transmitter transmits the data vector $\boldsymbol{s}_i$ with the precoding matrix $\boldsymbol{W}_i \in \mathbb{C}^{N\times r_i}$ ($r_i$ is the number of transmitted data streams of the corresponding transmitter.). Then its transmitted signal vector $\boldsymbol{x}_i$ can be written as

$$\boldsymbol{x}_i = \boldsymbol{W}_i \boldsymbol{s}_i \tag{1}$$

with the transmit power constraint $\mathbb{E}\{\|\boldsymbol{x}_i\|^2\} \leq \epsilon_i P_i^{(\mathrm{ref})}$ where $\epsilon_i \in \mathbb{R}^{+}$. The elements of $\boldsymbol{s}_i$ are independently distributed with zero mean and unit variance. Moreover, the self-interference cancellation ability/self-interference power constraint is defined as

$$\mathbb{E}\{\eta_j \|\boldsymbol{H}_{ij}\boldsymbol{x}_j\|^2\} \leq P_i^{(\mathrm{ref})}, \tag{2}$$

where $\eta_j \in \mathbb{R}^{+}$ denotes the ratio between the path-loss of the self-interference channel and the path-loss of the desired channel. The constraint (2) implies that only if the self-interference power is below a certain threshold, the receiver can work properly and the received signal at the $i$th receiver is written as

$$\boldsymbol{y}_i = \underbrace{\boldsymbol{H}_{ii}\boldsymbol{x}_i}_{\text{desired signal}} + \underbrace{\sqrt{\eta_j}\boldsymbol{H}_{ij}\boldsymbol{x}_j}_{\text{residual self-interference}} + \boldsymbol{n}_i \in \mathbb{C}^M \tag{3}$$

where $\boldsymbol{n}_i$ denotes the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise and $\mathbb{E}\{\boldsymbol{n}_i \boldsymbol{n}_i\} = \sigma_{\mathrm{n}}^2 \boldsymbol{I}_{M_{\mathrm{R}}}, \forall i$. If channel knowledge is available at the receiver, the residual self-interference can be subtracted since the transceiver knows its own transmitted signal. If we further ignore the remaining self-interference after the subtraction, the received signal is simplified as

$$\hat{\boldsymbol{y}}_i = \boldsymbol{H}_{ii}\boldsymbol{x}_i + \boldsymbol{n}_i \in \mathbb{C}^M \tag{4}$$

However, if the self-interference power constraint (2) is violated, we say that the receiver is saturated. Even though it still works, the system will suffer significantly from the self-interference and it is not worth enabling the FD mode.

Given the above system model and assuming perfect channel knowledge at the transmitter, in the following we develop linear transmit strategies which maximizes the system sum rate and discuss possible analytic solutions for the MIMO and the MISO setup, respectively.

## 3. OPTIMAL LINEAR TRANSMIT STRATEGIES

In this section we solve the sum rate maximization problem for the MISO and the MIMO setup separately.

### 3.1. MIMO setup

The sum rate maximization problem for the MIMO setup is formulated as:

$$\begin{aligned} \max_{\boldsymbol{Q}_i} \quad & \sum_{i=1}^{2} \log_2 \left( \left| \boldsymbol{I}_M + \frac{1}{\sigma_{\mathrm{n}}^2} \boldsymbol{H}_{ii} \boldsymbol{Q}_i \boldsymbol{H}_{ii}^{\mathrm{H}} \right| \right) \\ \text{subject to} \quad & \boldsymbol{Q}_i \succeq 0, \mathrm{Tr}\{\boldsymbol{Q}_i\} \leq \epsilon_i P_i^{(\mathrm{ref})} \\ & \mathrm{Tr}\{\eta_i \boldsymbol{H}_{ji} \boldsymbol{Q}_i \boldsymbol{H}_{ji}^{\mathrm{H}}\} \leq P_i^{(\mathrm{ref})} \end{aligned} \tag{5}$$

where $\boldsymbol{Q}_i = \boldsymbol{W}_i \boldsymbol{W}_i^{\mathrm{H}}$. Since in problem (5) the design of $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ is not coupled so that we can design them separately, i.e., $\forall i$, we solve

$$\begin{aligned} \min_{\boldsymbol{Q}_i} \quad & -\log \left( \left| \boldsymbol{I}_M + \frac{1}{\sigma_{\mathrm{n}}^2} \boldsymbol{H}_{ii} \boldsymbol{Q}_i \boldsymbol{H}_{ii}^{\mathrm{H}} \right| \right) \\ \text{subject to} \quad & \boldsymbol{Q}_i \succeq 0, \mathrm{Tr}\{\boldsymbol{Q}_i\} \leq \epsilon_i P_i^{(\mathrm{ref})} \\ & \mathrm{Tr}\{\eta_i \boldsymbol{H}_{ji} \boldsymbol{Q}_i \boldsymbol{H}_{ji}^{\mathrm{H}}\} \leq P_i^{(\mathrm{ref})}. \end{aligned} \tag{6}$$

According to [5], problem (6) is a convex problem since both the cost function and the constraints are convex. Thus, it can be solved using the interior-point algorithm in [5]. Define the EVD of $\boldsymbol{Q}_i = \boldsymbol{U}_i \boldsymbol{\Sigma}_i \boldsymbol{U}_i^{\mathrm{H}}$. The optimal precoding matrix is given by $\boldsymbol{W}_{i,\mathrm{opt}} = \boldsymbol{U}_i \boldsymbol{\Sigma}_i^{\frac{1}{2}}$.

Nevertheless, the interior-point algorithm only provides numerical solutions which do not give all the insights of the problem. If possible, an analytic solution is preferable. To this end, we analyze the active[1] and inactive constraints of problem (6).

We start by first pointing out that at least one of the constraints in (6) is active at the optimality. Otherwise, the optimal $\boldsymbol{Q}_{i,\mathrm{opt}}$ can be scaled up to satisfy at least one of the constraints with equality while increasing the objective function, which contradicts the optimality. Based on this fact, we are able to prove the following proposition.

**Proposition 1.** *At the optimality of problem (6), the following statements hold:*

*i) If the transmit power constraint is active while the self-interference power constraint is inactive. The analytic solution is obtained by using the SVD of $\boldsymbol{H}_{ii}$ together with the water-filling power allocation, i.e., the optimal solution for a HD point-to-point system. Two-fold gain is achievable;*

*ii) If the transmit power constraint is inactive while the self-interference power constraint is active. The analytic solution is given by the water-filling power allocation over the eigenmodes of the effective channel $\boldsymbol{H}_{ii}\boldsymbol{H}_{ji}^{+}$.*

[1]Active constraints means that at the optimality the constraints are satisfied with equality [5].

*Proof.* Case i): after dropping the inactive self-interference power constraint, the remaining problem is the same as the capacity achieving precoder design problem for a HD MIMO system [6]. Thereby, the solution is the well-known water-filling solution [6].

Case ii): Let us define $\hat{\boldsymbol{Q}}_i = \boldsymbol{H}_{ji} \boldsymbol{Q}_i \boldsymbol{H}_{ji}^{\mathrm{H}}$. If the Moore-Penrose pseudoinverse of $\boldsymbol{H}_{ji}^{+} = (\boldsymbol{H}_{ji}^{\mathrm{H}} \boldsymbol{H}_{ji})^{-1} \boldsymbol{H}_{ji}^{\mathrm{H}}$ ($M \geq N$ is required) exists, then we have $\boldsymbol{Q}_i = \boldsymbol{H}_{ji}^{+} \hat{\boldsymbol{Q}}_i \boldsymbol{H}_{ji}^{+^{\mathrm{H}}}$. Problem (6) is reformulated as

$$\begin{aligned} \min_{\hat{\boldsymbol{Q}}_i} \quad & -\log \left( \left| \boldsymbol{I}_M + \frac{1}{\sigma_{\mathrm{n}}^2} \boldsymbol{H}_{ii} \boldsymbol{H}_{ji}^{+} \hat{\boldsymbol{Q}}_i \boldsymbol{H}_{ji}^{+^{\mathrm{H}}} \boldsymbol{H}_{ii}^{\mathrm{H}} \right| \right) \\ \text{subject to} \quad & \hat{\boldsymbol{Q}}_i \succeq 0, \mathrm{Tr}\{\eta_i \hat{\boldsymbol{Q}}_i\} = P_i^{(\mathrm{ref})} \end{aligned} \tag{7}$$

Problem (7) has the same formulation as the MIMO capacity achieving problem for a HD system in [6] and thus it can be solved using the water-filling algorithm. This solution is called inverse water-filling in the rest of the paper. □

When both constraints are active, an analytic solution is in general difficult to obtain. A possible routine for obtaining an analytic solution for such a situation is to apply the Karush-Kuhn-Tucker (KKT) conditions [5] which are first-order necessary conditions for optimality. The intention behind this approach is that the solution obtained from the KKT conditions (if exists) is also globally optimal since our problem is a convex problem. For the case $M = N = 2$ an analytic solution is obtained but it is not included in this paper due to the space limitation.

### 3.2. MISO setup

The sum rate maximization problem for the MISO setup is given by:

$$\begin{aligned} \max_{\boldsymbol{w}_i} \quad & \sum_{i=1}^{2} \log_2 \left( 1 + \boldsymbol{w}_i^{\mathrm{H}} \boldsymbol{h}_{ii}^{\mathrm{H}} \boldsymbol{h}_{ii} \boldsymbol{w}_i \right) \\ \text{subject to} \quad & \boldsymbol{w}_i^{\mathrm{H}} \boldsymbol{w}_i \leq \epsilon_i P_i^{(\mathrm{ref})} \\ & \boldsymbol{w}_i^{\mathrm{H}} \eta_i \boldsymbol{h}_{ji}^{\mathrm{H}} \boldsymbol{h}_{ji} \boldsymbol{w}_i \leq P_i^{(\mathrm{ref})} \end{aligned} \tag{8}$$

where $\boldsymbol{w}_i \in \mathbb{C}^{N}$, $\boldsymbol{h}_{ii} \in \mathbb{C}^{1 \times N}$, $\boldsymbol{h}_{ji} \in \mathbb{C}^{1 \times N}$ are vector versions of $\boldsymbol{W}_i$, $\boldsymbol{H}_{ii}$, $\boldsymbol{H}_{ji}$, respectively. Similarly, problem (8) is equivalent to solving the following subproblem, $\forall i$,

$$\begin{aligned} \max_{\boldsymbol{w}_i} \quad & \boldsymbol{w}_i^{\mathrm{H}} \boldsymbol{h}_{ii}^{\mathrm{H}} \boldsymbol{h}_{ii} \boldsymbol{w}_i \\ \text{subject to} \quad & \boldsymbol{w}_i^{\mathrm{H}} \boldsymbol{w}_i \leq \epsilon_i P_i^{(\mathrm{ref})} \\ & \boldsymbol{w}_i^{\mathrm{H}} \eta_i \boldsymbol{h}_{ji}^{\mathrm{H}} \boldsymbol{h}_{ji} \boldsymbol{w}_i \leq P_i^{(\mathrm{ref})} \end{aligned} \tag{9}$$

Problem (9) is non-convex because the cost function is the maximization of a convex function. A common approach to solve (9) is to apply the semidefinite relaxation (SDR) to first obtain a convex problem [7]. That is, define $\boldsymbol{X}_i = \boldsymbol{w}_i \boldsymbol{w}_i^{\mathrm{H}}$ and get the following convex problem:

$$\begin{aligned} \max_{\boldsymbol{X}_i} \quad & \mathrm{Tr}\{\boldsymbol{h}_{ii}^{\mathrm{H}} \boldsymbol{h}_{ii} \boldsymbol{X}_i\} \\ \text{subject to} \quad & \boldsymbol{X}_i \succeq 0, \mathrm{Tr}\{\boldsymbol{X}_i\} \leq \epsilon_i P_i^{(\mathrm{ref})} \\ & \mathrm{Tr}\{\eta_i \boldsymbol{h}_{ji}^{\mathrm{H}} \boldsymbol{h}_{ji} \boldsymbol{X}_i\} \leq P_i^{(\mathrm{ref})} \end{aligned} \tag{10}$$

clearly, adding the rank-one constraint $\mathrm{rank}(\boldsymbol{X}_i) = 1$ into (10) yields the original problem (9). Thereby, a rank-one optimal solution of (10) is the solution to the original problem. Fortunately, according to *Corollary 3.4* in [7], problem (10) has always a rank-one solution.

Again, we investigate the possibility of an analytic solution to problem (9). For this purpose, we derive the following proposition.

**Proposition 2.** *At the optimality of problem (9), $\forall i$, the self-interference power constraint cannot be active while the transmit power constraint is inactive.*

*Proof.* The proof is omitted here due to the space limitation. □

Proposition 2 leads to the following statements.

**Proposition 3.** *At the optimality of problem (9), $\forall i$,*

*i) The transmit power constraint is always active;*

*ii) If the self-interference power constraint is inactive, the optimal transmit strategy is the maximum ratio transmission (MRT) scheme, i.e., $\boldsymbol{w}_{\mathrm{o},i} = \boldsymbol{h}_{ii}^{\mathrm{H}}$. A two-fold gain is achievable.*

*Proof.* The proof to Proposition 3 is evident. The statement i) is based on two other statements. First, due to the same reason as in a MIMO setup, at least one of the constraints has to be active at the optimality. Second, Proposition 2 is valid at the optimality. The statement ii) comes directly from the fact that if the self-interference power constraint is inactive, problem (9) degrades into a classical HD MISO setup. □

When both of the constraints in (9) are active, an analytic solution can be derived. This is again not included in this paper due to the space limitation.

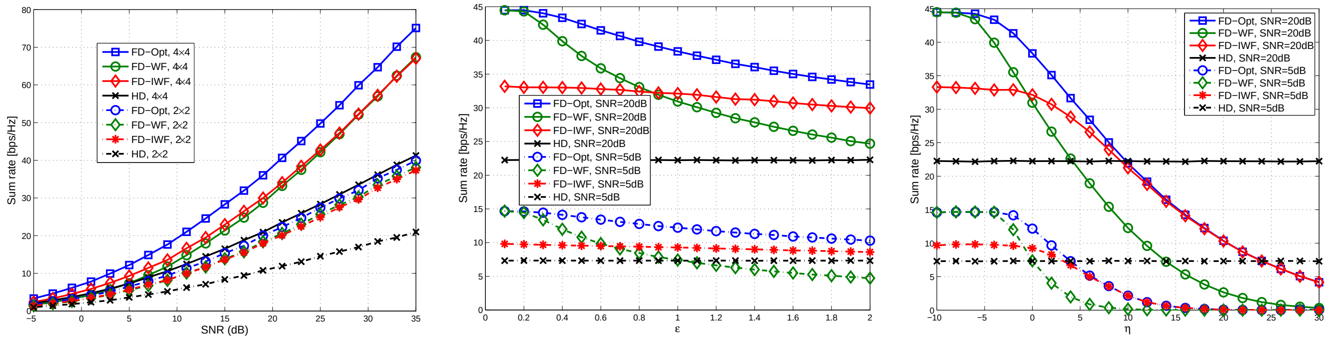
## 4. SIMULATION RESULTS

In this section the proposed algorithms are evaluated using Monte-Carlo simulations. The generated channels are uncorrelated Rayleigh flat fading. For simplicity, we have $\epsilon_i = \epsilon$, $\eta_i = \eta$, and $P_i^{(\mathrm{ref})} = P$, $\forall i$. The reference power level $P$ is set to 1 and the SNR is thus defined as $\mathrm{SNR} = \epsilon/\sigma_{\mathrm{n}}^2$. All the simulation results are averaged over 10000 channel realizations. To demonstrate whether and under which conditions a FD system with limited self-interference ability can achieve a two-fold gain compared to a HD system in terms of the sum rate, we define the HD baseline system. The HD baseline system has the same hardware configurations (e.g., the same number of Tx and Rx antennas, the same transmit power constraints, etc.) as the transceiver in the FD system. It works in a TDD HD mode, i.e., at each time slot, the HD system only receives or transmits the data. The applied transmit strategies are capacity achieving, i.e., MRT for the MISO setup and the SVD-based water-filling solution for the MIMO setup. Other than these optimal solutions, the performance of suboptimal transmit strategies for the FD system is also demonstrated in the simulations. For the MIMO case, we show the performance of the classical water-filling (WF) solution and the inverse water-filling (IWF) solution derived in Section 3.1. This is done by first calculating the WF or IWF solution and then scaling the obtained solution such that both constraints in (6) are satisfied. The same procedure is also applied to the selected suboptimal algorithms for the MISO setup. For a FD MISO setup, we also demonstrate the performance of the MRT and the zero-forcing (ZF) strategy[2].

Fig. 2 demonstrates the achievable sum rate of a FD MIMO system. The parameters $\epsilon$ and $\eta$ can be seen as the indicator of the direct (creating a negative copy of the transmitted signal at the receiver, e.g., using the axillary Tx chain in [4]) and indirect (artificially introducing path-loss, e.g., techniques in [2]) RF self-interference cancellation ability, respectively. The smaller the $\epsilon$ or the $\eta$ is, the higher is the RF cancellation ability of the FD system. Clearly, the three subfigures imply that compared to a HD MIMO system a two-fold gain in terms of the sum rate is only achievable in the high SNR regime and when $\epsilon$ or $\eta$ is small enough. It is also observed that the suboptimal solution WF and IWF is not far from the optimal solution. When the self-interference cancellation ability is weak, i.e., in the low SNR regime and when $\epsilon$ or $\eta$ is big, the optimal solution corresponds to the IWF algorithm and a two-fold gain is not obtainable. When the self-interference cancellation ability is strong, the optimal solution corresponds to the WF algorithm.

A similar observation can be obtained for the MISO setup in Fig. 3. That is, the suboptimal algorithms MRT and ZF have close to optimum performance. When the self-interference cancellation ability is strong, the MRT method corresponds to the optimal scheme. When the self-interference cancellation ability is weak, the ZF solution is close to the optimal solution. However, interestingly, it can be seen that as $\epsilon$ increases the gain of using a FD system is constant. Even though the gain of using a FD system decreases as $\eta$ increases, the degradation is quite small compared to the MIMO setup. This implies that for a MISO setup the transmit strategies act not only as an aid of the RF cancellation techniques but also as a replacement of them. This advan-

[2]The transmitter transmits into the null space of the self-interference channel

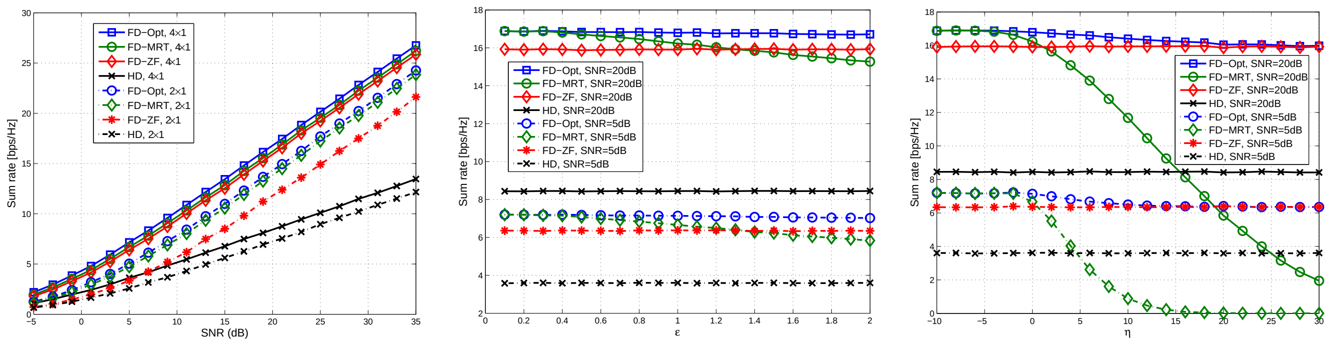



(a) Sum rate vs. SNR, $\epsilon = \eta = 1$.

(b) Sum rate vs. $\epsilon$, $M = N = 4$, $\eta = 1$.

(c) Sum rate vs $\eta$, $M = N = 4$, $\epsilon = 1$

**Fig. 2**. Achievable sum rate for a full duplex system with a MIMO setup. FD-Opt: optimal FD solution using the interior-point algorithm. FD-WF: WF solution. FD-IWF: IWF solution. HD: optimal HD solution.




(a) Sum rate vs. SNR, $\epsilon = \eta = 1$.

(b) Sum rate vs. $\epsilon$, $N = 4$, $\eta = 1$.

(c) Sum rate vs $\eta$, $N = 4$, $\epsilon = 1$

**Fig. 3**. Achievable sum rate for a full duplex system with a MISO setup. FD-Opt: optimal FD solution using the interior-point algorithm. FD-MRT: MRT solution. FD-ZF: zero-forcing solution. HD: optimal HD solution.

tage is due to the fact that in the MISO case the transmitter can allocate as much power as possible to the null space of the self-interference channel and this amount of power will also contribute to the sum rate maximization. However, this ability is limited in the MIMO setup due to the existence of the co-channel interference created by the multiple stream transmission.

## 5. CONCLUSION

The performance of a FD point-to-point MIMO system strongly depends on its self-interference cancellation ability since in practice it is difficult to have a self-interference free FD system. In this paper we have proposed a common system model for FD systems with partial self-interference cancellation ability. Afterwards, we have studied the achievable sum rate of the system and have developed optimal linear transmit strategies for the system with a MIMO and a MISO setup, respectively. Simulation results have demonstrated that compared to the HD baseline system a two-fold gain in terms of sum rate is achievable only if the self-interference cancellation ability is sufficiently strong.

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