

REDUCED-RANK DOA ESTIMATION BASED ON JOINT ITERATIVE SUBSPACE RECURSIVE OPTIMIZATION AND GRID SEARCH

Lei Wang, Rodrigo C. de Lamare

Communications Research Group,
University of York, United Kingdom
Email: {lw517,rcdl500}@ohm.york.ac.uk

Martin Haardt

Ilmenau University of Technology,
Communications Research Laboratory, Ilmenau, Germany.
Email: martin.haardt@tu-ilmenau.de

ABSTRACT

In this paper, we propose a reduced-rank direction of arrival (DOA) estimation algorithm based on joint and iterative subspace optimization (JISO) with grid search. The reduced-rank scheme includes a rank reduction matrix and an auxiliary reduced-rank parameter vector. They are jointly and iteratively optimized with a recursive least squares algorithm (RLS) to calculate the output power spectrum. The proposed JISO-RLS DOA estimation algorithm provides an efficient way to iteratively estimate the rank reduction matrix and the auxiliary reduced-rank vector. It is suitable for DOA estimation with large arrays and can be extended to arbitrary array geometries. It exhibits an advantage over MUSIC and ESPRIT when many sources exist in the system. A spatial smoothing (SS) technique is employed for dealing with highly correlated sources. Simulation results show that the JISO-RLS has a better performance than existing Capon and subspace-based DOA estimation methods.

Index Terms— DOA estimation, array processing, joint optimization, rank reduction.

1. INTRODUCTION

Direction of arrival (DOA) estimation algorithms have been extensively studied in the past due to their widespread applications [1]. Among the well known methods are Capon's methods [2], subspace-based [3, 4], and maximum likelihood (ML) techniques [8]. Capon [2] is a simple scheme that scans possible angles and locates peaks in the output power spectrum to estimate the DOAs of the signals. Its resolution strongly depends on the number of snapshots and the array size. MUSIC [3] and ESPRIT [4] are subspace-based algorithms that exploit the eigen-structure of the input covariance matrix for direction finding. Previously reported subspace-based methods suffer from correlated sources and a poor estimation of the input covariance matrix. In order to employ subspace-based algorithms and avoid a direct eigen-decomposition for the computation of the signal subspace, one can resort to subspace tracking algorithms [5] that exhibit an attractive trade-off between performance and complexity. The auxiliary vector (AV) [6] and conjugate gradient (CG) [7] algorithms are two subspace-based algorithms proposed recently. They utilize the AV basis vectors or CG residual vectors to form the signal subspace without the need for an eigen-decomposition. Both algorithms work well in severe conditions with low SNR for both correlated and uncorrelated sources. The maximum-likelihood (ML) algorithm [8] exhibits a superior resolution compared with the Capon and subspace-based methods but requires a high computational cost.

In this paper, we introduce a novel reduced-rank DOA estimation algorithm. The proposed JISO algorithm is robust against model-order selection and is suitable for DOA estimation with large arrays and a small number of snapshots. It is suited for problems of highly correlated sources or direction finding for a large number of users. The existing Capon and subspace-based algorithms cannot achieve a high resolution when many signals are present. Although the ML

technique is powerful under these conditions, with large arrays it results in an even higher complexity that is not acceptable in practice. The proposed JISO algorithm consists of a rank reduction matrix and an auxiliary reduced-rank parameter vector. They are jointly optimized according to the minimum variance (MV) design criterion for the calculation of the output power spectrum over the possible scanning angles. We also present a constrained adaptive recursive least squares (RLS) algorithm to iteratively estimate the rank reduction matrix and auxiliary reduced-rank parameter vector. The proposed JISO-RLS algorithm is more practical, in comparison with the existing methods, since its implementation is not limited by the array structure, does not require the eigen-decomposition, and works well without the knowledge of the number of sources. A spatial smoothing (SS) technique [9] is employed in the proposed DOA estimation algorithm to deal with the correlated sources. Simulation results illustrate the advantages of the proposed algorithms.

The rest of this paper is organized as follows: we outline the system model used for DOA estimation and state the problem in Section 2. Section 3 introduces the reduced-rank scheme and derives the proposed DOA estimation algorithm. The application of the SS technique in the proposed algorithm is introduced briefly in this part. Simulation results are provided and discussed in Section 4, and conclusions are drawn in Section 5.

2. SYSTEM MODEL AND PROBLEM STATEMENT

Let us suppose that q narrowband signals impinge on a uniform linear array (ULA) of M ($M \geq q$) sensor elements. The ULA is adopted here for simplicity. The proposed algorithm can be extended to arbitrary array structures and this will be sought in a future work. The i th snapshot's vector of sensor array outputs $\mathbf{x}(i) \in \mathbb{C}^{M \times 1}$ can be modeled as

$$\mathbf{x}(i) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(i) + \mathbf{n}(i), \quad i = 1, \dots, N \quad (1)$$

where $\boldsymbol{\theta} = [\theta_0, \dots, \theta_{q-1}]^T \in \mathbb{R}^{q \times 1}$ contains the DOAs of the signals, $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_0), \dots, \mathbf{a}(\theta_{q-1})] \in \mathbb{C}^{M \times q}$ is the matrix that contains the steering vectors $\mathbf{a}(\theta_k)$, where $\mathbf{a}(\theta_k) = [1, e^{-2\pi j \frac{d}{\lambda_c} \cos \theta_k}, \dots, e^{-2\pi j (M-1) \frac{d}{\lambda_c} \cos \theta_k}]^T \in \mathbb{C}^{M \times 1}$, ($k = 0, \dots, q-1$), λ_c is the wavelength, d ($d = \lambda_c/2$ in general) is the inter-element distance of the ULA, $\mathbf{s}(i) \in \mathbb{C}^{q \times 1}$ contains the source symbols, $\mathbf{n}(i) \in \mathbb{C}^{M \times 1}$ is the white sensor noise, which is assumed to be a zero-mean spatially uncorrelated and Gaussian process, N is the number of snapshots, and $(\cdot)^T$ denotes transpose. To avoid mathematical ambiguities, the steering vectors $\mathbf{a}(\theta_k)$ are considered to be linearly independent [11]. The input covariance matrix is

$$\mathbf{R} = \mathbb{E}[\mathbf{x}(i)\mathbf{x}^H(i)] = \mathbf{A}(\boldsymbol{\theta})\mathbf{R}_s\mathbf{A}^H(\boldsymbol{\theta}) + \sigma_n^2\mathbf{I} \quad (2)$$

where $\mathbf{R}_s = \mathbb{E}[\mathbf{s}(i)\mathbf{s}^H(i)]$ denotes the signal covariance matrix, which is diagonal if the sources are uncorrelated and is nondiagonal and nonsingular for partially correlated sources, and $\mathbb{E}[\mathbf{n}(i)\mathbf{n}^H(i)] =$

$\sigma_n^2 \mathbf{I}_{M \times M}$ with $\mathbf{I}_{M \times M}$ being the corresponding identity matrix. The matrix \mathbf{R} must be estimated in practice. In this work, we use a time-average estimate given by

$$\hat{\mathbf{R}}(i) = \frac{1}{N} \sum_{i=1}^N \mathbf{x}(i) \mathbf{x}^H(i) \quad (3)$$

The number of snapshots determines the estimation accuracy of the input covariance matrix, which significantly impacts the resolution of the DOA estimation of Capon's method and subspace-based algorithms. The use of large arrays could compensate this loss to a certain extent but increases the computational cost and is inefficient when highly correlated or a large number of sources are present [11].

3. PROPOSED REDUCED-RANK SCHEME AND DOA ESTIMATION ALGORITHM

In this section, we present a reduced-rank strategy to design the rank reduction matrix and the auxiliary reduced-rank parameter vector. We present a joint and iterative subspace optimization (JISO) technique for computing the rank reduction matrix and the auxiliary reduced-rank parameter vector. Then, we derive an RLS algorithm to compute the rank reduction matrix and the reduced-rank vector for obtaining the output power spectrum and the DOAs of the sources.

3.1. Proposed Reduced-Rank DOA Estimation Scheme

We introduce a rank reduction matrix $\mathbf{T}_r = [\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_r] \in \mathbb{C}^{M \times r}$, which is responsible for the dimensionality reduction. Basically, \mathbf{T}_r maps the $M \times 1$ input vector $\mathbf{x}(i)$ into a lower dimension yielding

$$\bar{\mathbf{x}}(i) = \mathbf{T}_r^H \mathbf{x}(i), \quad (4)$$

where $\mathbf{t}_l = [t_{1,l}, t_{2,l}, \dots, t_{M,l}]^T \in \mathbb{C}^{M \times 1}$, ($l = 1, \dots, r$) is the l th column of \mathbf{T}_r , $\bar{\mathbf{x}}(i) \in \mathbb{C}^{r \times 1}$ is the reduced-rank input vector, and the subscript r is the rank that is assumed to be less than M . The dimension is reduced and the key features of the original signal is retained in $\bar{\mathbf{x}}(i)$ according to the design criterion. In what follows, all r dimensional quantities are denoted with a "bar". The reduced-rank input vector is then processed by the auxiliary filter with the reduced-rank vector $\bar{\mathbf{g}}_\theta = [\bar{g}_{\theta,1}, \bar{g}_{\theta,2}, \dots, \bar{g}_{\theta,r}]^T \in \mathbb{C}^{r \times 1}$ to obtain the output power with respect to the current scanning angle. The computational cost for large arrays will be reduced if r is much less than M . The proposed reduced-rank method is formulated as the optimization problem

$$\hat{\theta} = \arg \min_{\theta} \bar{\mathbf{g}}_\theta^H \mathbf{T}_r^H \mathbf{R} \mathbf{T}_r \bar{\mathbf{g}}_\theta; \quad \text{subject to } \bar{\mathbf{g}}_\theta^H \mathbf{T}_r^H \mathbf{a}(\theta) = 1, \quad (5)$$

where we notice that the minimization problem (5) is equivalent to the joint optimization of the rank reduction matrix \mathbf{T}_r and the auxiliary reduced-rank vector $\bar{\mathbf{g}}_\theta$. These quantities are employed to obtain the output power spectrum for the possible directions and search the peaks that correspond to the DOAs of the sources.

To compute \mathbf{T}_r and $\bar{\mathbf{g}}_\theta$, we need to transform (5) into an unconstrained cost function with the method of Lagrange multipliers:

$$\mathcal{J} = \bar{\mathbf{g}}_\theta^H \mathbf{T}_r^H \mathbf{R} \mathbf{T}_r \bar{\mathbf{g}}_\theta + 2 \Re \{ \lambda [\bar{\mathbf{g}}_\theta^H \mathbf{T}_r^H \mathbf{a}(\theta) - 1] \}, \quad (6)$$

where λ is a scalar Lagrange multiplier and the operator $\Re(\cdot)$ selects the real part of the argument in (6).

Assuming $\bar{\mathbf{g}}_\theta$ is known and taking the gradient of (6) with respect to \mathbf{T}_r [10], we get

$$\nabla \mathcal{J}_{\mathbf{T}_r} = \mathbf{R} \mathbf{T}_r \bar{\mathbf{g}}_\theta \bar{\mathbf{g}}_\theta^H + \lambda \mathbf{T}_r \mathbf{a}(\theta) \bar{\mathbf{g}}_\theta^H. \quad (7)$$

Equating the gradient to a zero matrix and post multiplying the terms by $\bar{\mathbf{g}}_\theta$, we obtain

$$\mathbf{T}_r \bar{\mathbf{g}}_\theta = -\lambda \mathbf{T}_r \mathbf{R}^{-1} \mathbf{a}(\theta), \quad (8)$$

where for a small number of snapshots, \mathbf{R}^{-1} can in practice be calculated by either employing diagonal loading or the pseudo-inverse.

The matrix \mathbf{T}_r can be viewed as finding a solution to the linear equation $\mathbf{T}_r \bar{\mathbf{g}}_\theta = \mathbf{f}$ for $\mathbf{f} = -\lambda \mathbf{T}_r \mathbf{R}^{-1} \mathbf{a}(\theta)$. Given a $\bar{\mathbf{g}}_\theta \neq \mathbf{0}$, there exist multiple \mathbf{T}_r satisfying the linear equation in general. Thus, we derive the minimum Frobenius-norm solution for stability. Let us express the quantities involved by

$$\mathbf{T}_r = [\bar{\mathbf{t}}_1, \bar{\mathbf{t}}_2, \dots, \bar{\mathbf{t}}_M]^H; \quad \mathbf{f} = [f_1, f_2, \dots, f_M]^T. \quad (9)$$

where $\bar{\mathbf{t}}_j = [\bar{t}_{j,1}^*, \dots, \bar{t}_{j,r}^*]^T \in \mathbb{C}^{r \times 1}$ with $j = 1, \dots, M$.

The computation of the minimum Frobenius-norm solution is reduced to the following M subproblems:

$$\min \|\bar{\mathbf{t}}_j\|^2, \quad \text{subject to } \bar{\mathbf{t}}_j^H \bar{\mathbf{g}}_\theta = f_j. \quad (10)$$

The solution to (10) is the projection of $\bar{\mathbf{t}}_j$ onto the hyperplane $\mathcal{H}_j = \{\bar{\mathbf{t}}_j \in \mathbb{C}^{r \times 1} : \bar{\mathbf{t}}_j^H \bar{\mathbf{g}}_\theta = f_j\}$, which is given by

$$\bar{\mathbf{t}}_j = f_j^* \frac{\bar{\mathbf{g}}_\theta}{\|\bar{\mathbf{g}}_\theta\|^2}, \quad (11)$$

and thus the rank reduction matrix is

$$\mathbf{T}_r = \mathbf{f} \frac{\bar{\mathbf{g}}_\theta^H}{\|\bar{\mathbf{g}}_\theta\|^2}. \quad (12)$$

Substituting the definition of \mathbf{f} into (12) and incorporating the constraint in (5) to get $\lambda \mathbf{T}_r$, we have the expression of \mathbf{T}_r , which is

$$\mathbf{T}_r = \frac{\mathbf{R}^{-1} \mathbf{a}(\theta)}{\mathbf{a}^H(\theta) \mathbf{R}^{-1} \mathbf{a}(\theta)} \frac{\bar{\mathbf{g}}_\theta^H}{\|\bar{\mathbf{g}}_\theta\|^2}. \quad (13)$$

On the other hand, fixing \mathbf{T}_r , taking the gradient of (6) with respect to $\bar{\mathbf{g}}_\theta$, and equating it to a zero vector, we obtain

$$\nabla \mathcal{J}_{\bar{\mathbf{g}}_\theta} = \bar{\mathbf{R}} \bar{\mathbf{g}}_\theta + \lambda_{\bar{\mathbf{g}}_\theta} \mathbf{T}_r^H \mathbf{a}(\theta), \quad (14)$$

where $\bar{\mathbf{R}} = \mathbb{E}[\bar{\mathbf{x}}(i) \bar{\mathbf{x}}^H(i)] \in \mathbb{C}^{r \times r}$ is the reduced-rank covariance matrix. Following the same procedures as for the computation of \mathbf{T}_r , the auxiliary reduced-rank parameter vector is expressed by

$$\bar{\mathbf{g}}_\theta = \frac{\bar{\mathbf{R}}^{-1} \bar{\mathbf{a}}(\theta)}{\bar{\mathbf{a}}^H(\theta) \bar{\mathbf{R}}^{-1} \bar{\mathbf{a}}(\theta)}, \quad (15)$$

where $\bar{\mathbf{a}}(\theta) = \mathbf{T}_r^H \mathbf{a}(\theta) \in \mathbb{C}^{r \times 1}$ is the reduced-rank steering vector with respect to the current scanning angles. Note that the proposed auxiliary reduced-rank vector $\bar{\mathbf{g}}_\theta$ is more general when dealing with DOA estimation. Specifically, for $r = M$ and $\mathbf{T}_r = \mathbf{I}_{M \times M}$, it is equivalent to the minimum variance weight solution, and, for $1 < r < M$, it operates with a lower dimension to reduce the complexity and suppress the noise.

3.2. Proposed JISO RLS Algorithm

Using (13) and (15), the rank reduction matrix and auxiliary reduced-rank vector are jointly optimized. It is necessary to perform an initialization for the calculation. Here, we develop an adaptive RLS implementation of the proposed JISO reduced-rank scheme. As explained before, both \mathbf{R} and $\bar{\mathbf{R}}$ are not available but have to be estimated in practice. We employ the recursive form to estimate them,

which are $\hat{\mathbf{R}}(i) = \alpha\hat{\mathbf{R}}(i-1) + \mathbf{x}(i)\mathbf{x}^H(i)$ and $\hat{\bar{\mathbf{R}}}(i) = \alpha\hat{\bar{\mathbf{R}}}(i-1) + \bar{\mathbf{x}}(i)\bar{\mathbf{x}}^H(i)$, respectively, where α is a forgetting factor, which is a positive constant close to, but less than 1.

To avoid the matrix inversion and to reduce the complexity, we employ the matrix inversion lemma [12] to update $\hat{\mathbf{R}}^{-1}(i)$ iteratively. Defining $\hat{\Phi}(i) = \hat{\mathbf{R}}^{-1}(i)$ for concise presentation, the recursive estimation procedures are given by

$$\mathbf{k}(i) = \frac{\alpha^{-1}\hat{\Phi}(i-1)\mathbf{x}(i)}{1 + \alpha^{-1}\mathbf{x}^H(i)\hat{\Phi}(i-1)\mathbf{x}(i)}, \quad (16)$$

$$\hat{\Phi}(i) = \alpha^{-1}\hat{\Phi}(i-1) - \alpha^{-1}\mathbf{k}(i)\mathbf{x}^H(i)\hat{\Phi}(i-1), \quad (17)$$

where $\mathbf{k}(i) \in \mathbb{C}^{M \times 1}$ is the Kalman gain vector. We set $\hat{\Phi}(0) = \delta\mathbf{I}_{M \times M}$ where $\delta > 0$ is a scalar for numerical stability.

For the calculation of \bar{g}_θ , we define $\hat{\bar{\Phi}}(i) = \hat{\bar{\mathbf{R}}}(i)$ and thus

$$\bar{\mathbf{k}}(i) = \frac{\alpha^{-1}\hat{\bar{\Phi}}(i-1)\bar{\mathbf{x}}(i)}{1 + \alpha^{-1}\bar{\mathbf{x}}^H(i)\hat{\bar{\Phi}}(i-1)\bar{\mathbf{x}}(i)}, \quad (18)$$

$$\hat{\bar{\Phi}}(i) = \alpha^{-1}\hat{\bar{\Phi}}(i-1) - \alpha^{-1}\bar{\mathbf{k}}(i)\bar{\mathbf{x}}^H(i)\hat{\bar{\Phi}}(i-1), \quad (19)$$

where $\bar{\mathbf{k}} \in \mathbb{C}^{r \times 1}$ is the reduced-rank gain vector and the recursive procedures are performed by initializing $\hat{\bar{\Phi}}(0) = \bar{\delta}\mathbf{I}_{r \times r}$ for $\bar{\delta} > 0$.

Substituting the expressions of the rank reduction matrix and the auxiliary reduced-rank vector with respect to the possible scanning angles $\theta \in (0^\circ, 180^\circ)$ into (5), we have the corresponding output power spectrum for DOA estimation

$$P(\theta_n) = (\bar{\mathbf{a}}^H(\theta_n)\hat{\bar{\Phi}}\bar{\mathbf{a}}(\theta_n))^{-1}, \quad (20)$$

where $\hat{\bar{\Phi}}$ is the estimate of the input covariance matrix after all N snapshots are available, the scanning angles $\theta_n = n\Delta^\circ$, Δ° is the search step, and $n = 1, 2, \dots, 180^\circ/\Delta^\circ$. For a convenient search, we make $180^\circ/\Delta^\circ$ an integer. The output power in (20) is much higher if the scanning angle $\theta_n = \theta_k$ ($k = 0, \dots, q-1$), which corresponds to the transmitted sources, compared with other scanning angles that correspond to the noise level. By finding the peaks in the output power spectrum, we can determine the DOAs of the sources.

The proposed JISO-RLS DOA estimation algorithm for each scanning angle θ_n is summarized in Table 1. Since $\mathbf{T}_r(i)$ and $\bar{g}_\theta(i)$ depend on each other, it is necessary to initialize $\mathbf{T}_r(0)$ to start the iteration. The selection of δ and $\bar{\delta}$ depends on the input signal-to-noise ratio (SNR) [12]. The specific values will be given in the simulations. The proposed JISO-RLS algorithm provides an iterative exchange of information between the rank reduction matrix and the reduced-rank vector, which leads to an improved resolution.

For correlated sources, we use a forward-backward SS technique [9] in the JISO scheme with a least squares algorithm and call it JISO-SS. The SS preprocessing technique operates on $\mathbf{x}(i)$ to estimate the forward and backward covariance matrix. We divide the ULA into overlapping subarrays of size n , with elements $\{1, \dots, n\}$ forming the first subarray, elements $\{2, \dots, n+1\}$ forming the second one, and so on with $J = M - n + 1$ being the number of forward subarrays [9]. We carry out a similar but reverse procedure for the backward subarrays and then take the average of the estimated covariance matrices obtained with the forward and backward recursions [9]. Since it is a well-known technique for dealing with correlated sources, the details are omitted but can be found in [9].

In terms of the computational cost, the Capon, MUSIC, and ESPRIT algorithms work with $O(M^3)$, and the recent AV and CG methods have a higher complexity [7]. The subspace tracking method

Table 1. Proposed JISO-RLS algorithm for DOA estimation

Initialization:

$$\mathbf{T}_r(0) = [\mathbf{I}_r^T \mathbf{0}_{r \times (M-r)}^T]; \quad \delta, \bar{\delta} = \text{positive constants};$$

$$\hat{\Phi}(0) = \delta\mathbf{I}_{M \times M}; \quad \hat{\bar{\Phi}}(0) = \bar{\delta}\mathbf{I}_{r \times r}.$$

Update for each time instant $i = 1, \dots, N$

$$\bar{\mathbf{x}}(i) = \mathbf{T}_r^H(i-1)\mathbf{x}(i)$$

$$\bar{\mathbf{a}}(\theta_n) = \mathbf{T}_r^H(i-1)\mathbf{a}(\theta_n)$$

$$\bar{\mathbf{k}}(i) = \frac{\alpha^{-1}\hat{\bar{\Phi}}(i-1)\bar{\mathbf{x}}(i)}{1 + \alpha^{-1}\bar{\mathbf{x}}^H(i)\hat{\bar{\Phi}}(i-1)\bar{\mathbf{x}}(i)}$$

$$\hat{\bar{\Phi}}(i) = \alpha^{-1}\hat{\bar{\Phi}}(i-1) - \alpha^{-1}\bar{\mathbf{k}}(i)\bar{\mathbf{x}}^H(i)\hat{\bar{\Phi}}(i-1)$$

$$\bar{g}_\theta(i) = \frac{\hat{\bar{\Phi}}(i)\bar{\mathbf{a}}(\theta)}{\bar{\mathbf{a}}^H(\theta)\hat{\bar{\Phi}}(i)\bar{\mathbf{a}}(\theta)}$$

$$\mathbf{k}(i) = \frac{\alpha^{-1}\hat{\Phi}(i-1)\mathbf{x}(i)}{1 + \alpha^{-1}\mathbf{x}^H(i)\hat{\Phi}(i-1)\mathbf{x}(i)}$$

$$\hat{\Phi}(i) = \alpha^{-1}\hat{\Phi}(i-1) - \alpha^{-1}\mathbf{k}(i)\mathbf{x}^H(i)\hat{\Phi}(i-1)$$

$$\mathbf{T}_r(i) = \frac{\hat{\Phi}(i)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\hat{\Phi}(i)\mathbf{a}(\theta)} \frac{\bar{g}_\theta^H(i)}{\|\bar{g}_\theta(i)\|^2}$$

Output power

$$P(\theta_n) = 1/(\bar{\mathbf{a}}^H(\theta_n)\hat{\bar{\Phi}}\bar{\mathbf{a}}(\theta_n))$$

denoted as approximated power iteration (API) [5] for use with the MUSIC and ESPRIT algorithms requires $O(qM + q^3)$. The API is significantly simpler than the direct eigen-decomposition [5], however, its complexity can become very high as the number of sources q becomes large. For the proposed algorithm, the complexity is $O(M^2)$ due to the use of the matrix inversion lemma [12]. Besides, the cost of computing $\hat{\Phi}(i)$ is invariable for the grid search, namely, the result computed for the first scanning angle can be used for the rest. The complexity of the reduced-rank process is $O(r^2)$, which is much less than $O(M^2)$ if r is much less than M for large arrays.

4. SIMULATIONS

Simulations are performed for a ULA with half a wavelength interelement spacing. We compare the proposed JISO-RLS algorithm with Capon's method, subspace-based methods with the API implementation [5], the ML method [8], and carry out $K = 1000$ runs to get each curve. BPSK sources separated by 3° with powers $\sigma_s^2 = 1$ are considered and the noise is spatially and temporally white Gaussian. The search step is $\Delta^\circ = 0.5^\circ$. We suppose that the DOAs are resolved if $|\hat{\theta} - \theta_k| < |\theta_k - \theta_{k-1}|/2$ [6]. In the first experiment, there are $q = 2$ highly correlated sources in the system with correlation value $c = 0.9$, which are generated as follows:

$$s_1 \sim \mathcal{N}(0, \sigma_s^2) \quad \text{and} \quad s_2 = cs_1 + \sqrt{1-c^2}s_3, \quad (21)$$

where $s_3 \sim \mathcal{N}(0, \sigma_s^2)$. The array size is $M = 30$ and the number of snapshots $N = 10$ is fixed. We set $\alpha = 0.998$, $r = 4$, and $\delta = \bar{\delta} = 10^{-3}$. The SS technique is employed in the existing and proposed algorithms (except ML) to improve the resolution. The subarray size is $n = 26$ for the proposed method. The probability of resolution [6], [7], [11] is plotted against the input SNR values. The resolution of the ML algorithm is superior to that of other methods. However, the complicated implementation and heavy computational load prevent its use in practice for large arrays. The proposed JISO-SS algorithm outperforms other existing methods except ML for all input SNR values.

In Fig.2, we set the sources to be uncorrelated but increase the number of sources by setting $q = 10$. The number of snapshots is $N = 20$ and the array size is $M = 40$. From Fig.2, the curves between the proposed and the ESPRIT algorithms are shown to intersect when the input SNR values increase. The proposed JISO-RLS

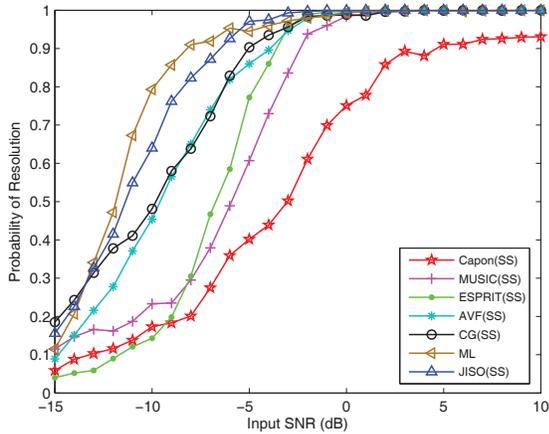


Fig. 1. Probability of resolution versus input SNR.

DOA estimation algorithm shows a better resolution with low SNRs. The complexity of the ESPRIT algorithm is high due to the eigen decomposition. The API approach estimates the signal subspace with a low-complexity to implement ESPRIT. This estimation is insufficient with a small number of snapshots and thus results in a poorer resolution, so does MUSIC(API). The recent AV and CG algorithms exhibit inferior performance with many sources.

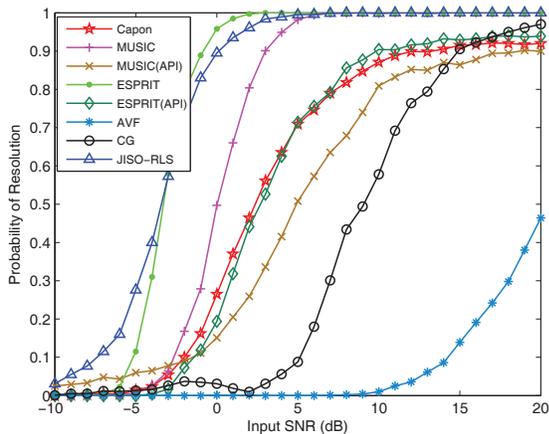


Fig. 2. Probability of resolution versus input SNR.

In Fig.3, we keep the scenario as that in Fig. 2 and assess the root-mean-squared error (RMSE) performance of the proposed and existing algorithms, and compare them with the Cramér-Rao bound (CRB). As can be seen from this figure, the RMSE values of the proposed algorithm are around 10 dB away from the CRB in the threshold region (low input SNR) and then closely follow the CRB curve with the increase of the SNR. The improved RMSE performance of the proposed algorithm over existing methods is evident.

5. CONCLUDING REMARKS

We have introduced a reduced-rank method based on a rank reduction matrix and an auxiliary reduced-rank parameter vector for DOA estimation. The rank reduction matrix maps the full-rank covariance matrix into a lower dimension and the auxiliary reduced-rank vector is combined to calculate the output power spectrum for each

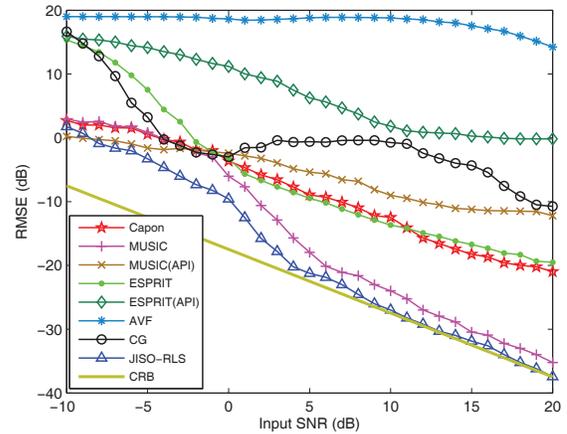


Fig. 3. RMSE versus input SNR.

scanning angle. We have derived an efficient algorithm to jointly update the rank reduction matrix and the auxiliary vector. The proposed JISO-RLS algorithm shows a superior resolution for large arrays with uncorrelated or correlated sources.

6. REFERENCES

- [1] H. Krim and M. Viberg, "Two decades of array signal processing research," *IEEE Sig. Proc. Magazine*, vol.13, pp. 67-94, July 1996.
- [2] J. Capon, "High resolution frequency-wavenumber spectral analysis," *IEEE Proc.*, vol.57, pp.1408-1418, Aug. 1969.
- [3] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas and Propagat.*, vol. 34, pp. 276-280, Mar. 1986.
- [4] R. H. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoustics, Speech, and Signal Processing*, vol. 37, pp. 984-995, Jul. 1989.
- [5] R. Bodeau, B. David, and G. Richard, "Fast approximated power iteration subspace tracking," *IEEE Trans. Signal Processing*, vol. 53, pp. 2931-2941, Aug. 2005.
- [6] R. Grover, D. A. Pados, and M. J. Medley, "Subspace direction finding with an auxiliary-vector basis," *IEEE Trans. on Signal Processing*, vol. 55, pp. 758-763, Feb. 2007.
- [7] H. Semira, H. Belkacemi, and S. Marcos, "High-resolution source localization algorithm based on the conjugate gradient," *EURASIP Journal on Adv. in Sig. Proc.*, vol. 2007, pp. 1-9, Mar. 2007.
- [8] I. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternating projection," *IEEE Trans. Acoustics, Speech, and Signal Processing*, vol. 36, pp. 1553-1560, Oct. 1988.
- [9] S. U Pillai and B. H. Kwon, "Forward/Backward Spatial Smoothing Techniques for Coherent Signal Identification," *IEEE Trans. on Acoustics, Speech and Sig. Proc.*, vol. 37, no.1, pp.8-15, Jan 1989.
- [10] L. Wang and R. C. de Lamare, "A new approach to reduced-rank DOA estimation based on joint iterative subspace optimization and grid search", *Proc. IEEE 16th International Conference on Digital Signal Processing*, July 2009, pp. 1 - 6.
- [11] H. L. Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, John Wiley & Sons, 2002.
- [12] S. Haykin, *Adaptive Filter Theory, 4rd ed.*, Englewood Cliffs, NJ: Prentice-Hall, 1996.