BEAMFORMING DESIGN FOR MULTI-USER TWO-WAY RELAYING WITH MIMO AMPLIFY AND FORWARD RELAYS

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ABSTRACT

Relays represent a promising approach to extend the cell coverage, combat the strong shadowing effects as well as guarantee the QoS in dense networks. Among the numerous existing relaying techniques, two-way relaying uses the radio resource in a particular efficient manner. Moreover, amplify and forward (AF) relays cause less delays and require lower hardware complexity. Therefore, we consider multi-user two-way relaying with MIMO AF relays where a base station (BS) and multiple users (UT) exchange messages via the relay in this paper. We propose three sub-optimal algorithms for computing the transmit and receive beamforming matrices at the BS as well as the amplification matrix at the relay. Simulations show that block diagonalization (BD) combined with the algebraic norm-maximizing (ANOMAX) transmit strategy provides the best balance between complexity and performance, the zero-forcing dirty paper coding (ZFDPC) based design can perform well when the system is heavily loaded, and the channel inversion (CI) based design yields the lowest complexity. All three algorithms outperform a recently proposed technique from the literature.

Index Terms— two-way relaying, amplify and forward (AF), MIMO, block diagonalization (BD), algebraic norm-maximizing (ANOMAX), zero-forcing dirty paper coding (ZFDPC)

1. INTRODUCTION

Recently, relays have received an increase interest due to their potential abilities of reducing the deployment cost, enhancing the network capacity, mitigating shadowing effects, and so on. When placed at the cell edge, relays can boost the coverage. In such applications, it is likely that each relay has to support multiple users. This motivates the development of multi-user MIMO relaying techniques, where the relay forwards data to and from multiple users. Prior work on multi-user relay channels focuses on one-way relaying [1], [2]. However, it is known that the two-way relaying technique can compensate the spectral efficiency loss in one-way relaying due to the half-duplex constraint of the relay and therefore uses the radio resources in a particular efficient manner [3]. To our knowledge, only a few references deal with multi-user two-way relaying, which include beamforming with an AF relay [4], beamforming with a decode and forward (DF) relay [5], and relaying protocols with repeaters [6]. Therefore, we consider the beamforming design for multi-user two-way relaying in our work. Moreover, we prefer the AF relays which retransmit an amplified version of their received signal since these cause less transmission delays and require lower hardware complexity than DF relays.

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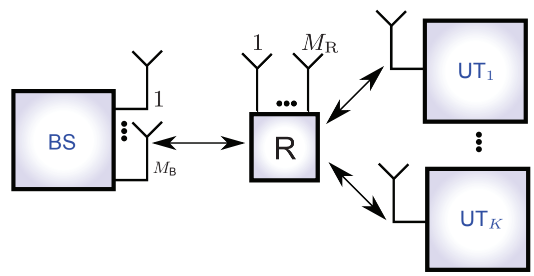

Fig. 1. Multi-user two-way relaying with a MIMO amplify and forward relay.

Finding the sum-rate optimal strategy involves a non-tractable optimization problem. To avoid this complex problem, we introduce three sub-optimal algorithms for computing the transmit and receive beamforming matrices at the BS as well as the amplification matrix at the relay. They are based on conventional channel inversion (CI), BD [7] combined with ANOMAX (BD ANOMAX) and ZFDPC (OWR ZFDPC). We also compare our algorithms with the algorithm in reference [4]. It turns out that BD ANOMAX provides the best balance between complexity and performance, OWR ZFDPC can still perform well for large loaded systems, while CI yields the lowest complexity.

Notation: Uppercase and lower case bold letters denote matrices and vectors, respectively. The expectation, trace of a matrix, transpose, Hermitian transpose, and Moore-Penrose pseudo inverse are denoted by $\mathrm{E}\{\cdot\}$, $\mathrm{Tr}\{\cdot\}$, $\{\cdot\}^{\mathrm{T}}$, $\{\cdot\}^{\mathrm{H}}$, and $\{\cdot\}^{+}$, respectively. The $m-$by$-m$ identity matrix is $\boldsymbol{I}_m$. The Euclidean norm of a vector and the Frobenius norm of a matrix is denoted by $\|\cdot\|$ and $\|\cdot\|_{\mathrm{F}}$, respectively. The Kronecker product is denoted by $\otimes$ and $\mathrm{blkdiag}\{\cdot\}$ is a block diagonal matrix containing several matrices. The rank of a matrix is denoted by $\mathrm{rank}\{\cdot\}$ and $\mathrm{vec}\{\cdot\}$ stacks the columns of a matrix into a vector.

2. SYSTEM MODEL

The scenario under investigation is shown in Figure 1. Due to the poor quality of the direct channel between the BS and the UTs, they can only communicate with each other with the help of the relay. Assume that we have K single antenna UTs. The BS is equipped with M_{B} antennas and the relay has M_{R} antennas. For notational simplicity, in the rest of our work we assume that $M_{\mathrm{B}} = K$. The channel is flat fading. The channel between the kth user and the relay is denoted by $\boldsymbol{h}_k \in \mathbb{C}^{M_\mathrm{R}}$. The channel between the base station and the relay is full rank and denoted by $\boldsymbol{H}_\mathrm{B} \in \mathbb{C}^{M_\mathrm{R} \times M_\mathrm{B}}$.

The two-way AF relaying protocol consists of two transmission phases: in the first phase all the users and the base station transmit their data simultaneously to the relay. Let the BS transmit the data symbol vector $\boldsymbol{d}_\mathrm{B} = [d_{\mathrm{B},1}, \ldots, d_{\mathrm{B},K}]^\mathrm{T} \in \mathbb{C}^K$ using the transmit beamforming matrix $\boldsymbol{F}_\mathrm{B} \in \mathbb{C}^{M_\mathrm{B} \times K}$. The data symbols in $\boldsymbol{d}_\mathrm{B}$ are independently distributed with zero mean and unit variance. Let us further assume that $d_{\mathrm{B},k}$ is the symbol transmitted from the BS to the kth UT and the relay knows the order of the data streams from the BS. The total power at the base station is denoted by P_B. The transmit power constraint can be written as

$$\mathrm{E}\{\|\boldsymbol{F}_\mathrm{B}\boldsymbol{d}_\mathrm{B}\|^2\} = \mathrm{Tr}\{\boldsymbol{F}_\mathrm{B}\boldsymbol{F}_\mathrm{B}^\mathrm{H}\} \leq P_\mathrm{B}. \tag{1}$$

Then, the received signal vector at the relay is given by

$$\boldsymbol{r} = \sum_{k=1}^{K} \boldsymbol{h}_k \cdot d_k + \boldsymbol{H}_\mathrm{B}\boldsymbol{F}_\mathrm{B}\boldsymbol{d}_\mathrm{B} + \boldsymbol{n}_\mathrm{R} \in \mathbb{C}^{M_\mathrm{R}}, \tag{2}$$

where d_k is the transmitted scalar from the kth user to the BS and $\boldsymbol{n}_\mathrm{R} \in \mathbb{C}^{M_\mathrm{R}}$ is the zero mean circularly symmetric complex Gaussian (ZMCSCG) noise with $\mathrm{E}\{\boldsymbol{n}_\mathrm{R}\boldsymbol{n}_\mathrm{R}^\mathrm{H}\} = \sigma_\mathrm{R}^2 \boldsymbol{I}_{M_\mathrm{R}}$. Moreover, we assume that each user has identical transmit power P_U and the transmit power constraint is equivalent to $\mathrm{E}\{|d_k|^2\} \leq P_\mathrm{U}$.

In the second phase, the relay amplifies the received signal and then forwards it to all the UTs as well as the BS. The signal transmitted by the relay can be expressed as

$$\bar{\boldsymbol{r}} = \gamma_0 \cdot \boldsymbol{G} \cdot \boldsymbol{r}. \tag{3}$$

where $\boldsymbol{G} \in \mathbb{C}^{M_\mathrm{R} \times M_\mathrm{R}}$ is the relay amplification matrix and $\gamma_0 \in \mathbb{R}^+$ is chosen such that the transmit power constraint at the relay is fulfilled, i.e.,

$$\begin{aligned}&\mathrm{E}\{\mathrm{Tr}\{\bar{\boldsymbol{r}}\bar{\boldsymbol{r}}^\mathrm{H}\}\} = \mathrm{Tr}\{\gamma_0^2 \cdot \boldsymbol{G}\{P_\mathrm{U}\boldsymbol{H}_\mathrm{U}\boldsymbol{H}_\mathrm{U}^\mathrm{H} \\ &+P_\mathrm{B}\boldsymbol{H}_\mathrm{B}\boldsymbol{F}_\mathrm{B}\boldsymbol{F}_\mathrm{B}^\mathrm{H}\boldsymbol{H}_\mathrm{B}^\mathrm{H} + \sigma_\mathrm{R}^2\boldsymbol{I}_{M_\mathrm{R}}\}\boldsymbol{G}^\mathrm{H}\} = P_\mathrm{R},\end{aligned} \tag{4}$$

where $\boldsymbol{H}_\mathrm{U} = [\boldsymbol{h}_1, \ldots, \boldsymbol{h}_K] \in \mathbb{C}^{M_\mathrm{R} \times K}$ is the concatenated channel matrix of all UTs.

For notational simplicity, we assume that the reciprocity assumption between the first- and second- phase channels is valid. This assumption is fulfilled in a TDD system if identical RF chains are applied. Then the received signal vector at the BS can be expressed as

$$\begin{aligned}\boldsymbol{y}_\mathrm{B} &= \boldsymbol{W}_\mathrm{B}(\boldsymbol{H}_\mathrm{B}^\mathrm{T}\bar{\boldsymbol{r}} + \boldsymbol{n}_\mathrm{B}) \\ &= \underbrace{\gamma_0\boldsymbol{W}_\mathrm{B}\boldsymbol{H}_\mathrm{B}^\mathrm{T}\boldsymbol{G}\boldsymbol{H}_\mathrm{U}\boldsymbol{d}_\mathrm{U}}_{\text{useful signal}} + \underbrace{\gamma_0\boldsymbol{W}_\mathrm{B}\boldsymbol{H}_\mathrm{B}^\mathrm{T}\boldsymbol{G}\boldsymbol{H}_\mathrm{B}\boldsymbol{F}_\mathrm{B}\boldsymbol{d}_\mathrm{B}}_{\text{self-interference}} \\ &\quad + \underbrace{\gamma_0\boldsymbol{W}_\mathrm{B}\boldsymbol{H}_\mathrm{B}^\mathrm{T}\boldsymbol{G}\boldsymbol{n}_\mathrm{R} + \boldsymbol{W}_\mathrm{B}\boldsymbol{n}_\mathrm{B}}_{\text{effective noise}} \in \mathbb{C}^{M_\mathrm{R}}\end{aligned} \tag{5}$$

where $\boldsymbol{d}_\mathrm{U} = [d_1, \ldots, d_K]^\mathrm{T} \in \mathbb{C}^K$ is the concatenated data vector of all the UTs and $\boldsymbol{n}_\mathrm{B} \in \mathbb{C}^{M_\mathrm{B}}$ is the ZMCSCG noise with $\mathrm{E}\{\boldsymbol{n}_\mathrm{B}\boldsymbol{n}_\mathrm{B}^\mathrm{H}\} = \sigma_\mathrm{B}^2\boldsymbol{I}_{M_\mathrm{B}}$. The receive beamforming matrix is denoted by $\boldsymbol{W}_\mathrm{B} \in \mathbb{C}^{K \times M_\mathrm{B}}$. It can be seen from (5) that the BS only experiences the self-interference caused by its own transmitted signal. If the BS has perfect channel knowledge, the self-interference can be subtracted.

On the other hand, the received scalar y_k at the kth UT can be written as

$$\begin{aligned}y_k &= \boldsymbol{h}_k^\mathrm{T}\bar{\boldsymbol{r}} + n_k = \underbrace{\gamma_0\boldsymbol{h}_k^\mathrm{T}\boldsymbol{G}\boldsymbol{H}_\mathrm{B}\boldsymbol{f}_{\mathrm{B},k}d_{\mathrm{B},k}}_{\text{useful signal}} \\ &\quad + \underbrace{\gamma_0\boldsymbol{h}_k^\mathrm{T}\boldsymbol{G}\boldsymbol{h}_k d_k}_{\text{self-interference}} + \underbrace{\sum_{\substack{m=1\\m\neq k}}^{K}\gamma_0\boldsymbol{h}_k^\mathrm{T}\boldsymbol{G}\boldsymbol{H}_\mathrm{B}\boldsymbol{f}_{\mathrm{B},m}d_{\mathrm{B},m}}_{\text{interference from other streams to other UTs}} \\ &\quad + \underbrace{\sum_{\substack{j=1\\j\neq k}}^{K}\gamma_0\boldsymbol{h}_k^\mathrm{T}\boldsymbol{G}\boldsymbol{h}_j d_j}_{\text{interference from other UTs}} + \underbrace{\gamma_0\boldsymbol{h}_k^\mathrm{T}\boldsymbol{G}\boldsymbol{n}_\mathrm{R} + n_k}_{\text{effective noise}}\end{aligned} \tag{6}$$

where $\boldsymbol{f}_{\mathrm{B},k}$ is the kth column of $\boldsymbol{F}_\mathrm{B}$ and n_k is ZMCSCG noise at each UT with identical variance σ_U^2. As can be seen from (6), unlike the BS, each UT experiences self-interference, interference caused by other UTs, and the interference caused by the signal which is transmitted from the BS but intended for another UT.

The overall sum rate of the system could be written as

$$R_\mathrm{sum} = R_\mathrm{U} + R_\mathrm{B} \tag{7}$$

where R_B and R_U are the achievable data rate at the BS and the cumulated achievable data rate at all UTs, respectively. The optimization problem to find the relay amplification matrix structure which maximizes (7) subject to transmit power constrains in (1) and (4) is non-convex. To avoid a non-tractable optimization problem, we resort to sub-optimal algorithms instead.

In [4], a linear beamforming is proposed such that

$$\begin{aligned}\boldsymbol{G} &= \gamma_1(\boldsymbol{H}_\mathrm{U}^\mathrm{T})^{-1}\boldsymbol{H}_\mathrm{U}^{-1} \\ \boldsymbol{F}_\mathrm{B} &= \gamma_2\boldsymbol{H}_\mathrm{B}^{-1}\boldsymbol{H}_\mathrm{U} \\ \boldsymbol{W}_\mathrm{B} &= \boldsymbol{H}_\mathrm{U}^\mathrm{T}(\boldsymbol{H}_\mathrm{B}^\mathrm{T})^{-1}\end{aligned} \tag{8}$$

where γ_1 and γ_2 are the normalizing coefficients satisfying the transmit power constraint at the relay and the BS, respectively.

However, it can be seen that the inverses of $\boldsymbol{H}_\mathrm{U}$ and $\boldsymbol{H}_\mathrm{B}$ do not always exist. Hence, this method can hardly be utilized since (8) requires that $M_\mathrm{R} = M_\mathrm{B} = K$. Our algorithms in Sections 3, 4, and 5 are applicable for a broader range of antenna configurations. We specify the corresponding dimensionality constraints below.

Moreover, to have a common framework for the proposed sub-optimal solutions, we decompose $\boldsymbol{G}$ into

$$\boldsymbol{G} = \boldsymbol{G}_\mathrm{T}\boldsymbol{G}_\mathrm{S}\boldsymbol{G}_\mathrm{R} \in \mathbb{C}^{M_\mathrm{R} \times M_\mathrm{R}} \tag{9}$$

3. CHANNEL INVERSION BASED DESIGN

In this section, we introduce a straightforward beamforming design based on channel inversion. Using this method, orthogonal channels are created between the BS and the UTs for interference free communication. This algorithm can efficiently eliminate the self-interference as well as the co-channel interference. However, the well-known disadvantage of it is the enhancement of the noise power.

Let us define $\boldsymbol{H} = [\boldsymbol{H}_\mathrm{B}\ \ \boldsymbol{H}_\mathrm{U}] \in \mathbb{C}^{M_\mathrm{R} \times (K+M_\mathrm{B})}$. Then the channel inversion receive beamforming is then given by

$$\boldsymbol{G}_\mathrm{R} = \boldsymbol{H}^+ = (\boldsymbol{H}^\mathrm{H}\boldsymbol{H})^{-1}\boldsymbol{H}^\mathrm{H} \tag{10}$$

and the transmit beamforming is given by $\boldsymbol{G}_{\rm T} = \boldsymbol{G}_{\rm R}^{\rm T}$.

In this case, the matrix $\boldsymbol{G}_{\rm S}$ is chosen to be a block matrix of the form $\boldsymbol{G}_{\rm S} = \boldsymbol{\Pi}_2 \otimes \boldsymbol{I}_K \in \mathbb{C}^{2\cdot K \times 2\cdot K}$, where $\boldsymbol{\Pi}_2 = \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$ is the exchange matrix which ensures that the BS and the UTs will not receive their own transmitted signals. Furthermore, for simplicity, we choose $\boldsymbol{F}_{\rm B} = \boldsymbol{W}_{\rm B} = \sqrt{\frac{P_{\rm B}}{M_{\rm B}}}\boldsymbol{I}_{M_{\rm B}}$. As can be seen from (10), this channel inversion method requires that $M_{\rm R} \geq 2K$. Moreover, compared to the other algorithms proposed in the following sections, the complexity for calculating the Moore-Penrose pseudo inverse is much lower.

4. BD COMBINED WITH ANOMAX

For simplicity, we again choose $\boldsymbol{F}_{\rm B} = \boldsymbol{W}_{\rm B} = \sqrt{\frac{P_{\rm B}}{M_{\rm B}}}\boldsymbol{I}_{M_{\rm B}}$ in this section. Let us further fix the order of the users such that the kth user communicates with the BS only via the kth antenna at the BS. Then this system can be regarded as multiple pairs of single-antenna users which communicate with each other with the help of the relay. This scheme is mathematically analogous to multi-pair two-way relaying in [8] or multi-operator two-way AF relaying in [9].

Although all proposed schemes in the above papers can be applied, we recommend BD combined with ANOMAX since according to our work in [9] for the multi-operator two-way relaying case (2 UTs per operator) it provides the best performance. Here we briefly extend this BD ANOMAX method to the multi-user two-way relaying case.

First, partition $\boldsymbol{G}_{\rm T}$, $\boldsymbol{G}_{\rm S}$, and $\boldsymbol{G}_{\rm R}$ as

$$\begin{aligned} \boldsymbol{G}_{\rm T} &= \left[\boldsymbol{G}_{\rm T}^{(1)}, \ \ldots, \ \boldsymbol{G}_{\rm T}^{(K)}\right] \in \mathbb{C}^{M_{\rm R} \times K M_{\rm R}} \\ \boldsymbol{G}_{\rm S} &= \text{blkdiag}\left\{\boldsymbol{G}_{\rm S}^{(1)}, \ \ldots, \ \boldsymbol{G}_{\rm S}^{(K)}\right\} \in \mathbb{C}^{K M_{\rm R} \times K M_{\rm R}} \\ \boldsymbol{G}_{\rm R} &= \left[\boldsymbol{G}_{\rm R}^{(1)^{\rm T}}, \ \ldots, \ \boldsymbol{G}_{\rm R}^{(K)^{\rm T}}\right]^{\rm T} \in \mathbb{C}^{K M_{\rm R} \times M_{\rm R}} \end{aligned} \tag{11}$$

where $\boldsymbol{G}_{\rm T}^{(k)}$, $\boldsymbol{G}_{\rm S}^{(k)}$, and $\boldsymbol{G}_{\rm R}^{(k)} \in \mathbb{C}^{M_{\rm R} \times M_{\rm R}}$.

BD ANOMAX consists of two steps. In the first step, the system is converted into K parallel independent sub-systems via the BD design of $\boldsymbol{G}_{\rm R}$ and $\boldsymbol{G}_{\rm T}$. Then, in the second step, for each single-pair two-way relaying sub-system, we use the ANOMAX algorithm to calculate $\boldsymbol{G}_{\rm S}^{(k)}$.

Let us define the combined channel matrix $\tilde{\boldsymbol{H}}^{(k)}$ for all UTs except for the kth UT as

$$\tilde{\boldsymbol{H}}^{(k)} = \left[\boldsymbol{H}^{(1)} \ \ldots \ \boldsymbol{H}^{(k-1)} \ \boldsymbol{H}^{(k+1)} \ \ldots \ \boldsymbol{H}^{(K)}\right], \tag{12}$$

where $\boldsymbol{H}^{(k)} = [\boldsymbol{h}_{{\rm B},k} \ \boldsymbol{h}_k]$ and $\boldsymbol{h}_{{\rm B},k}$ is the kth column of $\boldsymbol{H}_{\rm B}$.

Let $\tilde{L}^{(k)} = \text{rank}\{\tilde{\boldsymbol{H}}^{(k)}\}$ and calculate the singular value decomposition (SVD)

$$\tilde{\boldsymbol{H}}^{(k)} = [\tilde{\boldsymbol{U}}_{\rm s}^{(k)} \ \tilde{\boldsymbol{U}}_{\rm n}^{(k)}]\tilde{\boldsymbol{\Sigma}}^{(k)}\tilde{\boldsymbol{V}}^{(k)^{\rm H}}. \tag{13}$$

where $\tilde{\boldsymbol{U}}_{\rm n}^{(k)}$ contains the last $(M_{\rm R} - \tilde{L}^{(k)})$ left singular vectors. Thus, $\tilde{\boldsymbol{U}}_{\rm n}^{(k)}$ forms an orthogonal basis for the null space of $\tilde{\boldsymbol{H}}^{(k)}$. Therefore, we choose $\boldsymbol{G}_{\rm R}^{(k)} = \tilde{\boldsymbol{U}}_{\rm n}^{(k)}\tilde{\boldsymbol{U}}_{\rm n}^{(k)^{\rm H}} \in \mathbb{C}^{M_{\rm R} \times M_{\rm R}}$ which is a projection matrix that projects any matrix into the null space of $\tilde{\boldsymbol{H}}^{(k)}$. Due to the channel reciprocity, we can simply set $\boldsymbol{G}_{\rm T}^{(k)} = \boldsymbol{G}_{\rm R}^{(k)^{\rm T}}$.

Next, we define the matrix

$$\begin{aligned} \boldsymbol{K}_{\beta}^{(k)} = &\left[\beta((\boldsymbol{G}_{\rm R}^{(k)}\boldsymbol{h}_k) \otimes (\boldsymbol{G}_{\rm T}^{(k)^{\rm T}}\boldsymbol{h}_{{\rm B},k})),\right. \\ &\left.(1-\beta)((\boldsymbol{G}_{\rm R}^{(k)}\boldsymbol{h}_{{\rm B},k}) \otimes (\boldsymbol{G}_{\rm T}^{(k)^{\rm T}}\boldsymbol{h}_k)))\right]. \end{aligned} \tag{14}$$

which is needed to calculate the ANOMAX solution of $\boldsymbol{G}_{\rm S}^{(k)}$ [10]. The parameter $\beta \in [0, 1]$ is a weighting factor.

Then we compute the SVD of $\boldsymbol{K}_{\beta}^{(k)}$ as $\boldsymbol{K}_{\beta}^{(k)} = \boldsymbol{U}_{\beta}^{(k)}\boldsymbol{\Sigma}_{\beta}^{(k)}\boldsymbol{V}_{\beta}^{(k)^{\rm H}}$. Let the first column of $\boldsymbol{U}_{\beta}^{(k)}$, i.e., the dominant left singular vector of $\boldsymbol{K}_{\beta}^{(k)}$ be denoted by $\boldsymbol{u}_{\beta,1}^{(k)}$. According to the ANOMAX concept, the matrix $\boldsymbol{G}_{\rm S}^{(k)}$ is then obtained via

$$\boldsymbol{G}_{\rm S}^{(k)} = \text{unvec}_{M_{\rm R} \times M_{\rm R}}\{\boldsymbol{u}_{\beta,1}^{(k)^*}\}. \tag{15}$$

where the operator $\text{unvec}_{M_{\rm R} \times M_{\rm R}}\{\cdot\}$ inverts the $\text{vec}\{\cdot\}$ operation by forming a $M_{\rm R}$-by-$M_{\rm R}$ matrix $\boldsymbol{G}_{\rm S}^{(k)}$. In this paper we use equal weighting and therefore β is set to 0.5. This algorithm has the dimensionality constraint that $M_{\rm R} > (2K - 2)$.

5. ZFDPC BASED DESIGN

The multi-antenna BS has the ability of jointly encoding its transmitted data streams or jointly decoding of its received data streams. To further make use of this capability, we introduce the ZFDPC based beamforming design.

Let us partition $\boldsymbol{G}_{\rm R} = [\boldsymbol{G}_{\rm B}^{\rm T} \ \boldsymbol{G}_{\rm U}^{\rm T}]^{\rm T}$ and assume that $\boldsymbol{G}_{\rm T} = \boldsymbol{G}_{\rm R}^{\rm T}$. Moreover, let $L_{\rm U} = \text{rank}(\boldsymbol{H}_{\rm U})$ and define the SVD of $\boldsymbol{H}_{\rm U}$ as

$$\boldsymbol{H}_{\rm U} = [\boldsymbol{U}_{\rm U,s} \ \boldsymbol{U}_{\rm U,n}]\boldsymbol{\Sigma}_{\rm U}\boldsymbol{V}_{\rm U}^{\rm H} \in \mathbb{C}^{M_{\rm R} \times K}. \tag{16}$$

where $\boldsymbol{U}_{\rm U,n}$ contains the last $\bar{L}_{\rm U} = M_{\rm R} - L_{\rm U}$ left singular vectors. Thus, with the same reasoning as in Section 4, we choose $\boldsymbol{G}_{\rm B} = \boldsymbol{U}_{\rm U,n}\boldsymbol{U}_{\rm U,n}^{\rm H} \in \mathbb{C}^{M_{\rm R} \times M_{\rm R}}$.

Furthermore, let us define $\boldsymbol{G}_{\rm S} = \boldsymbol{\Pi}_2 \otimes \boldsymbol{I}_{M_{\rm R}} \in \mathbb{C}^{2\cdot M_{\rm R} \times 2\cdot M_{\rm R}}$ and $\boldsymbol{0}_{K \times K}$ to be the K-by-K matrix with all zero elements. Then the concatenated received signal at the BS and all UTs can be written as

$$\begin{aligned} \begin{bmatrix} \boldsymbol{y}_{\rm B} \\ \boldsymbol{y}_{\rm U} \end{bmatrix} &= \underbrace{\begin{bmatrix} \gamma_0 \boldsymbol{W}_{\rm B}\boldsymbol{H}_{\rm B}^{\rm T}\boldsymbol{G}\boldsymbol{H}_{\rm B}\boldsymbol{F}_{\rm B} & \boldsymbol{W}_B\boldsymbol{H}_B^{\rm T}\boldsymbol{G}_{\rm B}^{\rm T}\boldsymbol{G}_{\rm U}\boldsymbol{H}_{\rm U} \\ \boldsymbol{H}_U^{\rm T}\boldsymbol{G}_{\rm U}^{\rm T}\boldsymbol{G}_{\rm B}\boldsymbol{H}_{\rm B}\boldsymbol{F}_{\rm B} & \boldsymbol{0}_{K \times K} \end{bmatrix}}_{\boldsymbol{H}_{\rm eff}} \\ &\cdot \begin{bmatrix} \boldsymbol{d}_{\rm B} \\ \boldsymbol{d}_{\rm U} \end{bmatrix} + \tilde{\boldsymbol{n}} \in \mathbb{C}^{(M_{\rm B}+K)}. \end{aligned} \tag{17}$$

In equation (17), the first $M_{\rm B}$ rows represent the received signal at the BS ($\boldsymbol{y}_{\rm B}$). We further assume that the BS has perfect channel knowledge, and thus, the self-interference term which corressponds to the upper left block of $\boldsymbol{H}_{\rm eff}$ can be subtracted from $\boldsymbol{y}_{\rm B}$. Then, the system is further decomposed into two-sub systems where the upper right part is equivalent to the uplink of a one-way relay broadcast channel and the lower left part is equivalent to the downlink of a one-way relay multiple access channel. In the next step, we show how to design $\boldsymbol{G}_{\rm U}$, $\boldsymbol{F}_{\rm B}$, and $\boldsymbol{W}_{\rm B}$ using ZFDPC.

ZFDPC is a sub-optimal beamforming solution which has been used in several multi-user MIMO relaying references ([1], [2], [5]). Thus, we will also modify the ZFDPC design for our scenario.

First, we apply the QR decomposition and the SVD to the channel matrices $\boldsymbol{H}_{\rm U}^{\rm T}$ and $\boldsymbol{G}_{\rm B}\boldsymbol{H}_{\rm B}$ respectively,

$$\boldsymbol{H}_{\rm U}^{\rm T} = \boldsymbol{M}_{\rm U}\boldsymbol{Q}_{\rm U} \in \mathbb{C}^{K \times M_{\rm R}}, \tag{18}$$

where $\boldsymbol{M}_{\rm U}$ is a lower triangular matrix and $\boldsymbol{Q}_{\rm U}$ is a unitary matrix. The singular value decomposition of $\boldsymbol{G}_{\rm B}\boldsymbol{H}_{\rm B}$ is denoted by

$$\boldsymbol{G}_{\rm B}\boldsymbol{H}_{\rm B} = \boldsymbol{U}_{\rm B}\boldsymbol{\Sigma}_{\rm B}\boldsymbol{V}_{\rm B}^{\rm H} \in \mathbb{C}^{M_{\rm R} \times M_{\rm B}}. \tag{19}$$

Then the linear processing matrix $\boldsymbol{G}_{\rm U}$ can be expressed as:

$$\boldsymbol{G}_{\rm U} = \boldsymbol{U}_{\rm B}^*\boldsymbol{Q}_{\rm U}^* \in \mathbb{C}^{M_{\rm R} \times M_{\rm R}}. \tag{20}$$

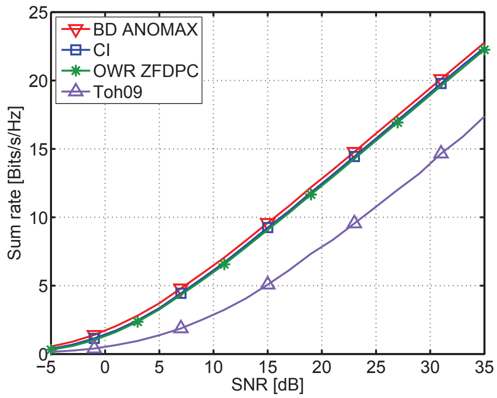

Fig. 2. Sum rate comparison for $M_\mathrm{R} = 8$ and $K = 2$.

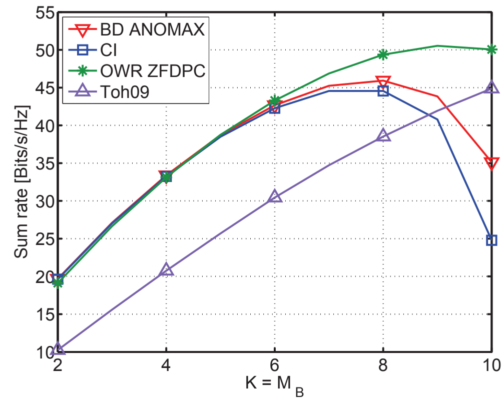

Fig. 3. Sum rate comparison for $M_\mathrm{R} = 20$ and $\mathrm{SNR} = 25$ dB.

Moreover, the precoding matrix $\boldsymbol{F}_\mathrm{B}$ is chosen as $\boldsymbol{F}_\mathrm{B} = \sqrt{\frac{P_\mathrm{B}}{M_\mathrm{B}}}\boldsymbol{V}_\mathrm{B}$ and the decoding matrix $\boldsymbol{W}_\mathrm{B}$ is constructed as $\boldsymbol{W}_\mathrm{B} = \boldsymbol{F}_\mathrm{B}^\mathrm{T} \in \mathbb{C}^{M_\mathrm{B} \times M_\mathrm{B}}$.

Inserting $\boldsymbol{G}_\mathrm{U}$, $\boldsymbol{F}_\mathrm{B}$ and $\boldsymbol{W}_\mathrm{B}$ into (17), the upper right matrix in $\boldsymbol{H}_\mathrm{eff}$ is converted into an upper-triangular matrix while the lower left part of it is converted into a lower-triangular matrix. Thus, for each UT, the interference can be canceled by applying a successive interference cancellation (SIC) receiver with perfect knowledge of the interfering signals. Assuming that the BS has also perfect knowledge of the interference signals, it can also utilize a SIC receiver to decode each data stream. Unfortunately, the ZFDPC design has also a dimensionality constraint, which means $M_\mathrm{R} \geq 2K$. Furthermore, since this is a non-linear algorithm, it has the highest computational complexity among the three proposed algorithms.

6. SUM RATE NUMERICAL RESULTS

In this section, the performance of the proposed algorithms is evaluated via Monte-Carlo simulations. The simulated MIMO flat fading channels $\boldsymbol{h}_k$ and $\boldsymbol{H}_\mathrm{B}$ are spatially uncorrelated Rayleigh fading channels. The SNRs at all nodes are defined as $\mathrm{SNR} = 1/\sigma_\mathrm{B}^2 = 1/\sigma_\mathrm{R}^2 = 1/\sigma_\mathrm{U}^2$. All the simulation results are obtained by averaging over 1000 channel realizations. “CI”, “BD ANOMAX”, “OWR ZFDPC”, and “Toh09” denote the algorithms in Section 3, 4, 5 and [4], respectively. Note that the curves labeled “Toh09” in our results are obtained by using the pseudo-inverse of $\boldsymbol{H}_\mathrm{B}$ and $\boldsymbol{H}_\mathrm{U}$ in (8).

As can be seen from Figure 2, “BD ANOMAX” provides the best performance and is 8 dB away form “Toh09” in the high SNR regime. The “OWR ZFDPC” curve is as good as “CI” and is close to “BD ANOMAX”. However, it should be noted that “OWR ZFDPC” has the highest complexity. Moreover, all the curves have the same slope at high SNRs which implies that they possess the same multiplexing gain.

Figure 3 show the system loading when $M_\mathrm{R} = 20$ and the SNRs at all nodes are 25 dB. It can be seen that due to the dimensionality constraint for “BD ANOMAX” and “CI”, there is an inflexion point after which increasing the number of UTs will decrease the system sum rate. For “OWR ZFDPC”, although there seems to be also an inflexion point when the system is heavily loaded (at $K = 9$), the sum rate does not drop as quickly as in the case of the other two algorithms.

7. CONCLUSION

In this paper, we consider multi-user two-way relaying with MIMO AF relays. We propose three different algorithms for computing the transmit and receive beamforming matrices at the base station as well as the linear amplification matrix at the relay. Among those “BD ANOMAX” provides the best balance between complexity and performance, “OWR ZFDPC” can still perform well in a heavily loaded system and “CI” yields the lowest complexity. All the algorithms outperform the recently proposed algorithm in [4].

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