

SUBSPACE-BASED ADAPTIVE WIDELY LINEAR BLIND CHANNEL ESTIMATION FOR CONSTRAINED MINIMUM VARIANCE CDMA RECEIVER

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ABSTRACT

We propose a subspace-based Widely Linear (WL) blind channel estimation scheme based on the iterative power method for the WL constrained minimum variance Code Division Multiple Access (CDMA) receiver. The novel technique approximates the noise subspace by using a matrix power and the WL processing fully exploits the second-order non-circularity of the signal. Two adaptive recursive least squares algorithms are developed using power iterations, which completely avoid the computationally intensive singular value decomposition. Simulation results show an improved performance of the proposed algorithms in terms of convergence and complexity as compared to their linear counterparts.

Index Terms— widely linear, non-circular, subspace, blind channel estimation, power method, constrained minimum variance

1. INTRODUCTION

In dynamic environments which experience Intra-/Inter-Symbol Interference (ISI) and Multi-User Interference (MUI), blind channel estimation schemes are quite attractive, since no transmission of the training symbols is required. Conventional subspace-based approaches to blind channel estimation mostly rely on the Singular Value Decomposition (SVD) to obtain the signal or noise subspace [1, 2, 3]. When a large processing gain of the system is applied, SVD-based methods for a large matrix, by contrast, will exhibit a very high computational complexity. Furthermore, these techniques also require the estimate of the subspace rank, e.g., by using information theoretic criteria such as the Akaike Information Criterion (AIC) or Minimum Description Length (MDL) criterion [4]. Erroneous rank estimates will also lead to a drastic performance degradation. Prior work in [5, 6, 7, 8, 9] proposed to approximate the noise subspace by replacing the SVD with a matrix power, which simplifies the optimization problem, but the channel estimate is still obtained by computing the SVD. An iterative power technique [10] has been proposed in [11], which completely eliminates the SVD and results in a substantial reduction of the computational complexity.

Channel estimation techniques often require the second-order statistics of the observation data vector \mathbf{r} , which can be fully described by its covariance matrix $\mathbf{R} = \mathbb{E}\{\mathbf{r}\mathbf{r}^H\}$ and its pseudo-covariance matrix $\tilde{\mathbf{R}} = \mathbb{E}\{\mathbf{r}\mathbf{r}^T\}$. In the situations when \mathbf{r} is second-order non-circular¹, i.e., $\tilde{\mathbf{R}} \neq \mathbf{0}$, Widely Linear (WL) processing can improve the performance as compared to the conventional linear approaches [12, 13, 14, 15, 16].

¹Non-circularity mentioned in this paper is constraint to the second-order case.

To enhance the estimation performance, several measurements of the received symbols are usually obtained at the receiver by means of oversampling or several sensors [5]. On the contrary, “virtual” measurements can be constructed by WL processing, which saves the hardware resources of the receiver front-end and more importantly leads to a significant performance gain for non-circular signals. Thus, WL subspace-based blind channel estimation becomes quite promising. Another advantage is that in the WL case, the inherent phase ambiguity that exists in conventional linear subspace methods, reduces to a sign ambiguity [17, 18]. Existing WL blind channel estimation schemes are based on the SVD [19, 20, 17, 18]. The complexity problem becomes more severe in the WL case, since both the original received signal \mathbf{r} and its complex conjugate \mathbf{r}^* have to be considered as measurements.

In this paper, we propose a new subspace-based WL blind channel estimation scheme based on the iterative power technique for a WL Constrained Minimum Variance (WL-CMV) receiver in a Direct Sequence Code Division Multiple Access (DS-CDMA) system. The proposed approach can also be applied to multiple-antenna and multi-carrier systems. The proposed scheme applies the matrix power in the WL sense to approximate the noise subspace, which fully exploits the advantages of non-circular signals. Instead of using the SVD, two WL adaptive Recursive Least Squares (RLS) algorithms are developed to obtain the channel estimate, namely Augmented-RLS (A-RLS) and a more efficient one Structured-RLS (S-RLS). We analyze the performance of the proposed algorithms for the WL-CMV receiver and compare them to their linear counterparts.

Notation: The superscripts T , H , and $*$ represent transpose, Hermitian transpose, and complex conjugation, respectively. We use a tilde above a variable to denote the associated augmented quantity. The trace of a matrix is denoted by $\text{tr}\{\cdot\}$ and the operation $\Re\{\cdot\}$ is to take the real part of a variable.

2. SYSTEM MODEL

We consider the uplink of a DS-CDMA system with K asynchronous users. In the complex baseband, the transmitted signal for the k -th user is given by

$$s_k^{(i)}(t) = b_k(i) \sum_{n=0}^{N-1} \sqrt{E_k} c_k(n) g(t - iT_b - nT_c), \quad (1)$$

where $b_k(i) \in \{\pm 1\}$ is the i -th Binary Phase Shift Keying (BPSK) symbol for the user k with unit variance, $g(t)$ is baseband reference pulse, T_b is the bit duration, E_k and $c_k(n) \in \{\pm 1/\sqrt{N}\}$ denote the

corresponding energy per bit and the multiple access code with chip interval T_c . The processing gain N is equal to T_b/T_c .

The impulse response of the multipath channel can be described by a tapped-delay line model and the channel vector can be written as $\mathbf{h}_k = [\alpha_k(0), \dots, \alpha_k(L-1)]^T \in \mathbb{C}^L$, where $\alpha_k(l)$ is the l -th complex channel tap for the k -th user and $\sum_{l=0}^{L-1} |\alpha_k(l)|^2 = 1$. After a pulse matched filter with the impulse response $g(T-t)$, for the i -th transmitted bit, the corresponding received vector of length $M = N + L - 1$ can be written as

$$\mathbf{r}(i) = \sqrt{E_1} b_1(i) \mathbf{C}_1 \mathbf{h}_1 + \mathbf{v}(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i), \quad (2)$$

including the signal from the desired user, the MUI part $\mathbf{v}(i)$, the ISI $\boldsymbol{\eta}(i)$, and the zero-mean, complex Additive White Gaussian Noise (AWGN) vector $\mathbf{n}(i)$ with a power spectral density N_0 . The code matrix for the k -th user $\mathbf{C}_k \in \mathbb{R}^{M \times L}$ is a Toeplitz matrix, which can be expressed as

$$\mathbf{C}_k^T = \begin{bmatrix} c_k(0) & \cdots & c_k(N) & 0 & \cdots & \cdots & 0 \\ 0 & c_k(0) & \cdots & c_k(N) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & c_k(0) & \cdots & c_k(N) \end{bmatrix}.$$

3. WIDELY LINEAR CHANNEL ESTIMATION

Figure 1 shows the block diagram of the WL receiver with blind channel estimation.

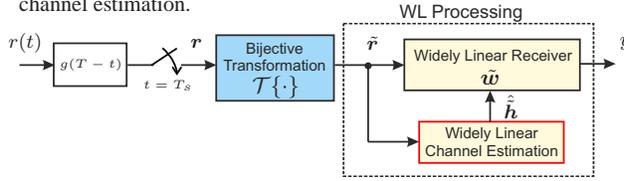


Fig. 1. Block diagram of the WL receiver with channel estimation.

In order to exploit the information contained in both second-order statistics, i.e., \mathbf{R} and $\tilde{\mathbf{R}}$, the received signal $\mathbf{r}(i)$ and its complex conjugate $\mathbf{r}^*(i)$ are combined into an augmented vector using a bijective transformation \mathcal{T} [21, 22]

$$\mathbf{r} \xrightarrow{\mathcal{T}} \tilde{\mathbf{r}} : \quad \tilde{\mathbf{r}} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{r}^T & \mathbf{r}^H \end{bmatrix}^T \in \mathbb{C}^{2M \times 1}, \quad (3)$$

where $1/\sqrt{2}$ is a scalar to normalize the augmented vector. The blind WL receiver can be obtained based on various criteria such as CMV, Constrained Constant Modulus (CCM) [23], Minimum Output Energy (MOE) [24] or other reduced-rank techniques [25, 26, 27].

3.1. WL-CMV Receiver

The output of a WL-CMV receiver is

$$\mathbf{y} = \tilde{\mathbf{w}}^H \tilde{\mathbf{r}}, \quad (4)$$

where the complex weight vector $\tilde{\mathbf{w}} \in \mathbb{C}^{2M}$ is calculated by solving the following constrained optimization problem

$$\begin{aligned} & \text{minimize } \mathbb{E}\{|y|^2\} = \tilde{\mathbf{w}}^H \tilde{\mathbf{R}} \tilde{\mathbf{w}} \\ & \text{s. t. } \tilde{\mathbf{w}}^H \tilde{\mathbf{C}}_1 \tilde{\mathbf{h}}_1 = 1, \end{aligned} \quad (5)$$

where $\tilde{\mathbf{C}}_1 = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_1 \end{bmatrix} / \sqrt{2}$ is the augmented code matrix and $\tilde{\mathbf{h}}_1$ is the augmented channel impulse response for the desired user.

The augmented covariance matrix $\tilde{\mathbf{R}}$ with a block structure is represented as

$$\tilde{\mathbf{R}} = \mathbb{E}\{\tilde{\mathbf{r}}(i)\tilde{\mathbf{r}}^H(i)\} = \frac{1}{2} \begin{bmatrix} \mathbf{R} & \tilde{\mathbf{R}} \\ \tilde{\mathbf{R}}^* & \mathbf{R}^* \end{bmatrix} \in \mathbb{C}^{2M \times 2M}. \quad (6)$$

For the non-circular data sources, $\tilde{\mathbf{R}} \neq \mathbf{0}$, which means that \mathbf{r} is second-order non-circular. The weight vector designed from (5) minimizes the output power while preserving the response of the desired user. The optimum solution is given by

$$\tilde{\mathbf{w}} = \frac{\tilde{\mathbf{R}}^{-1} \tilde{\mathbf{C}}_1 \tilde{\mathbf{h}}_1}{\tilde{\mathbf{h}}_1^H \tilde{\mathbf{C}}_1^H \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{C}}_1 \tilde{\mathbf{h}}_1}. \quad (7)$$

The corresponding maximum output Signal-to-Interference plus Noise Ratio (SINR) of the WL-CMV filter can be calculated as

$$\text{SINR}_{\text{max}}^{\text{WL-CMV}} = \frac{E_1 \tilde{\mathbf{h}}_1^H \tilde{\mathbf{C}}_1^H \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{C}}_1 \tilde{\mathbf{h}}_1}{1 - E_1 \tilde{\mathbf{h}}_1^H \tilde{\mathbf{C}}_1^H \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{C}}_1 \tilde{\mathbf{h}}_1}. \quad (8)$$

It has been shown in [21, 28] that when the data to be estimated is real, $\tilde{\mathbf{w}}$ follows the transformation defined in (3) such that $\tilde{\mathbf{w}} = [\tilde{\mathbf{w}}^T, \tilde{\mathbf{w}}^H]^T / \sqrt{2}$, where $\tilde{\mathbf{w}} \in \mathbb{C}^{M \times 1}$. Therefore, a key property of the WL filtering is the conjugate symmetry defined as $\tilde{\mathbf{w}}^H \tilde{\mathbf{r}} = \tilde{\mathbf{r}}^T \tilde{\mathbf{w}}^* = \Re\{\tilde{\mathbf{w}}^H \tilde{\mathbf{r}}\}$. Thus, $\tilde{\mathbf{w}}$ minimizes $\mathbb{E}\{|\Re\{y\}|^2\}$ and equivalently maximizes the output SINR:

$$\text{SINR}_{\text{max}}^{\text{WL-CMV}} = \frac{\mathbb{E}\{|\Re\{\tilde{\mathbf{w}}^H \tilde{\mathbf{s}}\}|^2\}}{\mathbb{E}\{|\Re\{\tilde{\mathbf{w}}^H \tilde{\mathbf{j}}\}|^2\}} \quad (9)$$

where $\tilde{\mathbf{s}}$ and $\tilde{\mathbf{j}}$ represent the augmented desired signal and the interference plus noise, respectively. We have shown in [26] that the maximum output SINRs of the WL-CMV and the L-CMV satisfy $\text{SINR}_{\text{max}}^{\text{WL-CMV}} \geq 2 \text{SINR}_{\text{max}}^{\text{L-CMV}}$, i.e., the WL-CMV provides at least 3 dB gain over the L-CMV.

3.2. Subspace-based Method

The WL-CMV solution in (7) requires the estimation of the channel impulse response from the desired user. In the following we will present the subspace-based method for the WL blind channel estimation. The augmented covariance matrix $\tilde{\mathbf{R}}$ can be decomposed by using the SVD as

$$\tilde{\mathbf{R}} = \begin{bmatrix} \tilde{\mathbf{U}}_s & \tilde{\mathbf{U}}_n \end{bmatrix} \begin{bmatrix} \tilde{\Lambda}_s + \frac{N_0}{2} \mathbf{I}_K & \mathbf{0} \\ \mathbf{0} & \frac{N_0}{2} \mathbf{I}_{2M-K} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{U}}_s & \tilde{\mathbf{U}}_n \end{bmatrix}^H, \quad (10)$$

where $\tilde{\Lambda}_s = \text{diag}\{\lambda_1, \dots, \lambda_K\}$ contains the singular values corresponding to the signal subspace and $\tilde{\mathbf{U}}_s$ as well as $\tilde{\mathbf{U}}_n$ span the WL augmented signal and noise subspaces, respectively. Due to the orthogonality of two subspaces, we can obtain $\tilde{\mathbf{U}}_n \tilde{\mathbf{C}}_1 \tilde{\mathbf{h}}_1 = \mathbf{0}$, which leads to the following optimization problem

$$\hat{\tilde{\mathbf{h}}}_1 = \arg \min_{\tilde{\mathbf{h}}_1} \tilde{\mathbf{h}}_1^H \tilde{\mathbf{C}}_1^H \tilde{\mathbf{U}}_n \tilde{\mathbf{U}}_n^H \tilde{\mathbf{C}}_1 \tilde{\mathbf{h}}_1, \quad \|\tilde{\mathbf{h}}_1\| = 1. \quad (11)$$

The estimated channel $\hat{\tilde{\mathbf{h}}}_1$ in the WL sense is the singular vector corresponding to the smallest singular value of the matrix $\mathbf{W} = \tilde{\mathbf{C}}_1^H \tilde{\mathbf{U}}_n \tilde{\mathbf{U}}_n^H \tilde{\mathbf{C}}_1$. To solve the above problems, i.e., obtaining the noise subspace $\tilde{\mathbf{U}}_n$ by estimating the rank of the dominant subspace and the recovery of the channel estimate, a computationally expensive SVD is required [2]. It has also been shown in [8] that a slight rank estimation error will result in a significant performance degradation.

Remark: Traditional linear subspace-based channel estimation suffers from a phase ambiguity. For the WL case the phase ambiguity reduces to a sign ambiguity [17, 18]. Such an ambiguity can be resolved by differential encoding or the partial knowledge of the channel coefficients, e.g., the first one or the one with the largest magnitude.

3.2.1. The Power of the R Concept in the WL Case

The power of the R concept was introduced for the linear case in [9]. For WL processing, to avoid estimating the noise subspace $\tilde{\mathbf{U}}_n$ directly, we introduce the following Lemma to approximate $\tilde{\mathbf{U}}_n$ by the power of the matrix $\tilde{\mathbf{R}}$.

Lemma: If the augmented covariance matrix $\tilde{\mathbf{R}}$ can be decomposed through the SVD as in (10), it holds that

$$\lim_{m \rightarrow \infty} \left(\frac{N_0}{2} \right)^m \tilde{\mathbf{R}}^{-m} = \tilde{\mathbf{U}}_n \tilde{\mathbf{U}}_n^H, \quad m = 1, 2, \dots \quad (12)$$

Proof: Applying (12) in (11) we can obtain

$$\begin{aligned} & \lim_{m \rightarrow \infty} \left(\frac{N_0}{2} \right)^m \tilde{\mathbf{R}}^{-m} \\ &= \lim_{m \rightarrow \infty} \tilde{\mathbf{U}}_s \text{diag} \left\{ \left(\frac{N_0/2}{\lambda_i + N_0/2} \right)^m \right\}_{i=1, \dots, K} \tilde{\mathbf{U}}_s^H + \tilde{\mathbf{U}}_n \tilde{\mathbf{U}}_n^H \\ &= \tilde{\mathbf{U}}_n \tilde{\mathbf{U}}_n^H, \end{aligned}$$

where the last equality holds since the term $\frac{N_0/2}{\lambda_i + N_0/2}$ is less than unity. Thus, the optimization problem in (11) can be simplified to

$$\hat{\mathbf{h}}_1 = \arg \min_{\tilde{\mathbf{h}}_1} \tilde{\mathbf{h}}_1^H \mathbf{W} \tilde{\mathbf{h}}_1, \quad \|\tilde{\mathbf{h}}_1\| = 1 \quad (13)$$

with

$$\mathbf{W} = \tilde{\mathbf{C}}_1^H \tilde{\mathbf{R}}^{-m} \tilde{\mathbf{C}}_1 \in \mathbb{C}^{L \times L}. \quad (14)$$

3.2.2. The Iterative Power Method in the WL Case

In [10], the iterative power method is used to estimate the largest singular value of a diagonalizable matrix. Following the idea in [11], instead of using the SVD we can apply the modified power method in the WL case to solve the optimization problem in (13) as follows:

$$\tilde{\mathbf{h}}_1(i) = \frac{(\mathbf{I}_{2L} - \beta \mathbf{W}) \tilde{\mathbf{h}}_1(i-1)}{\|(\mathbf{I}_{2L} - \beta \mathbf{W}) \tilde{\mathbf{h}}_1(i-1)\|}, \quad (15)$$

where $\beta = 1/\text{tr}\{\mathbf{W}\}$ and $\tilde{\mathbf{h}}_1(0)$ is orthogonal to \mathbf{h}_1 . Similarly as in [11], the recursion $\tilde{\mathbf{h}}_1(i)$ will converge to the augmented channel impulse response $\tilde{\mathbf{h}}_1$ with a sign ambiguity. It should be noted that by using the iteration in (15), we avoid the computation of the SVD to solve the problem of (13), which simplifies the implementation.

3.3. Adaptive WL Blind Channel Estimation Algorithms

Adaptive algorithms are of great interest due to their efficient implementation and good tracking performance in dynamic scenarios. In the adaptive case, the iterative power method in (15) should also be updated with

$$\mathbf{W}(i) = \tilde{\mathbf{C}}_1^H \tilde{\mathbf{R}}^{-m(i)} \tilde{\mathbf{C}}_1, \quad m = 1, 2, 3, \quad \beta(i) = \frac{1}{\text{tr}\{\mathbf{W}(i)\}}. \quad (16)$$

It has been shown in [11] that for the linear case, it is sufficient to use the power index up to $m = 3$. To estimate $\tilde{\mathbf{R}}^{-1}(i)$, two adaptive algorithms, namely A-RLS and S-RLS, are developed for the WL blind channel estimation based on the power method.

3.3.1. Augmented RLS (A-RLS)

One straightforward way is to apply the RLS adaptation based on the augmented received vector $\tilde{\mathbf{r}}(i)$, i.e., the A-RLS algorithm. The adaptations of both (15) and the WL-CMV solution (7) require estimating the inverse of a matrix. According to the matrix inversion lemma, for example, we can update $\tilde{\mathbf{R}}^{-1}(i)$ as

$$\tilde{\mathbf{R}}^{-1}(i) = \lambda^{-1} \tilde{\mathbf{R}}^{-1}(i-1) - \lambda^{-1} \mathbf{k}(i) \tilde{\mathbf{r}}^H(i) \tilde{\mathbf{R}}^{-1}(i-1), \quad (17)$$

where the gain vector is

$$\mathbf{k}(i) = \frac{\lambda^{-1} \tilde{\mathbf{R}}^{-1}(i-1) \tilde{\mathbf{r}}(i)}{1 + \lambda^{-1} \tilde{\mathbf{r}}^H(i) \tilde{\mathbf{R}}^{-1}(i-1) \tilde{\mathbf{r}}(i)} \quad (18)$$

and λ is the forgetting factor which is a positive constant close to but less than 1.

3.3.2. Structured RLS (S-RLS)

In A-RLS, the calculation of $\tilde{\mathbf{R}}^{-1}(i)$ requires the calculation of parameters with a dimension of $2M$, which is computationally inefficient especially when M is large. By exploiting the structured property of the augmented covariance matrix $\tilde{\mathbf{R}}$ as shown in (6), the adaptive estimation algorithm can be implemented in a much more efficient way [29]. Let us rewrite $\tilde{\mathbf{R}}^{-1}(i)$ as

$$\tilde{\mathbf{R}}^{-1}(i) = \begin{bmatrix} \mathbf{P}(i) & \mathbf{Q}(i) \\ \mathbf{Q}^*(i) & \mathbf{P}^*(i) \end{bmatrix}, \quad (19)$$

where it follows that $\mathbf{P} = \mathbf{P}^H$ and $\mathbf{Q} = \mathbf{Q}^T$. Thereby, the estimation of $\tilde{\mathbf{R}}^{-1}(i)$ can be broken down into the calculation of $\mathbf{P}(i)$ and $\mathbf{Q}(i)$, respectively, so as to reduce the computational complexity. By inserting (19) into (17), we can obtain

$$\mathbf{P}(i) = \lambda^{-1} \left(\mathbf{P}(i-1) - c^{-1}(i) \mathbf{x}(i) \mathbf{x}^H(i) \right) \quad (20)$$

$$\mathbf{Q}(i) = \lambda^{-1} \left(\mathbf{Q}(i-1) - c^{-1}(i) \mathbf{x}(i) \mathbf{x}^T(i) \right), \quad (21)$$

where

$$\mathbf{x}(i) = \mathbf{P}(i-1) \mathbf{r}(i) + \mathbf{Q}(i-1) \mathbf{r}^*(i) \quad (22)$$

$$c(i) = \lambda + 2 \cdot \Re \left\{ \mathbf{x}^H(i) \mathbf{r}(i) \right\}. \quad (23)$$

Moreover, inserting (19) into (16) as well as (15) and by using the property of conjugate symmetry, we obtain

$$\tilde{\mathbf{h}}_1(i) = \frac{(\mathbf{I}_L - \tilde{\beta}(i) \mathbf{C}_1^H \mathbf{P}(i) \mathbf{C}_1) \tilde{\mathbf{h}}_1(i-1) - \tilde{\beta}(i) \mathbf{C}_1^H \mathbf{Q}(i) \mathbf{C}_1 \tilde{\mathbf{h}}_1^*(i-1)}{\|(\mathbf{I}_L - \tilde{\beta}(i) \mathbf{C}_1^H \mathbf{P}(i) \mathbf{C}_1) \tilde{\mathbf{h}}_1(i-1) - \tilde{\beta}(i) \mathbf{C}_1^H \mathbf{Q}(i) \mathbf{C}_1 \tilde{\mathbf{h}}_1^*(i-1)\|}, \quad (24)$$

where $\tilde{\beta}(i) = 1/\Re \left\{ \text{tr} \left\{ \mathbf{C}_1^H \mathbf{P}(i) \mathbf{C}_1 \right\} \right\}$. The expression for $\tilde{\mathbf{h}}_1(i)$ breaks the calculation of $\mathbf{W}(i)$ in (16) from $2M$ down to M , which reduces the computational complexity as compared to the A-RLS. The augmented channel estimate follows $\tilde{\mathbf{h}}_1(i) = \mathcal{T} \left\{ \tilde{\mathbf{h}}_1(i) \right\}$.

Remark 1: The initializations for the proposed algorithms are chosen as $\tilde{\mathbf{R}}^{-1}(0) = \delta_a \mathbf{I}_{2M}$, $\mathbf{P}(0) = \delta_p \mathbf{I}_M$, $\mathbf{Q}(0) = \delta_q \mathbf{I}_M$, where $\delta_a, \delta_p, \delta_q$ are initialization scalars to ensure the numerical stability. In order to keep the conjugate structure of WL processing, the channel estimate is initialized as $\tilde{\mathbf{h}}_1(0) = \mathcal{T} \left\{ \tilde{\mathbf{h}}_1(0) \right\}$, where the first coefficient of $\tilde{\mathbf{h}}_1(0)$ is set to 1 and the others to zero.

Remark 2: In the adaptive algorithms, the updates of $\tilde{\mathbf{R}}^{-1}(i)$ and $\mathbf{P}(i), \mathbf{Q}(i)$ will also be used in (7) for the A-RLS and S-RLS versions of the receiver, respectively.

4. COMPLEXITY ANALYSIS

The computational complexity of the proposed blind channel estimation algorithms and their linear counterparts is estimated and compared in Table 1. Fig. 2 illustrates the total number of complex additions and multiplications per iteration per symbol as a function of M , where the channel length is chosen as $L = 3$ and the matrix power $m = 1, 2, 3$. It can be observed that the complexity of WL-A-RLS is higher than the L-RLS, since the most computational complex part is the computation of $\mathbf{W}(i)$. The WL-S-RLS has a lower complexity than the WL-A-RLS, while with $m = 1$ it exhibits a slightly lower complexity than the L-RLS with $m = 3$.

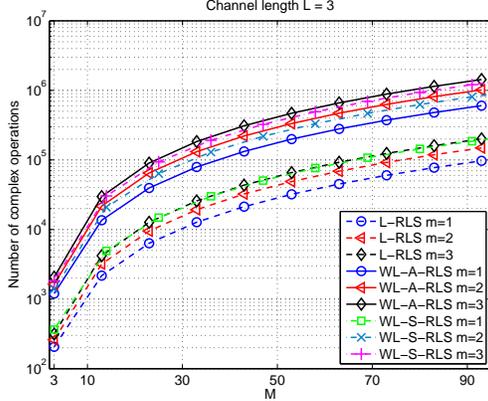


Fig. 2. Computational complexity in terms of complex additions and multiplications per iteration per symbol versus M .

Table 1. Computational Complexity Analysis

Algorithms	Additions	Multiplications
L-RLS	$(mL + 2)M^2 + (L^2 + 1)M + L^2 + 3L + 1$	$(mL + 3)M^2 + (L^2 + 2)M + 2L^2 + 2L$
WL-A-RLS	$8(mL + 1)M^2 + 2(4L^2 + 1)M + 4L^2 + 6L + 1$	$4(2mL + 3)M^2 + 4(4L^2 + 1)M + 8L^2 + 4L$
WL-S-RLS ($m = 1$)	$(2L + 4)M^2 + 2(L^2 + 1)M + L^2 + 4L + 1$	$(2L + 6)M^2 + (2L^2 + 3)M + 2L^2 + L + 1$
WL-S-RLS ($m = 2, 3$)	$(8mL + 4)M^2 + 2(4L^2 + 1)M + 4L^2 + 6L + 1$	$(8mL + 6)M^2 + (8L^2 + 3)M + 8L^2 + 4L + 1$

5. SIMULATIONS

We consider a CDMA system with $K = 12$ users and only strictly non-circular BPSK-modulated signals. Randomly generated sequences of length $N = 32$ are used as spreading codes. The multipath channel is assumed time-invariant (block fading) with a length of $L = 3$ and a power delay profile of $[0, -3, -6]$ dB. For the adaptive algorithms we set the forgetting factor to $\lambda = 0.998$. Fig. 3 shows the Mean Square Error (MSE) performance of the channel estimation for both the linear and WL schemes in a dynamic environment, where at bit 1000, 6 additional users enter the channel, having a power of 10 dB stronger. We can observe that the proposed WL algorithms outperform their linear counterparts and are less vulnerable to such dynamic situation. Increasing the power index m provides a better estimation performance and $m = 2$ is sufficient for the WL-A-RLS method, since almost no improvement can be obtained with $m = 3$. The WL-S-RLS algorithm with $m = 1$ shows

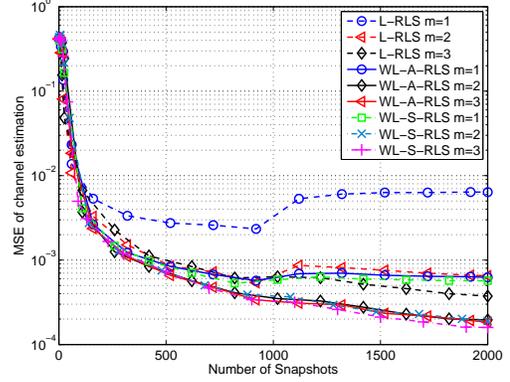


Fig. 3. The MSE performance for the proposed WL blind channel estimation methods at SNR = 12 dB.

a slightly better performance than the corresponding WL-A-RLS but has a much lower complexity.

The output SINR performance of the WL-CMV receiver using the proposed blind channel estimation methods is shown in Fig. 4, where no additional users enter the system. The maximum SINR with perfect channel state information is denoted as a reference for both linear and WL cases. We can see that the WL-CMV receiver that uses A-RLS/S-RLS algorithms provides at least a 3 dB gain over the L-CMV one.

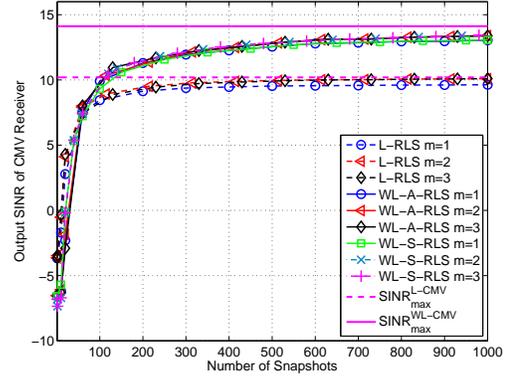


Fig. 4. The output SINR of the WL-CMV receiver using the proposed WL blind channel estimation methods at SNR = 12 dB.

6. CONCLUSIONS

In this work, we propose a novel subspace-based WL blind channel estimation scheme based on the iterative power technique for a WL-CMV CDMA receiver. The proposed technique completely avoids the computation of SVD in estimating the noise subspace as well as in obtaining the channel estimate and takes full advantage of the non-circular signals. The channel impulse response is adaptively updated by two developed RLS algorithms based on the iterative power method, namely A-RLS and S-RLS. Simulation results show that the proposed WL schemes outperform their linear counterparts and the S-RLS algorithm provides a good trade-off in terms of performance and complexity.

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