Are LAS-codes a miracle ?

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***Abstract*—Large Area Synchronized (LAS) - CDMA has been proposed to enhance third generation and fourth generation wireless systems. LAS-CDMA is based on multiple access codes that result from a combination of LA codes and pulse compressing LS codes. To reduce multiple access interference and inter-symbol interference in time dispersive channels, LS codes have perfect auto-correlation and cross-correlation functions in a certain vicinity of the zero shift. In this paper, we provide systematic methods and the underlying theory for the construction of such codes that go far beyond the examples revealed by LinkAir in [1].**

I. Introduction

Recently the Chinese-American start-up company LinkAir[1] has presented LAS-CDMA (Large Area Synchronized - Code Division Multiple Access). They claim that "LAS-CDMA significantly outperforms all proposed third generation (3G) standards"[2] and propose LAS-CDMA also as the basis for future all IP networks and the Fourth Generation (4G) of wireless communications. LAS-CDMA modes have been defined for the major IMT-2000 standards: UTRA FDD (W-LAS), UTRA TDD (TD-LAS), and cdma2000 (LAS-2000).

It is well known that current CDMA systems are interference limited. There is inter-symbol-interference (ISI) due to the non-zero auto-correlation sidelobes of the spreading sequences and multiple-access-interference (MAI) due to the non-zero cross-correlations. Consequently, to suppress both ISI and MAI, the sidelobes of the auto-correlation and the cross-correlations should be as small as possible. Thereby, the "near far effect" could be reduced significantly and the spectral efficiency of the system could be enhanced tremendously. In the ideal case, all the auto-correlation sidelobes and the cross-correlations are zero so that independent CDMA channels are created. However, it has been proven [2] that such "perfect" bipolar, and even unimodular complex sequences do not exist. Moreover, it is well known that the auto-correlations and the cross-correlations contradict one another so that smaller ISI leads to larger MAI and vice versa.

In case of the LAS-CDMA, the aperiodic auto-correlation sidelobes and the cross-correlations of the so-called LS-codes are zero within the interference free window (IFW), i.e., within the interval $\{-d,\ldots,+d\}$ of length $W = 2d+1$, where d is the maximum delay dispersion of the channel. To achieve this goal, it is necessary to insert "guard" intervals (or zero gaps) between sequences of length equal to the maximum dispersion delay of the channel. Unfortunately, these preventive measures lead to an undesired reduction of both spectral efficiency and energy efficiency of the sequences.

Another problem is that the number K of such perfect $\{0,\pm1\}$ sequences is bounded above by $L'/(d+1)$, where L' is the length of the sequences without the zero gap. Hence, for a code length $L' = 128$ and a relatively small interference free window of $W = 7$, only $K = 32$ LS codes can be found. In case of LAS-CDMA, the LS codes are combined with LA codes to give a set of LAS codes, which in general does not have the perfect behavior. All LA codes are derived from the so-called primary code, whose construction is based on methods used for optical codes ($\{0,1\}$ sequences) with small sidelobes of aperiodic correlation functions. Consequently, the primary code must have large intervals (zero gaps) between two adjacent pulses, where the minimal interval is equal to $L = L' + W_0$, where W_0 denotes the size of the zero gap in the middle of the sequences. This leads to a further reduction of the energy efficiency and very long sequences. For instance, if $L = 132$ and $d = 4$, LAS codes have the length 2559 with zero gaps between LS codes ranging from 4 to 40 zeros. The design of LA-codes (or optical codes) is beyond the scope of this paper.

The objective of this paper is to provide the general theory for the construction of sequences with perfect aperiodic correlation functions within the IFW, which goes far beyond the examples revealed by LinkAir [1]. We think that such a theory is necessary for the construction of large sets of sequences with small sidelobes within the IFW and a better energy efficiency than LAS codes.

II. Demands on spreading sequences

In a CDMA communication system, each of the K users, say user $k, 1 \le k \le K$, is assigned a distinct signature sequence $g_k := \{g_k(0),\ldots,g_k(L-1)\}$. This paper deals with sequences, whose components are equal to ±1 or 0. It is assumed that $|g_k(0)| = |g_k(L-1)| = 1$ and that all sequences have the same energy efficiency

$$\eta(\mathcal{S}) := \frac{E(\mathcal{S})}{L},\ E(\mathcal{S}) := E_1 =,\ldots,= E_K, \qquad (1)$$

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[1] http://www.linkair.com/

[2] http://www.linkair.com/technology/overview.html

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where $E_k = \sum_{i=0}^{L-1} |g_k(i)|^2$ is the energy of the sequence g_k and $\mathcal{S}$ is used to denote a set of the sequences $g_1, \ldots, g_K$. It is well known [3] that the ISI and the MAI in non-synchronous CDMA systems strongly depend on the aperiodic correlations of the spreading sequences. The j-th aperiodic cross-correlation $R_{g_k,g_l}(m) = R_{g_l,g_k}(-m)$ of the sequences g_k and g_l (not necessarily distinct) is given by

$$R_{g_k,g_l}(m) = \sum_{i=0}^{L-m-1} g_k(i)\, g_l(i+m),\ 0 \le m < L \qquad (2)$$

with $R_{g_k,g_l}(m) = 0$ if $|m| \ge L$. If $g_k = g_l$, $R_{g_k}(m) := R_{g_k,g_k}(m)$ is called the j-th aperiodic auto-correlation of the sequence g_k.

An inherent part of many wireless communication systems is the multipath channel, which, roughly speaking, causes the transmitted signal to be received via many different paths. Unfortunately, these signal components are usually prevented from arriving simultaneously at the receiver since they have to travel over different path lengths. Without loss of generality, let us suppose that the arrival time of the first path is 0. Consequently, the arrival time of the last path is equal to the maximum delay dispersion of the multipath channel, which we denote as T_d. On the uplink, there can be an additional time delay T_m due to the fact that only a coarse synchronization among spatially separated mobile users can be established. In our discussion, however, we will assume $T_m = 0$ since T_m is usually significantly smaller than T_d [3]. Consequently, the arrival time of the last significant path expressed in the number of chips is $\tau := \lfloor T_d/T_c \rfloor + 1$, where T_c denotes the chip duration. Apparently, we have $\tau = 1$ ($T_d = 0$) in synchronous CDMA (S-CDMA) systems, and $\tau >> L$ in the so-called asynchronous (A-CDMA) ones. As for the LAS-CDMA, all signals must arrive within the $IFW = \{-d, \ldots, d\}$ so we have $\tau = d+1$. If we assume that the users transmit symbols at the rate R using a 2^m-ary modulation constellation, we obtain $d/L \approx \frac{1}{m} T_d R$. Since d only accounts for a small fraction of L, modulation constellations of higher order ($m = 4, 5, 6$) must be employed to achieve high data rates. Alternatively, the multi-code technique could also be used, which, however, requires the availability of a large number of spreading sequences.

Note that, as long as $\tau > 1$, spreading sequences transmitted one after another without any guard interval will eventually overlap, which is independent of how small the relative delays are. Such overlapping results in the need of minimizing $\max_{1 \le j \le d} R_{g_k,g_l}(j - L)$ for every pair of sequences $g_k, g_l \in \mathcal{S}$. An important observation is that this quantity can never be zero since $|R_{g_k,g_l}(L-1)| = |R_{g_k,g_l}(1-L)| = 1$. Consequently, independent (orthogonal) CDMA channels can be established if and only if a guard interval (a zero gap) is inserted between any two consecutive sequences preventing them from overlapping. Of course, the length of this guard interval must be at least equal to d. In the sequel, we always assume such a guard interval.

[3]A good synchronization can be achieved through the use of the GPS-based common clock, or alternatively, the aid of higher-layers network protocols.

To assess the aperiodic correlation properties of sets of sequences, it is common practice [3] to use the maximum criteria. The proceeding discussion shows that the aperiodic correlations $R_{g_k,g_l}(m)$ with $|m| > d$ do not need to be considered. As a consequence, for a given fixed d, we need to find a set of sequences $\mathcal{S}$ that minimizes

$$0 \le C(d, \mathcal{S}) = \max\{C_a(d, \mathcal{S}), C_c(d, \mathcal{S})\}, \qquad (3)$$

where

$$\begin{aligned} C_a(d, \mathcal{S}) &= \max_{g_k \in \mathcal{S}} \left(\max_{1 \le m \le d} |R_{g_k}(m)| \right) \\ C_c(d, \mathcal{S}) &= \max_{g_k, g_l \in \mathcal{S}} \left(\max_{0 \le m \le d} |R_{g_k,g_l}(m)| \right), g_k \ne g_l. \end{aligned}$$

Note that the use of any set of sequences $\mathcal{S}$ with $C(d, \mathcal{S}) = 0$ completely eliminates the ISI and the MAI. In this case, independent CDMA channels are created so that the target signal-to-interference ratios can be met by increasing the signal power of the users. Otherwise, apart from minimizing the ISI and the MAI, we also have to maximize the energy efficiency $\eta(\mathcal{S})$ so that

$$0 \le \tilde{C}(d, \mathcal{S}) = \frac{C(d, \mathcal{S})}{\eta(\mathcal{S})}. \qquad (4)$$

should be taken as the basic measure of the aperiodic correlation properties of sequences.

III. LS Code Construction

In this section, we present systematic methods of constructing sets of sequences $\mathcal{S}$ such that $C(d, \mathcal{S}) = 0$ for some given d. We confine our attention to sequences that, like LS codes, result from the concatenation of shorter sequences with certain aperiodic correlation properties. We will first construct two sequences p and q such that $R_p(m) = R_q(m) = 0$, where $1 \le m \le d$, and $R_{p,q}(m) = 0$ for $0 \le m \le d$ [4]. In doing so, we require that the energy efficiency of the sequences is maximum. Let

$$\begin{aligned} p(n) &= c_0(n) + s_0(n - N - W_0) \qquad (5) \\ q(n) &= c_1(n) + s_1(n - N - W_0), \qquad (6) \end{aligned}$$

where c_0, c_1, s_0, s_1 are bipolar sequences (± 1) of length $N := N(d)$ and $W_0 := W_0(d) \in \mathbb{N}_0$. Note that $R_{p,q}(0) = 0$ is the orthogonality condition, which implies that $N > 1$ must be an *even* positive integer. Furthermore, one conjectures that sets of orthogonal bipolar sequences exist if and only if $N = 0 \,(\mathrm{mod}\, 4)$. It may be easily shown [2] that the aperiodic auto-correlations and the cross-correlations are given by

$$\begin{aligned} R_p(m) &= R_{c_0}(m) + R_{c_0,s_0}(m - N - W_0) \\ &+ R_{s_0}(m) + R_{s_0,c_0}(m + N + W_0). \qquad (7) \end{aligned}$$

[4]Note that we may confine our attention to $0 \le m \le d$

and

$$\begin{aligned} R_{p,q}(m) &= R_{c_0,c_1}(m) + R_{c_0,s_1}(m-N-W_0) \\ &+ R_{s_0,s_1}(m) + R_{c_1,s_0}(m+N+W_0), \quad (8) \end{aligned}$$

respectively. The aperiodic auto-correlation $R_q(m)$ is obtained in the same fashion as $R_p(m)$. Note that p and q are bipolar (± 1) sequences if and only if $W_0 = 0$ in (5) and (6). Otherwise, there is a gap of W_0 zeros between the sequences c_0 and s_0 as well as c_1 and s_1, which leads to a decrease of the energy efficiency $\eta_p = \eta_q$. Thus, it is desirable to keep W_0 as small as possible. Unfortunately, a careful examination of (7) and (8) reveals that bipolar sequences can never have the desired correlation properties. To see this, it is sufficient to consider the aperiodic auto-correlation $R_p(m)$ for $W_0 = 0$ and $1 \le m < 2N$. Since $R_{s_0,c_0}(m+N) = 0$ for $m \ge 0$, we actually have $R_p(m) = R_{c_0}(m) + R_{s_0}(m) + R_{c_0,s_0}(m-N)$ for $1 \le m < 2N$. Since N is even, it is well known that $R_{c_0}(m) = R_{s_0}(m) = R_{c_0,s_0}(m-N) = 1(\text{mod } 2)$ whenever $m = 1(\text{mod } 2), 1 \le m < 2N$. In other words, if m is odd, the aperiodic correlations of bipolar sequences of even length are also odd. Consequently, $R_p(m) = 1(\text{mod } 2) + 1(\text{mod } 2) + 1(\text{mod } 2) = 1(\text{mod } 2) \ne 0$ if m is an odd positive integer.

The basic idea of the LS-codes is the insertion of zeros to avoid that the sequences c_0 and c_1 overlap with the sequences s_0 and s_1. Since d is assumed to be the maximum delay dispersion of the channel, this is achieved if $W_0 \ge d$, and then $R_{c_0,s_0}(m-N-W_0) = R_{s_0,c_0}(m+N+W_0) = 0$ for $|m| \le d$. In order not to decrease the energy of the sequences, it is reasonable to let $W_0 = d$. Note that although the energy efficiency of such sequences is not the optimal one, the cost functional (4) can still attain its minimum if the numerator is equal to zero. To accomplish this, it follows from (7) and (8) for $W_0 = d$ that it is sufficient to construct a quadruplet of sequences c_0, c_1, s_0, s_1 of length N such that

$$\begin{aligned} C_0(z)C_0(z^{-1}) + S_0(z)S_0(z^{-1}) &= 2N \quad (9) \\ C_1(z)C_1(z^{-1}) + S_1(z)S_1(z^{-1}) &= 2N, \quad (10) \end{aligned}$$

and

$$C_0(z)C_1(z^{-1}) + S_0(z)S_1(z^{-1}) = 0 \quad (11)$$

in the Laurent polynomial ring $\mathbb{Z}[z, z^{-1}]$, where we used the more compact polynomial notation to denote the sequences c_0, c_1, s_0, s_1 [5]. Sequence pairs (c_0, s_0) and (c_1, s_1) that satisfy (9) and (10) with $d = N-1$ are known in the literature as complementary sequences (or Golay pairs) and were first considered in [4]. On the other hand, if (11) holds, (c_0, s_0) and (c_1, s_1) are referred to as cross-complementary sequence pairs. If (c_0, s_0) and (c_1, s_1) are both Golay pairs and cross-complementary sequence pairs, (c_1, s_1) is said to be a mate of (c_0, s_0) or, alternatively, we say that (c_0, s_0) and (c_1, s_1) are mates [5]. The assumption $N = 0(\text{mod}4)$ is not sufficient for the existence of Golay pairs. It is known that N of the form $N = 2^r \cdot 10^s \cdot 26^t$ is the length of some Golay sequence. The length 2 is easily realized since $A(z) = 1+z$ and $B(z) = 1-z$ is a Golay pair. The reader is referred to [6] to see how to realize the lengths 10 and 26. To obtain longer sequences, we can use the following well-known fact: if $A(z)$ and $B(z)$ form a Golay pair of length N, so also do the sequences $C(z)$ and $D(z)$ of length $2N$ given by

$$C(z) = A(z) + z^N B(z), \ D(z) = A(z) - z^N B(z) \quad (12)$$

or, alternatively,

$$C(z) = A(z^2) + zB(z^2), \ D(z) = A(z^2) - zB(z^2). \quad (13)$$

Consequently, applying t iterations of these rules to an arbitrary Golay pair of length N yields another Golay pair of length N^t. Now let us suppose that we have generated a Golay pair (c_0, s_0) of the desired length N. Then, (c_1, s_1) given by

$$C_1(z) = z^n S_0(z^{-1}), \ S_1(z) = -z^n C_0(z^{-1}) \quad (14)$$

with $n = N-1$ is a mate of (c_0, s_0) since

$$\begin{aligned} & C_0(z)C_1(z^{-1}) + S_0(z)S_1(z^{-1}) \\ =\ & C_0(z)z^{-n}S_0(z) - S_0(z)z^{-n}C_0(z) = 0. \end{aligned}$$

Let us consider an example for $N = 8$. We start with the sequences $1+z$ and $1-z$ and iteratively apply the rules (12) to them to obtain

$$\begin{aligned} C_0(z) &= 1 + z + z^2 - z^3 + z^4 + z^5 - z^6 + z^7 \\ S_0(z) &= 1 + z + z^2 - z^3 - z^4 - z^5 + z^6 - z^7, \end{aligned}$$

which are known in the literature as binary Rudin-Shapiro sequences [7]. According to (14), the mate (c_1, s_1) is

$$\begin{aligned} C_1(z) &= -1 + z - z^2 - z^3 - z^4 + z^5 + z^6 + z^7 \\ S_1(z) &= -1 + z - z^2 - z^3 + z^4 - z^5 - z^6 - z^7 \end{aligned}$$

The aperiodic auto-correlation of the sequences c_0, c_1, s_0, s_1 are depicted in figure 1 (above) showing that both (c_0, s_0) and (c_1, s_1) form Golay pairs. Moreover, the figure below shows that (c_0, s_0) and (c_1, s_1) are also mates since $R_{c_0,c_1}(m) + R_{s_0,s_1}(m) = 0$. It is important to mention that if (c_0, s_0) and (c_1, s_1) are mates with the relationship between them given by (14), so also are (c_0, c_1) and (s_0, s_1), which can be immediately proven by calculating the appropriate polynomials. Another remarkable fact is that $R_{c_0}(m) = R_{s_0}(m) = 0, m \ne 0$ and $R_{c_0,s_0}(m) = 0$ for each even integer m if c_0 and s_0 are the Rudin-Shapiro sequences or a mate of them. This fact can be used to construct a set of sequences whose all even aperiodic auto-correlation sidelobes are zero [8]. It is easy to show

[5] In this notation, any bipolar sequence $a = \{a(0), \ldots, a(N-1)\}$ is simply replaced by the polynomial $A(z) = \sum_{n=0}^{N-1} a(n)z^n$. In the Laurent polynomial ring, $A(z)A(z^{-1}) = N + \sum_m R_a(m)(z^m + z^{-m})$. Thus, given bipolar sequences a and b of length N, $A(z)A(z^{-1}) + B(z)B(z^{-1}) = 2N$ means $R_a(m) + R_b(m) = 0, 1 \le m < N$. Equivalently, $A(z)B(z^{-1}) + C(z)D(z^{-1}) = 0$ means $R_{a,b}(m) + R_{c,d}(m) = 0, 0 \le m \le N$ for any bipolar sequences a, b, c and d.

that, given an arbitrary Golay pair $A(z)$ and $B(z)$, the rule (13) always yields the so-called even-shift sequences (or QMF-sequences) whose aperiodic auto-correlation is zero for each even integer $m \neq 0$. However, in contrast to the Rudin-Shapiro sequences, the aperiodic cross-correlation of two even-shift sequences does not need to have this property.

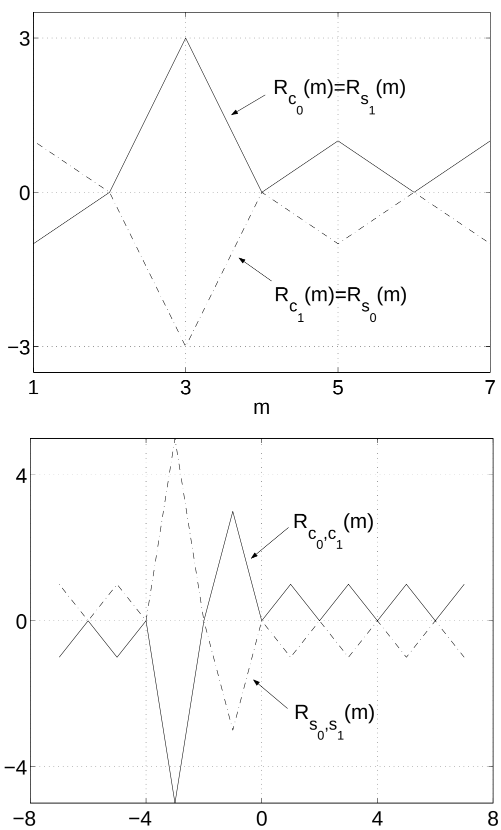

Fig. 1. The aperiodic auto-correlation ($1 \leq m \leq 7$) and the aperiodic cross-correlation of the Golay pairs (c_0, s_0) and (c_1, s_1)

The normalized aperiodic auto-correlation and cross-correlation of the sequences p and q given by (5) and (6) with $W_0 = 8$ are depicted in figure 2. The size of of the interference free window of the sequences p and q is equal to $2W_0 + 1$ and their energy efficiency is

$$\eta_p = \eta_q = \frac{\sum_{n=0}^{2N+W_0-1} |p(n)|^2}{2N + W_0} = \frac{1}{1 + \frac{d}{2N}} \leq 1, \qquad (15)$$

with equality if and only if $d = W_0 = 0$. Consequently, for a given $N > 1$, there is a trade-off between d and η_p. In the sequel, we will use the sequences c_0, c_1, s_0, s_1 described above, each of length N, to obtain a set of $K = 2^n, n \in \mathbb{N}$ sequences of length $L = KN + W_0$, each having perfect aperiodic correlation properties within the IFW. To this end, we need an arbitrary $p \times p$ Hadamard matrix $\mathbf{H}$, where $p = K/2$. It is conjectured that such matrices exist for each $p = 0 (\text{mod}\, 4)$. The reader is referred to [2] for more details. We use the vector

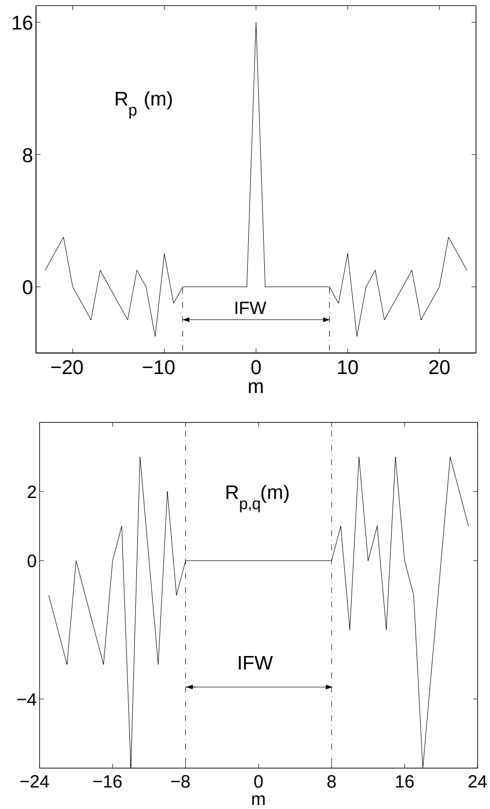

Fig. 2. The aperiodic auto-correlation and the aperiodic cross-correlation of the sequences p and q of length $2N + W_0 = 16 + 8 = 24$

$\pi = [\pi_1, \ldots, \pi_p], \pi_k \in \{0, 1\}$ to denote a binary expansion of an arbitrary integer $n, 0 \leq n < 2^p$ so that $n = \sum_i \pi_i 2^i$.

Proposition 1: Suppose that $\mathbf{H} = [h_{i,j}]$ is a $p \times p$ Hadamard matrix, and (c_2, s_2) is a mate of (c_1, s_1), where the sequences have the length N. We define the sequence $g_k, 1 \leq k \leq p$ of length $L = KN + W_0$ as

$$G_k(z) = \sum_{i=1}^{p} h_{k,i} \left[C_{\pi_i}(z) + z^{pN+W_0} S_{\pi_i}(z) \right] z^{(i-1)N},$$

where $K = 2p, W_0 \in \mathbb{N}$. Further, the sequences $g_{p+1}, \ldots, g_K$ are obtained when the binary expansion vector π in the formula above is replaced by its complement $\pi^* = [\pi_1^*, \ldots, \pi_p^*]$ with $\pi_k^* = \pi_k + 1 (\text{mod}\ 2)$ for each $1 \leq k \leq p$. Then,

$$R_{g_k, g_l}(m) = 0, \ \forall_{|m| \leq \min\{2N-1, 2W_0+1\}}$$

for every $1 \leq k, l \leq K$ and $m \neq 0$ if $k = l$.

The proposition is straightforward to verify by calculating the products $G_k(z) G_l(z^{-1}), 1 \leq k, l \leq K$ and using the theory provided in this section. Since the calculation is a little cumbersome, we omit the proof here and refer the reader to [8]. As an immediate consequence of the proposition 1, we have $W = \min\{2N - 1, 2W_0 + 1\}$ so that length of the IFW is proportional to W_0 if and only if $W_0 \leq N - 1$. Since the energy efficiency $\eta(\mathcal{S})$ of the sequences decreases when W_0 increases, it is reasonable to choose $W_0 \leq N-1$. On the other hand, $K = L'/N$ so that we have

$$K = \frac{L - W_0}{N} \leq \frac{2L'}{W+1} = \frac{L'}{d+1}, \qquad (16)$$

with equality if $W_0 = N-1$, where $L' = L - W_0$. Let us consider an example with the sequences c_0, c_1, s_0 and s_1 of length 8 used previously. Thus $W \leq 2 \cdot 8 - 1 = 15$ with equality if $W_0 = N-1$. For brevity, we assume $W_0 = N$. We want to construct four sequences of length $L = KN + W_0 = 4 \cdot 8 + 8 = 40$. We choose $\pi = [\pi_1, \pi_2] = [0, 1]$ so that its complement is $[1, 0]$. We take the 2×2 Hadamard matrix $\mathbf{H}$ with columns $\mathbf{h}_1 = [1\ 1]^T$ and $\mathbf{h}_2 = [1\ -1]^T$. According to proposition 1, the polynomial form of the sequences is

$$\begin{aligned}
G_1(z) &= C_0(z) + z^N C_1(z) + z^{N_2} S_0(z) + z^{N_3} S_1(z)\\
G_2(z) &= C_0(z) - z^N C_1(z) + z^{N_2} S_0(z) - z^{N_3} S_1(z)\\
G_3(z) &= C_1(n) + z^N C_0(z) + z^{N_2} S_1(z) + z^{N_3} S_0(z)\\
G_4(z) &= C_1(z) - z^N C_0(z) + z^{N_2} S_1(z) - z^{N_3} S_0(z),
\end{aligned}$$

where $N_k = kN + W_0$. Examples of the aperiodic auto-correlation and the cross-correlation are presented in figure 3.

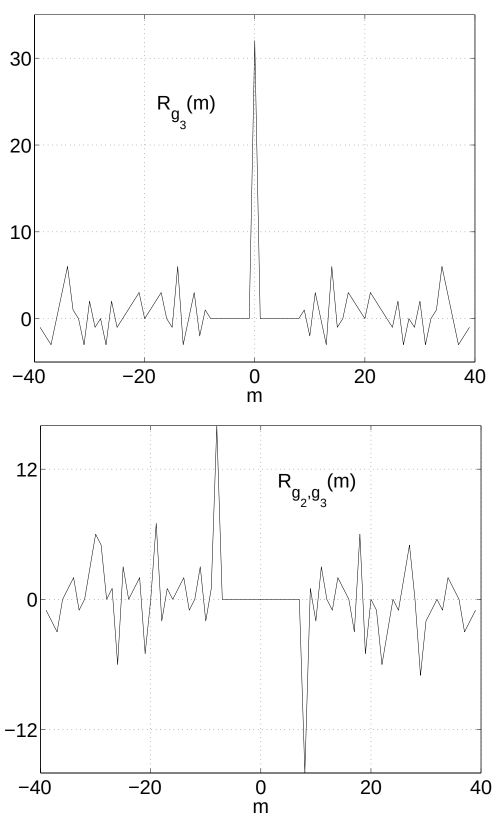

Fig. 3. The aperiodic auto-correlation and cross-correlation of the LS codes of length $L = 32 + 8 = 40$.

IV. Conclusions

The theory of constructing sets of orthogonal sequences is abundant. Unfortunately, the use of such sets in CDMA systems requires a perfect synchronization of received signal components, which is difficult to maintain in the multipath channel. On the other hand, systematic methods of constructing sequences with favorable aperiodic correlation properties that would effectively combat the ISI and the MAI in totally asynchronous CDMA systems are not known. In the LAS-CDMA, a coarse synchronization of signal components is established so that it is sufficient to optimize the aperiodic correlations of the sequences in a small window $\{-d, \ldots, d\}$ around the zero shift. In particular, if the aperiodic correlations of the sequences are zero in this window (the LS-codes have this property), independent CDMA channels are created. In [9], sequences are constructed whose *periodic* correlations have such a perfect behavior [6]. This, however, is not sufficient since the aperiodic correlations, and not the periodic ones determine the performance of CDMA systems. In this paper, we have provided a quite general theory behind the LS-codes. The theory is aimed to support current efforts directed towards discovering large sets of sequences with small aperiodic correlations in a given window. This is necessary since the maximum number of the LS-codes of length $L + W_0$ is equal to $L/(d+1)$. Consequently, the spectral efficiency of CDMA systems that employ such sequences cannot be high, which is especially true in the outdoor vehicular radio environments ($d \approx 3$). Furthermore, it is not clear if the desired gain in terms of the signal-to-interference-plus-noise ratio (SINR) is achieved through the insertion of the zeros. It would be interesting to know where and how many zeros should be placed in order to maximize the SINR.

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