

BLIND LINEAR INTERFERENCE SUPPRESSION BASED ON REDUCED-RANK LEAST-SQUARES CONSTRAINED CONSTANT MODULUS DESIGN FOR DS-CDMA SYSTEMS

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ABSTRACT

In this paper we present a blind interference suppression technique for DS-CDMA systems based on a reduced-rank decomposition of a code-constrained constant modulus design criterion. We describe a least-squares (LS) type design criterion for blind linear detectors using the constrained optimization of the constant modulus cost function subject to code constraints. Based on this design approach and using the Lanczos algorithm, a multistage decomposition in the Krylov subspace is devised for blind reduced-rank parameter estimation. A computationally efficient blind reduced-rank LS type algorithm is also developed and compared with existing methods. Numerical results show that the proposed full-rank and reduced-rank techniques outperform existing methods for the linear suppression of multi-access and intersymbol interference in DS-CDMA systems.

Index Terms— Least squares methods, CDMA, interference suppression, blind methods.

1. INTRODUCTION

Blind linear receivers [1, 2] for DS-CDMA systems are effective techniques for interference suppression as they offer an attractive trade-off between performance and complexity, and can be used in situations where a receiver loses track of the desired signal and a training sequence is not available. Some of the most successful blind techniques in the literature [3, 4, 5] are based on constrained optimization solutions that combine multipath components and suppress multi-access and intersymbol interference. In these works, the designer may resort to several estimation methods ranging from the low complexity stochastic gradient techniques to the complex and fast converging least-squares (LS) algorithms and sample matrix inversion approaches [6]. A major drawback of existing techniques is that when the number of elements M in the estimator is large, an LS algorithm requires an amount of data proportional to $2M$ to converge. This is particular relevant in dynamic environments and may lead to unacceptable convergence and tracking performance.

In order to accelerate the convergence and to improve the tracking of dynamic signals, one can utilize reduced-rank estimation methods [7]-[11]. These techniques are highly effective in short data support situations and can provide a significant performance advantage over conventional approaches. Several reduced-rank methods have been investigated and vary

from complex eigen-decomposition-based estimators to simpler approaches such as the promising multistage Wiener filter (MWF) of Goldstein *et al.* [9] based on orthogonal decompositions. Previous works on blind reduced-rank parameter estimation are very limited and rely on computationally expensive methods [8] or on an MWF version of the constrained minimum variance (CMV) design criterion [10], which is very sensitive to signature mismatch and does not tridiagonalize the covariance matrix as the supervised versions of the MWF. In this respect, blind constrained parameter estimators with the constant modulus criterion (CCM) [4, 5] have shown improved performance and increased robustness against signature mismatch over the CMV approach and there is no reduced-rank version of the CCM criterion available.

In this work, we develop a blind interference suppression technique for DS-CDMA systems based on a reduced-rank decomposition of the LS CCM design criterion. We devise a multistage decomposition for blind parameter estimation in the Krylov subspace using the LS CCM design criterion. A computationally efficient blind reduced-rank LS algorithm is also developed and compared with existing methods.

This paper is organized as follows. Section 2 describes a DS-CDMA system model and Section 3 briefly reviews LS CCM receivers. Section 4 introduces the reduced-rank version of the LS CCM design for linear receivers and details the multistage decomposition used to compute the reduced-rank projection matrix. Section 5 presents and discusses the simulation results, while Section 6 gives the conclusions.

2. DS-CDMA SYSTEM MODEL

Consider the uplink connection of a synchronous DS-CDMA system with K users, N chips per symbol and L_p paths. Assuming that the channel is constant during each symbol interval and the spreading codes are repeated from symbol to symbol, the received signal after filtering by a chip-pulse matched filter and sampled at chip rate yields the M -dimensional received vector

$$\mathbf{r}[i] = \sum_{k=1}^K A_k b_k[i] \mathbf{C}_k \mathbf{h}_k[i] + \boldsymbol{\eta}_k[i] + \mathbf{n}[i] \quad (1)$$

where $M = N + L_p - 1$, $\mathbf{n}[i] = [n_1[i] \dots n_M[i]]^T$ is the complex Gaussian noise vector with $E[\mathbf{n}[i]\mathbf{n}^H[i]] = \sigma^2 \mathbf{I}$, where $(\cdot)^T$ and $(\cdot)^H$ denotes transpose and Hermitian transpose, respectively. The operator $E[\cdot]$ stands for expected value, $b_k[i] \in$

$\{\pm 1 + j0\}$ is the symbol for user k with $j^2 = -1$, $\eta_k[i]$ represents the intersymbol interference for user k , the amplitude of user k is A_k , the $M \times L_p$ constraint matrix \mathbf{C}_k that contains one-chip shifted versions of the signature sequence $\mathbf{s}_k = [a_k(1) \dots a_k(N)]^T$ for user k and the $L_p \times 1$ vector $\mathbf{h}_k[i]$ with the multipath components are described by:

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix}, \mathbf{h}_k[i] = \begin{bmatrix} h_{k,0}[i] \\ \vdots \\ h_{k,L_p-1}[i] \end{bmatrix} \quad (2)$$

3. LINEARLY CONSTRAINED LEAST-SQUARES TYPE CONSTANT-MODULUS RECEIVER DESIGN

The CCM linear receiver design is equivalent to determining an FIR filter \mathbf{w}_k with M coefficients that provide an estimate of the desired symbol $\hat{b}_k[i] = \text{sgn}(\Re(\mathbf{w}_k^H[i]\mathbf{r}[i]))$, where $\text{sgn}(\cdot)$ is the signum function, $\Re(\cdot)$ selects the real component and \mathbf{w}_k is optimized according to the exponentially weighted LS CM cost function:

$$J_{CM}(\mathbf{w}_k) = \sum_{l=1}^i \lambda^{i-l} (|\mathbf{w}_k^H[i]\mathbf{r}[l]|^2 - 1)^2 \quad (3)$$

subject to the constraints given by $\mathbf{w}_k^H[i]\mathbf{p}_k[i] = \nu$, where λ is a forgetting factor, $\mathbf{p}_k[i] = \mathbf{C}_k\mathbf{h}_k[i]$, $\mathbf{h}_k[i]$ is the vector that contains the multipath gains and ν is a constant to ensure the convexity of (3)[5]. The CCM filter expression that iteratively solves the constrained optimization problem in (3) is

$$\mathbf{w}_k[i] = \mathbf{R}_k^{-1}[i] \left[\mathbf{d}_k[i] - \left(\mathbf{p}_k^H[i]\mathbf{R}_k^{-1}[i]\mathbf{p}_k[i] \right)^{-1} \cdot \left(\mathbf{p}_k^H[i]\mathbf{R}_k^{-1}[i]\mathbf{d}_k[i] - \nu \right) \mathbf{p}_k[i] \right] \quad (4)$$

where $z_k[i] = \mathbf{w}_k^H[i]\mathbf{r}[i]$, $\mathbf{R}_k[i] = \sum_{l=1}^i \lambda^{i-l} |z_k[l]|^2 \mathbf{r}[l]\mathbf{r}^H[l]$, $\mathbf{d}_k[i] = \sum_{l=1}^i \lambda^{i-l} z_k^*[l]\mathbf{r}[l]$. In order to blindly estimate the channel, we adopt here the blind channel estimation procedure based on the power method proposed by Doukopoulos and Moustakides [13], described by

$$\hat{\mathbf{h}}_k[i] = \arg \min_{\hat{\mathbf{h}}_k[i]} \hat{\mathbf{h}}_k^H[i] \mathbf{C}_k^H \mathbf{R}_k^{-1}[i] \mathbf{C}_k \hat{\mathbf{h}}_k[i] \quad (5)$$

subject to $|\hat{\mathbf{h}}_k[i]| = 1$, where $\mathbf{R}[i] = \sum_{l=1}^i \lambda^{i-l} \mathbf{r}[l]\mathbf{r}^H[l]$. The solution is the eigenvector of the $L_p \times L_p$ matrix corresponding to the minimum eigenvalue of $\mathbf{C}_k^H \mathbf{R}_k^{-1}[i] \mathbf{C}_k$ that can be obtained through a singular value decomposition (SVD).

4. REDUCED-RANK LEAST SQUARES CONSTRAINED CONSTANT-MODULUS DESIGN

In this section, we describe the reduced-rank CCM design based on a multistage decomposition of the CCM expression.

4.1. Reduced-Rank Receiver Design

We devise a reduced-rank algorithm that reduces the number of adaptive coefficients by projecting the received signal onto a lower dimension subspace. An illustration of the dimensionality reduction and corresponding receiver design is depicted in Fig. 1.

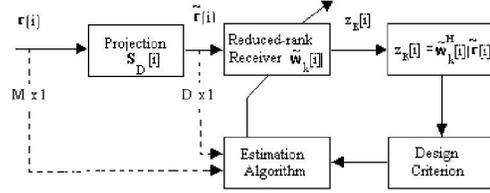


Fig. 1. Reduced-rank processing and receiver design.

Specifically, let $\mathbf{S}_D[i]$ be a $M \times D$ -dimensional matrix that accomplishes the dimensionality reduction as given by

$$\tilde{\mathbf{r}}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i] \quad (6)$$

where, in what follows, all D -dimensional quantities incorporate a ‘‘tilde’’. The reduced-rank CCM linear receiver design is equivalent to computing an FIR filter $\tilde{\mathbf{w}}_k$ with D elements that yield the desired symbol as

$$\hat{b}_k[i] = \text{sgn}(\Re[\tilde{\mathbf{w}}_k^H[i]\tilde{\mathbf{r}}[i]]) = \text{sgn}(\Re[z_k[i]]) \quad (7)$$

where the optimization criterion, i.e., the LS CM cost function is $J_{CM}(\tilde{\mathbf{w}}_k) = \sum_{l=1}^i \lambda^{i-l} (|\tilde{\mathbf{w}}_k^H[l]\tilde{\mathbf{r}}[l]|^2 - 1)^2$ and the set of constraints is $\tilde{\mathbf{w}}_k^H[i]\tilde{\mathbf{p}}_k[i] = \nu$, where $\tilde{\mathbf{p}}_k[i] = \mathbf{S}_D^H[i]\mathbf{C}_k\mathbf{h}_k[i]$. The reduced-rank CCM filter expression is

$$\tilde{\mathbf{w}}_k[i] = \tilde{\mathbf{R}}_k^{-1}[i] \left[\tilde{\mathbf{d}}_k[i] - \left(\tilde{\mathbf{p}}_k^H[i]\tilde{\mathbf{R}}_k^{-1}[i]\tilde{\mathbf{p}}_k[i] \right)^{-1} \cdot \left(\tilde{\mathbf{p}}_k^H[i]\tilde{\mathbf{R}}_k^{-1}[i]\tilde{\mathbf{d}}_k[i] - \nu \right) \tilde{\mathbf{p}}_k[i] \right] \quad (8)$$

where $z_k[i] = \tilde{\mathbf{w}}_k^H[i]\tilde{\mathbf{r}}[i]$, $\tilde{\mathbf{R}}_k[i] = \mathbf{S}_D^H[i]\mathbf{R}_k[i]\mathbf{S}_D[i]$, $\tilde{\mathbf{d}}_k[i] = \sum_{l=1}^i \lambda^{i-l} z_k^*[l]\tilde{\mathbf{r}}[l] = \mathbf{S}_D^H[i] \sum_{l=1}^i \lambda^{i-l} z_k^*[l]\mathbf{r}[l]$.

4.2. Multistage Decomposition and Projection Matrix

Here we detail the procedure to compute the projection matrix $\mathbf{S}_D[i]$ and the multistage decomposition. Let us rewrite the CCM expression of (4) in the following alternative form

$$\mathbf{w}_k[i] = \mathbf{R}_k^{-1}[i]\mathbf{q}_k[i] \quad (9)$$

where

$$\mathbf{q}_k[i] = \mathbf{d}_k[i] - \left(\mathbf{p}_k^H[i]\mathbf{R}_k^{-1}[i]\mathbf{p}_k[i] \right)^{-1} \left(\mathbf{p}_k^H[i]\mathbf{R}_k^{-1}[i]\mathbf{d}_k[i] - \nu \right) \mathbf{p}_k[i] \quad (10)$$

Following the diagram of Fig. 2, we wish to develop a multistage decomposition of the expression in (9) that computes

the projection matrix $\mathbf{S}_D[i]$. The first filter of the structure in Fig. 2, namely $\mathbf{f}_k^{(1)}[i]$, is the normalized version of $\mathbf{q}_k[i]$, i.e., $\mathbf{f}_k^{(1)}[i] = \frac{\mathbf{q}_k[i]}{\|\mathbf{q}_k[i]\|}$. In this proposed multistage decomposition, the d th filter $\mathbf{f}_k^{(d)}[i]$ maximizes the real part of the correlation between its output $z^{(d)}(i)$ and the output of the previous filters $z^{(d-1)}(i)$. By restricting the filters to be orthonormal, the d th filter can be computed via the optimization

$$\begin{aligned} \mathbf{f}_k^{(d)}[i] &= \arg \max_{\mathbf{f}_k} \sum_{l=1}^i \lambda^{i-l} \Re \left(z^{(d)}[l] z^{(d-1)*}[l] \right) \\ &= \arg \max_{\mathbf{f}_k} \sum_{l=1}^i \lambda^{i-l} \Re \left(\mathbf{f}_k^{(d), H}[i] \mathbf{r}[l] \mathbf{r}^H[l] \mathbf{f}_k^{(d-1)}[i] \right) \end{aligned} \quad (11)$$

subject to $\mathbf{f}_k^{(j), H}[i] \mathbf{f}_k^{(m)}[i] = 1$ for $m = j$ and $\mathbf{f}_k^{(j), H}[i] \mathbf{f}_k^{(m)}[i] = 0$ for $m \neq j$. A general solution to the optimization problem in (12) can be computed via the Arnoldi iteration [12], which is a numerical optimization algorithm to solve linear systems problems, and is described by

$$\mathbf{f}_k^{(d)}[i] = \frac{\left(\prod_{l=1}^{d-1} \mathbf{P}_l[i] \right) \mathbf{R}_k^{-1}[i] \mathbf{f}_k^{(d-1)}[i]}{\left\| \left(\prod_{l=1}^{d-1} \mathbf{P}_l[i] \right) \mathbf{R}_k^{-1}[i] \mathbf{f}_k^{(d-1)}[i] \right\|} \in \mathcal{C}^M \quad (12)$$

where the matrix $\mathbf{P}_l[i] = \mathbf{I}_M - \mathbf{f}_k^{(l)}[i] \mathbf{f}_k^{(l), H}[i]$ has the role of projecting the signal onto the space orthogonal to the filter $\mathbf{f}_k^{(l)}[i]$ and \mathbf{I}_M is the $M \times M$ identity matrix. Because $\mathbf{R}_k[i]$ is Hermitian, the designer can resort to the Lanczos algorithm, a simpler technique than the Arnoldi recursion and that can be used to solve symmetric systems of linear equations [12]. The method generates a sequence of tridiagonal matrices and parameters that are gradually better estimates of the desired solution as given by

$$\mathbf{f}_k^{(d)}[i] = \frac{\mathbf{P}_{d-1}[i] \mathbf{P}_{d-2}[i] \mathbf{R}_k^{-1}[i] \mathbf{f}_k^{(d-1)}[i]}{\left\| \mathbf{P}_{d-1}[i] \mathbf{P}_{d-2}[i] \mathbf{R}_k^{-1}[i] \mathbf{f}_k^{(d-1)}[i] \right\|}, \in \mathcal{C}^M \quad (13)$$

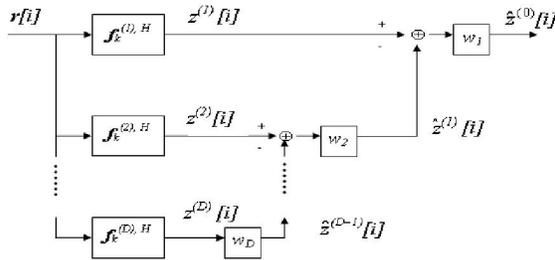


Fig. 2. Proposed reduced-rank CCM filterbank structure.

The Lanczos method above leads to a tridiagonal covariance matrix $\mathbf{R}_k^{(d)}$ for the projected vector $\mathbf{z}[i] = [z^{(1)} \dots z^{(d)}]^T$,

where the parameters $z^{(d)}$ are obtained after the application of each filter $\mathbf{f}_k^{(d)}[i]$. The scalar filters w_d , where $d = 1, \dots, D$, are utilized to estimate the output $z^{(d-1)}$ of the previous filter from an error signal $\epsilon[i]$. Following this procedure and Fig. 2, the reduced-rank CCM filter with rank D can be obtained by neglecting the signal $z^{(D)}[i]$. In this respect, the procedure described above details the computation of the projection matrix $\mathbf{S}_D[i]$ for user k , which has the following structure

$$\begin{aligned} \mathbf{S}_D[i] &= \left[\mathbf{f}_k^{(1)}[i], \dots, \mathbf{f}_k^{(D)}[i] \right], \in \mathcal{C}^{M \times D} \\ &= \left[\mathbf{q}_k[i], \mathbf{R}_k[i] \mathbf{q}_k[i], \dots, \mathbf{R}_k^{(D-1)}[i] \mathbf{q}_k[i] \right] \end{aligned} \quad (14)$$

By using the projection matrix $\mathbf{S}_D[i]$, the D -dimensional observation vector is expressed by

$$\tilde{\mathbf{r}}[i] = \mathbf{z}[i] = \mathbf{S}_D^H[i] \mathbf{r}[i] \quad (15)$$

and the reduced-rank CCM filter $\mathbf{w}_k^D[i] = [w_1[i] \dots w_D[i]]^T$ with rank D is

$$\begin{aligned} \tilde{\mathbf{w}}_k^D[i] &= \left(\mathbf{S}_D^H[i] \mathbf{R}_k[i] \mathbf{S}_D[i] \right)^{-1} \mathbf{S}_D^H[i] \mathbf{q}_k[i] \\ &= \tilde{\mathbf{R}}_k^{-1}[i] \tilde{\mathbf{q}}_k[i] \end{aligned} \quad (16)$$

The reduced-rank solution with rank D above projects the received signal onto a lower dimensional subspace, which corresponds to the D -dimensional Krylov subspace $\mathcal{K}^D(\mathbf{R}_k, \mathbf{q}_k)$, where $\mathcal{K}^D(\mathbf{A}, \mathbf{b}) = \text{span}[\mathbf{b}, \mathbf{A}\mathbf{b}, \dots, \mathbf{A}^{(D-1)}\mathbf{b}]$ and was defined for the problem at hand in (15). The complexity of the CCM-MWF-LS algorithm is $O(DM^2 + D^3)$.

5. SIMULATIONS

In this section we assess the bit error rate (BER) performance of the following LS type parameter estimation criteria, i.e., the supervised LS, the CMV-LS and the proposed CCM-LS. We also evaluate their corresponding full-rank and reduced-rank versions. The DS-CDMA system employs randomly generated sequences of length $N = 64$. The channels experienced by the users are different since we focus on an up-link scenario and the channel coefficients $h_{k,l}[i] = p_{k,l} \alpha_{k,l}[i]$, where $\alpha_{k,l}[i]$ ($l = 0, 1, \dots, L_p - 1$) are obtained with Clarke's model [14]. We show the results in terms of the normalized Doppler frequency $f_d T$ (cycles/symbol) and use three-path channels with relative powers given by 0, -3 and -6 dB, where in each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips. The channel estimator of [13] models the channel as an FIR filter and we employ a filter with 8 taps as an upper bound for the experiments. The phase ambiguity derived from the blind channel estimation method in [13] is eliminated in our simulations by using the phase of $\hat{\mathbf{h}}_k(0)$ as a reference to remove the ambiguity. The system has a power distribution amongst the users for each run that follows a log-normal distribution with associated standard deviation of 1.5 dB, all LS type estimators use $\lambda = 0.998$ to ensure good performance and all experiments are averaged over 200 runs.

In the first experiment, we evaluate the BER performance of the proposed and analyzed reduced-rank algorithms versus

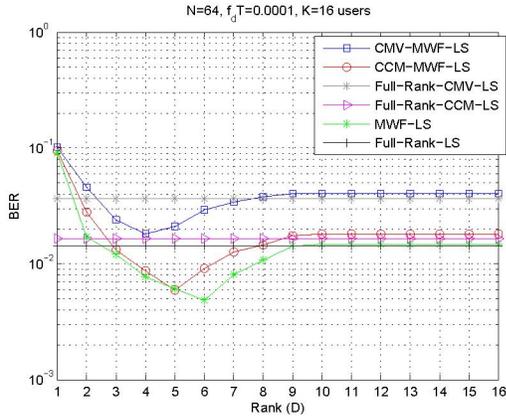


Fig. 3. BER performance versus D for data record of 800 symbols.

their associated rank D . This experiment is intended for setting the adequate rank D of the reduced-rank schemes for the remaining assessments. The full-rank performance is also included for comparison purposes. The results shown in Fig. 3 indicate that the best performance of the proposed reduced-rank CCM scheme is obtained with $D = 5$. It is interesting to note that the best D is usually much smaller than the number of elements in the received data M , which leads to significant computational savings. These optimized parameters for D will be used for the remaining numerical results.

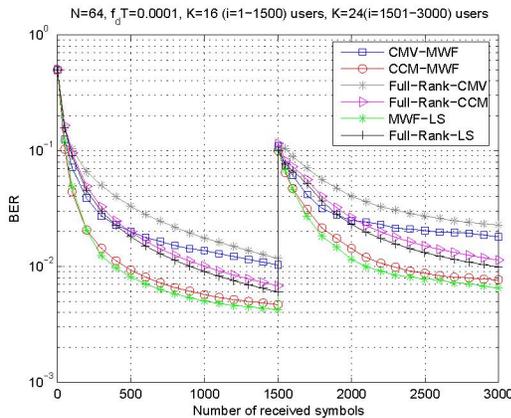


Fig. 4. BER convergence performance of LS algorithms at $E_b/N_0 = 12$ dB in a dynamic scenario.

The BER convergence performance of the analyzed algorithms is depicted in Fig. 4. We consider a non-stationary scenario, where the system starts with $K = 16$ users and at time $i = 1501$, 8 additional users enter the system, totalling $K = 24$ users, and the blind adaptive algorithms are subject to new interferers/users in the environment. The results show that the new reduced-rank algorithms based on the LS CCM design criterion can perform very close to the supervised reduced-rank algorithms, while they do not require training data. The convergence performance of the new method is significantly

better than all existing full-rank schemes and allows a faster acquisition and tracking of the desired signals.

6. CONCLUSIONS

This work proposed a multistage decomposition for blind parameter estimation in the Krylov subspace with the LS type CCM design criterion. Numerical results for an application of the proposed techniques to the suppression of multi-access and intersymbol interference in DS-CDMA systems have shown that the proposed method achieves a performance equivalent to the best known supervised reduced-rank approaches without the need for training data and has very fast convergence performance.

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