

An Efficient Transmission Strategy for the Multicarrier Multiuser MIMO Downlink

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Abstract—A new transmission strategy that consists of a spatial scheduling algorithm and two precoding algorithms is developed for multicarrier multiuser (MU) multiple-input–multiple-output (MIMO) systems. The scheduling algorithm, which is called efficient multicarrier ProSched (EMC-ProSched), adopts a novel and effective scheduling metric for each user and can efficiently search for a suitable group of users to be served at the same time on the same frequency. Two precoding techniques are then designed to handle different antenna configurations. For the case where the number of transmit antennas at the base station (BS) is not smaller than the total number of receive antennas at the user terminals (UTs), the linear-precoding-based geometric mean decomposition (LP-GMD) algorithm is proposed. It suppresses the MU interference (MUI) and enables an effective implementation of the same modulation and coding scheme (MCS) on all spatial streams of each user. Consequently, smaller signaling overhead is required compared with the case where a different MCS is applied on each spatial stream. When the total number of receive antennas at the UTs exceeds the number of transmit antennas at the BS, we propose the low-complexity coordinated beamforming (LoCCoBF) algorithm to accomplish the goal of the MUI mitigation and to achieve a high capacity. A system-level simulator with a link-to-system interface is further developed under the framework of the IEEE 802.11ac standard to evaluate the performance of the proposed transmission strategy. The simulation results indicate that a promising performance can be achieved by employing the proposed transmission strategy.

Index Terms—IEEE 802.11ac, linear precoding, multicarrier systems, multiple-input–multiple-output (MIMO) systems, multiuser downlink, space-division multiple access (SDMA).

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I. INTRODUCTION

MULTIUSER (MU) multiple-input–multiple-output (MIMO) systems have the potential of combining the high capacity achievable by MIMO processing with the benefits of space-division multiple access (SDMA) [1], [2]. Constrained by the processing ability and the quality-of-service requirement of standards, only a limited number of users can be served simultaneously. Hence, for the downlink of an MU MIMO system with a large number of users, spatial scheduling algorithms can be employed to select a suitable SDMA user group to be served simultaneously, and precoding algorithms can be then used to mitigate the MU interference (MUI) [3]–[7].

The main task of spatial scheduling algorithms is to separate users with a high spatial correlation into different SDMA groups. In [8], a scheduling algorithm is proposed, where the beamforming vectors are generated with a common codebook shared by the base station (BS) and the user terminals (UTs). However, this codebook is not available for some systems constructed under the framework of certain standards such as the IEEE 802.11ac. In the scheduling algorithm developed in [9], precoding matrices for each possible user group are required to be precalculated, which leads to unaffordable computational complexity. In [7], the ProSched scheme, which can avoid the precalculation of the precoding matrices by using a capacity-related scheduling metric, is initially developed for one subcarrier. In a multicarrier system, ProSched has been proposed to treat each subcarrier as a virtual user. However, it suffers from high complexity and large signaling overhead.

After determining the user group to be served, linear precoding (LP) algorithms, such as block diagonalization (BD) [3] and regularized BD (RBD) [4], [10], can be used to suppress the MUI and to tune the performance of each equivalent single-user transmission. It is known that singular value decomposition (SVD) can be used to decompose a single-user MIMO channel into multiple parallel subchannels to transmit multiple data streams. A water-filling algorithm can be employed to assign the transmit power of each subchannel to achieve the channel capacity [11]. It should be noted that these subchannels might have very different SNRs [12]. To achieve the channel capacity, different modulation and coding schemes (MCSs) should be used in different subchannels. In practice, unequal MCSs across subchannels of each user will introduce large signaling overhead. However, if the same MCS is used on all the subchannels, the packet-error-rate (PER) performance

will be dominated by the weakest subchannel. Therefore, to enable the implementation of the same MCS across all the subchannels and to fully utilize the transmit power, in [12] and [13], a geometric mean decomposition (GMD) technique has been employed for single-user systems to decompose the MIMO channel into multiple subchannels with identical SNRs. In [14], BD-GMD has been proposed to recursively decompose the MU MIMO channel such that the subchannels for each user have identical SNRs. However, the recursive procedure for the decomposition leads to high computational complexity. In addition, BD-GMD is only a decomposition algorithm, and it has to be combined with some precoding technique, such as Tomlinson–Harashima precoding (THP) [14].

It is known that many LP algorithms require that the number of transmit antennas at the BS is not smaller than the total number of receive antennas at the UTs [3]. In MU MIMO systems where the total number of users to be served is large and each user is equipped with multiple antennas, this constraint becomes a bottleneck of the precoding techniques. Although some existing algorithms, such as RBD [4], can relax this antenna constraint, its performance heavily degrades with an increasing aggregated number of receive antennas at the UTs [15]. The concept of coordinated beamforming was introduced in [3] to overcome this antenna constraint by initializing the receive beamforming matrix for each user using the dominant left singular vectors of the channel matrix from the BS to this user. The transmit beamforming matrices are then calculated using the resulting equivalent MU channel matrix. Since this scheme has only two steps (no iterative computations required), it has low complexity, however, at the price of degradation in the performance. The coordinated beamforming algorithm proposed in [16] jointly and iteratively optimizes the transmit and receive beamforming matrices. It achieves a good sum rate performance but suffers from high complexity. In addition, this algorithm allows only one spatial stream transmitted to each user. In [15], the flexible coordinated beamforming (FlexCoBF) algorithm is developed to iteratively calculate beamforming matrices for a subcarrier when the antenna constraint is not fulfilled. Although FlexCoBF allows transmitting multiple spatial streams to each user, it requires a large number of iterations. A closed-form expression for the coordinated beamforming was proposed in [17]. However, it can only be applied for a system with two users, and the number of data streams transmitted to each user is restricted to one.

In this paper, we develop an efficient transmission strategy for the multicarrier MU MIMO downlink. Our major focus is on a scenario where the total number of users is much larger than the maximum number of users that can be served at the same time and on the same frequency without any constraint on antenna configurations. The proposed transmission strategy consists of a scheduling stage and a precoding stage. In the scheduling stage, the efficient multicarrier ProSched (EMC-ProSched) algorithm determines the most suitable user group using an efficient tree-based sorting algorithm and a novel and effective scheduling metric inspired by the philosophy of the link-to-system mapping with well-calibrated parameters. A comparison in terms of computational complexity is made between the proposed EMC-ProSched and the ProSched [7]

that treats each subcarrier of each user as a virtual user. In the precoding stage, we distinguish two cases.

- **Case 1:** The total number of receive antennas of the UTs *does not exceed* the number of transmit antennas at the BS.
- **Case 2:** The total number of receive antennas of the UTs *exceeds* the number of transmit antennas at the BS.

Then, for **Case 1**, the LP-GMD algorithm is proposed to suppress the MUI at the BS and decompose each equivalent single-user MIMO channel into multiple parallel subchannels that have the same signal-to-interference-plus-noise ratio (SINR). Consequently, the implementation of the same MCS is enabled on different spatial streams of each user. This scheme is particularly suitable for IEEE 802.11ac systems where an equal MCS across the subchannels of each user is required [18]. In addition, an MMSE power loading strategy is designed to enable better power balancing between different users and to further improve the system performance. For **Case 2**, the low-complexity coordinated beamforming (LoCCoBF) algorithm is developed to jointly and iteratively optimize the transmit and receive beamformers. By exploiting the correlation of the channels of neighboring subcarriers, the LoCCoBF algorithm has an enhanced efficiency.

The new contributions beyond the previous conference papers [19], [20] include the following. First, a new framework of an efficient and flexible transmission strategy is proposed to accomplish the tasks of spatial scheduling and precoding for multicarrier MU MIMO downlink systems. It does not place any restrictions on the number of UTs or the number of antennas and provides solutions to the two aforementioned cases of the system configuration. In the EMC-ProSched algorithm, a novel scheduling metric for multicarrier systems is developed. In the LP-GMD algorithm, we elaborate how to obtain the decoding matrices at the UTs. Moreover, the construction of the link-to-system interface in the system-level simulator under the framework of the IEEE 802.11ac is described in detail. The importance of employing the MMSE-based power loading in practice is pointed out. Furthermore, a more detailed complexity analysis of the proposed algorithms is carried out. Finally, a more thorough evaluation of the proposed algorithms is performed via simulations, and completely new simulation results are presented.

This paper is organized as follows. Section II introduces the system model of multicarrier MU MIMO downlink transmissions. Then, the proposed EMC-ProSched algorithm is detailed in Section III. In Section IV, the LP-GMD algorithm is described for **Case 1** with the focus on two versions, i.e., LP-GMD-BD and LP-GMD-RBD. Section V presents the LoCCoBF scheme that is designed for **Case 2**. To examine the proposed transmission strategy in an efficient and flexible manner, a system-level simulator with a link-to-system interface is designed for the IEEE 802.11ac standard, as described in Section VI. The benefits of the proposed transmission strategy are demonstrated through numerical simulations in Section VII. Finally, the conclusions are drawn in Section VIII.

Notation: Matrices and vectors are denoted by boldfaced uppercase and lowercase letters, respectively. We use superscripts \cdot^T , \cdot^H , and \cdot^{-1} for transpose, Hermitian transpose, and

matrix inversion, respectively. $\text{trace}\{\cdot\}$ denotes the trace of a matrix. An m -by- m identity matrix is denoted by \mathbf{I}_m . The Frobenius norm of a matrix and the absolute value are denoted by $\|\cdot\|_F$ and $|\cdot|$, respectively. Operator $\text{blkdiag}\{\cdot\}$ creates a block-diagonal concatenation of its input matrices.

II. SYSTEM MODEL

We consider the downlink of an MU MIMO system with one BS equipped with M_T transmit antennas and N UTs. A scheduling algorithm is first employed to select K users out of N users and assign them to one SDMA group, i.e., these K users are served at the same time and on the same frequency. We set the indexes of the selected users as $i = 1, 2, \dots, K$ for the following introduction of the precoding process. The number of receive antennas of each user is then denoted by M_{R_i} , and the total number of receive antennas for one SDMA group is $M_R = \sum_{i=1}^K M_{R_i}$. We assume that the channel is modeled as a perfectly tuned orthogonal frequency-division multiplexing (OFDM) channel without any intercarrier interference and denote the number of data subcarriers by N_{SD} . It is worth noting that all the N_{SD} subcarriers are utilized for the transmissions of data to each of the selected K users. In other words, the BS transmits to all the K users on each of the N_{SD} subcarriers simultaneously, i.e., the selected K users are solely spatially multiplexed. At a certain time instant, the channel between the BS and the i th user on a certain subcarrier¹ is denoted by $\mathbf{H}_i \in \mathbb{C}^{M_{R_i} \times M_T}$. The joint channel matrix is given by

$$\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \dots, \mathbf{H}_K^T]^T \in \mathbb{C}^{M_R \times M_T}. \quad (1)$$

In this paper, perfect channel state information at the transmitter (CSIT) is always assumed. The transmitted signal for the i th user is defined as a d_i -dimensional vector \mathbf{x}_i . These vectors are stacked into vector $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_K^T]^T \in \mathbb{C}^{d \times 1}$ with $d = \sum_{i=1}^K d_i$, where $d \leq M_T$ and $d_i \leq M_{R_i}$ for $i = 1, 2, \dots, K$. For the i th user, precoding matrix $\mathbf{F}_i \in \mathbb{C}^{M_T \times d_i}$ is calculated at the BS, and the joint precoding matrix can be expressed as

$$\mathbf{F} = [\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_K] \in \mathbb{C}^{M_T \times d}. \quad (2)$$

The precoding matrix is employed to suppress the MUI while taking a transmit power constraint into account. At the UTs, decoding matrices $\mathbf{D}_i \in \mathbb{C}^{M_{R_i} \times d_i}$ are employed. The joint block-diagonal decoding matrix containing the decoding matrices of the users is denoted by \mathbf{D} . Finally, the d -dimensional joint received vector is given by

$$\mathbf{y} = \mathbf{D}^H (\mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}) \quad (3)$$

where $\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_K^T]^T$, $\mathbf{y}_i \in \mathbb{C}^{d_i \times 1}$ represents the received vector of the i th user, and $\mathbf{n} \in \mathbb{C}^{M_R \times 1}$ denotes the vector of the additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2 .

¹Note that the precoding is performed on a subcarrier basis. The index of subcarriers is then ignored in the notation of the channel matrices when describing the precoding algorithms. To detail the proposed scheduling technique in Section III dedicated to such a multicarrier system, the subcarrier indexes of the channel matrices are included.

The major challenge of the transmission strategy design for such a multicarrier MU MIMO downlink is to adopt efficient and effective spatial scheduling algorithms and precoding algorithms such that the SDMA benefits are fully exploited, the MUI at the UTs is mitigated, and most of the computations are carried out at the BS.

III. EFFICIENT MULTICARRIER PROSCHED

The proposed EMC-ProSched scheme consists of three major steps: 1) setting up the possible user groups; 2) calculating the scheduling metrics; and 3) selecting the user group with the maximum sum metric. In addition, to ensure the fairness of the scheduling, a proportional fairness-based extension is also developed for the EMC-ProSched algorithm. Instead of carrying out an exhaustive search of all possible user groups, the tree-based sorting algorithm [7] is utilized to produce candidate user groups featuring the maximum sum metric for all possible group sizes from 1 to the maximum supported size of the precoder or a certain communications standard (e.g., the maximum group size is 4, as in the IEEE 802.11ac specifications [18]). To avoid the precalculation of the precoding matrices, we employ the interpretation of the precoding process based on the orthogonal projection, as developed in [7]. Then, for every possible user group, a new scheduling metric design based on the exponential effective SINR mapping (EESM) [21] is calculated for each user (rather than for each subcarrier of one user as in [7]) in the presence of a possible user group. Finally, the sum metrics of the candidate user groups with different sizes are compared, and the group with the largest sum metric is selected and served. Note that the optimum group size is also determined.

A. Metric Calculation

Now, the metric calculation for the user selection will be detailed. We define the channel matrix for the i th user on the j th subcarrier as $\mathbf{H}_{i,j}$ and use $\mathbf{F}_{i,j} \in \mathbb{C}^{M_T \times d_i}$ to denote the precoding matrix for the i th user on the j th subcarrier. The number of users in the current user set \mathcal{S} as a candidate SDMA user group is denoted by $K^{(\mathcal{S})}$. Assuming that the BD algorithm [3] is employed as the precoding technique, the equivalent single-user channel matrix after the MUI elimination on the j th subcarrier for the i th user present in user set \mathcal{S} is represented as

$$\mathbf{H}_{i,j}^{(\text{eq})} = \mathbf{H}_{i,j} \tilde{\mathbf{V}}_{i,j}^{(0)}, \quad i = 1, \dots, K^{(\mathcal{S})}; \quad j = 1, \dots, N_{SD} \quad (4)$$

where the columns of $\tilde{\mathbf{V}}_{i,j}^{(0)}$ form an orthonormal basis of the common null space of all the other users present in user set \mathcal{S} .

It has been shown in [7] that, when the BD algorithm [3] is employed, the norm of the equivalent channel after precoding equals the norm of the projected channel, i.e.,

$$\left\| \mathbf{H}_{i,j} \tilde{\mathbf{V}}_{i,j}^{(0)} \right\|_F^2 = \left\| \mathbf{H}_{i,j} \tilde{\mathbf{P}}_{i,j} \right\|_F^2 \quad (5)$$

$$i = 1, \dots, K^{(\mathcal{S})}; \quad j = 1, \dots, N_{SD}$$

where $\tilde{\mathbf{P}}_{i,j}$ is a projection matrix that serves to project the channel matrix of the i th user on the j th subcarrier $\mathbf{H}_{i,j}$ onto

the common null space of the other co-channel users. When using other LP algorithms for the MU MIMO downlink aiming at the MUI mitigation, this relationship holds approximately. To further avoid the calculation of the projection matrices for all possible user combinations and thus achieve low complexity, we adopt the following approximation [22]:

$$\tilde{\mathbf{P}}_{i,j} \approx (\mathbf{P}_{1,j} \cdot \mathbf{P}_{2,j} \cdots \mathbf{P}_{i-1,j} \cdot \mathbf{P}_{i+1,j} \cdots \mathbf{P}_{K^{(S)},j})^p, \quad p \rightarrow \infty \quad (6)$$

where $\mathbf{P}_{i,j}$ is the projection matrix on the i th user's null space (on the j th subcarrier) which can be obtained by performing the SVD of the i th user's channel matrix on the j th subcarrier $\mathbf{H}_{i,j}$. Note that it is sufficient to choose the projection order² p between 1 and 3 [7].

Hence, by using such an orthogonal-projection-based interpretation of the precoding process and the repeated projection approximation, we obtain $\|\mathbf{H}_{i,j}^{(\text{eq})}\|_F$, which is the Frobenius norm of the equivalent single-user channel matrix on the j th subcarrier for the i th user present in user set \mathcal{S} without performing the precoding. With the perfect CSIT, an SVD-based transmission scheme can be applied to the equivalent single-user transmissions. Assume that the number of spatial streams transmitted to the i th user on the j th subcarrier, i.e., d_i as denoted previously, is chosen as the rank of $\mathbf{H}_{i,j}^{(\text{eq})}$. The channel gains on these spatial streams are then the d_i nonzero eigenvalues of $\mathbf{H}_{i,j}^{(\text{eq})} \mathbf{H}_{i,j}^{(\text{eq})H}$ when the SVD-based transmission scheme is adopted. Note that one of the goals of the proposed efficient scheduling algorithm is to avoid the calculation of the precoding matrices. The optimum power allocation on the spatial streams of each user cannot be determined without the knowledge of the channel gain on each spatial stream. Thus, for the spatial scheduling process, equal power is assumed allocated to each spatial stream for simplicity. Therefore, knowing that $\|\mathbf{H}_{i,j}^{(\text{eq})}\|_F$, we obtain the receive SINR averaged over the d_i spatial streams of the i th user present in user set \mathcal{S} on the j th subcarrier as

$$\rho_{i,j}^{(S)} = \frac{P_T \cdot \text{trace} \left\{ \mathbf{H}_{i,j}^{(\text{eq})} \mathbf{H}_{i,j}^{(\text{eq})H} \right\}}{K^{(S)} d_i^2 \sigma_n^2} = \frac{P_T \cdot \|\mathbf{H}_{i,j}^{(\text{eq})}\|_F^2}{K^{(S)} d_i^2 \sigma_n^2}, \quad i = 1, \dots, K^{(S)}; \quad j = 1, \dots, N_{\text{SD}} \quad (7)$$

where P_T is the transmit power allocated on each subcarrier. Note that equal power is assigned on the subcarriers, i.e., $P_T = P_{T, \text{tot}}/N_{\text{SD}}$, where $P_{T, \text{tot}}$ is the total transmit power. It is known that, using a link-to-system mapping scheme, a metric of the link quality can be acquired from a collection of data measuring the instantaneous channel state, such as the SINRs on all the data streams and across all the subcarriers in a multicarrier system. Inspired by this idea, we propose extracting an effective SINR as an estimate of the quality of transmissions involving a candidate user group and use it as the scheduling metric. Based on the EESM scheme, we represent the effective SINR for the transmission of the i th user in the presence of user set \mathcal{S} by using the averaged SINRs across

²In the simulations, the results of which are presented in Section VII, the projection order p is chosen as 1.

the spatial streams on each subcarrier obtained from (7) for simplicity, i.e.,

$$\rho_i^{(S)} = -\beta_2 \ln \left(\frac{1}{N_{\text{SD}}} \sum_{j=1}^{N_{\text{SD}}} \exp \left(-\frac{\rho_{i,j}^{(S)}}{\beta_1} \right) \right), \quad i = 1, \dots, K^{(S)} \quad (8)$$

where β_1 and β_2 are system-dependent parameters obtained from calibration. The detailed calibration procedures are presented in Section VI where the system-level simulator is developed. It is worth mentioning that, in the evaluation of the performance of multicarrier systems, parallel transmissions on different subcarriers are sometimes assumed independent for simplicity. Then, the performance is measured by averaging over certain metrics (e.g., rate or SINR) for all the subcarriers, and the averaged metric is used for scheduling. However, in some practical cases, for instance, when the channel shows obvious frequency selectivity and the information symbols at the transmitter are not independently coded (e.g., a convolutional encoder is applied), the aforementioned assumption no longer holds. It might also lead to an overoptimistic estimate of the system performance, which is not an effective metric for a scheduling algorithm. By comparison, our proposed scheduling metric utilizes well-calibrated parameters and provides a more accurate measure of the performance of the transmissions of each candidate user group in such a multicarrier system.

Thereby, based on (5)–(8), the scheduling metric for the i th user in the presence of a set of users \mathcal{S} , $i = 1, \dots, K^{(S)}$, can be calculated as

$$\rho_i^{(S)} = -\beta_2 \ln \left(\frac{1}{N_{\text{SD}}} \sum_{j=1}^{N_{\text{SD}}} \exp \left(-\frac{P_T \cdot \|\mathbf{H}_{i,j} \tilde{\mathbf{P}}_{i,j}\|_F^2}{\beta_1 K^{(S)} d_i^2 \sigma_n^2} \right) \right). \quad (9)$$

The sum metric of candidate user set \mathcal{S} is then

$$\eta^{(S)} = \sum_{i=1}^{K^{(S)}} \rho_i^{(S)}. \quad (10)$$

Note that the Frobenius norm of the projected channel $\|\mathbf{H}_{i,j} \tilde{\mathbf{P}}_{i,j}\|_F$ also indicates the spatial correlation among the users. A higher spatial correlation results in a smaller Frobenius norm of the projected channel and, hence, a smaller scheduling metric. Thus, the task of avoiding the allocation of users with a high spatial correlation into one SDMA group is fulfilled.

B. Fairness Consideration

To further take the fairness of the scheduling into account, the scheduling metric in (10) can be extended using one of the known fairness algorithms. Here, we present one extension of the EMC-ProSched scheduling metric based on the proportional fairness algorithm. The scheduling metric for the i th user in the presence of a set of users \mathcal{S} is then normalized by its long-term average metric. Accordingly, the sum metric of user set \mathcal{S} is written as

$$\eta^{(S)} = \sum_{i=1}^{K^{(S)}} \frac{\rho_i^{(S)}}{\bar{\rho}_i} \quad (11)$$

where $\bar{\rho}_i$ is the average of the user metrics of the previous m time slots. We define $\bar{\rho}_i(n)$ for the n th time slot as

$$\bar{\rho}_i(n) = \frac{1}{m} \sum_{\mu=n-m}^{n-1} \rho_i(\mu). \quad (12)$$

In the μ th time slot, if the i th user is selected, then $\rho_i(\mu)$ is set to its metric calculated for its presence in the selected group. Conversely, if the i th user is not selected in the μ th time slot, $\rho_i(\mu)$ is set to zero. With this proportional fairness-based extension, the scheduling fairness is enhanced by taking into account previous scheduling results while determining the users to be served in the present scheduling process.

C. Complexity Analysis

The complexity of the proposed scheduling algorithm is mainly determined by the calculation of the SVDs. This is due to the fact that the SVD for an N_{row} -by- N_{col} dimensional matrix experiences complexity proportional to $\min(N_{\text{row}} \cdot N_{\text{col}}^2, N_{\text{row}}^2 \cdot N_{\text{col}})$. In the proposed EMC-ProSched scheme, only $(N \cdot N_{\text{SD}})$ SVDs are required at the beginning of each scheduling process for all the possible user groups. By comparison, for the conventional spatial scheduling algorithms where the calculation of the precoding matrices for all the user combinations is required [9], [23], assuming that the SDMA group size is fixed to K , the number of SVDs needed to accomplish the same scheduling task is

$$N_{\text{SD}} \times [(N-1) \times 2 + (N-2) \times 3 + \dots + (N-K+1) \times K] \\ = N_{\text{SD}} \times \left[\left(\frac{1}{2} K(K+1) - 1 \right) N - \frac{1}{3} K(K^2 - 1) \right].$$

For example, in a system with $N = 16$ users, the number of data subcarriers N_{SD} is 234, as specified in the IEEE 802.11ac standard for an 80-MHz transmission [18], and the SDMA group size is set to $K = 4$ users. The number of SVDs required for EMC-ProSched is only 3744. On the other hand, for the conventional scheduling algorithms requiring the calculation of the precoding matrices for all the user combinations, the same scheduling task involves 29016 SVDs. Compared with these spatial scheduling algorithms, the proposed EMC-ProSched scheme has much lower complexity, particularly for multicarrier systems with a large number of data subcarriers.

Note that the ProSched algorithm [7] initially proposed for one subcarrier is also based on the orthogonal projections to avoid a prohibitive large number of required SVDs. However, when applied in a multicarrier system, the ProSched algorithm treats subcarriers as virtual users. It experiences unaffordable computational complexity to search for a suitable group of virtual users among the total $N \cdot N_{\text{SD}}$ virtual users. In addition, by treating the subcarriers as virtual users, the number of users served will be different from one subcarrier to another. This feature requires much more information exchange between the BS and the UTs, and huge signaling overhead is unavoidable. In addition, at each time slot, serving some selected subcarriers but not all subcarriers for one user is not compatible with the IEEE 802.11ac specifications [18].

D. Cross-Layer Aspects

EMC-ProSched, which is a spatial scheduling algorithm, is proposed as the first stage of the physical-layer (PHY) transmission scheme for the MU MIMO downlink, which is the major focus of this paper. It accomplishes the task of the SDMA user group selection mainly by exploring the spatial correlation of the users such that the MU diversity is better exploited, whereas the impacts of multicarrier transmissions are also taken into account. However, in practice, link layer retransmission mechanisms, such as automatic repeat request (ARQ) and hybrid ARQ (HARQ), are used to handle erroneous packet transmissions. This gives rise to the concern of considering possible effects of retransmissions in the scheduling process when a certain ARQ/HARQ scheme is employed. In this case, retransmissions and new transmissions are involved in the scheduling procedure. If a higher priority is granted to retransmissions, it can be reflected in the design of the scheduling metric. In [24], modifications to a scheduler are proposed by considering the retransmission information in the cases where simple ARQ and Chase combining are used, respectively. They provide some insights into a cross-layer extension of the proposed EMC-ProSched algorithm. Moreover, the impact of the SINR increment per retransmission to that of the first transmission is also worthy of attention since EMC-ProSched employs an SINR-based scheduling metric. (see the HARQ combining gain model proposed in [25] and its modified versions in [26] for both the Chase combining scheme and the incremental redundancy scheme).

Hence, to adapt EMC-ProSched to systems where retransmission mechanisms are considered as future work, the aforementioned SINR increment can be incorporated into an extension of the scheduling metric. In addition, a weighted version of the scheduling metric can be devised, where the weights are computed based on, for instance, estimates of the PERs for retransmissions and new transmissions [24]. Consequently, a higher priority can be provided to retransmissions as they might be more prone to experience a higher SINR due to the HARQ combining gain.

IV. LINEAR-PRECODING-BASED GEOMETRIC MEAN DECOMPOSITION

After selecting the user group to be served, LP algorithms are employed to mitigate the MUI. For **Case 1** where $M_T \geq M_R$, we propose the LP-GMD algorithm. Let us follow the generic philosophy of the calculation of the LP matrix as explained in [4]. The calculation can be divided into two steps, i.e., the MUI cancelation or suppression and the system performance optimization. Thus, precoding matrix \mathbf{F} can be expressed as

$$\mathbf{F} = \gamma \mathbf{F}_a \mathbf{F}_b \quad (13)$$

where $\mathbf{F}_a = [\mathbf{F}_{a_1}, \mathbf{F}_{a_2}, \dots, \mathbf{F}_{a_K}] \in \mathbb{C}^{M_T \times M_x}$ with $\mathbf{F}_{a_i} \in \mathbb{C}^{M_T \times M_{x_i}}$ serves to cancel or suppress the MUI, and the block-diagonal matrix $\mathbf{F}_b = \text{blkdiag}\{\mathbf{F}_{b_1}, \mathbf{F}_{b_2}, \dots, \mathbf{F}_{b_K}\} \in \mathbb{C}^{M_x \times d}$ with $\mathbf{F}_{b_i} \in \mathbb{C}^{M_{x_i} \times d_i}$ is used to further tune the system performance for each user separately. We define $M_x = \sum_{i=1}^K M_{x_i}$ and assume that $M_{x_i} \leq d$. Parameter γ is chosen

such that the total transmit power constraint is fulfilled. In the proposed LP-GMD algorithm, which follows this generic philosophy, matrix \mathbf{F}_a is first calculated by suitable MUI suppression algorithms to mitigate the MUI, and then, matrix \mathbf{F}_b is computed by using the GMD algorithm to enable an equal MCS implementation across the subchannels of each user. It should be noted that the MUI suppression algorithm can be flexibly selected in the LP-GMD algorithm, and two versions will be given in what follows, which are called LP-GMD-BD and LP-GMD-RBD. In addition, an MMSE-based power loading strategy is developed for the LP-GMD schemes to further enhance their performances.

A. LP-GMD-BD

First, we follow the strategy of the BD algorithm to calculate matrix \mathbf{F}_a , and we call this approach LP-GMD-BD. By substituting (13) into joint received vector (3), we can express the received vector for the i th user as

$$\mathbf{y}_i = \gamma \mathbf{D}_i^H \mathbf{H}_i \mathbf{F}_{a_i} \mathbf{F}_{b_i} \mathbf{x}_i + \bar{\mathbf{n}}_i + \sum_{j=1, j \neq i}^K \gamma \mathbf{D}_i^H \mathbf{H}_i \mathbf{F}_{a_j} \mathbf{F}_{b_j} \mathbf{x}_j \quad (14)$$

where $\bar{\mathbf{n}}_i = \mathbf{D}_i^H \mathbf{n}_i$, and the last term on the right-hand side of the equation represents the MUI. The design criterion for \mathbf{F}_a is to mitigate this interference term. It has been pointed out in [3] that, for the i th user \mathbf{F}_{a_i} can be calculated such that it lies in the common null space of the other users' channel matrices. Let us define matrix $\widetilde{\mathbf{H}}_i \in \mathbb{C}^{(M_R - M_{R_i}) \times M_T}$ as the joint matrix that consists of all the other users' channel matrices, which is given by

$$\widetilde{\mathbf{H}}_i = [\mathbf{H}_1^T, \dots, \mathbf{H}_{i-1}^T, \mathbf{H}_{i+1}^T, \dots, \mathbf{H}_K^T]^T. \quad (15)$$

The zero MUI constraint forces the precoding matrix of the i th user \mathbf{F}_{a_i} to lie in the null space of $\widetilde{\mathbf{H}}_i$. Assuming that the rank of $\widetilde{\mathbf{H}}_i$ is \widetilde{L}_i , by performing the SVD of $\widetilde{\mathbf{H}}_i$, we have

$$\widetilde{\mathbf{H}}_i = \widetilde{\mathbf{U}}_i \widetilde{\Sigma}_i \begin{bmatrix} \widetilde{\mathbf{V}}_i^{(1)} \\ \widetilde{\mathbf{V}}_i^{(0)} \end{bmatrix}^H \quad (16)$$

where $\widetilde{\mathbf{V}}_i^{(1)}$ contains the first \widetilde{L}_i right singular vectors, and $\widetilde{\mathbf{V}}_i^{(0)}$ contains the last $M_T - \widetilde{L}_i$ right singular vectors that form an orthogonal basis for the null space of $\widetilde{\mathbf{H}}_i$. Therefore, matrix \mathbf{F}_{a_i} for the i th user is identified as

$$\mathbf{F}_{a_i} = \widetilde{\mathbf{V}}_i^{(0)}. \quad (17)$$

It should be noted that this result leads to $\widetilde{\mathbf{H}}_i \mathbf{F}_{a_i} = \mathbf{0}$, i.e., $\mathbf{H}_j \mathbf{F}_{a_i} = \mathbf{0}$ with $j \neq i$. Hence, the received vector of the i th user can be then expressed as

$$\mathbf{y}_i = \gamma \mathbf{D}_i^H \mathbf{H}_i \mathbf{F}_{a_i} \mathbf{F}_{b_i} \mathbf{x}_i + \bar{\mathbf{n}}_i. \quad (18)$$

The equivalent channel of the i th user denoted by $\mathbf{H}_i^{(\text{eq})}$ is then expressed as

$$\mathbf{H}_i^{(\text{eq})} = \mathbf{H}_i \widetilde{\mathbf{V}}_i^{(0)} \in \mathbb{C}^{M_{R_i} \times (M_T - \widetilde{L}_i)}. \quad (19)$$

Note that $\mathbf{H}_i^{(\text{eq})}$ equivalently corresponds to the channel of a single-user system with $M_T - \widetilde{L}_i$ transmit antennas and M_{R_i} receive antennas, where $M_T - \widetilde{L}_i \geq M_{R_i}$. Hence, the GMD algorithm can be employed to the equivalent single-user MIMO transmissions. The GMD of the equivalent channel matrix of the i th user can be expressed as

$$\mathbf{H}_i^{(\text{eq})} = \mathbf{Q}_i \mathbf{R}_i \mathbf{Z}_i^H \quad (20)$$

where $\mathbf{Q}_i^H \mathbf{Q}_i = \mathbf{Z}_i^H \mathbf{Z}_i = \mathbf{I}_L$, assuming that the rank L matrix $\mathbf{H}_i^{(\text{eq})}$ has nonzero singular values λ_l , with $l = 1, \dots, L$. The L -by- L matrix \mathbf{R}_i is an upper triangular matrix with real-valued and equal entries on its diagonal, i.e., $r_{nn} = \bar{\lambda} = (\prod_{l=1}^L \lambda_l)^{1/L}$ [13], where $n = 1, \dots, L$. The GMD of $\mathbf{H}_i^{(\text{eq})}$ can be performed starting with the SVD of $\mathbf{H}_i^{(\text{eq})}$ given by

$$\mathbf{H}_i^{(\text{eq})} = \mathbf{U}_i^{(\text{eq})} \Sigma_i^{(\text{eq})} \mathbf{V}_i^{(\text{eq})H}. \quad (21)$$

The upper triangular matrix \mathbf{R}_i can be constructed, as explained in [13]

$$\mathbf{R}_i = \widetilde{\mathbf{Q}}_i \Sigma_i^{(\text{eq})} \widetilde{\mathbf{Z}}_i^H \quad (22)$$

where $\widetilde{\mathbf{Q}}_i \in \mathbb{C}^{L \times L}$, and $\widetilde{\mathbf{Z}}_i \in \mathbb{C}^{L \times L}$ are unitary matrices. Each of them equals the product of a sequence of orthogonal matrices constructed using a symmetric permutation and a pair of Givens rotations [13]. By combining (21) and (22), matrices \mathbf{Q}_i and \mathbf{Z}_i are expressed as

$$\mathbf{Q}_i = \mathbf{U}_i^{(\text{eq})} \widetilde{\mathbf{Q}}_i^H \quad (23)$$

$$\mathbf{Z}_i = \mathbf{V}_i^{(\text{eq})} \widetilde{\mathbf{Z}}_i^H. \quad (24)$$

Matrix \mathbf{F}_{b_i} in the LP-GMD scheme is defined as

$$\mathbf{F}_{b_i} = \mathbf{Z}_i \in \mathbb{C}^{(M_T - \widetilde{L}_i) \times L}. \quad (25)$$

Let us use $\widetilde{\mathbf{y}}_i$ as a short-hand notation for the signal received by the i th user before decoding, which is written as

$$\begin{aligned} \widetilde{\mathbf{y}}_i &= \gamma \mathbf{H}_i^{(\text{eq})} \mathbf{F}_{b_i} + \mathbf{n}_i \\ &= \gamma (\mathbf{Q}_i \mathbf{R}_i \mathbf{Z}_i^H) \mathbf{Z}_i \mathbf{x}_i + \mathbf{n}_i \\ &= \gamma \mathbf{Q}_i \mathbf{R}_i \mathbf{x}_i + \mathbf{n}_i. \end{aligned} \quad (26)$$

The equivalent channel in (26) identified as $\mathbf{Q}_i \mathbf{R}_i$ can be acquired by performing channel estimation at the i th user. Via a QR decomposition of this equivalent channel matrix³, \mathbf{Q}_i is obtained and employed as the decoding matrix for the i th user, i.e.,

$$\mathbf{D}_i = \mathbf{Q}_i \in \mathbb{C}^{M_{R_i} \times L}. \quad (27)$$

³The QR decomposition of invertible matrix $\mathbf{A} \in \mathbb{C}^{m \times m}$ written as $\mathbf{A} = \mathbf{Q}\mathbf{R}$ is unique when the diagonal elements of \mathbf{R} are required to be real and positive. Here, assume that the equivalent single-user channel matrix of the i th user $\mathbf{H}_i^{(\text{eq})}$ has full row rank, i.e., $L = M_{R_i}$. In addition, note that, in our case, the real-valued matrix \mathbf{R}_i is indeed required to have positive diagonal entries. Hence, assuming that the equivalent channel $\mathbf{Q}_i \mathbf{R}_i$ is perfectly estimated, \mathbf{Q}_i and \mathbf{R}_i are the unique outcome of the QR decomposition of this equivalent channel. In addition, the number of spatial streams transmitted to the i th user d_i is chosen to be the rank of the equivalent single-user channel $\mathbf{H}_i^{(\text{eq})}$ to achieve the full multiplexing gain.

Therefore, with (26) and (27), the received signal of the i th user expressed in (18) is further written as

$$\begin{aligned} \mathbf{y}_i &= \mathbf{Q}_i^H \tilde{\mathbf{y}}_i \\ &= \gamma \mathbf{Q}_i^H \mathbf{Q}_i \mathbf{R}_i \mathbf{x}_i + \tilde{\mathbf{n}}_i \\ &= \gamma \mathbf{R}_i \mathbf{x}_i + \tilde{\mathbf{n}}_i. \end{aligned} \quad (28)$$

Recall that \mathbf{R}_i is an upper triangular matrix with equal diagonal elements. By adopting a successive interference cancellation (SIC) receiver, all spatial streams for the i th user experience the same gain and the same noise variance resulting in the same SINR. The implementation of the same MCS is consequently enabled on the spatial streams of each user. As the number of receive antennas at a UT is rather limited, applying an SIC receiver does not induce much additional computational effort. Furthermore, it has been shown in [13] that, when the number of receive antennas at the UTs is moderate, the propagation error incurred by the SIC receiver can be ignored.

Note that the LP-GMD algorithm is proposed as one promising solution for MU MIMO downlink transmissions where it is desired (e.g., due to certain constraints by standardization or the pursuit of smaller signaling overhead) that the same MCS is used on all spatial streams for each user. In this paper, we treat the case where all possible spatial modes for each user are switched on, i.e., the number of spatial streams transmitted to each user equals the rank of the equivalent single-user channel after the MUI mitigation. As future work, it is also of interest to extend the LP-GMD scheme to the case where more than one but not all spatial modes are used and equal MCS is allowed on them.

B. LP-GMD-RBD

In what follows, we present LP-GMD-RBD, in which the calculation of matrix \mathbf{F}_a follows the philosophy of the RBD algorithm [4]. We start from the equivalent combined channel matrix of all users after the MUI mitigation, which is given by

$$\mathbf{H}\mathbf{F}_a = \begin{bmatrix} \mathbf{H}_1 \mathbf{F}_{a_1} & \mathbf{H}_1 \mathbf{F}_{a_2} & \cdots & \mathbf{H}_1 \mathbf{F}_{a_K} \\ \mathbf{H}_2 \mathbf{F}_{a_1} & \mathbf{H}_2 \mathbf{F}_{a_2} & \cdots & \mathbf{H}_2 \mathbf{F}_{a_K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_K \mathbf{F}_{a_1} & \mathbf{H}_K \mathbf{F}_{a_2} & \cdots & \mathbf{H}_K \mathbf{F}_{a_K} \end{bmatrix} \quad (29)$$

where the i th user's effective channel is represented as $\mathbf{H}_i \mathbf{F}_{a_i}$, and the interference generated to the other users is determined by $\tilde{\mathbf{H}}_i \mathbf{F}_{a_i}$ with $\tilde{\mathbf{H}}_i$ defined in (15). Here, matrix \mathbf{F}_{a_i} is chosen such that the off-diagonal block matrices of the equivalent combined channel matrix of all users after the MUI cancellation converge to zero as the SNR increases. According to [4], \mathbf{F}_{a_i} can be written as

$$\mathbf{F}_{a_i} = \tilde{\mathbf{V}}_i \left(\tilde{\Sigma}_i^T \tilde{\Sigma}_i + \frac{M_R \sigma_n^2}{P_T} \mathbf{I}_{M_T} \right)^{-1/2} \quad (30)$$

where $\tilde{\mathbf{V}}_i$ and $\tilde{\Sigma}_i$ are obtained from the SVD of $\tilde{\mathbf{H}}_i$ given by

$$\tilde{\mathbf{H}}_i = \tilde{\mathbf{U}}_i \tilde{\Sigma}_i \tilde{\mathbf{V}}_i^H. \quad (31)$$

Recall that P_T is the transmit power allocated on each subcarrier, and σ_n^2 is the noise power in the bandwidth of each subcarrier at the receiver. In the high-SNR regime, the off-diagonal block matrices of the equivalent combined channel matrix of all users $\mathbf{H}\mathbf{F}_a$ converge to zero, i.e., $\mathbf{H}_j \mathbf{F}_{a_i} \approx \mathbf{0}$ with $j \neq i, i, j = 1, 2, \dots, K$.

After determining matrix \mathbf{F}_a , the LP-GMD-RBD scheme also arrives at the equivalent single-user MIMO channels. The GMD algorithm can be then employed to obtain matrices \mathbf{F}_{b_i} as in (25). Note that the methods to obtain \mathbf{F}_a are not limited to the two approaches that we show here. Other linear MUI suppression algorithms fitted into the MU MIMO LP philosophy represented by (13) can be employed in the LP-GMD scheme.

C. Complexity Analysis

The proposed LP-GMD algorithm consists of two parts, which are the MUI suppression and the GMDs of the equivalent single-user MIMO channel matrices. For all the LP-GMD versions, the second step requires K times the calculation of the GMD. The complexity of the first step depends on the complexity of the chosen MUI suppression algorithm for MU MIMO systems. For example, in LP-GMD-BD, the complexity of the MUI suppression is mainly determined by the K calculations of SVDs of the $(M_R - M_{R_i})$ -by- M_T joint matrices expressed in (15). In LP-GMD-RBD, the MUI mitigation procedure requires not only K SVDs of the $(M_R - M_{R_i})$ -by- M_T joint matrices but also the calculation of the expressions shown in (30) for each user. Note that, for the work of BD-GMD combined with THP [14], the matrix decomposition of the K -user MIMO channel requires K times the calculation of the GMD and a recursion process, whereas the nonlinear THP algorithm experiences higher complexity than LP algorithms such as BD and RBD.

D. MMSE-Based Power Loading

Note that the goal of LP-GMD is to balance the power across the subchannels of each user. To further assign the power efficiently to all the users, an MMSE-based power loading strategy is developed here for further enhancement of the LP-GMD algorithm.

For the i th user, let us define diagonal matrix $\Sigma_i \in \mathbb{C}^{d_i \times d_i}$ such that the entries on its diagonal are the same as the diagonal elements of \mathbf{R}_i obtained from the GMD of the i th user's equivalent channel after the MUI mitigation. We further define matrix Σ_e as

$$\Sigma_e = \begin{bmatrix} \Sigma_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \Sigma_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \Sigma_K \end{bmatrix} \in \mathbb{R}^{d \times d}. \quad (32)$$

Matrix \mathbf{F}_b , after applying the proposed power loading strategy, is then identified as

$$\mathbf{F}_b = \text{blkdiag}\{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_K\} \cdot \Lambda \in \mathbb{C}^{M_x \times d} \quad (33)$$

where matrices \mathbf{Z}_i with $i = 1, 2, \dots, K$ are obtained by the GMD algorithm as in (25), and $\mathbf{\Lambda} \in \mathbb{R}^{d \times d}$ denotes the power loading matrix given by [4]

$$\mathbf{\Lambda} = \left(\sum_e^T \Sigma_e + \frac{M_R \sigma_n^2}{P_T} \mathbf{I}_d \right)^{-1} \cdot \Sigma_e^T \in \mathbb{R}^{d \times d}. \quad (34)$$

By employing such a power loading matrix, the weak users are enhanced, whereas the strong users are weakened. Consequently, the degradation caused by the weak users dominating the PER performance is avoided. Moreover, it should be noted that, in MU transmissions, in practice, under a certain standard framework such as the IEEE 802.11ac, the manner of the integration of the payload belonging to different users into the PHY packet must be taken into account. When the sizes of the payload of different users are on the same level to ensure fairness, unequal MCSs across different users will cause overhead due to the requirement of padding bits [18]. When this MMSE-based power loading strategy is employed, the power is balanced over all the users such that the probability that a lower MCS is applied to some users whereas a higher to others is reduced. The power efficiency is thereby enhanced by reducing the power loss caused by the overhead.

V. LOW-COMPLEXITY COORDINATED BEAMFORMING

In **Case 2**, where the total number of receive antennas at the K users selected by performing the EMC-ProSched algorithm is larger than the number of transmit antennas at the BS, the proposed precoding algorithm LoCCoBF is employed. Note that, for precoding algorithms such as BD, which achieve the MUI suppression by letting each user transmit in all the other users' common null space, under the condition that $M_R > M_T$, such a null space is, in general, empty. Therefore, we define an equivalent joint channel matrix \mathbf{H}_e for the K users, which is given by

$$\mathbf{H}_e = \begin{bmatrix} \mathbf{D}_1^H \mathbf{H}_1 \\ \vdots \\ \mathbf{D}_K^H \mathbf{H}_K \end{bmatrix} \in \mathbb{C}^{d \times M_T} \quad (35)$$

where \mathbf{H}_i , $i = 1, 2, \dots, K$, is the channel matrix for the i th user on a certain subcarrier at a certain time instant, as defined in Section II. Recall that $\mathbf{D}_i \in \mathbb{C}^{M_{R_i} \times d_i}$ and $d_i \leq M_{R_i}$, where $i = 1, 2, \dots, K$, is the receive beamforming matrix. Due to the fact that the number of spatial streams does not exceed the number of transmit antennas, i.e., $d \leq M_T$, the aforementioned dimension constraint is then relaxed by performing precoding on the equivalent channel matrix defined in (35). The main idea of coordinated beamforming algorithms is that the transmit and receive beamformers are iteratively updated based on an initialized receive beamforming matrix until a stopping criterion is satisfied to achieve the goal of the MUI suppression [15], [16]. In our proposed LoCCoBF algorithm, we further take into account the correlation among the channels of different subcarriers. The receive beamforming matrices calculated for the adjacent subcarrier are employed as the receive beamforming matrices for the initialization step of the current subcarrier to

reduce computational complexity. The LoCCoBF algorithm is summarized as follows.

- **Step 1:** Initialize the receive beamforming matrices $\mathbf{D}_i^{(0)}$ with $i = 1, 2, \dots, K$, set iteration index p to zero, and set threshold ϵ for the stopping criterion. If the current subcarrier is the first subcarrier, the receive beamforming matrices are generated randomly; otherwise, set the receive beamforming matrices as the receive beamforming matrices calculated for the previous subcarrier.
- **Step 2:** Set $p = p + 1$, and calculate the equivalent joint channel matrix $\mathbf{H}_e^{(p)}$ in the p th iteration as

$$\mathbf{H}_e^{(p)} = \left[\mathbf{H}_{e_1}^{(p)T}, \mathbf{H}_{e_2}^{(p)T}, \dots, \mathbf{H}_{e_K}^{(p)T} \right]^T \quad (36)$$

where $\mathbf{H}_{e_i}^{(p)} = \mathbf{D}_i^{(p-1)H} \mathbf{H}_i$ is the equivalent channel matrix for the i th user in the p th iteration.

- **Step 3:** Calculate the transmit beamforming matrix in the p th iteration using the equivalent joint channel matrix $\mathbf{H}_e^{(p)}$. For the i th user, let us first define matrix $\widetilde{\mathbf{H}}_{e_i}^{(p)}$ as

$$\widetilde{\mathbf{H}}_{e_i}^{(p)} = \left[\mathbf{H}_{e_1}^{(p)T}, \dots, \mathbf{H}_{e_{i-1}}^{(p)T}, \mathbf{H}_{e_{i+1}}^{(p)T}, \dots, \mathbf{H}_{e_K}^{(p)T} \right]^T \quad (37)$$

which contains the equivalent channel matrices of all the other users obtained in **Step 2**. Then, transmit beamforming matrix $\mathbf{F}_i^{(p)}$ in the p th iteration for the i th user is obtained as $\mathbf{F}_i^{(p)} = \widetilde{\mathbf{V}}_{e_i}^{(p,0)}$, where $\widetilde{\mathbf{V}}_{e_i}^{(p,0)}$ contains the d_i singular vectors that form an orthonormal basis for the null space of $\widetilde{\mathbf{H}}_{e_i}^{(p)}$.

- **Step 4:** Calculate the receive beamforming matrix in the p th iteration using the channel after precoding $\mathbf{H}_i \mathbf{F}_i^{(p)}$ for the i th user. When using the MMSE receiver, $\mathbf{D}_i^{(p)}$ then has the following form:

$$\mathbf{D}_i^{(p)} = \left(\mathbf{H}_i \mathbf{F}_i^{(p)} \mathbf{F}_i^{(p)H} \mathbf{H}_i^H + \sigma_n^2 \mathbf{I}_{M_{R_i}} \right)^{-1} \mathbf{H}_i \mathbf{F}_i^{(p)}. \quad (38)$$

- **Step 5:** Calculate the term $\text{MUI}(\mathbf{H}_e^{(p)} \mathbf{F}^{(p)})$ defined as

$$\text{MUI}(\mathbf{H}_e^{(p)} \mathbf{F}^{(p)}) = \left\| \text{off} \left(\mathbf{H}_e^{(p)} \mathbf{F}^{(p)} \right) \right\|_F^2 \quad (39)$$

where $\text{off}(\cdot)$ indicates an operation of keeping all off-diagonal elements of its input matrix while setting its diagonal elements to zero. This term measures the residual MUI and the interstream interference. If $\text{MUI}(\mathbf{H}_e^{(p)} \mathbf{F}^{(p)}) < \epsilon$, the convergence is achieved, and the iterative procedure comes to the end. Otherwise, go back to **Step 2**.

Via numerical investigations as in [27], it can be shown that there exist rare cases where the convergence of LoCCoBF is not achieved. For most of these cases, although the residual MUI still stays above threshold ϵ after a number of iterations, the value is already on a level that leads to a performance as good as in the case where the convergence is achieved. Therefore, the maximum number of iterations is set to handle these cases. The iterative procedure is manually terminated if the residual MUI does not fall below the threshold when the number of iterations reaches this maximum number. Moreover, other stopping criteria, such as the change of the transmit

TABLE I
PROPOSED TRANSMISSION STRATEGY

Stage 1: Scheduling	
Step 1	Set the SDMA group size as $K^{(S)} = 1$ Calculate the sum metric defined in (10) for all possible user groups Select the best one identified as $\mathcal{S}_1^{(\text{opt})}$ with the maximum sum metric
Step 2	Set $K^{(S)} = K^{(S)} + 1$, and add one of the remaining users to the selected user group with the group size $(K^{(S)} - 1)$ Calculate the sum metric using (10) for all the new possible user groups Select the best one identified as $\mathcal{S}_{K^{(S)}}^{(\text{opt})}$ with the maximum sum metric
Step 3	Compare $K^{(S)}$ with the total number of UTs in the system N and the maximum allowed SDMA group size K_{\max} <ul style="list-style-type: none"> • If $K^{(S)} < \min(N, K_{\max})$, go back to Step 2 • If $K^{(S)} = \min(N, K_{\max})$, compare the sum metrics of the optimum user groups with different sizes and select the one with the maximum sum metric
Stage 2: Precoding	
Step 1	Compare the number of transmit antennas at the BS M_T with the total number of receive antennas of the UTs in the user group selected in Stage 1 M_R
Step 2	Calculate the precoding matrices <ul style="list-style-type: none"> • If Case 1 ($M_T \geq M_R$), employ LP-GMD-BD detailed in Section IV-A or LP-GMD-RBD in Section IV-B • If Case 2 ($M_T < M_R$), employ LoCCoBF described in Section V

beamforming matrices expressed as $\|\mathbf{F}^{(p)} - \mathbf{F}^{(p-1)}\|_F^2$ [16] can be also employed.

It is worth mentioning that, in the LoCCoBF algorithm, by jointly updating the precoding and decoding matrices at the BS, we obtain the precoding matrices in the case where $M_R > M_T$. It is not required that the UTs are informed of their decoding matrices. The UTs can obtain the CSI via the channel estimation and use this to tune their decoding matrices.

In addition, the proposed LoCCoBF algorithm presents in a more general sense a framework of coordinated beamforming, and more flexibility is allowed in computing the transmit and receive beamforming matrices. For instance, in **Step 3** of LoCCoBF, the precoding matrix is calculated to mitigate the MUI, and the number of spatial streams transmitted to each user is chosen as the rank of the resulting equivalent single-user channel to achieve the full spatial multiplexing gain. One might also incorporate transmit processing techniques (beamforming techniques and spatial multiplexing techniques) originally designed for single-user MIMO transmissions into the calculation of the precoding matrix of LoCCoBF. Consequently, the number of spatial streams for each user can be adapted according to the transmission qualities in terms of, e.g., the spatial rank of the equivalent single-user channel after the MUI suppression and the SNR level.

To this end, the proposed transmission strategy is summarized in Table I.

VI. IEEE 802.11AC SYSTEM-LEVEL SIMULATOR

In this paper, a MATLAB system-level simulator using a link-to-system interface is developed under the IEEE 802.11ac

[18] framework. With the aid of this simulator, the system and user throughput of multicarrier MU MIMO downlink transmissions on one of the IEEE 5-GHz bands with a bandwidth of 20, 40, or 80 MHz can be demonstrated. For each independent run, a new channel realization is generated with the parameters concerning the channel and the antennas. The generation of the channel is based on the MATLAB implementation package of the IEEE 802.11n channel models [28] and follows the modification instructions [29] for the IEEE 802.11ac MU scenarios. A scheduling algorithm first selects a user group to be served, and precoding is then performed on the selected users. Using the calculated precoding and decoding matrices, the equivalent channel matrix and the equivalent noise covariance matrix are obtained. The SINRs are computed on each subcarrier across all the spatial streams.

With the knowledge of the SINRs on the i th spatial stream and j th data subcarrier, which is denoted by $\rho_{i,j}$, $i = 1, 2, \dots, N_{\text{SS}}$ and $j = 1, 2, \dots, N_{\text{SD}}$, the main idea of the link-to-system mapping is to first map all these SINRs to a scalar value known as the link quality metric (LQM). In our case, the effective SINR denoted by ρ_{eff} is employed as the LQM, and it is then used to estimate the PER for this specific channel state according to the AWGN link-level performance. In light of this statement, an accurate effective SINR mapping has to fulfill the following approximate equivalence:

$$P_e^{\text{fading}}(\{\rho_{1,1}, \dots, \rho_{i,j}, \dots, \rho_{N_{\text{SS}}, N_{\text{SD}}}\}) \approx P_e^{\text{AWGN}}(\rho_{\text{eff}}) \quad (40)$$

where P_e stands for the PER. Here, the EESM scheme is adopted. This method was originally used in [30] and has been later applied in the construction of other system-level simulators such as in [31]. Consequently, the expression of the effective SINR is given by

$$\rho_{\text{eff}} = -\beta_2 \ln \left(\frac{1}{N_{\text{SS}} \cdot N_{\text{SD}}} \sum_{j=1}^{N_{\text{SD}}} \sum_{i=1}^{N_{\text{SS}}} \exp \left(-\frac{\rho_{i,j}}{\beta_1} \right) \right) \quad (41)$$

where β_1 and β_2 depend on the MCS and the channel model. Hence, they need to be calibrated. In [30], these two parameters are treated as equal to each other, i.e., $\beta_1 = \beta_2 = \beta$. For each MCS, link-level simulations are performed using N_c channel realizations. For the k th channel realization, a PER denoted by $P_e^{(k)}$ is obtained, with $k = 1, 2, \dots, N_c$. By mapping $P_e^{(k)}$ onto a PER versus SNR curve of transmissions over the AWGN channel with the same MCS applied, an equivalent ‘‘measured’’ SINR $\rho_m^{(k)}$ for the k th channel realization is acquired. Meanwhile, according to (41) with a certain value of β , an effective SINR $\rho_{\text{eff}}^{(k)}(\beta)$ can be calculated using the SINRs on different subcarriers across all the spatial streams for the k th channel realization. Seeking the approximate equivalence in (40), β is then chosen as the numerical solution of the following optimization problem:

$$\beta_{\text{opt}} = \arg \min_{\beta} \left\{ \sum_{k=1}^{N_c} \left| \log \left(\rho_{\text{eff}}^{(k)}(\beta) \right) - \log \left(\rho_m^{(k)} \right) \right|^2 \right\}. \quad (42)$$

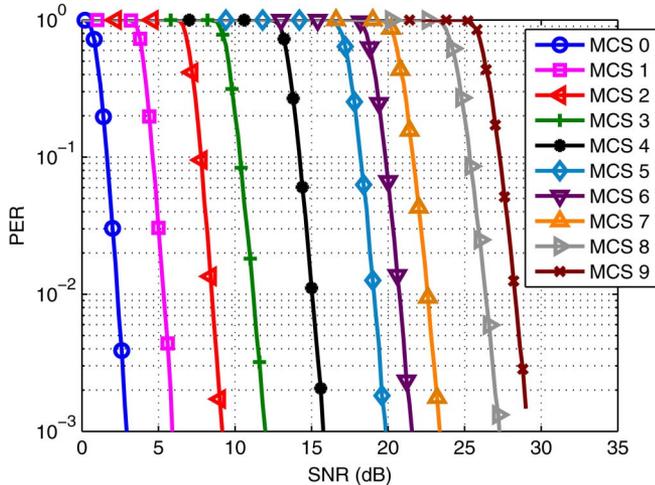


Fig. 1. PER versus SNR curves for single-input-single-output transmissions under AWGN channel for the ten IEEE 802.11ac MCSs.

Parameter β obtained from these calibration procedures guarantees that the approximate equivalence expressed in (40) is fulfilled, leading to an accurate estimate of the link performance in terms of the PER.

In the IEEE 802.11ac, 10MCSs (coding rate from 1/2 to 5/6, BPSK to 256-QAM) are specified [18]. To further enhance the accuracy of the effective SINR mapping for higher-order modulation formats, such as 64-QAM and 256-QAM, we carry out a two-dimensional calibration, turning (42) into

$$\begin{aligned}
 (\beta_1^{(\text{opt})}, \beta_2^{(\text{opt})}) &= \arg \min_{(\beta_1, \beta_2)} \\
 &\left\{ \sum_{k=1}^{N_c} \left| \log(\rho_{\text{eff}}^{(k)}(\beta_1, \beta_2)) - \log(\rho_m^{(k)}) \right|^2 \right\}. \quad (43)
 \end{aligned}$$

The PER versus SNR curves for all the ten IEEE 802.11ac MCSs under the AWGN channel are obtained via link-level simulations and are shown in Fig. 1. These curves play an important role in the parameter calibration of the EESM scheme and the link-to-system mapping itself.

In addition, a fast link adaptation procedure is adopted in our simulator such that one of the ten specified MCSs [18] is selected according to the link quality reflected by the estimates of the PER. Here, the threshold PER for the MCS switching is set to 0.01 [21], [32]. Finally, the PHY throughput with the PHY preamble taken into account is calculated by using the estimate of the PER for a channel realization with the suitable MCS applied.

VII. SIMULATION RESULTS

In the following, simulation results of 80-MHz transmissions are shown. Table II presents the corresponding OFDM parameters [18]. The noise power density is calculated assuming room temperature of 290 K as the noise temperature. The IEEE 802.11ac channel Model D that represents an indoor scenario [33] is adopted. The BS is equipped with eight transmit antennas [18], i.e., $M_T = 8$. The transmit power is set to 25 dBm for all the plots of the complementary cumulative distribution

TABLE II
IEEE 802.11ac OFDM PARAMETERS (80-MHz TRANSMISSION)

Parameter	Value
carrier frequency	5.21 GHz
total bandwidth	80 MHz
number of data subcarriers	234
subcarrier spacing	0.3125 MHz
symbol duration	4 μ s
guard interval	800 ns

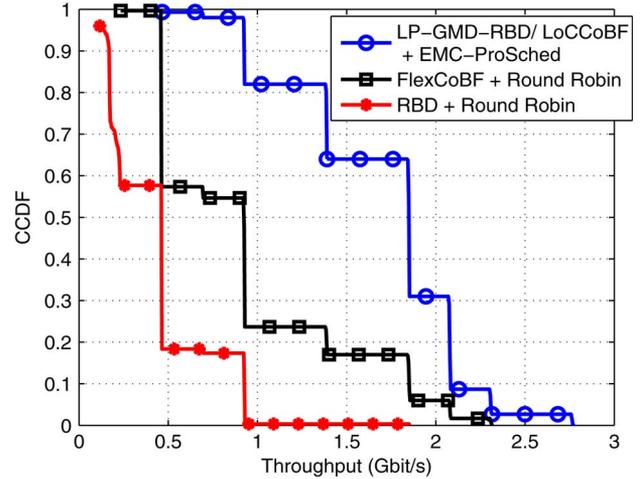


Fig. 2. CCDF of the system throughput of a 16-user scenario with the PSDU size of 500 000 B when the proposed transmission strategy is employed.

function (CCDF) of the throughput. If not specifically stated, the size of the PHY service data unit (PSDU) that is the PHY payload is set to 500 000 B in the simulations.

First, a 16-user scenario is considered. Here, the maximum allowed SDMA group size is 4 [18], i.e., $K_{\text{max}} = 4$. The BS transmits two spatial streams to each of the K users selected by the scheduling algorithm, i.e., $d_i = 2$, where $i = 1, 2, \dots, K$. Among the 16 users, there are 12 users, each of which is equipped with two receive antennas, whereas two of the other users have three receive antennas each, and the remaining two have four receive antennas each. Note that, under such an antenna configuration, the relation between the number of transmit antennas at the BS M_T and the total number of receive antennas at the selected users M_R is arbitrary. In this scenario, we compare our proposed transmission strategy, i.e., EMC-ProSched combined with LP-GMD or LoCCoBF to another two strategies, i.e., the round-robin scheduling scheme combined with the FlexCoBF algorithm and that combined with the RBD algorithm. Note that, in the proposed transmission strategy, as summarized in Table I, after the EMC-ProSched algorithm determines the user group that will be served, LP-GMD is employed to perform the precoding when $M_T \geq M_R$, whereas the LoCCoBF algorithm deals with the opposite case, i.e., $M_T < M_R$. Fig. 2 presents the CCDF of the system throughput for the aforementioned three transmission strategies. We observe that the proposed transmission strategy yields the best performance. This is because the proposed EMC-ProSched is a much more effective scheduling algorithm by applying a novel scheduling metric inspired by the link-to-system mapping. Together with well-calibrated parameters, it contributes to an accurate

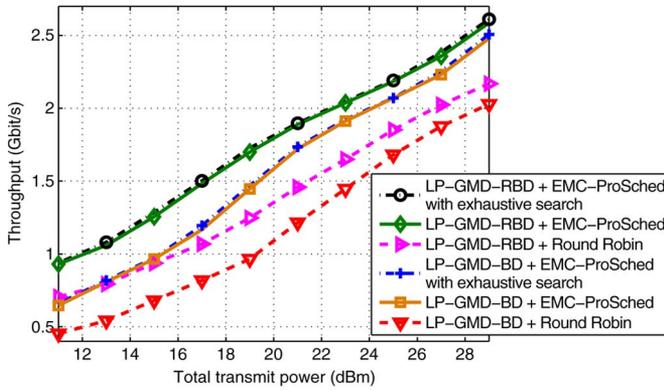


Fig. 3. System throughput of a 16-user scenario with the PSDU size of 500 000 B where EMC-ProSched, EMC-ProSched with exhaustive search, and the round-robin scheme are compared.

prediction of the performances of multicarrier transmissions. By comparison, the round-robin scheme selects a fixed number of users (here set to $K_{\max} = 4$) every time slot by cycling through the 16 users one by one in the system without taking into account the spatial correlation of the users. Moreover, the benefits of the proposed transmission strategy partially come from the precoding stage as well. The LP-GMD algorithm enables an effective equal MCS implementation over the spatial streams of each user, and a joint optimization of the transmit and receive beamformers is involved in the LoCCoBF algorithm. The two proposed algorithms are superior to their counterparts in the other two transmission strategies. Furthermore, the dynamic selection of the precoding algorithm also counts as one of the advantages of our developed transmission strategy. In addition, the fairness extension of the scheduling metric expressed in (11) is adopted. Numerical results show that, by taking into account the scheduling results in the previous 20 time slots, the scheduling fairness is achieved in the sense that, for all the users, the probabilities of being served are almost equal.

In the second and third experiments, we concentrate on the scheduling stage of the transmission strategy. Note that, in the EMC-ProSched algorithm, a suboptimum tree-based sorting scheme is employed to avoid the prohibitive complexity of the optimum exhaustive search among all possible user combinations. Here, a comparison is made between the proposed EMC-ProSched and a “special” version of EMC-ProSched where the candidate user groups are determined by an exhaustive search while keeping the calculation of the scheduling metric unchanged. In addition, the round-robin scheme is also considered in the comparison. A 16-user scenario is simulated, and the scheduling algorithms serve to select four users at a time. Each of the 16 users is equipped with two receive antennas, i.e., $M_{R_i} = 2$, $i = 1, 2, \dots, 16$. Here, two spatial streams are transmitted to each of the four users selected by the scheduling algorithm. The two versions of the LP-GMD algorithm, i.e., LP-GMD-BD and LP-GMD-RBD, are employed as the precoding algorithm. Fig. 3 shows the performance comparison in terms of the system throughput versus the transmit power. It can be seen that the gap between our EMC-ProSched with the low-complexity tree-based sorting scheme and the version

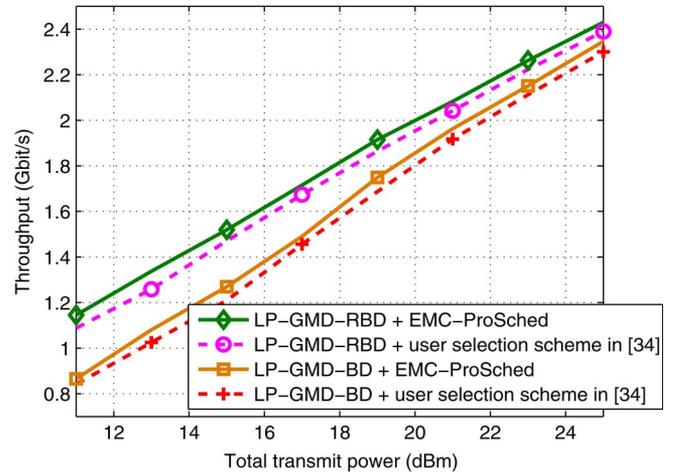


Fig. 4. System throughput of a 64-user scenario with the PSDU size of 500 000 B where EMC-ProSched and the user selection scheme in [34] are compared.

with the exhaustive search is very small, indicating that the EMC-ProSched algorithm finds a good tradeoff between complexity and performance. Moreover, this gap on the other hand implies the effectiveness of the designed scheduling metric for multicarrier systems. In addition, we can see that the proposed EMC-ProSched algorithm significantly outperforms the round-robin scheme. It is worth mentioning that, in this scenario, the original ProSched algorithm [7] cannot be employed as a total of 3744 (16 users with 234 data carriers) virtual users need to be considered, and the complexity is unaffordable in practice. In addition, ProSched assigns subcarriers rather than real users into an SDMA group, and this feature is not compatible with the IEEE 802.11ac standard [18].

We further make a comparison of EMC-ProSched and another scheduling algorithm proposed in [34] (in Table II, the antenna selection procedure is not considered) that selects user groups also based on the spatial correlation of the users. To distinguish with the previously presented examples, a 64-user scenario is simulated. EMC-ProSched and the scheduling scheme in [34] are employed to select suitable SDMA user groups, whereas LP-GMD-BD and LP-GMD-RBD are used as the precoding algorithm. The other simulation parameters are the same as in the second experiment. We plot the system throughput with respect to different transmit power levels in Fig. 4. It can be observed that the proposed EMC-ProSched algorithm provides a better performance compared with the user selection scheme developed in [34]. As both schemes take into account the spatial correlation of the users when selecting a user group, the superiority of EMC-ProSched to a large extent results from the fact that it uses a more effective scheduling metric for multicarrier systems. On the other hand, EMC-ProSched employs the orthogonal projection such that the complexity caused by computing SVDs is avoided when analyzing the spatial correlation of users in candidate SDMA user groups.

To further evaluate the performance of the proposed LP-GMD scheme, we consider the same 16-user scenario in Fig. 3 but fix the scheduling algorithm to EMC-ProSched. It serves to select four users out of the total 16 users, and precoding

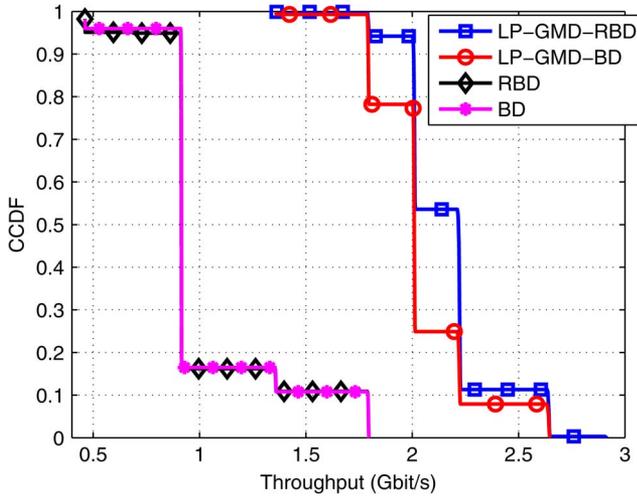


Fig. 5. CCDF of the system throughput when LP-GMD is employed for a 16-user scenario with the PSDU size of 100 000 B and EMC-ProSched as the scheduling algorithm.

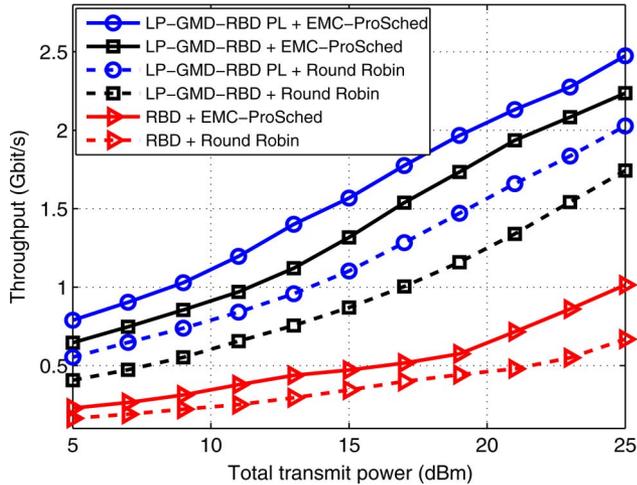


Fig. 6. System throughput when LP-GMD-RBD is employed for a 32-user scenario with the PSDU size of 500 000 B with and without power loading (“LP-GMD-RBD PL” represents LP-GMD-RBD with the MMSE-based power loading in the legend).

is performed on the selected user group by employing BD [3], RBD [4], and the two proposed versions of the LP-GMD algorithm, i.e., LP-GMD-BD and LP-GMD-RBD, respectively. The PSDU size is set to 100 000 B in this example. As shown in Fig. 5, we compare the CCDF of the system throughput when these precoding algorithms are applied. It can be found that the LP-GMD schemes provide much higher system throughput compared with BD and RBD. The maximum throughput that can be achieved when LP-GMD is employed is around 2.7 Gb/s.

We now continue to examine the MMSE-based power loading strategy developed for the LP-GMD schemes. In a 32-user scenario, EMC-ProSched and the round-robin scheme, respectively, are employed to assign four users to an SDMA group. Each of the 32 users is equipped with two receive antennas. The BS transmits two spatial streams to each of the four selected users. Fig. 6 shows the system throughput when LP-GMD-RBD with equal power allocation across the users, MMSE-

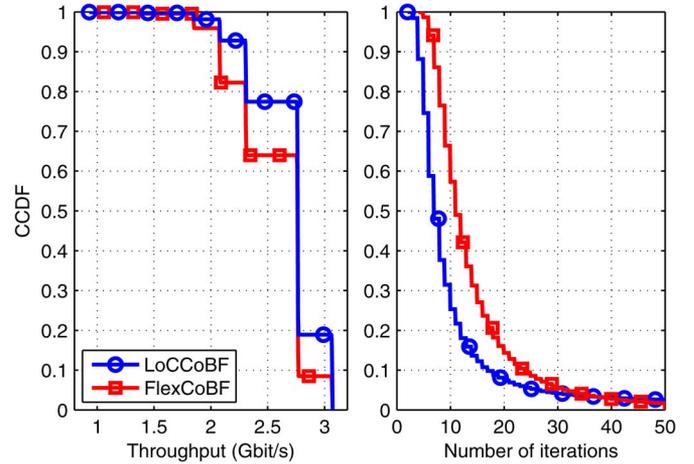


Fig. 7. CCDF of the system throughput and the number of iterations in a 16-user scenario when LoCCoBF is employed with the PSDU size of 500 000 B and EMC-ProSched as the scheduling algorithm.

based power loading (represented by “LP-GMD-RBD PL” in the legend of Fig. 6), and the RBD algorithm is employed. It can be seen that the power loading provides a noticeable gain over the LP-GMD-RBD scheme with equal power allocation. As already pointed out in Section IV-D, the proposed MMSE-based power loading strategy helps reduce the amount of padding bits due to the implementation of unequal MCSs across different users by balancing the power assigned to these users. Therefore, the performance gain of the LP-GMD schemes with the MMSE power loading comes from the reduction of the power loss due to the overhead, i.e., an improvement of the power utilization efficiency. Similarly, as observed in Fig. 5, the proposed LP-GMD-RBD greatly outperforms the RBD algorithm. In addition, in such a 32-user scenario, the superiority of EMC-ProSched over the round-robin scheme is even more pronounced.

In the last experiment, with the scheduling algorithm fixed to EMC-ProSched, a comparison between the proposed LoCCoBF and another iterative LP algorithm FlexCoBF [15] is presented. The EMC-ProSched algorithm serves to select four users out of the total 16 users at a time. The BS transmits two spatial streams to each of the four users selected by EMC-ProSched. Each user has four receive antennas. Thus, in this scenario, the total number of receive antennas of the users served at a time is 16 and exceeds the number of transmit antennas at the BS. The CCDF of the system throughput for FlexCoBF (BD as the precoding algorithm and MMSE receiver is adopted) [15] and the proposed LoCCoBF, both employed in combination with the proposed EMC-ProSched algorithm, is presented on the left-hand side of Fig. 7. It can be observed that the LoCCoBF algorithm outperforms FlexCoBF. The reason is that, instead of randomly initializing the receive beamforming matrices as in FlexCoBF, the proposed LoCCoBF algorithm uses the receive beamforming matrices calculated for the previous subcarrier and thus exploits the correlation of neighboring subcarriers.

On the right-hand side of Fig. 7, we further show the comparison between the proposed LoCCoBF algorithm and the FlexCoBF algorithm in terms of the CCDF of the number of

iterations. The maximum number of iterations is set to 50 in this experiment. The average number of iterations required by LoCCoBF is 9, whereas for FlexCoBF, it is 14. Since the computational complexity per iteration is similar for LoCCoBF and FlexCoBF, we can see that the LoCCoBF algorithm requires lower complexity while providing a better performance.

VIII. CONCLUSION

In this paper, an efficient and flexible transmission strategy for multicarrier MU MIMO downlink systems has been proposed, which consists of a spatial scheduling algorithm and two precoding algorithms. The spatial scheduling algorithm EMC-ProSched is able to assign users efficiently by using a novel and effective scheduling metric based on the philosophy of link-to-system mapping. An LP algorithm, which is called LP-GMD, is proposed to allow the transmission of multiple data streams to each user with the same MCS and thereby reduces the required signaling overhead compared with the case where unequal MCSs are employed on different data streams. BD and RBD versions of the LP-GMD algorithm are developed, whereas other suitable MUI mitigation schemes can be also flexibly adopted in LP-GMD. An MMSE-based power loading algorithm is further introduced in the LP-GMD schemes, which assigns transmit power across different users to achieve higher power efficiency. The second proposed precoding algorithm LoCCoBF also contributes to the flexibility of the proposed transmission strategy as it is able to suppress the MUI in scenarios where the number of transmit antennas at the BS is smaller than the total number of receive antennas at the UTs. To examine the proposed transmission strategy, a system-level simulator is developed under the IEEE 802.11ac framework with a link-to-system interface and a fast link adaptation procedure included. Simulation results indicate that the proposed transmission strategy outperforms the analyzed state-of-the-art transmission strategies and can achieve a very high system throughput.

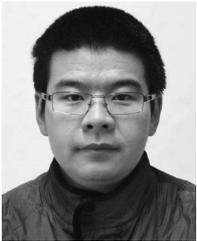
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