Energy Efficient Two-Way Non-Regenerative Relaying for Relays with Multiple Antennas

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***Abstract*—Energy efficient beamforming is studied for a MIMO two-way non-regenerative relaying system with multi-antenna users and a multi-antenna relay. The resulting optimization problem is non-convex. A Dinkelbach based alternating maximization (DAM) method is proposed to achieve an iterative design of the relay amplification matrix and the beamforming vectors at the users.**

***Index Terms*—Convex optimization, energy efficiency, MIMO, two-way relaying.**

I. Introduction

ENERGY EFFICIENCY (EE) is one of the key requirements of future wireless communication systems [1]. One way to save energy is to consider the minimization of the transmit power subject to quality of service (QoS) constraints. However, a more accurate measure of the EE is to optimize the ratio between the achievable sum rate and the consumed energy, i.e., the bit/Joule EE. Optimal resource allocation for the bit/Joule EE optimization in single hop communications has been thoroughly studied and a good summary is found in [2]. For two-hop communications with relays and especially for multi-antenna networks, energy efficient resource allocation solutions are rather limited. Recently, the EE has been studied in [3], [4], and [5] for a one-way relaying network with a multi-antenna amplify-and-forward (AF) relay and multi-antenna source and destination nodes. Optimal and suboptimal source precoding matrices and relay amplification matrices have been derived. When two-way relaying (TWR) networks are considered, major contributions to EE optimization deal with power allocation [6]–[10]. More specifically, researches in [6]–[9] study an AF TWR network with only single antenna nodes. Only the work in [10] considers a TWR network with multiple single antenna relays. However, the relay transmit strategies are fixed to suboptimal solutions and only the power allocation problem is studied. Moreover, all the previous works on the EE of TWR networks solve the bit/Joule EE optimization problem indirectly. That is, first the relationships between EE optimization and spectral efficiency optimization, e.g., [10], or transmit power minimization, e.g., [9], are derived. Then iterative solutions for optimizing the EE are devised. In summary, a direct EE optimization for a AF TWR network with multi-antenna nodes has not been well studied.

We study the EE optimization problem for a TWR system with a multi-antenna AF relay and multi-antenna users. A single stream transmission per user is considered. To maximize the EE of the system, a joint design of the beamforming vectors of the users and the relay amplification matrix is required. The resulting optimization problem is non-convex. Realizing that EE optimal user beamforming can be derived when fixing the relay amplification matrix, and vice versa, we propose a Dinkelbach based alternating maximization (DAM) scheme to address this joint design of the user and relay transmit strategies to optimize the EE.

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II. System Model

We consider a TWR system where two multi-antenna users, namely UT1 and UT2, communicate with each other via the help of a multi-antenna relay with $M_{\rm R}$ antennas. Each user has $M_{\rm U}$ antennas. We assume that the channel is i.i.d. frequency flat and quasi-static block fading. The channel matrices from UT1 to the relay and from UT2 to the relay are denoted by $\boldsymbol{H}_1 \in \mathbb{C}^{M_{\rm R}\times M_{\rm U}}$ and $\boldsymbol{H}_2 \in \mathbb{C}^{M_{\rm R}\times M_{\rm U}}$, respectively. An AF TWR strategy is used at the relay. The transmission takes two time slots. We assume that reciprocity holds for the channels. This is valid in an ideal reciprocal time-division duplex (TDD) system. Similarly as in [11], the received signal $\boldsymbol{y}_m$ $(m=\{1,2\})$ at the users is written as

$$\begin{aligned}\boldsymbol{y}_m &= \boldsymbol{H}_m^{\rm T}\underbrace{(\boldsymbol{G}(\boldsymbol{H}_m\boldsymbol{x}_m+\boldsymbol{H}_{3-m}\boldsymbol{x}_{3-m}+\boldsymbol{n}_{\rm R}))}_{\text{transmitted signal of the relay}}+\boldsymbol{n}_m\\ &= \underbrace{\boldsymbol{H}_m^{\rm T}\boldsymbol{G}\boldsymbol{H}_m\boldsymbol{x}_m}_{\text{self-interference}}+\underbrace{\boldsymbol{H}_m^{\rm T}\boldsymbol{G}\boldsymbol{H}_{3-m}\boldsymbol{x}_{3-m}}_{\text{desired signal}}+\underbrace{\boldsymbol{H}_m^{\rm T}\boldsymbol{G}\boldsymbol{n}_{\rm R}+\boldsymbol{n}_m}_{\text{effective noise}},\end{aligned}$$

where $\boldsymbol{x}_m = \boldsymbol{f}_m s_m$ denotes the transmitted data vector and the transmit powers at the users that satisfy $\mathbb{E}\{\|\boldsymbol{x}_m\|^2\}\le P_{\rm U}$. The vectors $\boldsymbol{f}_m \in \mathbb{C}^{M_{\rm U}}$, $\forall m$, are the transmit beamforming vectors of the users and $s_m\in\mathbb{C}, \forall m$, are the data symbols of the users with zero-mean and unit variance. Here we consider only a single stream transmission per user. The zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise at the relay and at the users is denoted as $\boldsymbol{n}_{\rm R}\in\mathbb{C}^{M_{\rm R}}$ and $\boldsymbol{n}_m\in\mathbb{C}^{M_{\rm U}}$ such that $\mathbb{E}\{\boldsymbol{n}_{\rm R}\boldsymbol{n}_{\rm R}^{\rm H}\}=\sigma_{\rm R}^2\boldsymbol{I}_{M_{\rm R}}$ and $\mathbb{E}\{\boldsymbol{n}_m\boldsymbol{n}_m^{\rm H}\}=\sigma_{\rm n}^2\boldsymbol{I}_{M_{\rm U}}, \forall m$. The relay amplifies its received signal with a relay amplification matrix $\boldsymbol{G}\in\mathbb{C}^{M_{\rm R}\times M_{\rm R}}$. A transmit power constraint at the relay has to be fulfilled such that $\mathbb{E}\{\|\boldsymbol{G}(\boldsymbol{H}_m\boldsymbol{x}_m+\boldsymbol{H}_{3-m}\boldsymbol{x}_{3-m}+\boldsymbol{n}_{\rm R})\|^2\}\le P_{\rm R}$.

Assume that perfect channel knowledge is available. Then the self-interference term can be subtracted. Define $\boldsymbol{g}=\operatorname{vec}\{\boldsymbol{G}\}\in\mathbb{C}^{M_{\rm R}^2}$, where $\operatorname{vec}\{\cdot\}$ stacks the columns of a matrix into a vector. Utilizing the fact that $\operatorname{vec}\{\Gamma_1\boldsymbol{W}\Gamma_2\}=(\Gamma_2^{\rm T}\otimes\Gamma_1)\operatorname{vec}\{\boldsymbol{W}\}$ and following a similar procedure as in [12], the SNR $\gamma_m, m=1,2$, at the m-th user is given by

$$\gamma_m = \frac{\boldsymbol{g}^{\mathrm{H}}\boldsymbol{B}_m\boldsymbol{g}}{\boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}_m\boldsymbol{g} + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2} \tag{1}$$

where $\boldsymbol{B}_m = ((\boldsymbol{H}_{3-m}\boldsymbol{f}_{3-m})^*(\boldsymbol{H}_{3-m}\boldsymbol{f}_{3-m})^{\mathrm{T}}) \otimes (\boldsymbol{H}_m^*\boldsymbol{H}_m^{\mathrm{T}})$ and $\boldsymbol{C}_m = \sigma_{\mathrm{R}}^2(\boldsymbol{I}_{M_{\mathrm{R}}} \otimes (\boldsymbol{H}_m^*\boldsymbol{H}_m^{\mathrm{T}}))$. Similarly, the transmit power constraint of the relay is reformulated as $\boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{g} \leq P_{\mathrm{R}}$, where we have $\boldsymbol{C} = \sum_{m=1}^{2}(((\boldsymbol{H}_m\boldsymbol{f}_m)^*(\boldsymbol{H}_m\boldsymbol{f}_m)^{\mathrm{T}}) \otimes \boldsymbol{I}_{M_{\mathrm{R}}}) + \sigma_{\mathrm{R}}^2\boldsymbol{I}_{M_{\mathrm{R}}^2}$. Finally, the overall sum rate of the system is calculated as $R_{\mathrm{sum}} = \sum_{m=1}^{2}\frac{1}{2}\log_2(1+\gamma_m)$, where the factor 1/2 is due to the two transmission phases.

Our goal is to design the user beamforming vectors $\boldsymbol{f}_m$ and the relay amplification matrix $\boldsymbol{G}$ such that the bits/Joule EE, i.e., the ratio between the achievable sum rate and the consumed power in the system, is maximized.

III. Energy Efficiency Via Alternating Maximization

The EE maximization problem is formulated as

$$\begin{aligned} \max_{\boldsymbol{f}_1,\boldsymbol{f}_2,\boldsymbol{g}} \quad & \eta_{\mathrm{EE}} = \frac{\sum_{m=1}^{2}\frac{1}{2}\log_2(1+\gamma_m)}{\boldsymbol{f}_1^{\mathrm{H}}\boldsymbol{f}_1 + \boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{f}_2 + \boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{g} + P_{\mathrm{c}}} \\ \text{s.t.} \quad & \boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{g} \leq P_{\mathrm{R}},\ \boldsymbol{f}_1^{\mathrm{H}}\boldsymbol{f}_1 \leq P_{\mathrm{U}}, \boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{f}_2 \leq P_{\mathrm{U}} \end{aligned} \tag{2}$$

where P_{c} denotes the fixed circuit power to operate the users and the relay [3], [13]. Dropping the constant 1/2 and utilizing a natural logarithm instead, problem (2) simplifies to

$$\begin{aligned} \max_{\boldsymbol{f}_1,\boldsymbol{f}_2,\boldsymbol{g}} \quad & \frac{\sum_{m=1}^{2}\log(1+\gamma_m)}{\boldsymbol{f}_1^{\mathrm{H}}\boldsymbol{f}_1 + \boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{f}_2 + \boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{g} + P_{\mathrm{c}}} \\ \text{s.t.} \quad & \boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{g} \leq P_{\mathrm{R}},\ \boldsymbol{f}_1^{\mathrm{H}}\boldsymbol{f}_1 \leq P_{\mathrm{U}},\ \boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{f}_2 \leq P_{\mathrm{U}}. \end{aligned} \tag{3}$$

Clearly, problem (3) is non-convex with respect to the optimization variables $(\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{g})$. It is difficult to perform a joint user beamforming and relay beamforming design because it might result in an intractable optimization problem. To avoid this, we propose to optimize $(\boldsymbol{f}_1, \boldsymbol{f}_2)$ and $\boldsymbol{g}$ iteratively. Our design concept is based on the fact that for fixed $\boldsymbol{g}$ EE optimal solutions of $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ can be derived, and vice versa. In the following we present the developed DAM algorithm.

A. Optimal User Beamforming via Dinkelbach algorithm

When the relay amplification matrix $\boldsymbol{G}$ is fixed, the original problem (3) simplifies to

$$\begin{aligned} \max_{\boldsymbol{f}_1,\boldsymbol{f}_2} \quad & \frac{\log(1+\boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{D}_1\boldsymbol{f}_2) + \log(1+\boldsymbol{f}_1^{\mathrm{H}}\boldsymbol{D}_2\boldsymbol{f}_1)}{\boldsymbol{f}_1^{\mathrm{H}}(\boldsymbol{I}_{M_{\mathrm{U}}}+\boldsymbol{E}_1)\boldsymbol{f}_1 + \boldsymbol{f}_2^{\mathrm{H}}(\boldsymbol{I}_{M_{\mathrm{U}}}+\boldsymbol{E}_2)\boldsymbol{f}_2 + b_{\mathrm{n}} + P_{\mathrm{c}}} \\ \text{s.t.} \quad & \boldsymbol{f}_1^{\mathrm{H}}\boldsymbol{E}_1\boldsymbol{f}_1 + \boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{E}_2\boldsymbol{f}_2 + b_{\mathrm{n}} \leq P_{\mathrm{R}} \\ & \boldsymbol{f}_1^{\mathrm{H}}\boldsymbol{f}_1 \leq P_{\mathrm{U}},\ \boldsymbol{f}_2^{\mathrm{H}}\boldsymbol{f}_2 \leq P_{\mathrm{U}}, \end{aligned} \tag{4}$$

where $\boldsymbol{D}_m = \boldsymbol{H}_{3-m}^{\mathrm{H}}\boldsymbol{G}^{\mathrm{H}}\boldsymbol{H}_m^*\boldsymbol{H}_m^{\mathrm{T}}\boldsymbol{G}\boldsymbol{H}_{3-m}/(\sigma_{\mathrm{R}}^2\|\boldsymbol{H}_m^{\mathrm{T}}\boldsymbol{G}\|_{\mathrm{F}}^2 + \sigma_{\mathrm{n}}^2 M_{\mathrm{U}})$, $\boldsymbol{E}_m = \boldsymbol{H}_m^{\mathrm{H}}\boldsymbol{G}^{\mathrm{H}}\boldsymbol{G}\boldsymbol{H}_m$, and $b_{\mathrm{n}} = \sigma_{\mathrm{R}}^2\|\boldsymbol{g}\|^2$. Problem (4) is a quadratically constrained quadratic programming (QCQP) problem with respect to $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$, which is in general non-convex. A traditional way to deal with non-convex QCQP problems is to apply the semidefinite relaxation (SDR) method [14]. That is, defining $\boldsymbol{X}_1 = \boldsymbol{f}_1\boldsymbol{f}_1^{\mathrm{H}}$ and $\boldsymbol{X}_2 = \boldsymbol{f}_2\boldsymbol{f}_2^{\mathrm{H}}$, we rewrite problem (4) as

$$\begin{aligned} \max_{\boldsymbol{X}_1,\boldsymbol{X}_2} \quad & \frac{\log(1+\mathrm{Tr}\{\boldsymbol{D}_1\boldsymbol{X}_2\})+\log(1+\mathrm{Tr}\{\boldsymbol{D}_2\boldsymbol{X}_1\})}{\mathrm{Tr}\{(\boldsymbol{I}_{M_{\mathrm{U}}}+\boldsymbol{E}_1)\boldsymbol{X}_1\}+\mathrm{Tr}\{(\boldsymbol{I}_{M_{\mathrm{U}}}+\boldsymbol{E}_2)\boldsymbol{X}_2\}+b_{\mathrm{n}}+P_{\mathrm{c}}} \\ \text{s.t.} \quad & \mathrm{Tr}\{\boldsymbol{E}_1\boldsymbol{X}_1\} + \mathrm{Tr}\{\boldsymbol{E}_2\boldsymbol{X}_2\} + b_{\mathrm{n}} \leq P_{\mathrm{R}} \\ & \mathrm{Tr}\{\boldsymbol{X}_1\} \leq P_{\mathrm{U}}, \mathrm{Tr}\{\boldsymbol{X}_2\} \leq P_{\mathrm{U}}, \boldsymbol{X}_1 \succeq 0,\ \boldsymbol{X}_2 \succeq 0, \end{aligned} \tag{5}$$

where $\mathrm{Tr}\{\cdot\}$ stands for the trace operation. If problem (5) is feasible and the optimal $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are rank-1 matrices, then an optimal solution to problem (4) is obtained. Otherwise, rank-1 extraction techniques have to be used [14]. Notice that the log-trace expressions in the objective function of (5) are concave according to the composition rules of convex functions in [15]. Then the objective function of (5) is a quasiconcave function with respect to $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ because its numerator is concave and its denominator can be seen as affine functions [15]. Optimal solutions to such a fractional programming problem can be obtained using iterative algorithms [16]. To be computationally more efficient, we propose to use a parametric approach which is known as the Dinkelbach algorithm [17]. To apply the Dinkelbach algorithm, we first rewrite (5) into a parametric programming problem, i.e.,

$$\begin{aligned} f(\lambda) = \max_{\boldsymbol{X}_1,\boldsymbol{X}_2} \quad & \log(1+\mathrm{Tr}\{\boldsymbol{D}_1\boldsymbol{X}_2\})+\log(1+\mathrm{Tr}\{\boldsymbol{D}_2\boldsymbol{X}_1\}) \\ & - \lambda\left(\sum_{m=1}^{2}\mathrm{Tr}\{(\boldsymbol{I}_{M_{\mathrm{U}}}+\boldsymbol{E}_m)\boldsymbol{X}_m\}+b_{\mathrm{n}}+P_{\mathrm{c}}\right) \\ \text{s.t.} \quad & \sum_{m=1}^{2}\mathrm{Tr}\{\boldsymbol{E}_m\boldsymbol{X}_m\}+b_{\mathrm{n}} \leq P_{\mathrm{R}} \\ & \mathrm{Tr}\{\boldsymbol{X}_1\} \leq P_{\mathrm{U}}, \mathrm{Tr}\{\boldsymbol{X}_2\} \leq P_{\mathrm{U}}, \boldsymbol{X}_1 \succeq 0, \boldsymbol{X}_2 \succeq 0, \end{aligned} \tag{6}$$

where parametric implies that we consider the solution of (6) for various values of λ. This reformulation is especially useful if the objective of the maximization problem (6), which defines $f(\lambda)$, is a concave function for a fixed λ. In our case, problem (6) is a convex semidefinite programming (SDP) problem, which can be solved using the interior-point algorithm in [15]. Moreover, if $f(\lambda) = 0$, the corresponding $(\boldsymbol{X}_1, \boldsymbol{X}_2)$ is also the optimal solution to problem (5) [17]. Thus, this gives rise to finding the roots of the equation $f(\lambda) = 0$. To find the roots, the Dinkelbach algorithm suggests that at the p-th iteration $\lambda^{(p)}$ is calculated using $\lambda^{(p)} = \lambda^{(p-1)} - f(\lambda^{(p-1)})/\partial_\lambda f(\lambda^{(p-1)})$. The term $\partial_\lambda f(\lambda^{(p-1)})$ is the subderivative of $f(\lambda)$ at $\lambda^{(p-1)}$ and it is derived as $-\sum_{m=1}^{2}\mathrm{Tr}\{(\boldsymbol{I}_{M_{\mathrm{U}}}+\boldsymbol{E}_m)\boldsymbol{X}_m^{(p-1)}\} - b_{\mathrm{n}} - P_{\mathrm{c}}$, where $\boldsymbol{X}_m^{(p-1)}, \forall m$, is the optimal solution of (6) at the $(p-1)$-th step. This is analogous to Newton's method for finding the roots. Moreover, problem (6) has $K = 2$ non-zero solutions and $M = 3$ semidefinite constraints such that $M < K + 2$. This satisfies Theorem 3.2 in [18] and thus a rank-1 solution is guaranteed. Numerical results also demonstrate that the obtained solutions to (6) are rank-1 matrices almost surely. Hence, optimal $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ are computed using the eigenvalue decomposition (EVD) method [19]. The proposed Dinkelbach algorithm for solving (4) is summarized in Algorithm 1 Although problem (6) is not a traditional parametric formulation as in [17], by following the proof in [20] it is straightforward to conclude that Algorithm 1 converges at least superlinearly to the optimal solution of (4).

Algorithm 1 The Dinkelbach algorithm for solving (4)

1: **Initialize:** set an initial $\lambda^{(0)}$, $p = 1$ and a threshold μ.
2: **Main step:**
3: **repeat**
4: Obtain $(\boldsymbol{X}_1^{(p-1)}, \boldsymbol{X}_2^{(p-1)})$ by solving (6).
5: Calculate $\lambda^{(p)}$ using
$\lambda^{(p)} = \frac{\sum_{m=1}^{2} \log(1+\mathrm{Tr}\{\boldsymbol{D}_m \boldsymbol{X}_{3-m}^{(p-1)}\})}{\sum_{m=1}^{2} \mathrm{Tr}\{(\boldsymbol{I}_{M_{\mathrm{U}}}+\boldsymbol{E}_m)\boldsymbol{X}_m^{(p-1)}\}+b_{\mathrm{n}}+P_{\mathrm{c}}}$
6: $p = p + 1$
7: **Until** $|f(\lambda^{(p-1)})| \leq \mu$
8: Return $\boldsymbol{X}_{\mathrm{opt},m}$. Calculate the economy-size EVD as $\boldsymbol{X}_{\mathrm{opt},m} = \boldsymbol{u}_m s_m \boldsymbol{u}_m^{\mathrm{H}}$, where $\boldsymbol{u}_m$ and s_m are the dominant eigenvector and the corresponding dominant eigenvalue.
9: Output: $\boldsymbol{f}_{\mathrm{opt},m} = \sqrt{s_m}\boldsymbol{u}_m$, $m = 1, 2$.

B. Energy Efficient Relay Amplification Matrix

When $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ are fixed, the original problem (3) simplifies to

$$\max_{\boldsymbol{g}} \frac{\sum_{m=1}^{2} \log(\boldsymbol{g}^{\mathrm{H}}\boldsymbol{A}_m\boldsymbol{g} + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2) - \sum_{m=1}^{2} \log(\boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}_m\boldsymbol{g} + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2)}{\|\boldsymbol{f}_1\|^2 + \|\boldsymbol{f}_2\|^2 + \boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{g} + P_{\mathrm{c}}}$$
$$\text{s.t.} \quad \boldsymbol{g}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{g} \leq P_{\mathrm{R}}, \tag{7}$$

where $\boldsymbol{A}_m = \boldsymbol{B}_m + \boldsymbol{C}_m$, $\forall m$. Problem (7) is a non-convex QCQP problem with respect to $\boldsymbol{g}$ and its objective function is neither quasiconcave nor quasiconvex because its numerator is neither convex nor concave. To solve (7), we still propose to use the SDR together with a parametric approach. To this end, we define $\boldsymbol{X} = \boldsymbol{g}\boldsymbol{g}^{\mathrm{H}}$. Then the parametric formulation of problem (7) is given by

$$\begin{aligned} f(\nu) = \max_{\boldsymbol{X}} & \sum_{m=1}^{2} \log(\mathrm{Tr}\{\boldsymbol{A}_m\boldsymbol{X}\} + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2) - \sum_{m=1}^{2} \log(\mathrm{Tr}\{\boldsymbol{C}_m\boldsymbol{X}\} \\ & + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2) - \nu(\|\boldsymbol{f}_1\|^2 + \|\boldsymbol{f}_2\|^2 + \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} + P_{\mathrm{c}}) \\ \text{s.t.} \quad & \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} \leq P_{\mathrm{R}}, \ \boldsymbol{X} \succeq 0. \end{aligned} \tag{8}$$

If problem (8) can be solved optimally, we check whether $f(\nu) = 0$. If not, ν is updated using the Dinkelbach algorithm in a similar way as in Section III-A and problem (8) is solved again. This procedure runs iteratively until $f(\nu) = 0$ is achieved. Then, if the corresponding optimal $\boldsymbol{X}_{\mathrm{opt}}$ is a rank-1 solution, an optimal $\boldsymbol{g}$ is obtained for problem (7). Otherwise, rank-1 extraction algorithms in [14] should be used. From now on, given a fixed ν, we devise an algorithm which provides at least a local optimum to problem (8).

Due to the concave log-trace expressions, the structure of the objective function corresponds to the difference of concave functions (DC). Therefore, problem (8) is a DC problem which is known to be non-convex and NP-hard to solve [21]. Nevertheless, by replacing the concave elements with minus signs by scalar variables, the objective function is turned into a concave function and an equivalent reformulation is given by

$$\begin{aligned} \max_{\boldsymbol{X}, \beta_m, t_m, \forall m} & \sum_{m=1}^{2} \log(\mathrm{Tr}\{\boldsymbol{A}_m\boldsymbol{X}\} + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2) - \sum_{m=1}^{2} t_m \\ & - \nu(\|\boldsymbol{f}_1\|^2 + \|\boldsymbol{f}_2\|^2 + \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} + P_{\mathrm{c}}) \\ \text{s.t.} \quad & \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} \leq P_{\mathrm{R}}, \boldsymbol{X} \succeq 0, \\ & \mathrm{Tr}\{\boldsymbol{C}_m\boldsymbol{X}\} + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2 = \beta_m, \ m = 1, 2, \\ & \log(\beta_m) \leq t_m, \ m = 1, 2. \end{aligned} \tag{9}$$

Compared to (8), the non-convexity in (9) is localized in the inequality constraints $\log(\beta_m) \leq t_m$, $\forall m$. To deal with the non-convex constraints, we propose to linearize the logarithmic function using its first order Taylor expansion, i.e., the first order Taylor polynomial approximation of $\log(\beta_m)$ at $\beta_{0,m}$ is defined as $\log(\beta_m) \approx \log(\beta_{0,m}) + (\beta_m - \beta_{0,m})/\beta_{0,m}$. Using this linear approximation, problem (9) can be rewritten as

$$\begin{aligned} \max_{\boldsymbol{X}, \beta_m, t_m, \forall m} & \sum_{m=1}^{2} \log(\mathrm{Tr}\{\boldsymbol{A}_m\boldsymbol{X}\} + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2) - \sum_{m=1}^{2} t_m \\ & - \nu(\|\boldsymbol{f}_1\|^2 + \|\boldsymbol{f}_2\|^2 + \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} + P_{\mathrm{c}}) \\ \text{s.t.} \quad & \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} \leq P_{\mathrm{R}}, \boldsymbol{X} \succeq 0, \\ & \mathrm{Tr}\{\boldsymbol{C}_m\boldsymbol{X}\} + M_{\mathrm{U}}\sigma_{\mathrm{n}}^2 = \beta_m, \ m = 1, 2, \\ & \log(\beta_{0,m}) + \frac{\beta_m - \beta_{0,m}}{\beta_{0,m}} \leq t_m, \ m = 1, 2. \end{aligned} \tag{10}$$

For fixed $\beta_{0,m}$, $\forall m$, problem (10) is a convex SDP problem and thus can be solved efficiently using the interior-point algorithm if it is feasible [15]. It is worth mentioning that similar approaches have been used in [19], [22] to solve the sum rate maximization problem. Since the best $\beta_{0,m}$, $\forall m$, is unknown, it is natural to update $\beta_{0,m}$ iteratively. Let $\beta_m^{(p-1)}$ be the optimal values which are obtained by solving (10) at the $(p-1)$-th step. Then the initial values of $\beta_{0,m}$ at the p-th step are set to $\beta_{0,m}^{(p)} = \beta_m^{(p-1)}$, $\forall m$. In this way it is guaranteed that the optimal value of (10) at the p-th step $f_{\mathrm{opt}}^{(p)}$ is always greater or equal to the optimal value $f_{\mathrm{opt}}^{(p-1)}$ obtained at the $(p-1)$-th step. Otherwise, it would be contradictory to the objective function. Moreover, according to [18, Corollary 3.4] a rank-1 solution is guaranteed for problem (10) because there are only 3 semidefinite constraints. Numerical results also show that the obtained solutions to (10) are almost always rank-1. Hence, an optimal $\boldsymbol{g}_{\mathrm{opt}}$ can be obtained by using the EVD method as in Algorithm 1. In summary, the proposed algorithm for solving the subproblem (8) is described in Algorithm 2.

Algorithm 2 The iterative approach for solving (8)

1: **Initialize:** input: $\boldsymbol{B}_m, \boldsymbol{C}_m, \boldsymbol{C}, \boldsymbol{f}_m, \nu$, set feasible $\beta_{0,m}^{(1)}$, $\forall m$, $f_{\mathrm{opt}}^{(0)} = 0$, $p = 1$, and a threshold value ϵ.
2: **Main step:**
3: **repeat**
4: Solve problem (10) for finding $f_{\mathrm{opt}}^{(p)}$, $\boldsymbol{X}_{\mathrm{opt}}^{(p)}$, and $\beta_m^{(p)}$
5: $\beta_{0,m}^{(p+1)} = \beta_m^{(p)}$
6: $p = p + 1$
7: **until** $|f_{\mathrm{opt}}^{(p-1)} - f_{\mathrm{opt}}^{(p-2)}| \leq \epsilon$
8: **Output:** $\boldsymbol{X}_{\mathrm{opt}}$

In each iteration of Algorithm 2, a convex problem is solved. Moreover, a finite number of iterations is required since β_m is bounded[1]. Therefore, Algorithm 2 provides a polynomial time solution to the DC problem (8). Moreover, solutions of Algorithm 2 have the following property.

Lemma 1: The solutions generated by Algorithm 2 converge to the Karush-Kuhn-Tucker (KKT) point of problem (8).

Proof: This conclusion comes straightforwardly from Proposition 3.2 of [23]. □

[1]This is because $\boldsymbol{X}$ is bounded due to the transmit power constraint.

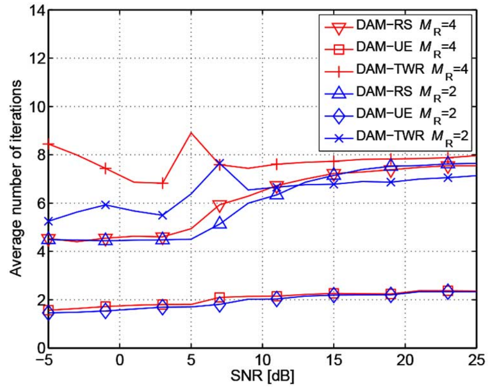

Fig. 1. Average number of iterations vs. SNR. $M_{\mathrm{U}}=2$, $P_{\mathrm{c}}=5$ W.

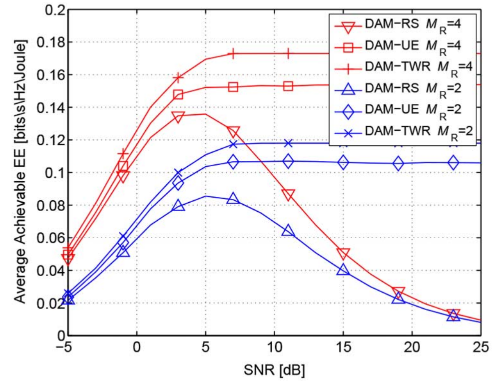

Fig. 2. Average EE vs. SNR. $M_{\mathrm{U}}=2$, $P_{\mathrm{c}}=5$ W.

Clearly, the initial values of $\beta_{0,m}^{(1)}$, $\forall m$, have to be feasible. One simply way to achieve this is to generate $\beta_{0,m}^{(1)}$ randomly. Let $\boldsymbol{a}\in\mathbb{C}^{M_{\mathrm{R}}^2}$ be a random vector generated from the ZMCSCG distribution, i.e., $\boldsymbol{a}\sim\mathcal{CN}(\mathbf{o},\boldsymbol{I}_{M_{\mathrm{R}}^2})$. Then $\beta_{0,m}^{(1)}$ is calculated as $\beta_{0,m}^{(1)}=\alpha\mathrm{Tr}\{\boldsymbol{C}_m\boldsymbol{a}\boldsymbol{a}^{\mathrm{H}}\}+M_{\mathrm{U}}\sigma_{\mathrm{n}}^2$, where $\alpha=P_{\mathrm{R}}/\mathrm{Tr}\{\boldsymbol{C}\boldsymbol{a}\boldsymbol{a}^{\mathrm{H}}\}$ ensures that the transmit power limit at the relay is satisfied with equality. Finally, our proposed Dinkelbach based algorithm for solving (7) is summarized in Algorithm 3 Note that although Algorithm 2 does not provide a globally optimal solution to (8) in general, numerically we observe that Algorithm 3 always converges.

Algorithm 3 Dinkelbach based algorithm for solving (7)

1: **Initialize:** set an initial $\nu^{(0)}$, $q=1$, and a threshold μ.
2: **Main step:**
3: **repeat**
4: Solve problem (8) using Algorithm 2 to find $\boldsymbol{X}_{\mathrm{opt}}^{(q-1)}$.
5: Update $\nu^{(q)}$ using the Dinkelbach method as

$$\nu^{(q)}=\frac{\sum\limits_{m=1}^{2}\log\frac{\mathrm{Tr}\{\boldsymbol{A}_m\boldsymbol{X}_{\mathrm{opt}}^{(q-1)}\}+M_{\mathrm{U}}\sigma_{\mathrm{n}}^2}{\mathrm{Tr}\{\boldsymbol{C}_m\boldsymbol{X}_{\mathrm{opt}}^{(q-1)}\}+M_{\mathrm{U}}\sigma_{\mathrm{n}}^2}}{\|\boldsymbol{f}_1\|^2+\|\boldsymbol{f}_2\|^2+\mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}_{\mathrm{opt}}^{(q-1)}\}+P_{\mathrm{c}}}$$

6: $q=q+1$
7: **Until** $|f(\nu^{(q-1)})|\le\mu$
8: Compute $\boldsymbol{g}_{\mathrm{opt}}$ using the EVD method.
9: **Output:** $\boldsymbol{g}_{\mathrm{opt}}$

C. Joint Optimization Via Alternating Maximization

Since optimal solutions can be obtained for the simplified problem (4) when $\boldsymbol{G}$ is fixed and for the simplified problem (7) when $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ are fixed, we propose to use an alternating method to solve the original problem (3) inspired by [24]. The proposed Dinkelbach based alternating maximization (DAM) method for solving problem (3) is described in Algorithm 4.

Remark 1: When additional constraints are added, e.g., the rate requirements of the users as in [3] and [4], the DAM algorithm can be still applied. However, these additional constraints violate the required conditions for obtaining guaranteed rank-1 solutions in [18]. Thus, the SDR will not be exact and the randomization procedure has to be used to get rank-1 approximations of the optimal solutions.

Remark 2: When each user transmits multiple streams, it is easy to verify that Algorithm 1 is still applicable and it provides optimal energy efficient precoders for the users. However, Algorithm 3 will fail to provide an energy efficient $\boldsymbol{G}$. This is because the linear approximation based iterative method in Algorithm 2 cannot be applied.

Algorithm 4 The DAM algorithm for solving (3)

1: **Initialize:** input: $\boldsymbol{B}_m$, $\boldsymbol{C}_m$, $\boldsymbol{C}$, set the j-th element of $\boldsymbol{f}_m^{(0)}$ as $f_{m,j}^{(0)}=\sqrt{P_{\mathrm{U}}/M_{\mathrm{U}}}e^{j\theta_{m,j}}$, where $\theta_{m,j}$ ($j=\{1,\cdots,M_{\mathrm{U}}\}$) is uniformly distributed between 0 and 2π, i.e., this corresponds to equal gain transmission (EGT) [25], set $f_{\mathrm{opt}}^{(0)}$, $\eta_{\mathrm{EE}}^{(0)}$, feasible $\beta_{0,m}^{(0)}$, $\forall m$, and threshold values ϵ, μ, δ. Let $n=0$.
2: **Main step:**
3: **repeat**
4: Given $\boldsymbol{f}_m^{(n)}$, solve problem (7) to find $\boldsymbol{g}_{\mathrm{opt}}^{(n+1)}$.
5: Given $\boldsymbol{g}_{\mathrm{opt}}^{(n+1)}$, solve problem (4) to obtain $\boldsymbol{f}_m^{(n+1)}$, $\forall m$.
6: **until:** $|\eta_{\mathrm{EE}}^{(n+1)}-\eta_{\mathrm{EE}}^{(n)}|\le\delta$.
7: **Output:** $\boldsymbol{g}_{\mathrm{opt}}$, $\boldsymbol{f}_{\mathrm{opt},1}$, $\boldsymbol{f}_{\mathrm{opt},2}$

IV. Simulation Results and Concluding Remarks

The proposed algorithm is evaluated using Monte-Carlo simulations. All the generated channels are uncorrelated Rayleigh fading channels. The noise powers at all nodes are set to unity, i.e., $\sigma_{\mathrm{R}}^2=\sigma_{\mathrm{n}}^2=1$ and the available transmit powers are identical, i.e., $P_{\mathrm{R}}=P_{\mathrm{U}}=P$. The SNR is defined as SNR $=P$. We set $\mu=\delta=10^{-4}$ and $\epsilon=10^{-6}$. "DAM-RS", "DAM-UE", and "DAM-TWR" stand for Algorithms 3, 1, and 4, respectively. Moreover, when Algorithms 3 is used standalone, $\boldsymbol{f}_m$, $\forall m$, is selected using the EGT method as in Algorithm 4. When Algorithms 1 is used standalone, $\boldsymbol{G}$ is calculated using the sum rate achievable design in [19] assuming that the users use EGT beamformers. All the simulation results are averaged over 1000 channel realizations.

Fig. 1 shows the average number of iterations for the proposed algorithms. In each iteration one SDP problem, i.e., (6) or (10), is solved. The proposed algorithms and especially Algorithms 1 and 3 converge only in a few iterations. Moreover, the convergence speed is almost independent of the SNR. Fig. 2 illustrates the achievable EE as a function of the SNR. It can be seen that the major portion of the achieved EE is contributed by adjusting the beamforming vectors of the users especially in the high SNR regime. The achievable EE increases as M_{R} increases.

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