Linear Detection Schemes for MIMO UW-OFDM

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***Abstract*—A new signaling concept, known as unique word - orthogonal frequency division multiplexing (UW-OFDM), has been introduced recently. It has been shown that UW-OFDM has better spectral properties and also a superior bit error ratio (BER) performance over conventional cyclic prefix (CP) based OFDM. In UW-OFDM, the CPs are replaced by deterministic sequences, the so-called unique words (UWs). This is achieved by adding redundancy in the frequency domain with the help of a code generator matrix at the transmitter which introduces correlation between the data symbols. A superior BER performance is achieved if the code generator matrix is utilized efficiently in the detection procedure. In wireless communication systems multiple-input multiple-output (MIMO) techniques are widely used to improve the overall spectral and energy efficiency by exploiting the multiple antennas. So far, the performance of UW-OFDM has been well investigated for single-input single-output (SISO) systems. In this work, we expand our investigation of UW-OFDM to MIMO systems and propose two detection approaches. We show that similar to CP-OFDM, a subcarrier wise detection is also possible for UW-OFDM. But in addition an efficient code generator demodulator is required to take advantage of the correlation introduced in the data symbols. The results show that UW-OFDM outperforms CP-OFDM significantly but requires a higher computational complexity.**

***Index Terms*—Orthogonal Frequency Division Multiplexing (OFDM), Cyclic Prefix (CP), Unique Word (UW), Multiple-Input Multiple-Output (MIMO)**

I. Introduction

In conventional orthogonal frequency division multiplexing (OFDM), subsequent symbols are separated by a cyclic prefix (CP). The main purpose of the CP is to transform the linear convolution of the transmit signal with the channel impulse response into a cyclic convolution such that the fast Fourier transform (FFT) can diagonalize the channel in the frequency domain. Since the CP is a random sequence and varies from OFDM symbol to OFDM symbol, it is removed at the receiver and is typically not utilized for any other purpose. In [1], [2], a new OFDM based signaling technique was introduced where the usual CP is replaced by a known deterministic sequence, the unique word (UW). The UWs not only transform the linear convolution into a cyclic convolution but can additionally be utilized for synchronization and channel estimation purposes.

One of the fundamental differences between UW-OFDM and CP-OFDM, and also to other related signaling concepts such as known symbol padded (KSP)-OFDM [3], is that the UW is a part of the FFT interval whereas the CP/KS is not. The generation of the UW within the FFT interval is achieved by adding redundancy in the frequency domain. This introduces correlation among the subcarriers which can advantageously be exploited at the receiver to improve the bit error ratio (BER) performance. If dedicated redundant subcarriers are used, the choice of their positions has a significant influence on the redundant energy. Therefore, in [4], the authors proposed a heuristic method to calculate the optimum or near optimum redundant subcarrier positions. However, even with the optimum choice, this so called systematic approach suffers from a high energy contribution of the redundant subcarriers. Hence, in [5], the authors introduced a non-systematic approach where the idea of the dedicated redundant subcarriers is abandoned, and the redundancy is distributed across all subcarriers. The results show that UW-OFDM based on the non-systematic approach not only outperforms the systematic UW-OFDM but also the conventional CP-OFDM [5].

One of the reasons for the popularity of CP-OFDM is its simple and low complex frequency domain equalization where usually a single-tap equalization is applied on each subcarrier. Since the code generator matrix in UW-OFDM introduces correlation between the subcarriers, more sophisticated detection schemes are required for UW-OFDM. Various linear and non-linear detection schemes have been studied for UW-OFDM in [6], [7]. So far, these detection procedures have been investigated for single-input single-output (SISO) systems only. In this work, we extend our investigations of UW-OFDM to multiple-input multiple-output (MIMO) systems. The main contribution of this work is to propose linear detection procedures for MIMO UW-OFDM. For this purpose, we propose two approaches where we take advantage of the correlation introduced by the code generator matrix. In the first approach, we present a model to employ joint detection which is similar to the model presented for SISO systems [7] but requires a quite high computational complexity. In the second approach, we propose a two step procedure where a subcarrier wise equalization is performed in the first step and in the second step, a code generator demodulator matrix is employed.

The organization of the remaining paper is as follows. Section II describes the overall system model. A brief overview of the procedures to derive the code generator matrix is also discussed. We present the linear detection procedures for MIMO UW-OFDM in Section III. Section IV shows the simulation results and quantifies the system performance in terms of the BER using parameters comparable to IEEE 802.11n. The performance of UW-OFDM is not only compared for the proposed approaches but also with CP-OFDM. The contributions are summarized at the end in Section V.

Notation: We use lower-case bold face letters $(\boldsymbol{a}, \boldsymbol{b}, ...)$ to indicate vectors and upper-case bold face letters $(\boldsymbol{A}, \boldsymbol{B}, ...)$

to indicate matrices. The superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ represent complex conjugate, matrix transpose, and complex conjugate transpose (Hermitian), respectively. The $\mathrm{vec}\{\cdot\}$ operator stacks the columns of a matrix into a vector while the $\mathrm{unvec}\{\cdot\}$ operator stands for the inverse operation of $\mathrm{vec}\{\cdot\}$. The Kronecker product is represented by $\otimes$ and the $\mathrm{tr}\{\cdot\}$ operator defines the trace of a matrix.

II. System Model

In UW-OFDM, the unique word is added in the time domain by adding redundancy in the frequency domain. Let $\boldsymbol{x}_u \in \mathbb{C}^{N_u\times 1}$ be a unique word of length N_u which is added at the tail of the time domain OFDM symbol such that

$$\boldsymbol{x}' = \begin{bmatrix}\boldsymbol{x}_d\\ \mathbf{0}\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \boldsymbol{x}_u\end{bmatrix} = \begin{bmatrix}\boldsymbol{x}_d\\ \boldsymbol{x}_u\end{bmatrix} \in \mathbb{C}^{N\times 1} \tag{1}$$

where, in the first step, a UW-OFDM symbol $\boldsymbol{x} = \begin{bmatrix}\boldsymbol{x}_d^T & \mathbf{0}\end{bmatrix}^T$ is generated with a zero UW. Then in the second step the desired UW is added in the time domain [2].

Consider a MIMO system with M_T transmit antennas and M_R receive antennas where $\boldsymbol{S} \in \mathbb{C}^{N_d\times M_T}$ contains the $N_d \times M_T$ data symbols to be transmitted from M_T transmit antennas. The time domain signal is given by

$$\mathbf{X} = \boldsymbol{F}_N^H \boldsymbol{B}\boldsymbol{G}\boldsymbol{S} \in \mathbb{C}^{N\times M_T} \tag{2}$$

before adding the UW, where $\boldsymbol{F}_N$ is the N-point DFT matrix with its elements $[\boldsymbol{F}_N]_{k,l} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}kl}$ for $k,l = 0,1,2,\ldots,N-1$, $\boldsymbol{B} \in \{0,1\}^{N\times N_m}$ inserts the zero subcarriers, and $\boldsymbol{G} \in \mathbb{C}^{N_m\times N_d}$ is the complex valued code generator matrix. The $\boldsymbol{G}$ matrix is designed in such a way that it fulfills the following constraint

$$\boldsymbol{F}_N^H \boldsymbol{B}\boldsymbol{G} = \begin{bmatrix}*\\ \mathbf{0}\end{bmatrix}. \tag{3}$$

Some redundancy is added in the frequency domain with the help of the $\boldsymbol{G}$ matrix to get the zero tail at the output of the IDFT such that $N_m = N_d + N_r$. The parameter N_r accounts for the redundancy. The $\boldsymbol{G}$ matrix is not unique since any matrix that fulfills the constraint in Eq. (3) could in principle be chosen as a code generator matrix. It has been shown in [7] that different $\boldsymbol{G}$ matrices lead to different performances of UW-OFDM for SISO systems. Since the $\boldsymbol{G}$ matrix is defined only once for the system and does not depend on the time varying propagation channel, the $\boldsymbol{G}$ matrices designed for a SISO system can also be utilized for MIMO systems.

Let $\boldsymbol{U} = \left[\boldsymbol{F}_N^H \cdot \boldsymbol{B}\right]_{\text{last } N_u \text{ rows}}$ be the $N_u \times N_m$ matrix containing the N_u last rows of $\boldsymbol{F}_N^H \cdot \boldsymbol{B}$. Then the constraint in (3) can be reformulated as

$$\boldsymbol{U}\boldsymbol{G} = \mathbf{0}, \tag{4}$$

which implies that the columns of $\boldsymbol{G}$ have to lie in the null space of the matrix $\boldsymbol{U}$. One solution is a matrix that contains an orthonormal basis of the null space. This can be achieved by computing the singular value decomposition of $\boldsymbol{U}$. In this paper, we denote the code generator matrix as $\boldsymbol{G}'$ which is computed in this way.

Various other approaches for the calculation of the $\boldsymbol{G}$ matrix have been suggested in [2], [5]. For example, in [5], it is suggested to derive optimum code generator matrices by minimizing the trace of the error covariance matrices of the best linear unbiased estimator (BLUE) or the linear minimum mean square error (LMMSE) estimator in the AWGN channel and for a fixed signal-to-noise ratio. The optimization problems are unconstrained problems and are solved with the steepest descent method. Moreover, different initializations to solve these problems result in different code generator matrices for the same problem [5]. In this work, we use a code generator matrix $\boldsymbol{G}''$ that has been obtained by a random initialization and a minimization of the trace of the LMMSE estimator.

III. Detection Schemes

The introduction of redundancy during the UW-OFDM symbol generation can be exploited at the receiver to get better estimates of the data vectors. This has been well investigated for SISO systems in [6] where it has been shown that more sophisticated linear estimators, which are based on joint detection, are required to exploit the benefits of this redundancy. This is in contrast to CP-OFDM where a single-tap estimator is usually employed. In the following, we present two approaches for MIMO UW-OFDM, as shown in Fig. 1. In the first scheme, we show that the same linear estimators obtained for SISO systems can be employed for MIMO UW-OFDM by using joint detection. But they suffer from a high computation complexity. In the second scheme, we show that a subcarrier wise detection is also possible where the redundancy is advantageously exploited by using an LMMSE based code generator demodulator.

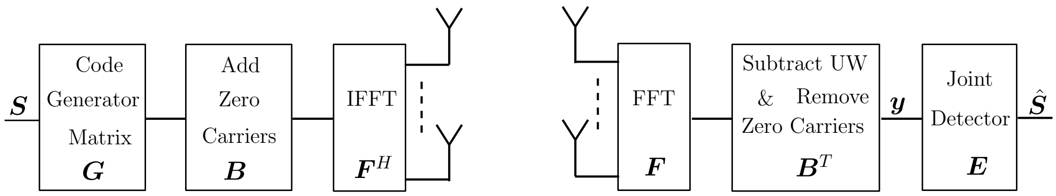
A. Approach 1 (Joint Detection)

The data received by the ith receive antenna in the frequency domain after subtracting the UW and removing the zero carriers is given by

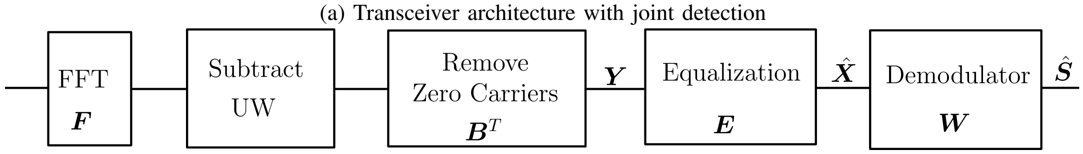
$$\begin{aligned}\boldsymbol{y}_i &= \sum_{j=1}^{M_T} \boldsymbol{B}^T \boldsymbol{F}_N \boldsymbol{H}_{i,j} \boldsymbol{F}_N^H \boldsymbol{B}\boldsymbol{G}\boldsymbol{s}_j + \boldsymbol{n}_i\\ &= \sum_{j=1}^{M_T} \hat{\boldsymbol{H}}_{i,j} \boldsymbol{G}\boldsymbol{s}_j + \boldsymbol{n}_i \quad \forall \quad i = 1,\ldots,M_R\end{aligned} \tag{5}$$

where $\boldsymbol{H}_{i,j}$ is the circulant channel convolutional matrix containing the channel impulse response, $\hat{\boldsymbol{H}}_{i,j} = \boldsymbol{B}^T \boldsymbol{F}_N \boldsymbol{H}_{i,j} \boldsymbol{F}_N^H \boldsymbol{B} \in \mathbb{C}^{N_m\times N_m}$ is a diagonal matrix containing the channel frequency response from the jth transmit antenna to the ith receive antenna on its main diagonal. The data received over all antennas can be written as

$$\begin{bmatrix}\boldsymbol{y}_1\\ \vdots\\ \boldsymbol{y}_{M_R}\end{bmatrix} = \begin{bmatrix}\hat{\boldsymbol{H}}_{1,1} & \cdots & \hat{\boldsymbol{H}}_{1,M_T}\\ \vdots & \ddots & \vdots\\ \hat{\boldsymbol{H}}_{M_R,1} & \cdots & \hat{\boldsymbol{H}}_{M_R,M_T}\end{bmatrix} (\boldsymbol{I}_{M_T} \otimes \boldsymbol{G}) \begin{bmatrix}\boldsymbol{s}_1\\ \vdots\\ \boldsymbol{s}_{M_T}\end{bmatrix} + \begin{bmatrix}\boldsymbol{n}_1\\ \vdots\\ \boldsymbol{n}_{M_R}\end{bmatrix}. \tag{6}$$



(a) Transceiver architecture with joint detection



(b) Two step detection approach

Fig. 1: Transceiver architecture with linear detection schemes for MIMO UW-OFDM

We can rewrite the above equation as

$$\boldsymbol{y} = \boldsymbol{H}_{\text{eff}}\boldsymbol{G}_{\text{eff}}\tilde{\boldsymbol{s}} + \boldsymbol{n} \in \mathbb{C}^{(N_m.M_R)\times 1}, \tag{7}$$

where $\boldsymbol{H}_{\text{eff}} \in \mathbb{C}^{(N_m.M_R)\times(N_m.M_T)}$ contains the overall channel frequency response as shown in Eq. (6), $\boldsymbol{G}_{\text{eff}} = \boldsymbol{I}_{M_T} \otimes \boldsymbol{G} \in \mathbb{C}^{(N_m.M_T)\times(N_d.M_T)}$ is the modified code generator matrix with a block diagonal structure, and $\tilde{\boldsymbol{s}} \in \mathbb{C}^{(N_d.M_T)\times 1}$ contains the transmitted data symbols and is related to $\boldsymbol{S}$ in Eq. (2) via $\tilde{\boldsymbol{s}} = \text{vec}(\boldsymbol{S})$. The optimum linear estimators such as BLUE or LMMSE can be used to estimate the data symbols [7]. These equalizer weight matrices are calculated as

$$\boldsymbol{E}_{\text{BLUE}} = \left(\boldsymbol{G}_{\text{eff}}^H\boldsymbol{H}_{\text{eff}}^H\boldsymbol{H}_{\text{eff}}\boldsymbol{G}_{\text{eff}}\right)^{-1}\boldsymbol{G}_{\text{eff}}^H\boldsymbol{H}_{\text{eff}}^H,$$
$$\boldsymbol{E}_{\text{MMSE}} = \left(\boldsymbol{G}_{\text{eff}}^H\boldsymbol{H}_{\text{eff}}^H\boldsymbol{H}_{\text{eff}}\boldsymbol{G}_{\text{eff}} + \frac{\sigma_n^2}{\sigma_d^2}\boldsymbol{I}_{(N_d.M_t)}\right)^{-1}\boldsymbol{G}_{\text{eff}}^H\boldsymbol{H}_{\text{eff}}^H,$$

with $\boldsymbol{E} \in \mathbb{C}^{(N_d.M_T)\times(N_m.M_R)}$, $\boldsymbol{I}_{(N_d.M_t)}$ is the identity matrix of size $N_d \cdot M_T$, σ_n^2 is the variance of the noise samples, and σ_d^2 is the variance of the data symbols. Note that the computational complexity increases significantly with an increased number of transmit antennas since a square matrix of size $N_d \cdot M_T$ has to be inverted. The estimated data symbols are then obtained from

$$\hat{\boldsymbol{S}} = \text{unvec}(\boldsymbol{E}\boldsymbol{y}) \in \mathbb{C}^{N_d\times M_T}. \tag{8}$$

B. Approach 2 (Subcarrier wise Detection)

The simplicity of CP-OFDM comes from per subcarrier operations where each data subcarrier is treated as a single narrowband carrier in the frequency domain. Since the $\boldsymbol{G}$ matrix introduces correlation between the data symbols by adding redundancy, it is difficult to relate the actual data symbols to the output of the $\boldsymbol{G}$ matrix. Therefore we propose a two step detection procedure for UW-OFDM where in the first step, similar to CP-OFDM, a subcarrier wise linear estimator can be employed. In the second step, a code generator demodulator is used as shown in Fig. 1b.

After taking the DFT, subtracting the UW, and removing the zero carriers, the received signal on the lth subcarrier is given as

$$\boldsymbol{y}_l = \boldsymbol{H}_l\boldsymbol{S}^T[\boldsymbol{G}^T]_l + \boldsymbol{n}_l \in \mathbb{C}^{M_R\times 1}, \qquad l = 1, ..., N_m \tag{9}$$

where $\boldsymbol{H}_l \in \mathbb{C}^{M_R\times M_T}$ is the channel matrix of the lth subcarrier in the frequency domain and $[\boldsymbol{G}^T]_l$ is the lth column of the matrix $\boldsymbol{G}^T$. Let $\boldsymbol{c}_l = \boldsymbol{S}^T[\boldsymbol{G}^T]_l \in \mathbb{C}^{M_T\times 1}$ represent the transmit signal on lth subcarrier, then

$$\boldsymbol{y}_l = \boldsymbol{H}_l\boldsymbol{c}_l + \boldsymbol{n}_l \in \mathbb{C}^{M_R\times 1}. \tag{10}$$

*1) **Linear Detectors**:* We first equalize the channel on the lth subcarrier by performing,

$$\hat{\boldsymbol{c}}_l = \boldsymbol{E}_l\boldsymbol{y}_l \in \mathbb{C}^{M_T\times 1}, \tag{11}$$

where $\boldsymbol{E}_l \in \mathbb{C}^{M_T\times M_R}$ contains the equalizer filter weights and can be calculated using the zero forcing (ZF) or the LMMSE criterion.

a) ZF Detector: The constraint for the ZF detector is given by

$$\boldsymbol{E}_l\boldsymbol{H}_l = \boldsymbol{I}_{M_T} \tag{12}$$

We get the same solution as in CP-OFDM which is given by

$$\boldsymbol{E}_{l,\text{ZF}} = (\boldsymbol{H}_l^H\boldsymbol{H}_l)^{-1}\boldsymbol{H}_l^H. \tag{13}$$

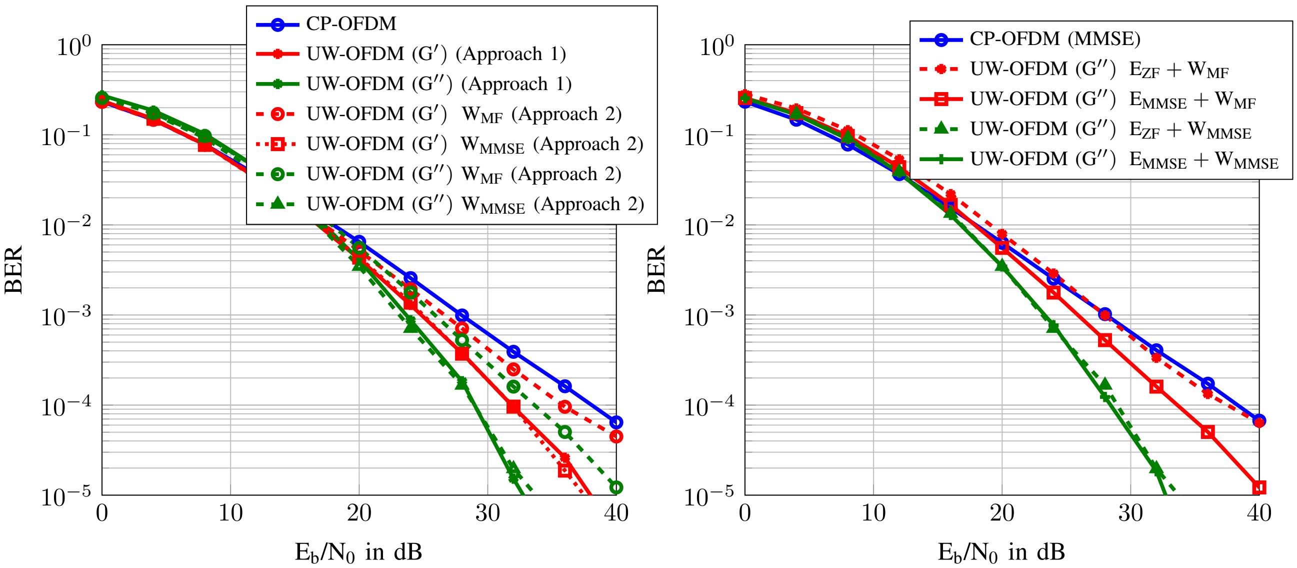
b) LMMSE Detector: Since ZF receivers are known for the noise amplification, a linear MMSE estimator can be employed which counteracts this problem. The filter weights using the MMSE criterion are calculated for the following unconstrained problem

$$\text{minimize} \quad \mathbb{E}\left\{\| \boldsymbol{E}_l\boldsymbol{y}_l - \boldsymbol{c}_l \|^2\right\}, \tag{14}$$

where $\mathbb{E}$ is the expectation operator. This leads to the solution

$$\boldsymbol{E}_{l,\text{MMSE}} = \boldsymbol{R}_{c_lc_l}\boldsymbol{H}_l^H(\boldsymbol{H}_l\boldsymbol{R}_{c_lc_l}\boldsymbol{H}_l^H + \boldsymbol{R}_{n_ln_l})^{-1}, \tag{15}$$

where $\boldsymbol{R}_{n_ln_l}$ is the noise covariance matrix and $\boldsymbol{R}_{c_lc_l}$ is given as



(a) Comparison of both approaches using MMSE equalization

(b) Different combinations of equalizer and demodulator for subcarrier wise detection

Fig. 2: Performance comparison of MIMO UW-OFDM with linear detection schemes

$$\begin{aligned}
\boldsymbol{R}_{c_l c_l} &= \mathbb{E}\left\{\boldsymbol{c}_l {\boldsymbol{c}_l}^H\right\} \\
&= \mathbb{E}\left\{(\boldsymbol{S}^T[\boldsymbol{G}^T]_l)(\boldsymbol{S}^T[\boldsymbol{G}^T]_l)^H\right\} \\
&= \mathbb{E}\{(([\boldsymbol{G}^T]_l^T \otimes \boldsymbol{I}_{M_T})\mathrm{vec}(\boldsymbol{S}^T)) \\
&\quad (([\boldsymbol{G}^T]_l^T \otimes \boldsymbol{I}_{M_T})\mathrm{vec}(\boldsymbol{S}^T))^H\} \\
&= ([\boldsymbol{G}^T]_l^T \otimes \boldsymbol{I}_{M_T})\mathbb{E}\left\{\mathrm{vec}(\boldsymbol{S}^T)\mathrm{vec}(\boldsymbol{S}^T)^H\right\} \\
&\quad ([\boldsymbol{G}^T]_l^T \otimes \boldsymbol{I}_{M_T})^H \\
&= \sigma_d^2([\boldsymbol{G}^T]_l^T \otimes \boldsymbol{I}_{M_T})\boldsymbol{I}_{M_T N_d}([\boldsymbol{G}^T]_l^T \otimes \boldsymbol{I}_{M_T})^H \\
&= \sigma_d^2([\boldsymbol{G}^T]_l^T[\boldsymbol{G}^T]_l^*) \otimes \boldsymbol{I}_{M_T} \\
&= \sigma_d^2 \parallel [\boldsymbol{G}^T]_l \parallel_2^2 \boldsymbol{I}_{M_T} \\
&= \sigma_d^2 \beta_l \boldsymbol{I}_{M_T},
\end{aligned}$$

where β_l is the norm of the lth row of the $\boldsymbol{G}$ matrix and can be precalculated. By inserting these values and by applying the matrix inversion lemma [8], it can be easily shown that

$$\boldsymbol{E}_{l,\mathrm{MMSE}} = ({\boldsymbol{H}_l}^H \boldsymbol{H}_l + \sigma_n^2/(\sigma_d^2\beta_l)\boldsymbol{I}_{M_T})^{-1}\boldsymbol{H}_l^H. \tag{16}$$

2) ***Code Generator Demodulator Matrix****:* In the second step, all equalized symbols are jointly multiplied by a code generator demodulator matrix to get the estimated data symbols

$$\hat{\boldsymbol{S}} = \boldsymbol{W}\hat{\boldsymbol{C}} \in \mathbb{C}^{N_d \times M_T}, \tag{17}$$

where $\hat{\boldsymbol{C}} = [\hat{\boldsymbol{c}}_1, ..., \hat{\boldsymbol{c}}_{N_m}]^T \in \mathbb{C}^{N_m \times M_T}$ and $\boldsymbol{W}$ is the code generator demodulator matrix. Various type of demodulator matrices can be defined.

a) Matched Filter based Demodulator: A matched filter based demodulator offers the simplest solution since the used code generator matrices fulfill the property $\boldsymbol{G}^H\boldsymbol{G} = \boldsymbol{I}_{N_d}$. Therefore, a matched filter based demodulator is given by

$$\boldsymbol{W}_{\mathrm{MF}} = \boldsymbol{G}^H. \tag{18}$$

b) LMMSE based Demodulator: A subcarrier wise estimation does not take full advantage of the correlation introduced by the $\boldsymbol{G}$ matrix if detection is used in conjunction with a matched filter based demodulator. Assuming that the propagation channel is perfectly equalized ($\boldsymbol{E}_l\boldsymbol{H}_l = \boldsymbol{I}_{M_T}$), which is exactly true if Eq. (13) is used and approximately true if Eq. (16) is used, $\hat{\boldsymbol{C}}$ in Eq. (17) can be written as

$$\hat{\boldsymbol{C}} = \boldsymbol{G}\boldsymbol{S} + \hat{\boldsymbol{N}}, \tag{19}$$

where $\hat{\boldsymbol{N}} = [\boldsymbol{E}_1\boldsymbol{n}_1, ..., \boldsymbol{E}_{N_m}\boldsymbol{n}_{N_m}]^T \in \mathbb{C}^{N_m \times M_T}$ is the noise at the output of the detector and is not white anymore. The LMMSE based optimization problem to calculate the demodulator filter weights can be written as

$$\text{minimize} \quad \mathbb{E}\left\{\parallel \boldsymbol{W}\hat{\boldsymbol{C}} - \boldsymbol{S} \parallel^2\right\}. \tag{20}$$

The solution to this problem leads to

$$\boldsymbol{W}_{\mathrm{MMSE}} = \boldsymbol{G}^H(\boldsymbol{G}\boldsymbol{G}^H + \sigma_n^2/\sigma_d^2\boldsymbol{Z})^{-1}, \tag{21}$$

where $\boldsymbol{Z} \in \mathbb{R}^{N_m \times N_m}$ is a diagonal matrix and contains $\mathrm{tr}(\boldsymbol{E}_l\boldsymbol{E}_l^H) \quad \forall \quad l = 1, \ldots, N_m$ on its diagonal. Note that the equalizer weights $\boldsymbol{E}_l$ are already calculated in the first step. Finally, the data symbols at the kth subcarrier can also be represented as

$$\hat{\boldsymbol{s}}_k = [\boldsymbol{W}]_k\hat{\boldsymbol{C}} \qquad k = 1, ..., N_d. \tag{22}$$

IV. SIMULATION RESULTS

In this section, we evaluate the performance of both approaches which are presented in Section III. Moreover, we also compare the performance of UW-OFDM with CP-OFDM. To this end, we have used the IEEE 802.11n parameters for a 40 MHz bandwidth as shown in Table I. The symbol duration of UW-OFDM is 3.2 μs while CP-OFDM has a duration of 4 μs. This is due to the fact that the guard interval in UW-OFDM is a part of the FFT interval. Moreover, the IEEE 802.11n channel model B is used for the simulations. We have assumed a perfect synchronization and channel estimation in this work.

	CP-OFDM	**UW-OFDM**
Modulation Scheme	QPSK	
Bandwidth	40 MHz	
M_T	2	
M_R	2	
Data Carriers (N_d)	108	82
FFT length (N)	128	
CP/UW duration	800 ns	
DFT Period	3.2 μs	
Total Symbol Period	4 μs	3.2 μs
Channel	IEEE 802.11n Channel B	

TABLE I: Simulation parameters

First, we compare the performance of both approaches for two code generator matrices $(\boldsymbol{G}', \boldsymbol{G}'')$ as shown in Fig. 2a. The results are shown with LMMSE equalization. Moreover, this LMMSE equalization is combined with a matched filter or an LMMSE based code generator demodulator for approach 2. Since the correlation is efficiently exploited in joint detection, it provides the best performance at the cost of a higher computational complexity. A subcarrier wise detection does not match the performance of joint detection, if the equalization is performed in conjunction with a matched filter based demodulator. But when it is combined with an LMMSE based code generator demodulator, it practically matches the performance of joint detection. This is due to the fact that a matched filter based demodulator does not exploit the statistics of the noise at the output of the equalizer which is not white any more. Clearly a subcarrier wise detection using an LMMSE based demodulator is a better option than the joint detection because of its lower complexity. However, it has a higher complexity compared to CP-OFDM because we have to calculate an $N_m \times N_m$ code generator demodulator matrix inverse (e.g., once per burst in WLAN systems) and have to apply $\boldsymbol{W}$ for every UW-OFDM symbol. Furthermore, we also observe that the two $\boldsymbol{G}$ matrices yield different results where UW-OFDM with $\boldsymbol{G}''$ outperforms the $\boldsymbol{G}'$ matrix. Note that $\boldsymbol{G}''$ has been optimized for the LMMSE cost function assuming an AWGN channel while $\boldsymbol{G}'$ has been calculated by computing the SVD of $\boldsymbol{U}$. Although both generator matrices form an orthonormal basis of the null space of $\boldsymbol{U}$, this does not necessarily mean that they perform equally well in frequency selective environments. However, they should perform equally well in AWGN environments.

In Fig. 2b, we show the results for different combinations of equalization and code generator demodulation techniques for the $\boldsymbol{G}''$ matrix using approach 2. The results show that when a ZF estimator is employed in conjunction with a matched filter, it has the worst performance but it still matches the performance of CP-OFDM. We get a performance enhancement if we use an LMMSE estimator even with a matched filter based demodulator. The best performance is achieved when an LMMSE based demodulator is used. Clearly, UW-OFDM with the $\boldsymbol{G}''$ matrix and an LMMSE based demodulator outperforms CP-OFDM significantly. This is also in line with [9] where it has been shown analytically for a SISO system that UW-OFDM achieves the maximum diversity order if $\text{rank}(\boldsymbol{BG}) = N_d$ while CP-OFDM requires precoding to achieve the maximum diversity. Note that there is no performance difference between ZF or LMMSE estimators if they are used in conjunction with an LMMSE based demodulator.

V. CONCLUSION

In this work, we have proposed two linear detection schemes for MIMO UW-OFDM. We have shown that the redundancy, which is introduced to generate a valid UW-OFDM symbol, can be exploited by a clever estimator design. For SISO systems, it has been shown that joint detection exploits this redundancy in an efficient manner. Therefore, we extend this design to a MIMO UW-OFDM system and investigate joint detection. Although this approach outperforms CP-OFDM significantly, it has a significantly higher computational complexity since a square matrix of size $N_d \cdot M_T \times N_d \cdot M_T$ has to be inverted. Hence, we propose an alternative scheme where the detection is performed in two steps. In the first step a subcarrier wise equalization is performed while in the subsequent step a matched filter or LMMSE based code generator demodulator is employed. The results show that subcarrier wise detection with an LMMSE based demodulator practically matches the performance of joint detection but with a considerably reduced computational complexity.

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