Iterative GFDM Receiver based on the PARATUCK2 Tensor Decomposition

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***Abstract*—Generalized Frequency Division Multiplexing (GFDM) is one of the multi-carrier transmission techniques considered as an alternative to orthogonal frequency division multiplexing (OFDM) for 5G wireless communication systems. GFDM is a flexible multi-carrier scheme that spreads the data symbols in a time-frequency block. Compared to OFDM, in GFDM each subcarrier is additionally filtered with a circular pulse shaping filter. Tensor algebra efficiently describes multidimensional signals, preserves their structure and provides improved identifiability. Moreover, in the past communication systems have been modeled using tensor algebra and often showed a tensor gain compared to matrix based receivers. Therefore, we model the GFDM system using tensor algebra and tensor decompositions. In this paper, we present a tensor model for the GFDM transmit signal for single and multiple antennas based on the PARATUCK2 decomposition. Furthermore, based on this model we design an iterative receiver that simultaneously estimates the channel and the transmitted data. It significantly outperforms the Least Squares (LS) receiver. The proposed iterative receiver has a comparable performance with the state-of-the-art Linear Minimum Mean Square Error (LMMSE) receivers while having a significantly lower computational complexity.**

I. Introduction

Currently, orthogonal frequency division multiplexing (OFDM) is the standard waveform for multi-carrier communication systems. OFDM requires a significant signaling overhead due to its strict synchronization requirements, which is a major drawback for the application scenarios being considered for 5G systems. Therefore, several new waveforms with less stringent synchronization requirements have been proposed for the 5G air interface. One of the candidate waveforms that fulfils the 5G requirements is Generalized Frequency Division Multiplexing (GFDM) [1].

GFDM is a flexible multi-carrier scheme that spreads the data symbols in a time-frequency block. GFDM is more complex than OFDM because additionally each subcarrier is filtered with a circular pulse shaping filter. Moreover GFDM introduces Inter Symbol Interference (ISI) due to the fact that (unlike OFDM) not all symbols are transmitted on orthogonal subcarriers. Therefore, especially for frequency selective channels, ISI cancellation has to be included, as presented in [2].

Moreover, Space Time Block Codes for MIMO-GFDM were presented in [3], whereas pilot aided channel estimation for MIMO-GFDM systems was proposed in [4]. For instance, in [3] it is shown that when STBCs are applied directly to the data symbols, the linear GFDM demodulator cannot decouple the subcarriers and subsymbols because of the multipath propagation channel leading to a severe performance loss. Because of this reason Time Reverse - Space Time Codes (TR-STC) are recommended for GFDM when space-time coding is applied on blocks of GFDM samples. In [4] where Least Squares (LS) and Linear Minimum Mean Square Error (LMMSE) channel estimators are presented it is shown that the ISI still has an impact on the estimates and additional pre-canceling of the interference is required.

The ISI in GFDM is a result of the fact that the subcarriers are not strictly orthogonal, consequently leading to a slight overlap of data symbols that are spread in time-frequency blocks. The spreading of the data symbols is defined using the GFDM modulation matrix. Hence the modulation matrix of GFDM has a special structure that was exploited in [5] for a design of low complexity transceivers for GFDM.

Tensors provide a useful tool for the analysis of multidimensional data. A comprehensive review of tensor concepts is provided in [6]. Tensors have a very broad range of applications such as compressed sensing, processing of big data, blind source separation and many more [7]. Moreover, tensors and tensor decompositions have been used to describe various communication systems as discussed in [8], [9], [10]. In these works wireless communication systems are modelled using the PARATUCK2 or the generalized PARATUCK2 decomposition. Similarly, in this paper we show that the GFDM transmit signal can also be defined as a PARATUCK2 model.

The goal of this paper is to model the GFDM transmit signal for single and multiple antennas based on the PARATUCK2 tensor decomposition. Moreover, we present a very simple iterative receiver that has comparable performance with the state-of-the-art LMMSE receivers.

We use the following notation. Scalars are denoted either as capital or lower-case italic letters, A, a. Vectors and matrices, are denoted as bold-face capital and lower-case letters, $\boldsymbol{a}, \boldsymbol{A}$, respectively. Tensors are represented by bold-face calligraphic letters $\boldsymbol{\mathcal{A}}$. The following superscripts, ^{T}, ^{H}, ${}^{-1}$, and ${}^+$ denote transposition, Hermitian transposition, matrix inversion and Moore-Penrose pseudo matrix inversion, respectively. The

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outer product, Kronecker product, and Khatri-Rao product are denoted as $\circ$, $\otimes$, and $\diamond$, respectively. The operators $||.||_{\mathrm{F}}^2$ and $||.||_{\mathrm{H}}^2$ denote the Frobenius norm and the Higher order norm, respectively. Moreover, an n-mode product between a tensor $\boldsymbol{\mathcal{A}} \in \mathbb{C}^{I_1 \times I_2 \ldots \times I_N}$ and a matrix $\boldsymbol{B} \in \mathbb{C}^{J \times I_n}$ is defined as $\boldsymbol{\mathcal{A}} \times_n \boldsymbol{B}$, for $n = 1, 2, \ldots N$ [6]. A super-diagonal or identity N-way tensor of dimensions $R \times R \ldots \times R$ is denoted as $\boldsymbol{\mathcal{I}}_{N,R}$. The n-mode unfolding of a tensor $\boldsymbol{\mathcal{A}}$ is denoted as $[\boldsymbol{\mathcal{A}}]_{(n)}$, and the n-th 3-mode slice is denoted as $\boldsymbol{\mathcal{A}}_{(.,.,n)}$.

The rest of the paper is organized as follows. In Section II we present the PARATUCK2 tensor decomposition. In Section III we use the PARATUCK2 tensor decomposition to describe the GFDM system model. Based on this model in Section IV we present an iterative receiver for MIMO-GFDM and its performance is shown via simulation results in Section V. Finally, in Section VI we conclude this paper.

II. Tensor Algebra

Let $\boldsymbol{A} \in \mathbb{C}^{I \times P}$ and $\boldsymbol{B} \in \mathbb{C}^{J \times Q}$ be two matrices containing the elements $a_{i,p}$ and $b_{j,q}$ and representing two different sets of latent components, respectively. The PARATUCK2 tensor decomposition of a tensor $\boldsymbol{\mathcal{L}} \in \mathbb{C}^{I \times J \times K}$ containing these matrices is defined as

$$\boldsymbol{\mathcal{L}}_{(.,.,k)} = \boldsymbol{A} \boldsymbol{\mathcal{D}}^{(A)}_{(.,.,k)} \boldsymbol{R} \boldsymbol{\mathcal{D}}^{(B)}_{(.,.,k)} \boldsymbol{B}^T, \quad \forall k = 1, 2, \ldots, K$$

where the matrix $\boldsymbol{R} \in \mathbb{C}^{P \times Q}$ indicates the interaction between the two different sets of latent components. The three mode slices of the two tensors $\boldsymbol{\mathcal{D}}^{(A)} \in \mathbb{C}^{P \times P \times K}$ and $\boldsymbol{\mathcal{D}}^{(B)} \in \mathbb{C}^{Q \times Q \times K}$ are diagonal matrices with diagonal elements equal to $d^{(A)}_{p,k}$ and $d^{(B)}_{q,k}$, respectively. The PARATUCK2 tensor decomposition is illustrated in Fig. 1. Equivalently the element-wise PARATUCK2 tensor decomposition of this tensor is

$$\boldsymbol{\mathcal{L}}_{(i,j,k)} = \sum_{p=1}^{P} \sum_{q=1}^{Q} a_{i,p} d^{(A)}_{p,k} r_{p,q} d^{(B)}_{q,k} b_{j,q} \tag{1}$$

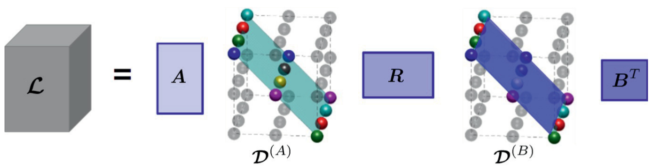

Fig. 1: PARATUCK2 tensor decomposition.

III. GFDM system model based on PARATUCK2 decomposition

A. GFDM-SISO model

For simplicity, we first present a tensor model for GFDM SISO systems, then in the subsequent section we will extend this model to MIMO systems. The GFDM modulated signal is defined as follows [1].

$$x_n = \sum_{k=1}^{K} \sum_{m=1}^{M} d_{k,m} p_{k,n} g_{m,n}, \quad \forall\ n = 1, \ldots, N, \tag{2}$$

where, M is the number of complex time subsymbols to be transmitted on K subcarriers, $N = K \cdot M$ is the block length, and $d_{k,m}$ are the complex modulated data symbols. The data symbols are filtered with the filter coefficients $g_{m,n}$ and are accordingly shifted to the corresponding subcarrier $p_{k,n} = \exp\left(j 2\pi \frac{k}{N} n\right)$, as explained in [1].

By comparing equation (1) and (2), we observe that the PARATUCK2 tensor decomposition can be used to describe the GFDM transmit signal. Accordingly, we define the GFDM transmit signal as

$$\boldsymbol{\mathcal{X}}_{(1,1,n)} = \boldsymbol{1}_K^T \cdot \boldsymbol{\mathcal{P}}_{(.,.,n)} \cdot \boldsymbol{D} \cdot \boldsymbol{\mathcal{G}}_{(.,.,n)} \cdot \boldsymbol{1}_{\boldsymbol{M}}, \quad \forall\ n = 1, \ldots, N$$

where, $\boldsymbol{\mathcal{X}} \in \mathbb{C}^{1 \times 1 \times N}$ is a 3-mode vector containing the GFDM modulated block of N samples. Moreover, $\boldsymbol{\mathcal{P}} \in \mathbb{C}^{K \times K \times N}$ and $\boldsymbol{\mathcal{G}} \in \mathbb{C}^{M \times M \times N}$ have diagonal slices with diagonal elements equal to $p_{k,n}$ and $g_{m,n}$, respectively. Furthermore, these diagonal elements can be combined as $\boldsymbol{P} \in \mathbb{C}^{K \times N}$ and $\boldsymbol{G} \in \mathbb{C}^{M \times N}$. Therefore, we have $\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{I}}_{3,M} \times_3 \boldsymbol{G}^T$ and similarly $\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{I}}_{3,K} \times_3 \boldsymbol{P}^T$. The data matrix $\boldsymbol{D} \in \mathbb{C}^{K \times M}$ contains the complex-valued data symbols, $d_{k,m}$. Finally, the two vectors $\boldsymbol{1}_K \in \mathbb{C}^{K \times 1}$ and $\boldsymbol{1}_M \in \mathbb{C}^{M \times 1}$ are vectors of ones and they represent the summation over the subcarriers and over the time subsymbols. The system model is illustrated in Fig. 2.

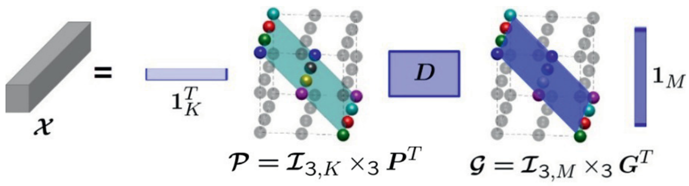

Fig. 2: GFDM transmit signal based on PARATUCK2 tensor decomposition.

Let us define a tensor $\boldsymbol{\mathcal{T}}_{(.,.,n)} = \boldsymbol{\mathcal{P}}_{(.,.,n)} \cdot \boldsymbol{D} \cdot \boldsymbol{\mathcal{G}}_{(.,.,n)}$, $n = 1, \ldots, N$. Then the GFDM modulated block of data is

$$\boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{T}} \times_1 \boldsymbol{1}_K^T \times_2 \boldsymbol{1}_M^T.$$

Note that the tensor $\boldsymbol{\mathcal{T}}$ represents the GFDM modulated block before the summation. It can be used to investigate how the data symbols are spread over time and frequency. The tensor $\boldsymbol{\mathcal{T}}$, for $M = 4$ and $K = 7$ is visualized in Fig. 3. The transpose of the 3-mode unfolding of $\boldsymbol{\mathcal{X}}$ is

$$[\boldsymbol{\mathcal{X}}]^T_{(3)} = (\boldsymbol{1}_M^T \otimes \boldsymbol{1}_K^T)[\boldsymbol{\mathcal{T}}]^T_{(3)} \in \mathbb{C}^{1 \times N}, \tag{3}$$

where $[\boldsymbol{\mathcal{T}}]^T_{(3)} \in \mathbb{C}^{K \cdot M \times N}$ defines the transpose of the 3-mode unfolding of $\boldsymbol{\mathcal{T}}$. After an analysis of the structure of $\boldsymbol{\mathcal{T}}$ it is easy to show that the n-th column of $[\boldsymbol{\mathcal{T}}]^T_{(3)}$ is equal to $\left(\mathrm{diag}(\boldsymbol{G}_{(.,n)}) \otimes \mathrm{diag}(\boldsymbol{P}_{(.,n)})\right) \cdot \mathrm{vec}(\boldsymbol{D})$. Accordingly, $\boldsymbol{G}_{(.,n)}$ and $\boldsymbol{P}_{(.,n)}$ are the n-th column of $\boldsymbol{G}$ and $\boldsymbol{P}$ or the diagonal elements of the n-th 3-mode slice of $\boldsymbol{\mathcal{G}}$ and $\boldsymbol{\mathcal{P}}$, respectively. Furthermore, using the property $\mathrm{diag}(\boldsymbol{a})\boldsymbol{b} = \mathrm{diag}(\boldsymbol{b})\boldsymbol{a}$ we have that

$$\begin{aligned}\left(\mathrm{diag}(\boldsymbol{G}_{(.,n)}) \otimes \mathrm{diag}(\boldsymbol{P}_{(.,n)})\right) \cdot \mathrm{vec}(\boldsymbol{D}) = \\ \mathrm{diag}\left(\mathrm{vec}(\boldsymbol{D})\right) \cdot \left(\boldsymbol{G}_{(.,n)} \otimes \boldsymbol{P}_{(.,n)}\right).\end{aligned}$$

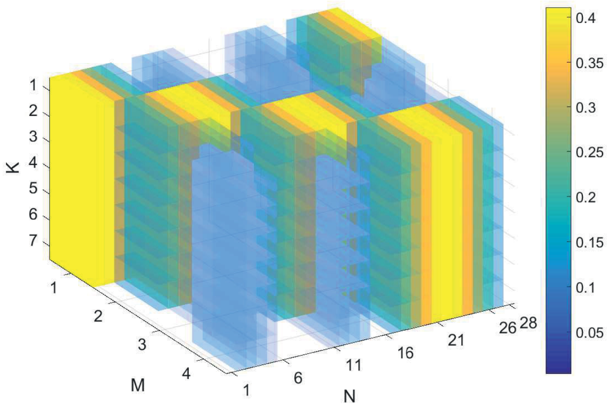

Fig. 3: Visualization of the tensor $\mathcal{T}$ as a function of the M subsymbols, K subcarriers and N samples.

Taking into account all N columns of $[\mathcal{T}]^T_{(3)}$, i.e,

$$\operatorname{diag}(\operatorname{vec}(\boldsymbol{D})) \cdot \left[\left(\boldsymbol{G}_{(.,1)} \otimes \boldsymbol{P}_{(.,1)}\right) \quad \ldots \quad \left(\boldsymbol{G}_{(.,N)} \otimes \boldsymbol{P}_{(.,N)}\right) \right],$$

and the fact that the Khatri-Rao product is the column-wise Kronecker product, it follows that

$$[\mathcal{X}]^T_{(3)} = (\boldsymbol{1}^T_M \otimes \boldsymbol{1}^T_K) \cdot \operatorname{diag}(\operatorname{vec}(\boldsymbol{D})) \cdot (\boldsymbol{G} \diamond \boldsymbol{P}). \qquad (4)$$

After applying the property $\operatorname{vec}\left(\boldsymbol{A}\operatorname{diag}(\boldsymbol{x})\boldsymbol{B}^T\right) = (\boldsymbol{B} \diamond \boldsymbol{A})\boldsymbol{x}$ and applying the $\operatorname{vec}(.)$ operator to equation (4) the GFDM modulated transmit block is

$$\boldsymbol{x} = \left((\boldsymbol{G} \diamond \boldsymbol{P})^T \diamond (\boldsymbol{1}^T_M \otimes \boldsymbol{1}^T_K) \right) \operatorname{vec}(\boldsymbol{D}). \qquad (5)$$

We can also define the selection vector $\boldsymbol{s} = \boldsymbol{1}^T_M \otimes \boldsymbol{1}^T_K$ that specifies which data symbols are transmitted during which time subsymbol and on which subcarriers. Using a more general structure of $\boldsymbol{s}$ leads to a more flexible GFDM model, where the particular subcarriers and time subsymbol can be selected. If all M time subsymbols and all K subcarriers are used, the selection vector is a row vector of ones, then equation (5) simplifies to

$$\boldsymbol{x} = (\boldsymbol{G} \diamond \boldsymbol{P})^T \operatorname{vec}(\boldsymbol{D}).$$

Let us assume a SISO system with a frequency selective channel, $\boldsymbol{h} \in \mathbb{C}^{L\times 1}$, with L taps. The received signal after the removal of the Cyclic Prefix (CP) can be defined as a multiplication along the third mode with a circular channel matrix $\boldsymbol{H} \in \mathbb{C}^{N\times N}$, where the first column is a zero padded version of $\boldsymbol{h}$.

$$\mathcal{Y} = \mathcal{X} \times_3 \boldsymbol{H}$$

To convert the GFDM block of N samples into the frequency domain we multiply with a discrete Fourier transform (DFT) matrix $\boldsymbol{F}_N \in \mathbb{C}^{N\times N}$ along the third mode.

$$\tilde{\mathcal{X}} = \mathcal{X} \times_3 \boldsymbol{F}_N$$

After the removal of the CP, the received GFDM block in the frequency domain is

$$\tilde{\mathcal{Y}} = \tilde{\mathcal{X}} \times_3 \tilde{\boldsymbol{H}}_D,$$

where $\tilde{\boldsymbol{H}}_D = \operatorname{diag}(\boldsymbol{F}_L \cdot \boldsymbol{h})$ is a diagonal matrix and $\boldsymbol{F}_L$ is a DFT matrix containing only the first L columns of $\boldsymbol{F}_N$. Moreover, we can build a 3-mode vector of the channel coefficients in the frequency domain $\tilde{\mathcal{H}}_{(.,.,n)} = \tilde{\boldsymbol{H}}_{D(n,n)} \in \mathbb{C}^{1\times 1\times N}$. Therefore, we have

$$\tilde{\mathcal{Y}}_{(1,1,n)} = \tilde{\mathcal{H}}_{(1,1,n)} \cdot \tilde{\mathcal{X}}_{(1,1,n)},$$

for this SISO system. This equation corresponds to the element wise or Hadamard product between the two 3-mode vectors.

B. GFDM-MIMO model

Next, let us assume a GFDM-MIMO system with M_T transmit and M_R receive antennas [4], [3]. The GFDM modulated signal for each transmit antenna $m_T = 1, \ldots, M_T$ and for each sample $n = 1, \ldots, N$ is defined as

$$x^{(m_T)}_n = \sum_{k=1}^{K} \sum_{m=1}^{M} d^{(m_T)}_{k,m} p_{k,n} g_{m,n}.$$

where $d^{(m_t)}_{k,m}$ are the complex modulated data symbols for each transmit antenna m_T.

Based on the PARATUCK2 model the transmit GFDM signal for each transmit antenna m_T in the time domain is given by

$$\mathcal{X}_{(1,1,n,m_T)} = \boldsymbol{1}^T_K \cdot \mathcal{P}_{(.,.,n)} \cdot \boldsymbol{D}^{(m_T)} \cdot \mathcal{G}_{(.,.,n)} \cdot \boldsymbol{1}_M.$$

Where $\boldsymbol{D}^{(m_T)}$ is the data matrix for each transmit antenna, and $\mathcal{X}_{(1,1,n,m_T)}$ is the 3-mode transmit vector for each antenna. Therefore the GFDM transmit signal using multiple antennas is a 4-D array with two singleton dimensions. Let us combine the second and the fourth dimension such that the transmit GFDM signal in the time domain is

$$\mathcal{X}_{(1,.,n)} = \boldsymbol{1}^T_K \cdot \mathcal{P}_{(.,.,n)} \cdot \boldsymbol{D}^{(M_T)} \cdot \mathcal{G}^{(M_T)}_{(.,.,n)} \cdot (\boldsymbol{I}_{M_T} \otimes \boldsymbol{1}_{\boldsymbol{M}}),$$

for each sample n, where $\mathcal{X} \in \mathbb{C}^{1\times M_T \times N}$, $\boldsymbol{D}^{(M_T)} = \left[\boldsymbol{D}^{(1)}, \ldots, \boldsymbol{D}^{(M_T)}\right]$ contains the transmit symbols from each transmit antenna. Moreover, $\mathcal{G}^{(M_T)} = \operatorname{blkdiag}(\mathcal{G}, \ldots, \mathcal{G})$ is a block diagonal tensor with dimensions $M_T \cdot M \times M_T \cdot M \times N$, or $\mathcal{G}^{(M_T)} = \mathcal{I}_{3,M\cdot M_T} \times_3 \left[\, \boldsymbol{G}^T \quad \ldots \quad \boldsymbol{G}^T \,\right]$. The GFDM transmit signal for two transmit antennas is illustrated in Fig. 4.

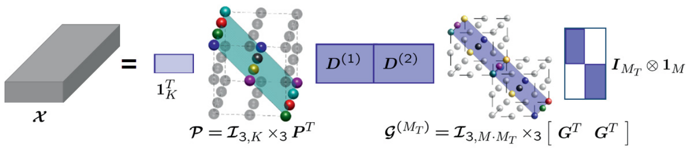

Fig. 4: GFDM transmit signal for two transmit antennas based on PARATUCK2 model.

Using a more compact notation, the GFDM transmit signal in the time domain is

$$\boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{T}} \times_1 \mathbf{1}_K^T \times_2 (\boldsymbol{I}_{M_T} \otimes \mathbf{1}_M^T), \tag{6}$$

with $\boldsymbol{\mathcal{T}}_{(.,.,n)} = \boldsymbol{\mathcal{P}}_{(.,.,n)} \cdot \boldsymbol{D}^{(M_T)} \cdot \boldsymbol{\mathcal{G}}^{(M_T)}_{(.,.,n)} \in \mathbb{C}^{K \times M \cdot M_T \times N}$. Accordingly, the GFDM block for all transmit antennas in the frequency domain is

$$\tilde{\boldsymbol{\mathcal{X}}} = \boldsymbol{\mathcal{T}} \times_1 \mathbf{1}_K^T \times_2 (\boldsymbol{I}_{M_T} \otimes \mathbf{1}_M^T) \times_3 \boldsymbol{F}_N, \tag{7}$$

where $\boldsymbol{F}_N \in \mathbb{C}^{N \times N}$ is a DFT matrix. The three unfoldings of the tensor $\tilde{\boldsymbol{\mathcal{X}}}$ are

$$\begin{aligned}
[\tilde{\boldsymbol{\mathcal{X}}}]_{(1)} &= \mathbf{1}_K^T \cdot [\boldsymbol{\mathcal{T}}]_{(2)} \cdot (\boldsymbol{F}_N \otimes (\boldsymbol{I}_{M_T} \otimes \mathbf{1}_{\boldsymbol{M}}{}^T))^T \\
[\tilde{\boldsymbol{\mathcal{X}}}]_{(2)} &= (\boldsymbol{I}_{M_T} \otimes \mathbf{1}_M^T) \cdot [\boldsymbol{\mathcal{T}}]_{(1)} \cdot (\boldsymbol{F}_N \otimes \mathbf{1}_K^T)^T \\
[\tilde{\boldsymbol{\mathcal{X}}}]_{(3)} &= \boldsymbol{F}_N \cdot [\boldsymbol{\mathcal{T}}]_{(3)} \cdot (\mathbf{1}_K^T \otimes (\boldsymbol{I}_{M_T} \otimes \mathbf{1}_{\boldsymbol{M}}{}^T))^T
\end{aligned}$$

Similarly, to the SISO model, taking the transpose of the 3-mode unfolding and exploiting the structure of the tensor $\boldsymbol{\mathcal{T}}$, the GFDM modulated blocks in the frequency domain are

$$[\tilde{\boldsymbol{\mathcal{X}}}]_{(3)}^T \triangleq \tilde{\boldsymbol{X}}^T = \boldsymbol{S} \cdot \operatorname{diag}\left(\operatorname{vec}(\boldsymbol{D}^{(M_T)})\right) \cdot (\boldsymbol{G}^{(M_T)} \diamond \boldsymbol{P}) \cdot \boldsymbol{F}_N,$$

where the selection matrix $\boldsymbol{S} = \mathbf{1}_K^T \otimes (\boldsymbol{I}_{M_T} \otimes \mathbf{1}_{\boldsymbol{M}}{}^T)$ defines which symbol is sent to which antenna, which time subsymbol and on which subcarrier. We would also like to point out that Space-Time Block Codes, or Space-Frequency Block codes can be directly implemented using this selection matrix. Moreover, $\boldsymbol{G}^{(M_T)} = \left[\boldsymbol{G}^T, \ldots, \boldsymbol{G}^T\right]^T \in \mathbb{C}^{M_T \cdot M \times N}$ has N columns equal to the diagonal elements of the tensor $\boldsymbol{\mathcal{G}}^{(M_T)} = \boldsymbol{\mathcal{I}}_{M \cdot M_T} \times_3 \boldsymbol{G}^{(M_T)T}$. Finally, after applying the vec operator, the resulting vector of transmit symbols is

$$\operatorname{vec}(\tilde{\boldsymbol{X}}^T) = \left[\left(\boldsymbol{F}_N (\boldsymbol{G}^{(M_T)} \diamond \boldsymbol{P})^T\right) \diamond \boldsymbol{S}\right] \operatorname{vec}(\boldsymbol{D}^{(M_T)}). \tag{8}$$

Note that equation (8) can be transformed into the time domain by leaving out the DFT matrix $\boldsymbol{F}_N$. Equation (8) can be used for GFDM demodulation. The GFDM samples for multiple antennas in the time domain are

$$\operatorname{vec}(\boldsymbol{X}^T) = \left[\left(\boldsymbol{G}^{(M_T)} \diamond \boldsymbol{P}\right)^T \diamond \boldsymbol{S}\right] \operatorname{vec}(\boldsymbol{D}^{(M_T)}).$$

Moreover, note that the GFDM modulated signal transmitted via multiple antennas has the same format as the transmit signal for one antenna presented in (5). Similarly, if all M_T transmit antenna, M subsymbols and K subcarriers are used for data transmission, the GFDM transmit signal in the frequency and time domain is

$$\tilde{\boldsymbol{X}}^T = \boldsymbol{F}_N (\boldsymbol{G}^{(M_T)} \diamond \boldsymbol{P})^T \left[\operatorname{vec}(\boldsymbol{D}^{(1)}), \ldots, \operatorname{vec}(\boldsymbol{D}^{(M_T)})\right]$$

and

$$\boldsymbol{X}^T = (\boldsymbol{G}^{(M_T)} \diamond \boldsymbol{P})^T \left[\operatorname{vec}(\boldsymbol{D}^{(1)}), \ldots, \operatorname{vec}(\boldsymbol{D}^{(M_T)})\right],$$

respectively.

If we assume that the GFDM signal is transmitted via a frequency selective channel with L taps, the channel impulse response between each transmit and receive antenna can be defined as a tensor $\boldsymbol{\mathcal{H}} \in \mathbb{C}^{M_R \times M_T \times L}$. Accordingly, the channel transfer function can also be defined as a tensor after the multiplication along the 3-mode with a DFT matrix $\boldsymbol{F}_L \in \mathbb{C}^{N \times L}$, $\tilde{\boldsymbol{\mathcal{H}}} = \boldsymbol{\mathcal{H}} \times_3 \boldsymbol{F}_L \in \mathbb{C}^{M_R \times M_T \times N}$. After the removal of the CP, the received signal is

$$\tilde{\boldsymbol{\mathcal{Y}}}_{(.,.,n)} = \tilde{\boldsymbol{\mathcal{H}}}_{(.,.,n)} \tilde{\boldsymbol{X}}^T_{(.,.,n)} \in \mathbb{C}^{M_R \times 1 \times N}.$$

Using the transpose of the 1-mode unfolding of $\tilde{\boldsymbol{\mathcal{Y}}}$ we can derive a more compact equation of the received signal as

$$[\tilde{\boldsymbol{\mathcal{Y}}}]_{(1)}^T \triangleq \tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{X}}_D \cdot [\tilde{\boldsymbol{\mathcal{H}}}]_{(1)}^T.$$

From $\tilde{\boldsymbol{\mathcal{H}}} = \boldsymbol{\mathcal{H}} \times_3 \boldsymbol{F}_L$ it follows that

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{X}}_D \cdot (\boldsymbol{F}_L \otimes \boldsymbol{I}_{M_T}) \cdot [\boldsymbol{\mathcal{H}}]_{(1)}^T, \tag{9}$$

where $\tilde{\boldsymbol{X}}_D$ is block diagonal matrix

$$\tilde{\boldsymbol{X}}_D = \left[\operatorname{diag}(\tilde{\boldsymbol{\mathcal{X}}}_{(1,1,.)}), \ldots, \operatorname{diag}(\tilde{\boldsymbol{\mathcal{X}}}_{(1,M_T,.)})\right].$$

Moreover, the matrix $\tilde{\boldsymbol{X}}_D$ can be defined using tensor notation as

$$\tilde{\boldsymbol{X}}_D = \left[\boldsymbol{\mathcal{I}}_{3,N} \times_3 \tilde{\boldsymbol{X}}^T\right]_{(1)} = (\tilde{\boldsymbol{X}}^T \diamond \boldsymbol{I}_N)^T. \tag{10}$$

Finally, the received signal for M_R received antennas is

$$\tilde{\boldsymbol{Y}} = (\tilde{\boldsymbol{X}}^T \diamond \boldsymbol{I}_N)^T \cdot (\boldsymbol{F}_L \otimes \boldsymbol{I}_{M_T}) \cdot [\boldsymbol{\mathcal{H}}]_{(1)}^T.$$

From equation (9) the Least Squares (LS) estimate of the channel impulse response $\boldsymbol{\mathcal{H}}$ can be obtained, provided that $N \geq M_T \cdot L$ and $\tilde{\boldsymbol{X}}_D$ is known.

If we use the $\operatorname{vec}(.)$ operator on the transpose of the second unfolding of $\tilde{\boldsymbol{\mathcal{Y}}}$ the received signal in frequency domain is

$$\operatorname{vec}(\tilde{\boldsymbol{Y}}) = \tilde{\boldsymbol{H}}_D \cdot \operatorname{vec}(\tilde{\boldsymbol{X}}) \in \mathbb{C}^{N \cdot M_R \times 1}, \tag{11}$$

where

$$\tilde{\boldsymbol{H}}_D = \begin{bmatrix} \operatorname{diag}(\tilde{\boldsymbol{\mathcal{H}}}_{(1,1,:)}) & \ldots & \operatorname{diag}(\tilde{\boldsymbol{\mathcal{H}}}_{(1,2,:)}) \\ \vdots & \vdots & \vdots \\ \operatorname{diag}(\tilde{\boldsymbol{\mathcal{H}}}_{(M_R,1,:)}) & \ldots & \operatorname{diag}(\tilde{\boldsymbol{\mathcal{H}}}_{(M_R,M_T,:)}) \end{bmatrix}$$

has a block diagonal structure. Equation (11) allows us to perform a frequency domain equalization in a LS sense by using the pseudo inverse of the matrix $\tilde{\boldsymbol{H}}_D$ when the channel is known to the receiver, provided that $M_R \geq M_T$.

$\tilde{\boldsymbol{H}}_D$ can also be defined by means of tensor algebra if we define a new tensor $\tilde{\boldsymbol{\mathcal{H}}}_D \in \mathbb{C}^{M_R \times M_T \times N \times N}$ as

$$\tilde{\boldsymbol{\mathcal{H}}}_{D(m_R,m_T,.,.)} = \operatorname{diag}(\tilde{\boldsymbol{\mathcal{H}}}_{(m_R,m_T,.)}).$$

Alternatively the tensor can be represented as

$$\tilde{\boldsymbol{\mathcal{H}}}_D^{(1)} = \boldsymbol{\mathcal{I}}_{3,N} \times_1 [\tilde{\boldsymbol{\mathcal{H}}}]_{(3)} \in \mathbb{C}^{M_R \cdot M_T \times N \times N}.$$

IV. Iterative Receiver for GFDM

GFDM systems suffer from ISI due to the frequency selective channels and additional filtering at the transmitter. Therefore, it has to be taken into account in the equalization. Thus they require interference cancellation techniques [2] or sophisticated receivers such as a LMMSE receivers [4]. As an alternative we propose an iterative receiver that jointly estimates the channel and the data symbols using pilot symbols as an initialization. If we assume that certain elements of the matrix $\boldsymbol{D}^{(M_T)}$ are pilots and the rest are data symbols, we can define

$$\boldsymbol{D}^{(M_T)} = \boldsymbol{D}_p^{(M_T)} + \boldsymbol{D}_d^{(M_T)} \in \mathbb{C}^{K \times M \cdot M_T},$$

where $\boldsymbol{D}_p^{(M_T)}$ is the piloting sequence which contains zeros at the positions of the data symbols, which are collected in the matrix $\boldsymbol{D}_d^{(M_T)}$, that contains zeros at the position of the pilot symbols.

At the receiver side the pilot sequence is used to initialize the channel estimate. As previously mentioned, from equation (9) we can directly estimate the channel impulse response. The initial channel estimate is

$$[\hat{\boldsymbol{\mathcal{H}}}]_{(1)}^T = \left(\tilde{\boldsymbol{X}}_{D,p} \cdot (\boldsymbol{F}_L \otimes \boldsymbol{I}_{M_T})\right)^+ \tilde{\boldsymbol{Y}},$$

where $\tilde{\boldsymbol{X}}_{D,p}$ is calculated according to (8) and (10), but taking into account only the pilot symbols, $\boldsymbol{D}_p^{(M_T)}$. The channel equalization in the frequency domain can be performed with the initial channel state information. Therefore, in each iteration we estimate the symbols using equation (11), i.e,

$$\mathrm{vec}(\hat{\tilde{\boldsymbol{X}}}) = \hat{\tilde{\boldsymbol{H}}}_D^+ \mathrm{vec}(\tilde{\boldsymbol{Y}}). \tag{12}$$

Additionally, from equation (8) the data symbols can be estimated according to

$$\mathrm{vec}(\boldsymbol{D}^{(M_T)}) = \left[\left(\boldsymbol{F}_N(\boldsymbol{G}^{(M_T)} \diamond \boldsymbol{P})^T\right) \diamond \boldsymbol{S}\right]^+ \mathrm{vec}(\hat{\tilde{\boldsymbol{X}}}^T). \tag{13}$$

Based on the symbol estimates we calculate an improved estimate of the channel impulse response using

$$[\hat{\boldsymbol{\mathcal{H}}}]_{(1)}^T = \left(\hat{\tilde{\boldsymbol{X}}}_D \cdot (\boldsymbol{F}_L \otimes \boldsymbol{I}_{M_T})\right)^+ \tilde{\boldsymbol{Y}}.$$

Accordingly, the following iteration starts with the improved channel estimate in order to enhance the symbol estimates. The iterative algorithm is stopped either by reaching a certain number of iterations, or when the estimated received signal reaches a given error tolerance of $||\tilde{\boldsymbol{Y}} - \hat{\tilde{\boldsymbol{Y}}}||_{\mathrm{F}}^2$. The number of iterations can be reduced and at the same time the performance of the algorithm can be enhanced when a hard decision demodulation is performed to the estimated data symbols in equation (13).

V. Simulation Results

For simulation purposes we have considered a 2×2 GFDM system with frequency selective channels, more precisely the 3GPP Pedestrian A channel (Ped A). The data symbols are modulated using QPSK, the sampling frequency is chosen as 7.68 MHz. The modulated data symbols are transmitted on 32 subcarriers ($K = 32$) with subcarrier spacing of 240 kHz and 15 subsymbols ($M = 15$). The filter is a root raised cosine with roll off factor 0.3 and the CP duration is 32 samples. The performance of the iterative receiver proposed in this paper is compared in terms of the symbol error rate (SER) with a Least Squares (LS) receiver with and without perfect channel state information and the LMMSE receiver proposed in [4]. Moreover, we assume a perfect synchronization and no coding. Furthermore, the normalized channel estimation error for the iterative, LS and LMMSE receiver is compared. The normalized channel estimation error is calculated according to

$$e = \frac{||\boldsymbol{\mathcal{H}} - \hat{\boldsymbol{\mathcal{H}}}||_{\mathrm{H}}^2}{||\boldsymbol{\mathcal{H}}||_{\mathrm{H}}^2}.$$

We assume pilot symbols that are distributed on random positions. The iterative algorithm is stopped either when the error between the received signal and the estimated signal is smaller then 10^{-4} or when the number of iterations exceeds five.

First we compare the receivers when 14 % of the data are pilot symbols. The results are presented in Fig. 5. The iterative receiver outperforms the LS receiver, its performance is even comparable to the LS receiver with perfect channel state information. The proposed iterative receiver has even a slightly better performance than the LMMSE receiver, even though the LMMSE receiver assumes a priori knowledge of the channel and the noise covariance matrices.

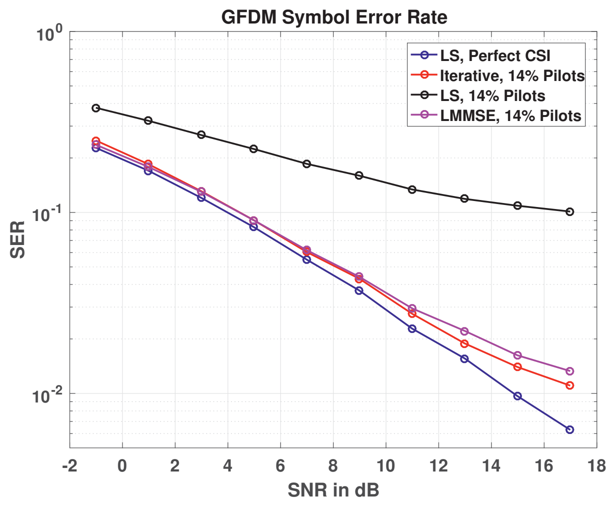

Fig. 5: SER for GFDM-MIMO 2×2 system with 14 % pilot symbols.

Second we compare the receivers when only 7% of the data are used as pilot symbols, where the SER is depicted in Fig. 6.

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In this case it can be noticed that the performance of the iterative receiver has decreased, however it still outperforms the LS receiver. The decrease in the performance is expected because there is less a priori information, i.e., less pilot symbols, for channel estimation. On the other hand, the LMMSE receiver has a better performance because it still has the channel covariance matrix as an additional a priori knowledge even though there are less pilot symbols. The iterative receiver is more sensible to the amount of pilot symbols because this is the only a priori knowledge available to the receiver.

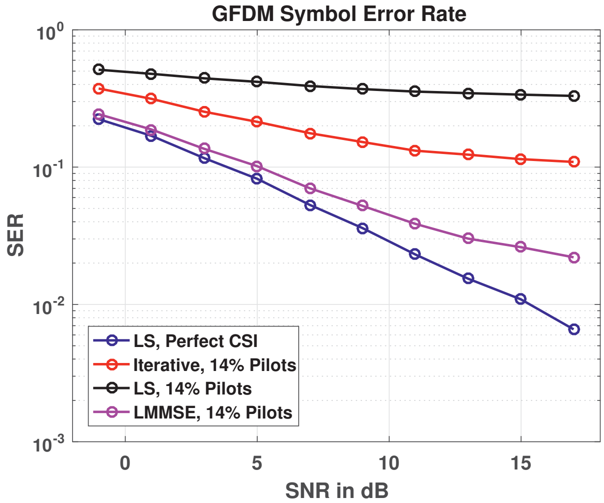

Fig. 6: SER for GFDM-MIMO 2×2 system with 7 % pilot symbols.

Finally, the normalized channel estimation error is depicted in Fig. 7 when 14% of the data are used as pilot symbols. Based on the channel estimation error, once again it can be confirmed that the iterative receiver outperforms the LS receiver. Moreover, the channel estimation error implies that the performance of the iterative receiver is strictly dependent on the SNR, whereas this is not true for the LMMSE receiver, because the LMMSE estimator takes the noise variance into account.

Note that the number of iterations was limited to five. An increase of the number of iterations will directly lead to a better performance of the iterative receiver, but also increased computational complexity. Moreover, the proposed iterative receiver does not necessarily have to use LS estimates. We have chosen the LS estimator because of the low computational complexity. However, if accuracy is more important for the system, than the iterative receiver can be combined with the LMMSE estimator.

VI. Conclusion

In this paper we have shown that the GFDM transmit signal can be modeled based on the PARATUCK2 tensor decomposition. Moreover, this model can be also extended to GFDM systems with multiple antennas. This tensor model provides new opportunities for the GFDM filter bank, such as finding

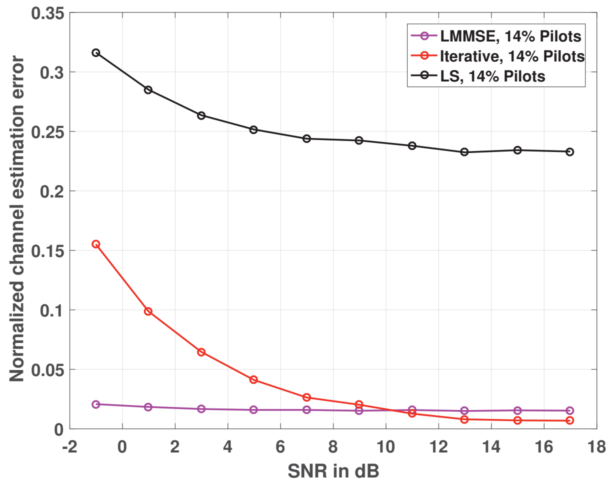

Fig. 7: Channel estimation error GFDM-MIMO 2×2 system with 14 % pilot symbols.

the best pilot sequences while studying the structure of the tensor $\mathcal{T}$, investigating more general GFDM systems when not all carriers or subsymbols are used for data transmission, and investigating new closed form solutions. Furthermore, we have presented a simple iterative LS receiver. By comparison with a LS receiver with and without perfect channel state information and LMMSE, it was shown that the proposed iterative receiver is able to estimate the channel impulse response and the data symbols within only a few iterations if sufficient pilot data is available. The proposed iterative receiver has a comparable performance with the state-of-the-art LMMSE receivers while having significantly lower computational complexity and without assuming any additional a priori information regarding the channel or noise statistics.

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