Randomized Multiple Candidate Iterative Hard Thresholding Algorithm for Direction of Arrival Estimation

Yuneisy E. García Guzmán[1], Rodrigo C. de Lamare [1,2] and Martin Haardt [3]
Center for Telecommunications Studies (CETUC)
[1] Pontifical Catholic University of Rio de Janeiro, RJ, Brazil.
[2] Department of Electronics, University of York, UK
[3] Ilmenau University of Technology, Germany
Emails: {yuneisy.garcia, delamare}@cetuc.puc-rio.br, martin.haardt@tu-ilmenau.de

***Abstract*—The sparse recovery problem by ℓ_0 minimization which is of central importance in compressed sensing (CS)-based algorithms for direction of arrival (DoA) estimation has attracted considerable interest recently. This paper proposes a greedy algorithm called randomized multiple candidate iterative hard thresholding (RMC-IHT) which generates a set of potential candidates using the iterative hard thresholding algorithm and selects the best candidate based on the a priori knowledge of the distribution of the signal and noise matrices. We also consider the case of correlated sources and develop a version of RMC-IHT for this scenario. Simulation results illustrate the improvement achieved by RMC-IHT.**

***Keywords*—Sensor array signal processing, Compressive Sensing (CS), Direction of Arrival estimation (DoA), sparse recovery, Iterative Hard Thresholding algorithm.**

I. Introduction

Direction of arrival (DoA) estimation has been an active research area in the last several decades and is key in modern applications of wireless communications, radar, sonar systems, acoustic signal processing, medical imaging and seismology.

Among the classic algorithms for DoA estimation are the multiple signal classification (MUSIC) [1], its extension Root-MUSIC [2] and the estimation of signal parameters via rotational invariance techniques (ESPRIT) [3], which exploit subspace techniques to achieve high-resolution [4]–[6].

Recent studies in [7], [8] have developed an approach to DoA estimation based on compressed sensing (CS) by formulating it as a sparse recovery problem. Several sparse recovery algorithms have been developed in the literature for solving this problem such as iterative hard thresholding (IHT) [9] [10], orthogonal matching pursuit (OMP) [11], basis pursuit denoising (BPDN) [12] and others.

In this paper, we propose a greedy algorithm called RMC-IHT that based on IHT and the incorporation of a priori knowledge of the signal and noise distributions selects the best solution for the sparse recovery problem. The performance of RMC-IHT is better than IHT and the other subspaces-based methods especially in scenarios with short data records, i.e., a few number of snapshots, and RMC-IHT also achieves a better performance in case of correlated sources.

The paper is organized as follows. In Section II, we briefly review the signal model for DoA estimation for a Uniform Linear Array (ULA). In Section III, we present the signal model used to formulate the DoA estimation problem as a sparse representation problem based on CS. Section IV describes how the DoAs can be estimated by using the IHT algorithm. In Section V, we propose the RMC-IHT algorithm and Section VI presents its version for the case of correlated sources. Section VII presents the results of the simulations in order to examine the DoA estimation performance and illustrate the effectiveness of the proposed RMC-IHT. Finally, Section VIII concludes the paper.

II. Signal Model

Let us assume that K uncorrelated narrowband zero mean signals $u_k(t), k = 1, 2, \dots, K$ from far-field sources impinge on a ULA of M $(M > K)$ sensor elements with inter-element spacing of half a wavelength $(d = \lambda/2)$ from directions $\theta_k \in [-90^\circ, 90^\circ)$ corresponding to the spatial frequency $\mu_k = -\pi \sin\theta_k$. At time instant $t, t = 1, 2, \dots, N$ where N is the total number of available snapshots, the received signal at the mth sensor can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{u}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{u}(t) = [u_1(t), \dots, u_K(t)]^T \in \mathbb{C}^K$ represents the zero-mean source data vector and $\mathbf{n}(t) \in \mathbb{C}^M$ is the vector of white circular complex Gaussian noise with zero mean and variance σ_n^2. The Vandermonde matrix $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{M\times K}$, known as the array manifold, contains the array steering vectors $\mathbf{a}(\theta_k)$ corresponding to the kth source, which can be expressed as:

$$\mathbf{a}(\theta_k) = [1, e^{j2\pi\frac{d}{\lambda}\sin\theta_k}, \dots, e^{j2\pi(M-1)\frac{d}{\lambda}\sin\theta_k}]^T. \tag{2}$$

III. Compressed Sensing for DoA estimation

DoA estimation using CS consists of formulating the source localization problem as a sparse representation problem through the introduction of an overcomplete representation of the matrix $\mathbf{A}$ in terms of all possible angles of interest [7].

Let $\boldsymbol{\theta} = [\theta_1, \dots, \theta_n], n = 1, 2, \dots, P$ be a sampling grid of all source locations of interest where P is equal to the number of potential sources which must be much greater than the number of sources K or even the number of sensors M, then the signal field can be represented by a vector $\mathbf{s}(t) = [s_1(t), \dots, s_n(t)]^T \in \mathbb{C}^P$ whose nth element is equal to $u_k(t)$ if the kth source comes from direction θ_n and zero otherwise [7], as described by

$$s_n(t) = \begin{cases} u_k(t) & \text{if } \theta_n = \theta_k, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The measurement model for one snapshot or a single measurement vector (SMV) $\mathbf{x}(t) = [x_1(t), \dots, x_M(t)]^T \in \mathbb{C}^M$ at the time t can be written as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{4}$$

where the matrix $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_P)] \in \mathbb{C}^{M\times P}$ contains the steering vectors corresponding to each potential source localization as its columns.

This representation allows to exchange the problem of parameter estimation of $\boldsymbol{\theta}$ for the sparse recovery problem of obtaining a K-sparse estimate $\hat{\mathbf{s}}(t) = [\hat{s}_1(t), \dots, \hat{s}_P(t)]^T \in \mathbb{C}^P$ for the true K-sparse vector $\mathbf{s}(t)$ from the measurements $\mathbf{x}(t)$ with the array steering

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matrix $\mathbf{A}$ as the measurement matrix where finally the non-zero indices of $\hat{\mathbf{s}}(t)$ determine the estimates $\hat{\theta}_k$ of the DoAs of the K sources.

The model in (4) can be extended to the multiple measurement vector (MMV) model as given by

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}, \tag{5}$$

where the matrix of measurements is $\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_N)] \in \mathbb{C}^{M \times N}$, the signal matrix is $\mathbf{S} = [\mathbf{s}(t_1), \ldots, \mathbf{s}(t_N)] \in \mathbb{C}^{P \times N}$ and the noise matrix is $\mathbf{N} = [\mathbf{n}(t_1), ..., \mathbf{n}(t_N)] \in \mathbb{C}^{M \times N}$. In this case the signal matrix $\mathbf{S}$ is row K-sparse and its non-zero rows indices correspond to the DoAs of the K sources.

IV. Iterative Hard Thresholding

The greedy IHT algorithm solves the sparse recovery problem presented above as an optimization problem which is formulated as

$$\hat{\mathbf{S}} = \arg \min_{\tilde{\mathbf{S}} \in \mathbb{C}^{P \times N}} \parallel \mathbf{A}\tilde{\mathbf{S}} - \mathbf{X} \parallel_F^2 \text{ s.t. } \parallel \tilde{\mathbf{S}} \parallel_{p,0} \leq K, \tag{6}$$

where the constraint ensures that the estimate $\hat{\mathbf{S}}$ is row K-sparse.

The Frobenius norm is defined as:

$$\|\mathbf{X}\|_F = \sqrt{\mathrm{Tr}(\mathbf{X}^H \mathbf{X})} = \|\mathrm{vec}(\mathbf{X})\|_2 \,,$$

where $\mathrm{vec}(\mathbf{X})$ is a vector formed by stacking the columns of $\mathbf{X}$ on top of each other and the mixed $\ell_{p,0}$ norm of $\mathbf{S}$ with rows $\mathbf{s}^i, i = 1, 2, \ldots, P$, is defined as:

$$\|\mathbf{S}\|_{p,0} = \left\| \left[\left\|\mathbf{s}^1\right\|_p, \left\|\mathbf{s}^2\right\|_p, \ldots, \left\|\mathbf{s}^P\right\|_p \right] \right\|_0 . \tag{7}$$

Each iteration of IHT consists of two steps, a gradient descent step and a hard thresholding step [8]. In the first step the consistency-enforcing objective is reduced by computing the gradient descent step described by

$$\check{\mathbf{S}}^{(i+1)} = \hat{\mathbf{S}}^{(i)} + \mu \mathbf{A}^H (\mathbf{X} - \mathbf{A}\hat{\mathbf{S}}^{(i)}), \tag{8}$$

where μ is the step size. The second step is to apply the hard thresholding operator $H_K(.)$ to the resulting matrix $\check{\mathbf{S}}^{(i+1)}$, which sets all but the K rows with the largest l_2 norm to $\mathbf{0}^T$ to get the new estimate $\hat{\mathbf{S}}^{(i+1)} = H_K(\check{\mathbf{S}}^{(i+1)})$. This step ensures that the constraint is fulfilled [8]. Then the indices of the non-zero rows of $\hat{\mathbf{S}}^{(i)}$, i.e., $\mathbf{\Gamma} = \mathrm{rsupp}(\hat{\mathbf{S}}^{(i)}) \subseteq \mathbb{R}^K$ finally determine the estimated angles $\hat{\theta}_k$ for the DoAs of the K sources. The IHT algorithm is described in Algorithm 1.

Algorithm 1: IHT (Iterative Hard Thresholding)

Input $\mathbf{X}, \mathbf{A}, K, \mu, \boldsymbol{\theta} = [\theta_1, \ldots, \theta_P]$
Initialization: $\hat{\mathbf{S}}^{(0)} = \mathbf{0}, i = 0$
while stopping criterion is not met **do**
  $\hat{\mathbf{S}}^{(i+1)} = H_K(\hat{\mathbf{S}}^{(i)} + \mu \mathbf{A}^H (\mathbf{X} - \mathbf{A}\hat{\mathbf{S}}^{(i)})$
  $i = i + 1$
end
$\mathbf{\Gamma} = \mathrm{rsupp}(\hat{\mathbf{S}}^{(i)})$
Output: DoAs: $[\hat{\theta}_1, \ldots, \hat{\theta}_K] = \boldsymbol{\theta}(\mathbf{\Gamma})$

V. Proposed Algorithm: Randomized Multiple Candidate Iterative Hard Thresholding

In the case when some prior knowledge about the distribution of the sparse matrix is available, it would make sense to incorporate that prior knowledge into the estimation process. Bayesian methods, which view the unknown sparse matrix as random, provide a systematic framework for doing that. By making use of Bayes' rule, these methods update the prior knowledge about the sparse matrix in accordance with the new evidence or observations [13]. In this section we present the proposed greedy algorithm called randomized multiple candidate iterative hard thresholding (RMC-IHT) which estimates the row sparse matrix $\mathbf{S}$ using a Bayesian framework.

RMC-IHT is based on the idea of selecting the best candidate at each iteration from a set of Q candidates which are obtained using the hard thresholding strategy. The criterion used for doing this selection is based on the computation of the maximum a posteriori probability (MAP) estimator of the signal for a given candidate support $\mathbf{\Gamma}$.

Let $\mathbf{S}_{(\mathbf{\Gamma})}$ denote the $K \times N$ matrix restricted to those K rows of $\mathbf{S}$ that are indexed by the support set $\mathbf{\Gamma}$. Then $\mathbf{S}_{(\bar{\mathbf{\Gamma}})} = \mathbf{0}$ by definition, where $\bar{\mathbf{\Gamma}}$ is the complement of $\mathbf{\Gamma}$. The row support $\mathbf{\Gamma}$ has a known fixed cardinality K and the restriction $\mathbf{A}_{\mathbf{\Gamma}}$ represents the sub-matrix of $\mathbf{A}$ obtained by selecting the columns of $\mathbf{A}$ indexed by $\mathbf{\Gamma}$.

Then the elements of $\mathbf{S}_{(\mathbf{\Gamma})}$ are assumed to be independent and identically distributed (i.i.d) complex normal random variables with zero mean and known variance σ_s^2 so the probability density function (pdf) is given by [14]

$$p(\mathbf{S}_{(\mathbf{\Gamma})}|\mathbf{\Gamma}) = \frac{1}{(\pi\sigma_s^2)^{KN}} \exp\left(-\frac{1}{\sigma_s^2} \parallel \mathbf{S}_{(\mathbf{\Gamma})} \parallel_F^2\right). \tag{9}$$

Then we compute $p(\mathbf{S}_{(\mathbf{\Gamma})}|\mathbf{X}, \mathbf{\Gamma})$ using the Bayes's rule as follows:

$$p(\mathbf{S}_{(\mathbf{\Gamma})}|\mathbf{X}, \mathbf{\Gamma}) = \frac{p(\mathbf{X}|\mathbf{S}_{(\mathbf{\Gamma})}, \mathbf{\Gamma}) p(\mathbf{S}_{(\mathbf{\Gamma})}|\mathbf{\Gamma})}{p(\mathbf{X}|\mathbf{\Gamma})}, \tag{10}$$

where $p(\mathbf{X}|\mathbf{\Gamma})$ is a normalizing constant for fixed $\mathbf{X}$ and $\mathbf{\Gamma}$. Then, $\mathbf{X}|\mathbf{S}_{(\mathbf{\Gamma})}, \mathbf{\Gamma} = \mathbf{A}_{\mathbf{\Gamma}}\mathbf{S}_{(\mathbf{\Gamma})} + \mathbf{N}$ for a given $\mathbf{S}_{(\mathbf{\Gamma})}$ is a Gaussian random matrix of mean $\mathbf{A}_{\mathbf{\Gamma}}\mathbf{S}_{(\mathbf{\Gamma})}$ and matrix covariance $\sigma_n^2 \mathbf{I}_M$, so we obtain

$$p(\mathbf{X}|\mathbf{S}_{(\mathbf{\Gamma})}, \mathbf{\Gamma}) = \frac{1}{(\pi\sigma_n^2)^{MN}} \exp\left(-\frac{1}{\sigma_n^2} \parallel \mathbf{X} - \mathbf{A}_{\mathbf{\Gamma}}\mathbf{S}_{(\mathbf{\Gamma})} \parallel_F^2\right). \tag{11}$$

Ignoring the normalizing constant $p(\mathbf{X}|\mathbf{\Gamma})$ and using the expressions (9) and (11) we can rewrite (10) as [14]:

$$p(\mathbf{S}_{(\mathbf{\Gamma})}|\mathbf{X}, \mathbf{\Gamma}) \propto \exp\left(-\frac{\parallel \mathbf{S}_{(\mathbf{\Gamma})} \parallel_F^2}{\sigma_s^2} - \frac{\parallel \mathbf{X} - \mathbf{A}_{\mathbf{\Gamma}}\mathbf{S}_{(\mathbf{\Gamma})} \parallel_F^2}{\sigma_n^2}\right). \tag{12}$$

Therefore, the MAP estimator of $\mathbf{S}_{(\mathbf{\Gamma})}$ for a given support $\mathbf{\Gamma}$ will be equal to [15]:

$$\begin{aligned}
\hat{\mathbf{S}}_{(\mathbf{\Gamma})_{\text{MAP}}} &= \arg \max_{\mathbf{S}_{(\mathbf{\Gamma})} \in \mathbb{C}^{K \times N}} p(\mathbf{S}_{(\mathbf{\Gamma})}|\mathbf{X}, \mathbf{\Gamma}) \\
&= \arg \max_{\mathbf{S}_{(\mathbf{\Gamma})} \in \mathbb{C}^{K \times N}} \log p(\mathbf{S}_{(\mathbf{\Gamma})}|\mathbf{X}, \mathbf{\Gamma}) \\
&= \arg \max_{\mathbf{S}_{(\mathbf{\Gamma})} \in \mathbb{C}^{K \times N}} \left(-\frac{\parallel \mathbf{S}_{(\mathbf{\Gamma})} \parallel_F^2}{\sigma_s^2} - \frac{\parallel \mathbf{X} - \mathbf{A}_{\mathbf{\Gamma}}\mathbf{S}_{(\mathbf{\Gamma})} \parallel_F^2}{\sigma_n^2}\right).
\end{aligned} \tag{13}$$

The above convex optimization problem can be solved by setting the gradient of the objective function to a zero matrix and solving the resulting set of equations which yields:

$$\hat{\mathbf{S}}_{(\mathbf{\Gamma})_{\text{MAP}}} = \left(\mathbf{A}_{\mathbf{\Gamma}}^H \mathbf{A}_{\mathbf{\Gamma}} + \frac{1}{\gamma}\mathbf{I}_K\right)^{-1} \mathbf{A}_{\mathbf{\Gamma}}^H \mathbf{X}, \tag{14}$$

where $\gamma = \sigma_s^2 / \sigma_n^2$.

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Then the proposed algorithm based on the results obtained before, first generates a set $\boldsymbol{\Upsilon} = [\boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2, \ldots, \boldsymbol{\Gamma}_Q]$ of Q potential candidates using the hard thresholding operator as was explained in the previous section. After that the MAP estimator of $\mathbf{S}_{(\boldsymbol{\Gamma})}$ is computed for each candidate support $\boldsymbol{\Gamma}_i$ and the best candidate support is selected based on the following criterion:

$$\boldsymbol{\Gamma}^* = \min_{\boldsymbol{\Gamma}_i \in \boldsymbol{\Upsilon}} \left\| \mathbf{X} - \mathbf{A}\hat{\mathbf{S}}_{(\boldsymbol{\Gamma}_i)_{\text{MAP}}} \right\|_F^2, \tag{15}$$

where $\boldsymbol{\Gamma}^*$ is the best support selected among the Q candidates in the current iteration.

Finally, RMC-IHT keeps the best signal estimates and the corresponding residual to initialize the next iteration. The main steps of RMC-IHT are described in Algorithm 2.

Algorithm 2: Randomized Multiple Candidate IHT (RMC-IHT)

```
Input: X, A, K, μ, θ = [θ_1, ...., θ_P], γ, Q,
Initialization:
Ŝ^(0) = 0, ii = 0, Γ = rsupp(H_K(A^H X)),
R^(0) = X, P = diag([p_i]) = diag([1/‖ a_i ‖_2^2]),
Γ_1 = Γ
while the stopping criterion is not met do
    for j ← 2 to Q do
        Ŝ^(j) = H_K(Ŝ^(j-1) + P A^H R^(j-1))
        Γ_i = rsupp(Ŝ^(j))
        R^(j) = X − A^H Ŝ^(j)
    end
    for i ← 1 to Q do
        Ŝ_{Γ_i} = (A_{Γ_i}^H A_{Γ_i} + (1/γ) I_K)^(-1) A_{Γ_i}^H X
        Ŝ_{Γ̄_i} = 0
        Γ* = min_{Γ_i ∈ Υ} ‖X − A Ŝ_{Γ_i}‖_F^2
    end
    keep Γ* as the first candidate for the next iteration
    keep the signal estimate Ŝ_{Γ*} and the to corresponding
      residual to inicialize the next iteration
    ii = ii + 1
end
Output: DoAs: [θ̂_1, ..., θ̂_K] = θ(Γ*)
```

VI. RMC-IHT for Correlated Sources

In the case of a scenario with correlated sources the covariance matrix of the signal becomes non diagonal. Then we can no longer assume that it is equal to $\sigma_s^2 \mathbf{I}_K$. Taking into account this consideration, the MAP estimator of $\mathbf{S}_{(\boldsymbol{\Gamma})}$ is now given by

$$\hat{\mathbf{S}}_{(\boldsymbol{\Gamma})_{\text{MAP}}} = \left(\frac{1}{\sigma_n^2} \mathbf{A}_{\boldsymbol{\Gamma}}^H \mathbf{A}_{\boldsymbol{\Gamma}} + \mathbf{K}_s \right)^{-1} \frac{1}{\sigma_n^2} \mathbf{A}_{\boldsymbol{\Gamma}}^H \mathbf{X} \tag{16}$$

when the covariance matrix of the signal is given by

$$\mathbf{K}_s = \mathbf{R}_s = E[\mathbf{S}_{(\boldsymbol{\Gamma})} \mathbf{S}_{(\boldsymbol{\Gamma})}^H], \tag{17}$$

and the mean of the signals is equal to zero resulting in equivalent covariance and correlation matrices.

In this case, we need some approximation of the correlation matrix of $\mathbf{S}_{(\boldsymbol{\Gamma})}$ for computing equation (16). To this end, we propose to obtain the K-term approximation $\mathbf{S}_{(\boldsymbol{\Gamma})}$ over $\boldsymbol{\Gamma}$ by a least-squares (LS) minimization as follows:

$$\mathbf{S}_{(\boldsymbol{\Gamma})} = \arg \min_{\text{rsupp}(\mathbf{Z}) \subseteq \boldsymbol{\Gamma}} \|\mathbf{X} - \mathbf{A}_{\boldsymbol{\Gamma}} \mathbf{Z}\|_F^2. \tag{18}$$

This minimization can be simply be performed by standard LS techniques, i.e. $\mathbf{S}_{(\boldsymbol{\Gamma})} = \mathbf{A}_{\Gamma}^{\dagger} \mathbf{X}$ [16]. Substituting the obtained approximation of $\mathbf{S}_{(\boldsymbol{\Gamma})}$ in (17) we have the following result:

$$\mathbf{R}_s = E[\mathbf{S}_{(\Gamma)} \mathbf{S}_{(\Gamma)}^H] = E[\mathbf{A}_{\Gamma}^{\dagger} \mathbf{X} \mathbf{X}^H (\mathbf{A}_{\Gamma}^{\dagger})^H] = \mathbf{A}_{\Gamma}^{\dagger} \mathbf{R}_{xx} (\mathbf{A}_{\Gamma}^{\dagger})^H. \tag{19}$$

Then we obtain an approximation of the correlation matrix of the signal as a function of the measurement matrix $\mathbf{A}_{\boldsymbol{\Gamma}}$ and the correlation matrix of the received matrix $\mathbf{X}$. Therefore, substituting the equation (19) in equation (16) we obtain the MAP estimator for scenarios with correlated sources:

$$\hat{\mathbf{S}}_{(\boldsymbol{\Gamma})_{\text{MAP}}} = \left(\frac{1}{\sigma_n^2} \mathbf{A}_{\boldsymbol{\Gamma}}^H \mathbf{A}_{\boldsymbol{\Gamma}} + \mathbf{A}_{\Gamma}^{\dagger} \mathbf{R}_{xx} (\mathbf{A}_{\Gamma}^{\dagger})^H \right)^{-1} \frac{1}{\sigma_n^2} \mathbf{A}_{\boldsymbol{\Gamma}}^H \mathbf{X}. \tag{20}$$

VII. Simulations Results

The simulation results have been obtained using a sampling grid of P= 1024 angles and $\theta_i = \arcsin(\frac{2}{P}(i-1)-1)$ corresponding to the equally spaced spatial frequencies. The measurement noise samples are drawn from an i.i.d complex Gaussian random process with zero mean and variance σ_n^2 [8]. The SNR in dB is defined as:

$$\text{SNR} = 10 \log_{10} \left(\frac{K\sigma_s^2}{\sigma_n^2} \right) \ \text{dB}. \tag{21}$$

The DoA estimation performance is measured in terms of the Root Mean Square Error (RMSE) between the DoAs θ_k of the sources $k = 1, 2, \ldots, K$ and their estimates $\hat{\theta}_{r,k}$ in R=100 Monte Carlo runs $r = 1, 2, \ldots, R$

$$\text{RMSE}_\theta = \sqrt{\left(\frac{1}{RK} \right) \sum_{r=1}^{R} \sum_{k=1}^{K} |\hat{\theta}_{k,r} - \theta_k)|^2}. \tag{22}$$

Fig. 1 represents a scenario with K=2 sources which are assumed uncorrelated complex Gaussian random variables with zero mean and variance σ_s^2 =1 and located at the angles $\theta_1 = 4.93°$ and $\theta_2 = 10.01°$, the RMSE is plotted over the SNR (signal-to-noise ratio) with the number of antenna elements $M = 15$ for $N = 2$, the size of the set of candidates is set to 4 and the number of iterations of the IHT algorithm is set to 10. In addition to the RMSE of the considered DoAs estimation methods, we also show the deterministic Cramér-Rao lower bound (CRB) [17].

As we can be noticed the proposed RMC-IHT algorithm shows a better performance than the conventional IHT and the subspaces-based methods MUSIC and ESPRIT. RMC-IHT is able to estimate the DoAs with a lower RMSE and a small number of snapshots.

Fig. 2 depicts a scenario with correlated sources where the coefficient of correlation $\rho = 0.8$, $M = 10$, $N = 50$ and the sources are assumed correlated complex Gaussian random variables with zero mean and variance σ_s^2 =1 and are located at the angles $\theta_1 = 4.93°, \theta_2 = 9.8969°, \theta_3 = 14.9403°$. The correlation matrix of the signal in this case can be represented by

$$\mathbf{R}_s = \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix} \tag{23}$$

Then for confirming the accuracy of our approximation we plot the RMC-IHT algorithm using the exact value of the covariance matrix given by equation (23) and the approximation that we obtain using the equation (19).

We can notice in Fig. 2 that our approximation is very close to the exact value of the correlation matrix of the signal and the performance of RMC-IHT is better than MUSIC, ESPRIT, IHT and IHT using the forward backward spatial smoothing (FBSS) technique [18], [19].

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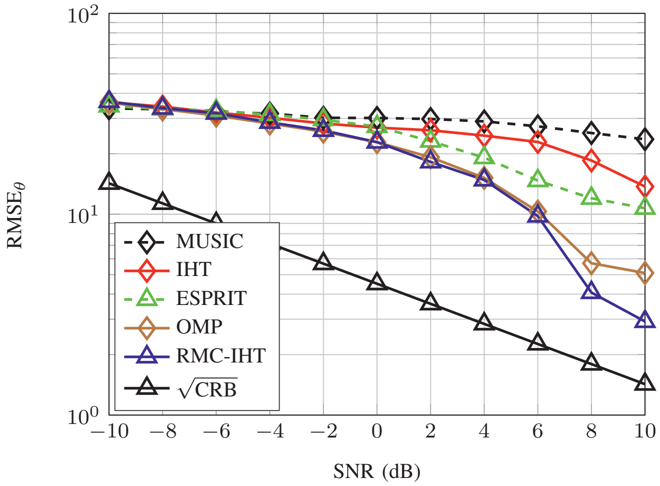

Fig. 1. RMSE vs. SNR for M=15 and N=2

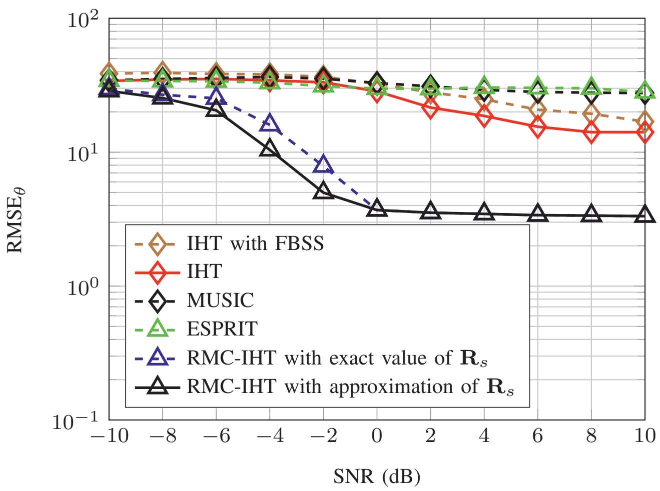

Fig. 2. RMSE vs. SNR for M=10, N=50 for correlated sources with $\rho = 0.8$

VIII. Conclusion

In this paper, we have developed the RMC-IHT algorithm based on the principles of IHT and statistical knowledge of the signal distribution for achieving a better performance. We have also developed an extension of the algorithm for the case of a scenario with correlated sources. The benefits of using RMC-IHT are mainly that it is able to estimate the DoAs with a lower RMSE than previously reported techniques in scenarios with a large-scale antenna array and only a few available snapshots. Moreover, this is also noticed in the case of correlated sources in which it performs better than IHT.

References

[1] R. Schmidt, “Multiple emitter location and signal parameter estimation,” *IEEE Transactions on Antennas and Propagation*, vol. 34, pp. 276–280, 1986.

[2] B. D. Rao and K. Hari, “Performance analysis of Root-MUSIC,” *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 37, no. 12, pp. 1939–1949, Dec, 1989.

[3] R. Roy and T. Kailath, “ESPRIT-estimation of signal parameter via rotational invariance techniques,” *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 37, no. 7, pp. 984–995, July, 1989.

[4] J. Steinwandt, R. C. Lamare, and M. Haardt, “Beamspace direction finding based on the conjugate gradient and the auxiliary vector filtering algorithms,” *Signal Processing*, vol. 93, no. 4, pp. 641–651, 2013.

[5] L. Wang, R. C. de Lamare, and M. Haardt, “Direction finding algorithms based on joint iterative subspace optimization,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, pp. 2541–2553, Oct, 2014.

[6] L. Qiu, Y. Cai, R. C. de Lamare, and M. Zhao, “Reduced-rank doa estimation algorithms based on alternating low-rank decomposition,” *IEEE Signal Processing Letters*, 2016.

[7] D. Malioutov, M. Cetin, and A. Willsky, “A sparse signal reconstruction perspective for source localization with sensor arrays,” *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 3010–3022, Aug, 2005.

[8] C. Stöckle, J. Munir, A. Mezghani, and J. A. Nossek, “DoA estimation performance and computational complexity of subspace and compressed sensing-based methods,” in *WSA 2015; 19th International ITG Workshop on Smart Antennas*, Ilmenau, Germany, March, 2015.

[9] T. Blumensath and M. E. Davies, “Iterative hard thresholding for compressed sensing,” *Applied and Computational Harmonic Analysis*, vol. 27, pp. 265–274, 2009.

[10] J. D. Blanchard, M. Cermak, D. Hanle, and Y. Jing, “Greedy algorithms for joint sparse recovery,” *IEEE Transactions on Signal Processing*, vol. 62, no. 7, pp. 1694–1704, April 2014.

[11] J. A. Tropp and A. C. Gilbert, “Signal recovery from random measurements via orthogonal matching pursuit,” *IEEE Transactions on Information Theory*, vol. 53, no. 12, pp. 4655–4666, Dec, 2007.

[12] E. den Berg and M. P. Friedlander, “Sparse optimization with least squares constraints,” *IEEE Transactions on Information Theory*, vol. 21, no. 4, pp. 1201–1229, 2011.

[13] A. Ejaz, “Algorithms for sparse signal recovery in compressed sensing,” Master thesis, Aalto university school of electrical engineering, Finland, May, 2015.

[14] A. Ejaz, E. Ollila, and V. Koivunen, “Randomized simultaneous orthogonal matching pursuit,” in *23rd European Signal Processing Conference EUSIPCO*, Nice, France, Aug, 2015.

[15] H. V. Trees, *Detection, Estimation and Modulation Theory , Part I*. J.Wiley and Sons, 2002.

[16] S. Huang and J. Zhu, “Recovery of sparse signals using omp and its variants: convergence analysis based on rip,” *Inverse Problems*, vol. 27, no. 3, 2011.

[17] P. Stoica and A. Nehorai, “MUSIC, Maximum likelihood, and Cramer-Rao bound: further results and comparisons,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 12, pp. 2140–2150, Dec 1990.

[18] R. T. Williams, S. Prasad, A. K. Mahalanabis, and L. H. Sibul, “An improved spatial smoothing technique for bearing estimation in a multipath environment,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 36, no. 4, pp. 425–432, Apr 1988.

[19] A. Thakre, M. Haardt, and K. Giridhar, “Single snapshot spatial smoothing with improved effective array aperture,” *IEEE Signal Processing Letters*, vol. 16, no. 6, June, 2009.

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