# Constrained Tensor Decompositions for Semi-blind MIMO Detection

*Liana Khamidullina[1,2], Yao Cheng[1], and Martin Haardt[1]*
[1]Communication Research Laboratory, Ilmenau University of Technology, Ilmenau, Germany
[2]German-Russian Institute of Advanced Technologies, Kazan National Research Technical University, Kazan, Russia

***Abstract*—In this paper we introduce a complex valued PARAllel FACtor analysis 2 (PARAFAC2) tensor decomposition to perform a semi-blind data detection and channel estimation in Multiple Input Multiple Output (MIMO) communication systems. We represent the received data in the form of a three-way tensor, with receive antennas on the first mode, data symbols in each packet on the second, and packets on the third mode. Factorizing the resulting received data tensor via the proposed complex valued PARAFAC2 decomposition enables a simultaneous estimation of the channel and the transmitted data. Moreover, the use of a few training symbols allows to find a rotation matrix resolving the ambiguity inherent in the decomposition. Extensive numerical simulations have been conducted to thoroughly evaluate the performance of the proposed semi-blind MIMO detection scheme.**

***Index Terms*—PARAFAC2, MIMO Detection, Tensor Factorization, Semi-Blind Channel Estimation**

## I. Introduction

Multiple antennas at the transmitter and at the receiver significantly improve the transmission performance over the wireless channels in comparison with the conventional systems [1], [2]. Such multi-antenna systems allow to combat the problems of fading, multi-user interference, and to achieve high data rates. The performance of MIMO systems depends vastly on the availability of the knowledge of the channel. Thus, a channel estimation is of crucial importance in MIMO signal processing. There are several training-based algorithms such as least squares (LS), maximum likelihood (ML), and minimum mean square error (MMSE) estimation that perform the MIMO channel estimation based on known pilots [3]. In contrast, there are also blind channel estimation algorithms such as those proposed in [4], [5] that provide a better spectral efficiency. The combination of the training-based and blind algorithms, i.e., semi-blind estimation techniques can enhance the performance of the channel estimation [6]. With a small number of training symbols, semi-blind techniques allow to solve ambiguity and convergence problems of the blind methods. Moreover, the use of the available information yields an improved channel estimation.

The use of tensor decompositions to factorize the receive data enables a new effective scheme for the channel and data estimation [7]. In this paper we use the PARAFAC2 [8] decomposition of the three-way data tensor of MIMO systems for the data and channel estimation. Such a semi-blind method of channel and source signals estimation has been already applied in [9]. In contrast to the mentioned paper, we extend the real valued PARAFAC2 decomposition to the application in complex valued data, and use it for the simultaneous channel estimation and data detection in MIMO communication systems with additional constraints. We conducted extensive numerical simulations considering a variety of settings, e.g., higher-order modulation schemes, various numbers of antennas and packets. A comparison with state-of-the-art training based channel estimation schemes is presented as well. We also investigate further requirements of the system setting, e.g., $M_T$, $M_R$, number of symbols and packets, to guarantee the identifiability of the PARAFAC2 decomposition. Similar as in [9], it is required that the number of receive antennas is equal or greater than the number of transmit antennas, $M_T \leq M_R$. However, in contrast to [9], our approach enforces no constraint on the modulation scheme used in the MIMO system. For instance, quadrature amplitude modulation (QAM) can be employed, opening up the potential of using higher order modulation formats and therefore a higher data rate.

The remainder of the paper is organized as follows: we first introduce the notation and present the complex valued PARAFAC2 decomposition as well as its computation algorithm. Then the system model of a MIMO system is provided. It is described in detail how the three-way received data tensor is constructed and how a semi-blind source and channel estimation is realized via the complex valued PARAFAC2 decomposition. In addition, further constraints for the PARAFAC2 model are addressed. Numerical results are shown, before conclusions are drawn in the end.

The following notation is used throughout the paper: matrices and vectors are denoted by upper-case and lower-case bold-faced letters, respectively. Bold faced calligraphic letters denote tensors. Superscripts $\{\cdot\}^{\mathrm{T}}$ and $\{\cdot\}^{\mathrm{H}}$ denote the transpose and Hermitian transpose, respectively, whereas $\mathrm{diag}\{\cdot\}$ is the operation of constructing a diagonal matrix with diagonal elements being the entries of the input vector. The $i$-th row and the $j$-th column of a matrix $\boldsymbol{A} \in \mathbb{C}^{I \times J}$ is represented by $\boldsymbol{A}(i,:) \in \mathbb{C}^{J}$ and $\boldsymbol{A}(:,j) \in \mathbb{C}^{I}$, respectively, where $i = 1,\ldots,I$ and $j = 1,\ldots,J$. The Khatri-Rao product between matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ is denoted as $\boldsymbol{A} \diamond \boldsymbol{B}$. Additionally, we denote the higher-order norm of a tensor $\boldsymbol{\mathcal{A}}$ by $\|\boldsymbol{\mathcal{A}}\|_{\mathrm{H}}$. It is defined as the square root of the sum of the squared magnitude of all elements in $\boldsymbol{\mathcal{A}}$. The $r$-mode unfolding of the tensor $\boldsymbol{\mathcal{A}}$ is denoted as $[\boldsymbol{\mathcal{A}}]_{(r)}$ The matrix $\boldsymbol{I}_d$ represents an $d \times d$ identity matrix.

ISBN 978-3-8007-4939-3  

## II. Complex valued PARAFAC2 decomposition

The PARAFAC2 decomposition of a complex valued three-way tensor $\boldsymbol{\mathcal{X}} \in \mathbb{C}^{I\times J\times K}$, can be represented as

$$\boldsymbol{X}_k = \boldsymbol{A} \cdot \mathrm{diag}\left\{\boldsymbol{C}(k,:)\right\} \cdot \boldsymbol{B}_k^{\mathrm{T}} + \boldsymbol{N}_k \in \mathbb{C}^{I\times J}, \quad (1)$$

where $\boldsymbol{X}_k \in \mathbb{C}^{I\times J}$ is the $k$-th frontal slice (also called a three mode slice) of $\boldsymbol{\mathcal{X}}$, $\boldsymbol{A} \in \mathbb{C}^{I\times R}$, $\boldsymbol{C} \in \mathbb{C}^{K\times R}$, $\boldsymbol{B}_k \in \mathbb{C}^{J\times R}$ for $k = 1,\ldots,K$, $R$ is the model order, and $\boldsymbol{N}_k \in \mathbb{C}^{I\times J}$ collects the residuals. Figure 1 presents an illustration of the PARAFAC2 decomposition. In this work we extend the Harshman constraint [10], that guarantees the uniqueness of the PARAFAC2 decomposition, to the complex-valued case: $\boldsymbol{B}_k^{\mathrm{H}} \cdot \boldsymbol{B}_k = \boldsymbol{H} \in \mathbb{C}^{R\times R}$ ($k = 1,\ldots,K$). It can be reformulated [8] such that $\boldsymbol{B}_k^{\mathrm{T}} = \boldsymbol{F}^{\mathrm{T}} \cdot \boldsymbol{V}_k$, where $\boldsymbol{F} \in \mathbb{C}^{R\times R}$, $\boldsymbol{F}^{\mathrm{H}} \cdot \boldsymbol{F} = \boldsymbol{H}$, and $\boldsymbol{V}_k \in \mathbb{C}^{R\times J}$ has unitary rows for $k = 1,\ldots,K$, satisfying $\boldsymbol{V}_k \cdot \boldsymbol{V}_k^{\mathrm{H}} = \boldsymbol{I}_R$. Now we can introduce a tensor $\widetilde{\boldsymbol{\mathcal{X}}}_0 \in \mathbb{C}^{I\times R\times K}$ where the $k$-th frontal slice of $\widetilde{\boldsymbol{\mathcal{X}}}_0$ is written as

$$\widetilde{\boldsymbol{X}}_k = \boldsymbol{A} \cdot \mathrm{diag}\left\{\boldsymbol{C}(k,:)\right\} \cdot \boldsymbol{F}^{\mathrm{T}}, \quad (2)$$

indicating that $\boldsymbol{A}$, $\boldsymbol{C}$, and $\boldsymbol{F}$ are the factor matrices of the PARAFAC decomposition of $\widetilde{\boldsymbol{\mathcal{X}}}_0$. In a zero-residual case, the following equality holds

$$\widetilde{\boldsymbol{X}}_k = \boldsymbol{X}_k \cdot \boldsymbol{V}_k^{\mathrm{H}}. \quad (3)$$

Based on these facts, we have extended the Direct Fitting algorithm [7] to the complex valued case. The resulting computation algorithm for the complex valued PARAFAC2 decomposition is summarized as follows:

- **Step 1**: Initialize $\boldsymbol{A}$, $\boldsymbol{C}$, and $\boldsymbol{F}$.
- **Step 2**: Reconstruct $\widetilde{\boldsymbol{\mathcal{X}}}_0$ with $\boldsymbol{A}$, $\boldsymbol{C}$, and $\boldsymbol{F}$ according to (2), and update $\boldsymbol{V}_k$ ($k = 1,\ldots,K$) with the extended generalized solution of the complex-valued Orthogonal Procrustes Problem (OPP) based on [11] .
- **Step 3**: Update $\widetilde{\boldsymbol{\mathcal{X}}}_0$ according to (3) with $\boldsymbol{V}_k$ ($k = 1,\ldots,K$) computed in **Step 2**. Then compute its PARAFAC decomposition to update $\boldsymbol{A}$, $\boldsymbol{C}$, and $\boldsymbol{F}$ via an alternating least squares procedure.
- **Step 4**: Compute the reconstructed tensor $\hat{\boldsymbol{\mathcal{X}}}$ and then calculate the Squared Reconstruction Error (SRE) defined via $E_{\mathrm{R}} = \left(\left\|\hat{\boldsymbol{\mathcal{X}}} - \boldsymbol{\mathcal{X}}\right\|_{\mathrm{H}}^2\right)/\|\boldsymbol{\mathcal{X}}\|_{\mathrm{H}}^2$. Repeat **Step 2** – **Step 4** until the change of the residual quantified by $\Delta E_{\mathrm{R}} = \left(E_{\mathrm{R}}^{\mathrm{old}} - E_{\mathrm{R}}\right)/E_{\mathrm{R}}^{\mathrm{old}}$ is smaller than a predefined threshold, implying the convergence of the algorithm, where $E_{\mathrm{R}}^{\mathrm{old}}$ represents the residual in the previous iteration.

## III. System Model

Let us consider a flat fading MIMO channel with $M_T$ transmit and $M_R$ receive antennas. The received signal for $N$ snapshots can be written as [9]

$$\boldsymbol{X} = \boldsymbol{H}\boldsymbol{S} + \boldsymbol{Z}, \quad (4)$$

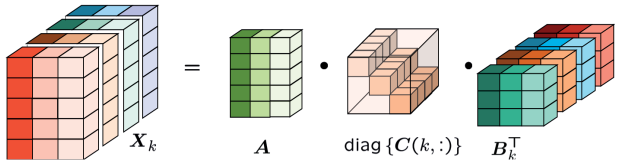


Figure 1: PARAFAC2 decomposition.

where $\boldsymbol{X} = [\boldsymbol{x}_1,\ldots,\boldsymbol{x}_{M_R}]^{\mathrm{T}} \in \mathbb{C}^{M_R\times N}$ contains the received vectors at each time snapshot, $\boldsymbol{H} \in \mathbb{C}^{M_R\times M_T}$ is the unknown MIMO channel, composed of the channel gains $h_{ij}$ from transmit antenna $j$ to receive antenna $i$, $\boldsymbol{S} = [\boldsymbol{s}_1,\ldots,\boldsymbol{s}_{M_T}] \in \mathbb{C}^{M_T\times N}$ contains the transmitted signals, and $\boldsymbol{Z} \in \mathbb{C}^{M_R\times N}$ corresponds to the independently and identically distributed (i.i.d.) zero mean spatially and temporally white additive noise with variance $\sigma_n^2$.

Assuming that the MIMO channel $\boldsymbol{H}$ has a quasi-static behaviour, i.e., the channel stays constant during the transmission of several packets each containing $N$ samples including the training sequence and the message. Consequently, we can form the received signals of $K$ packets as a three-way tensor with modes corresponding to receive antennas, samples in each packet, and packets. Then the $k$-th frontal slice of the resulting tensor $\boldsymbol{\mathcal{X}}$, related to the $k$-th packet can be written as follows

$$\boldsymbol{X}_k = \boldsymbol{H} \cdot \boldsymbol{S}_k + \boldsymbol{Z}_k \in \mathbb{C}^{I\times J}, \quad (5)$$

where $\boldsymbol{S}_k$ corresponds to the $k$-th packet passing through the channel. Jointly examining (1) and (5) reveals the link between the complex valued PARAFAC2 model and the system model of the MIMO transmission. We summarize it as follows: $\boldsymbol{A} \in \mathbb{C}^{I\times R}$ is a scaled and permuted version of the MIMO flat fading channel $\boldsymbol{H}$, $\boldsymbol{C} \in \mathbb{C}^{K\times R}$ corresponds to the third mode factor matrix of $\boldsymbol{\mathcal{X}}$, and $\mathrm{diag}\left\{\boldsymbol{C}(k,:)\right\}$ contains the elements related to the power of the symbols (scaling factors) for the corresponding source in the $k$-th packet, $\boldsymbol{B}_k^{\mathrm{T}} \in \mathbb{C}^{R\times J}$ corresponds to the matrix $\boldsymbol{S}_k$ and represents the temporal samples in each packet. The dimensions of the tensor $\boldsymbol{\mathcal{X}}$ in the PARAFAC2 model, $I$, $J$ and $K$ correspond to the number of receive antennas $M_R$, the number of samples in each temporal packet $N$, and the number of packets $K$, respectively. The PARAFAC2 model order $R$ represents the number of transmit antennas $M_T$ in the system.

Assuming the transmission of the large packets, the covariance matrix of the transmitted signals in each packet $\boldsymbol{S}_k^{\mathrm{H}} \cdot \boldsymbol{S}_k$ is approximately equal to the constant matrix $p\boldsymbol{I}_J$, where $p$ is a scaling. This guaranties that the Harshman constraint [10] is satisfied, and allow as to use the PARAFAC2 decomposition to estimate the channel and the transmitted symbols subject to permutation and scaling. We investigate an impact of the number of symbols per packet on the performance in Section V.

## IV. Semi-blind MIMO Detection and Channel Estimation Scheme

For the channel estimation and MIMO detection problems, in this paper we assume that the number of receive antennas

ISBN 978-3-8007-4939-3

$M_R$ is equal to or greater than the number of transmit antennas $M_T$ and the transmitted symbols of each transmit antenna are independent of other transmit antenna symbols. We also assume that the transmit power of all $K$ packets is constant. This assumption allows us to enforce the following constraint on the PARAFAC2 model: $\mathrm{diag}\{\boldsymbol{C}(k,:)\}$ $(k = 1,\ldots,K)$ are an identity matrices, i.e., the third mode matrix $\boldsymbol{C}$ is an all-one matrix.

In addition to the quadrature phase-shift keying (QPSK) modulation employed in [9], we also consider higher-order QAM modulation schemes, e.g., 8QAM and 16QAM, can also be used. Therefore, contrary to the mentioned above paper, we do not restrict the entries of the second mode factor matrix $\boldsymbol{B_k}$ to have a unit absolute value.

Taking into account the aforementioned constraints, the PARAFAC2 based MIMO detection algorithm can be simplified. It is described as follows:

- **Step 1**: Initialize $\boldsymbol{H}$ and $\boldsymbol{F}$. $\boldsymbol{C}$ is fixed and equal to matrix of all ones.
- **Step 2**: Reconstruct $\widetilde{\boldsymbol{\mathcal{X}}}_0$ with $\boldsymbol{H}$, $\boldsymbol{C}$, and $\boldsymbol{F}$ according to (2), and update $\boldsymbol{V}_k$ $(k = 1,\ldots,K)$ with the extended generalized solution of the complex-valued Orthogonal Procrustes Problem (OPP) based on [11] .
- **Step 3**: Update $\widetilde{\boldsymbol{\mathcal{X}}}_0$ according to (3) with $\boldsymbol{V}_k$ $(k = 1,\ldots,K)$ computed in **Step 2** and update $\boldsymbol{H}$ and $\boldsymbol{F}$ as follows

$$\boldsymbol{H} = [\boldsymbol{\mathcal{X}}]_{(1)} \left((\boldsymbol{F} \diamond \boldsymbol{C})^{+}\right)^{\mathrm{T}}, \tag{6}$$

$$\boldsymbol{F} = [\boldsymbol{\mathcal{X}}]_{(2)} \left((\boldsymbol{C} \diamond \boldsymbol{H})^{+}\right)^{\mathrm{T}}, \tag{7}$$

- **Step 4**: Compute the reconstructed tensor $\hat{\boldsymbol{\mathcal{X}}}$ and then calculate the Squared Reconstruction Error (SRE) defined via $E_{\mathrm{R}} = \left(\left\|\hat{\boldsymbol{\mathcal{X}}} - \boldsymbol{\mathcal{X}}\right\|_{\mathrm{H}}^{2}\right) / \|\boldsymbol{\mathcal{X}}\|_{\mathrm{H}}^{2}$. Repeat **Step 2** – **Step 4** until the change of the residual quantified by $\Delta E_{\mathrm{R}} = \left(E_{\mathrm{R}}^{\mathrm{old}} - E_{\mathrm{R}}\right) / E_{\mathrm{R}}^{\mathrm{old}}$ is smaller than a predefined threshold, implying the convergence of the algorithm, where $E_{\mathrm{R}}^{\mathrm{old}}$ represents the residual in the previous iteration.

Compared to the algorithm described in Section II, this algorithm to calculate PARAFAC2 with the additional constraints does not require a computation of the PARAFAC decomposition in **Step 3**. Knowing $\boldsymbol{C}$, $\boldsymbol{H}$ and $\boldsymbol{F}$ can be updated in a least squares manner.

According to our observations, it takes two iterations for the algorithm to converge. This implies that it can be regarded as a two-step algorithm and thus does not suffer from a high complexity of iterative schemes.

Using the complex valued PARAFAC2 decomposition, the transmitted signals and the channel can be estimated simultaneously, subject to ambiguities, e.g., permutation and scaling. This can be resolved by sending training symbols, which are known by the transmitter and the receiver. To resolve the ambiguities, we attach the training sequence to the first data packet and find the rotation matrix $\boldsymbol{P}_r$ as in [9]:

$$\boldsymbol{P}_r = (\hat{\boldsymbol{S}}_t \boldsymbol{S}_t^{+})^{-1}, \tag{8}$$

where $\hat{\boldsymbol{S}}_t$ and $\boldsymbol{S}_t$ are the estimated training sequence matrix and the original training sequence matrix, respectively. The ambiguities in the estimated received data matrix $\boldsymbol{S}_k$ and channel matrix $\boldsymbol{H}$ are resolved using rotation matrix $\boldsymbol{P}_r$ as follows

$$\hat{\boldsymbol{S}}_k = \boldsymbol{P}_r \breve{\hat{\boldsymbol{S}}}_k, \tag{9}$$

$$\hat{\boldsymbol{H}} = \breve{\hat{\boldsymbol{H}}} \boldsymbol{P}_r^{-1}, \tag{10}$$

where $\hat{\boldsymbol{S}}_k$ and $\hat{\boldsymbol{H}}$ are the estimated data matrix and channel matrix after resolving the ambiguities, respectively, and $\breve{\hat{\boldsymbol{S}}}_k$ and $\breve{\hat{\boldsymbol{H}}}$ are the estimated transmitted data and channel matrix, respectively, obtained by applying the PARAFAC2 decomposition.

## V. Simulation Results

In this section we evaluate the performance of the PARAFAC2 based MIMO detection and channel estimation algorithm considering various system parameters. For the simulations we use the estimation scheme described in Section IV. In the first step of the algorithm the matrices $\boldsymbol{H}$ and $\boldsymbol{F}$ are initialized randomly. To assess the results we compute a bit error rate among all data packets with respect to the signal-to-noise ratio (SNR), which is defined as $\mathrm{SNR} = 1/\sigma_{\mathrm{n}}^2$, where $\sigma_n^2$ is the noise variance. We also calculate the relative mean squared error (RMSE) for the estimated channel as follows

$$\mathrm{RMSE} = \frac{\|\hat{\boldsymbol{H}} - \boldsymbol{H}\|_{\mathrm{H}}^2}{\|\boldsymbol{H}\|_{\mathrm{H}}^2}, \tag{11}$$

where $\hat{\boldsymbol{H}}$ is the estimated channel matrix after resolving the ambiguities according to (10). For all simulations, the channel matrix $\boldsymbol{H}$ is generated randomly, such that its entries follow a zero mean i.i.d. circularly symmetric complex Gaussian distribution. In the following simulations we compare our proposed method with other channel and data estimation schemes. We also investigate the impact of different parameters, such as modulation schemes, number of antennas, packets and symbols on the performance of the algorithm.

In the first simulation we consider a MIMO system consisting of $M_T = 6$ transmit and $M_R = 6$ receive antennas. The number of packets $K$ is set to 10. Each packet contains $N = 1000$ QPSK symbols, including 10 training symbols. We compare the performance of the complex valued PARAFAC2 decomposition with the MMSE [3] and ZF [12] methods to estimate the transmitted data. Figure 2 demonstrates the averaged bit error rate among all data packets for different estimation schemes as a function of the signal-to-noise ratio. The MMSE and ZF schemes are applied separately for each received data packet assuming that the channel is known at the receive side. In contrast, the semi-blind PARAFAC2-based method estimates both the transmitted data and the unknown

ISBN 978-3-8007-4939-3

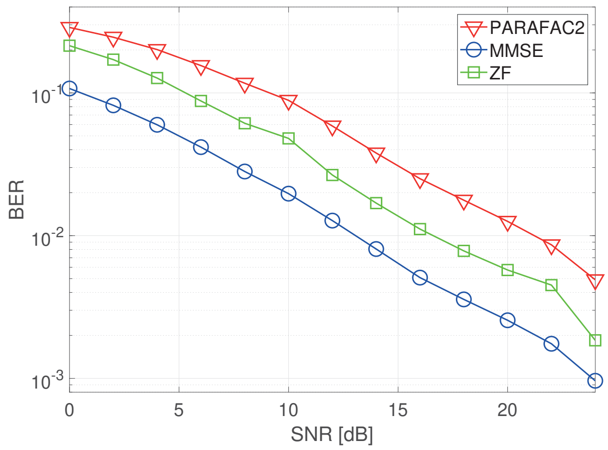


Figure 2: BER vs. SNR. Data estimation based on PARAFAC2, MMSE and ZF. $N = 1000$ QPSK symbols, including 10 training symbols, $M_T = 6$, $M_R = 6$, $K = 10$. The results are averaged over 1000 channel realizations.

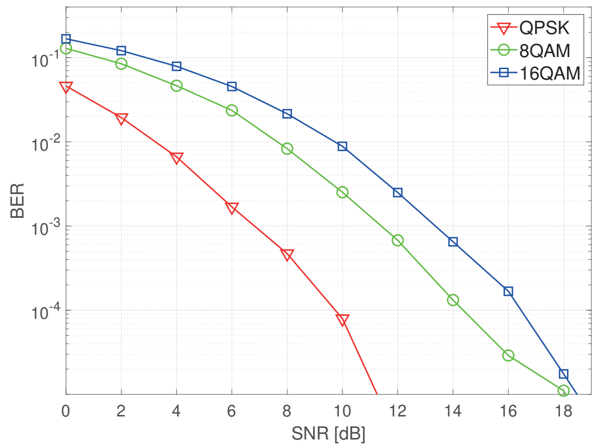


Figure 4: BER vs. SNR. PARAFAC2 based MIMO detection for different modulation schemes. 1000 QPSK, 8QAM and 16QAM symbols, including 10 training symbols, $M_T = 3$, $M_R = 7$, $K = 5$. The number of realizations is 1000.

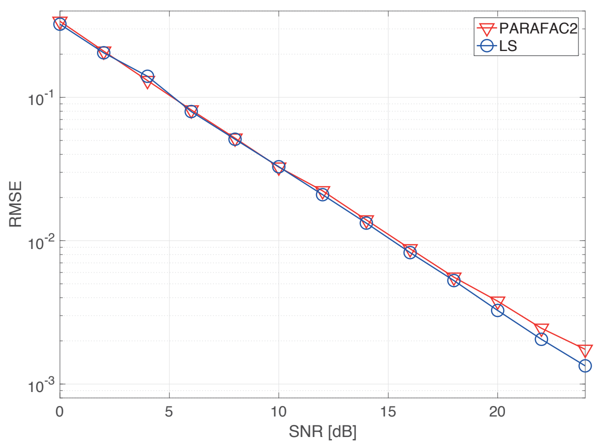


Figure 3: RMSE vs. SNR. Channel estimation based on PARAFAC2 and LS. $N = 1000$ QPSK symbols, including 10 training symbols, $M_T = 6$, $M_R = 6$, $K = 10$. The results are averaged over 1000 channel realizations.

channel, which explains the higher bit error rate for the PARAFAC2 approach.

Figure 3 displays the channel estimation error. The results are averaged over the 1000 trials for each SNR value, respectively. As it can be seen the proposed tensor based algorithm shows a similar accuracy as the training based LS approach [3]. The advantage of our approach is the ability to recover the symbols as well, in addition to estimating the channel. By comparison, the LS approach estimates the channel, and then a receiving strategy has to be employed to estimate the symbols.

In the next simulation we assess the performance of the proposed algorithm for different modulation schemes. For this simulation, the number of transmit antennas $M_T$, receive antennas $M_R$, and the number of packets $K$ are set to 3, 7, and 5, respectively. Each packet contains $N = 1000$ QPSK, 8QAM or 16QAM symbols, including 10 training symbols. Figure 4 presents the bit error rate curves for the QPSK and two higher order modulation schemes 8QAM and 16QAM. The results are averaged over the 1000 realizations for each SNR value. It is observed that proposed PARAFAC2 approach, contrary to [9], can be used not only for QPSK, but also for other modulation schemes.

Now we want to investigate the impact of the number of symbols per packet on the performance. In Figure 5 we depict the channel estimation error for systems with a different number of QPSK symbols for each packet. The number of antennas and packets is the same as in previous simulation. The number of symbols $N$ is set to 100, 1000 and to 5000, including 10 training symbols. We observe that for the packets with a large number of symbols the proposed algorithm provides more accurate estimates, especially at high SNRs. This is explained by the fact that the Harshman constraint [10] $\boldsymbol{B}_k^{\mathrm{H}} \cdot \boldsymbol{B}_k = \boldsymbol{H} \in \mathbb{C}^{R \times R}$, where $\boldsymbol{B}_k^{\mathrm{T}}$ corresponds to the transmitted symbols matrix $\boldsymbol{S}_k$ in our considered MIMO system, is better fulfilled for the matrices with a large number of symbols.

In Figures 6 and 7 we depict the bit error rate and the channel estimation error for the systems with different number of receive antennas and packets. The number of transmit antennas is set to 4. Each packet contains $N = 1000$ QPSK symbols, including 10 training symbols. As it can be seen the error depends vastly on the chosen number of receive antennas. For the better performance, the number of receive antennas $M_R$, should be larger than the number of transmit antennas $M_T$. The impact of increasing the number of packets from 4 to 7 on the performance is negligible in the considered scenarios.

ISBN 978-3-8007-4939-3

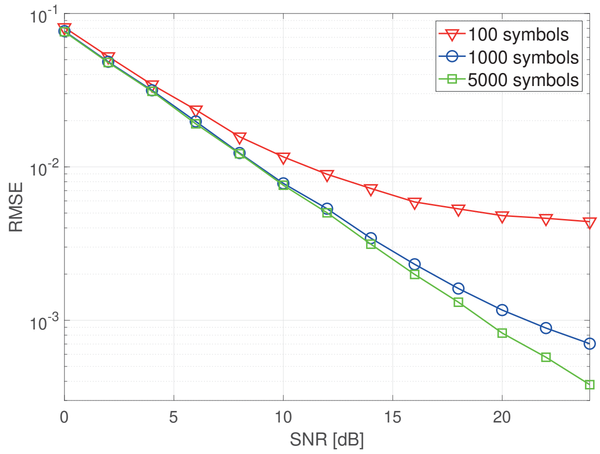


Figure 5: RMSE vs. SNR. Performance of the PARAFAC2 based channel estimation for the different number of QPSK symbols. Number of training symbols is 10, $M_T = 3$, $M_R = 7$, $K = 5$. The number of realizations is 1000.

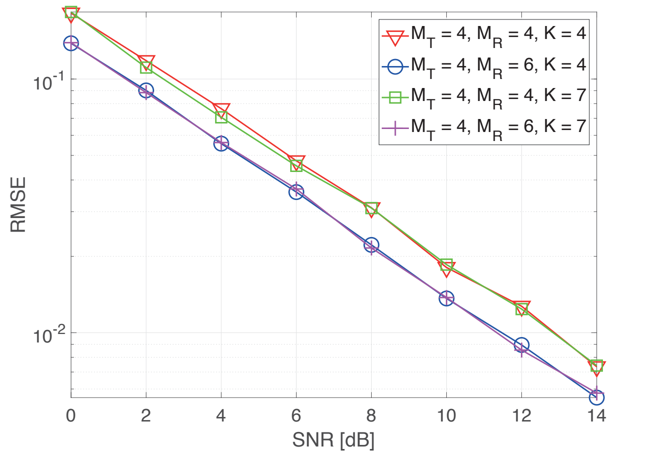


Figure 7: RMSE vs. SNR. Channel estimation error for the different number of receive antennas and packets. 1000 QPSK symbols, including 10 training symbols. The results are averaged over 1000 channel realizations.

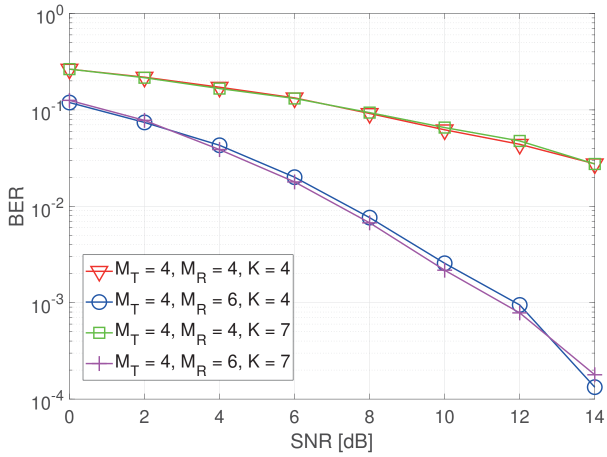


Figure 6: BER vs. SNR. PARAFAC2 based channel estimation for the different number of receive antennas and packets. 1000 QPSK symbols, including 10 training symbols. The results are averaged over 1000 channel realizations.

## VI. Conclusions

In this paper the complex valued PARAFAC2 tensor decomposition is applied for semi-blind data and channel estimation in MIMO communication systems. We have shown that the considered system satisfies the PARAFAC2 model and guaranties the uniqueness of the solution. The proposed algorithm enables a joint estimation of the both channel and symbol matrices and does not need channel information. The ambiguities are resolved by using the training sequences which are known by the transmitter and the receiver. Finally, some numerical simulations have been presented to evaluate the proposed method. Moreover, the PARAFAC2 based MIMO detection method has been compared to MMSE and ZF estimation methods. Also various parameter settings have been investigated. It is observed that the PARAFAC2 based semi-blind MIMO detection and channel estimation scheme provides an accurate channel and data estimation for different modulation schemes, assuming the transmission of a large number of samples per packet.

For future work, we will consider the orthogonal space-time block codes (OSTBC) coded MIMO systems [7] and will design a semi-blind data and channel estimation schemes. Also a power allocation scheme for transmitted packets will be investigated.

## References

[1] R. S. Kshetrimayum, *Fundamentals of MIMO Wireless Communications*. Cambridge University Press, 2017.

[2] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, 1st ed. New York, NY, USA: Cambridge University Press, 2008.

[3] M. Biguesh and A. B. Gershman, "Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals," *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 884–893, March 2006.

[4] F. Gao, A. Nallanathan, and C. Tellambura, "Blind channel estimation for cyclic-prefixed single-carrier systems by exploiting real symbol characteristics," *IEEE Transactions on Vehicular Technology*, vol. 56, no. 5, pp. 2487–2498, Sep. 2007.

[5] X. Cai and A. N. Akansu, "A subspace method for blind channel identification in OFDM systems," in *Proc. 2000 IEEE International Conference on Communications. ICC 2000. Global Convergence Through Communications. Conference Record*, vol. 2, June 2000, pp. 929–933.

[6] Y. Zeng, W. H. Lam, and T. S. Ng, "Semiblind channel estimation and equalization for MIMO space-time coded OFDM," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 53, no. 2, pp. 463–474, Feb. 2006.

[7] F. Roemer, N. Sarmadi, B. Song, M. Haardt, M. Pesavento, and A. B. Gershman, "Tensor-based semi-blind channel estimation for MIMO OSTBC-coded systems," in *Proc. 2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers*, Nov. 2011, pp. 449–453.

[8] H. A. L. Kiers, J. M. F. ten Berge, and R. Bro, "PARAFAC2 - part I. A direct fitting algorithm for the PARAFAC2 model," *Journal of Chemometrics*, 1999.

ISBN 978-3-8007-4939-3  

[9] B. M. Abadi, A. Sarrafzadeh, D. Jarchi, V. Abolghasemi, and S. Sanei, “Semi-blind signal separation and channel estimation in MIMO communication systems by tensor factorization,” in *Proc. 2009 IEEE/SP 15th Workshop on Statistical Signal Processing*, Aug. 2009, pp. 305–308.
[10] R. A. Harshman, “PARAFAC2: Mathematical and technical notes,” *UCLA Working Papers in Phonetics*, vol. 22, pp. 30 – 44, 1972.
[11] P. H. Schoenemann, “A generalized solution of the Orthogonal Procrustes Problem,” *Psychometrika*, vol. 31, no. 1, 1966.
[12] C. Wang, E. K. S. Au, R. D. Murch, W. H. Mow, R. S. Cheng, and V. Lau, “On the performance of the mimo zero-forcing receiver in the presence of channel estimation error,” *IEEE Transactions on Wireless Communications*, vol. 6, no. 3, pp. 805–810, March 2007.

ISBN 978-3-8007-4939-3