Non-linear precoding for the downlink of FBMC/OQAM based multi-user MIMO systems

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***Abstract*—In this contribution, we present new progress on the design of non-linear precoding techniques for filter bank based multi-carrier with offset quadrature amplitude modulation (FBMC/OQAM) based multi-user multiple-input-multiple-out (MIMO) downlink systems. The user terminals are allowed to have multiple antennas, and multi-stream transmissions for each user are enabled. First, the Tomlinson Harashima precoder (THP) based approach is generalized to such scenarios. Then, we discuss how to further adapt it to the case where the total number of receive antennas of the users exceeds the number of transmit antennas. An iterative THP scheme is devised such that the THP based non-linear precoding can be applied flexibly without suffering from the dimensionality constraint. The promising performance of the two non-linear precoding algorithms for FBMC/OQAM based multi-user MIMO downlink settings is demonstrated via numerical results.**

I. Introduction

Featuring a low out-of-band radiation and a high spectral efficiency, filter bank based multi-carrier with offset quadrature amplitude modulation (FBMC/OQAM) has found applications in a variety of communications scenarios as a candidate replacement of orthogonal frequency division multiplexing with the cyclic prefix insertion (CP-OFDM) [1], [2], [3], [4]. In contrast to CP-OFDM, the insertion of the CP is not required in FBMC/OQAM systems [5]. Moreover, with a well-contained spectrum, FBMC/OQAM is regarded as a suitable multi-carrier modulation scheme for systems where spectrum fragments need to be utilized effectively or robustness against loss of synchronization is preferred.

In the context of the multi-user multiple-input-single-output (MISO) and multiple-input-multiple-out (MIMO) downlink, there have been a number of linear precoding schemes that are tailored for FBMC/OQAM, such as the block diagonalization (BD) based precoder [6] and the coordinated beamforming based transmission strategies [7]. On the other hand, non-linear precoding has only been explored in [8] where a spatial Tomlinson Harashima precoder (STHP) based algorithm has been proposed for FBMC/OQAM based multi-user MISO downlink settings. The existence of the intrinsic interference in FBMC/OQAM systems is taken into account in the design of the STHP scheme where a linear precoding step is combined with the original THP technique [9]. However, it is assumed in [8] that each user terminal is equipped with a single antenna and thus receives only one spatial stream. In addition to this restriction on the STHP algorithm, the fact that non-linear precoding generally leads to a superior performance over linear precoding also motivates further investigations on non-linear precoding for FBMC/OQAM based multi-user downlink systems.

Therefore, we first propose an extension of the STHP scheme in this paper, and the resulting enhanced version can be employed when each user terminal is equipped with multiple antennas. Moreover, we devise an even more flexible THP based algorithm, which is also a combination of linear precoding and non-linear precoding. By employing an iterative procedure inspired by the coordinated beamforming designs in [10], [7], the proposed method is applicable in multi-user MIMO settings where the total number of receive antennas exceeds the number of transmit antennas at the base station. Extensive simulations have been performed to evaluate the performance of the proposed schemes. The numerical results that we have obtained show that these THP based algorithms significantly outperform state-of-the-art linear precoding techniques. In addition, the convergence behavior of the iterative THP algorithm is also investigated. Although compared to linear precoding, non-linear precoding is in general less immune to imperfect channel state information at the transmitter and has a higher computational complexity, robustified and efficient versions can be developed for FBMC/OQAM systems in the future similarly as for CP-OFDM systems or narrrowband systems [11], [12].

The remainder of the paper is organized as follows: Section II provides an overview of the data model for the downlink of FBMC/OQAM based multi-user MIMO systems where linear and/or non-linear precoding is applied. Then in Section III-A we describe our extension of the STHP scheme in detail, whereas Section III-B is dedicated to the iterative THP algorithm. Numerical results are presented in Section IV, before conclusions are drawn in Section V.

II. System model

In a multi-user MIMO downlink system where space division multiple access (SDMA) is employed, one base station equipped with $M_{\mathrm{T}}^{(\mathrm{BS})}$ transmit antennas transmits to Q users at the same time and on the same frequency. The number of receive antennas of the qth user is denoted by M_{R_q}, and the total number of receive antennas of all users served simultaneously is then $M_{\mathrm{R}}^{(\mathrm{tot})} = \sum_{q=1}^{Q} M_{\mathrm{R}_q}$. Assuming that the channel on each subcarrier can be treated as flat fading [6], [7], [8], the combined receive vector on the kth subcarrier at the nth time instant is denoted by $\boldsymbol{y}_k[n] =$

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	$n-3$	$n-2$	$n-1$	n	$n+1$	$n+2$	$n+3$
$k-1$	$0.043\jmath$	-0.125	$-0.206\jmath$	0.239	$0.206\jmath$	-0.125	$-0.043\jmath$
k	-0.067	0	0.564	1	0.564	0	-0.067
$k+1$	$-0.043\jmath$	-0.125	$0.206\jmath$	0.239	$-0.206\jmath$	-0.125	$0.043\jmath$

Table I. COEFFICIENTS $c_{i\ell}$ DETERMINED BY THE SYSTEM IMPULSE OF THE SYNTHESIS AND ANALYSIS FILTERS [5]

$\begin{bmatrix} \boldsymbol{y}_{1,k}^{\mathrm{T}}[n] & \boldsymbol{y}_{2,k}^{\mathrm{T}}[n] & \cdots & \boldsymbol{y}_{Q,k}^{\mathrm{T}}[n] \end{bmatrix}^{\mathrm{T}} \in \mathbb{C}^{M_{\mathrm{R}}^{(\mathrm{tot})}}$ where the received signals of all Q users are stacked. It has the following form

$$\boldsymbol{y}_k[n] = \boldsymbol{H}_k[n]\boldsymbol{f}\left(\boldsymbol{d}_k[n]\right) + \sum_{i=n-3}^{n+3}\sum_{\ell=k-1}^{k+1} \boldsymbol{H}_\ell[i]c_{i\ell}\boldsymbol{f}\left(\boldsymbol{d}_\ell[i]\right) + \boldsymbol{n}_k[n], \quad (\ell,i) \neq (k,n). \tag{1}$$

Here $\boldsymbol{H}_k[n] \in \mathbb{C}^{M_{\mathrm{R}}^{(\mathrm{tot})} \times M_{\mathrm{T}}^{(\mathrm{BS})}}$ denotes the combined channel matrix of all Q users[1] and is written as

$$\boldsymbol{H}_k[n] = \begin{bmatrix} \boldsymbol{H}_{1,k}^{\mathrm{T}}[n] & \boldsymbol{H}_{2,k}^{\mathrm{T}}[n] & \cdots & \boldsymbol{H}_{Q,k}^{\mathrm{T}}[n] \end{bmatrix}^{\mathrm{T}}, \tag{2}$$

where $\boldsymbol{H}_{q,k}[n] \in \mathbb{C}^{M_{\mathrm{R}_q} \times M_{\mathrm{T}}^{(\mathrm{BS})}}$ represents the channel frequency response between the base station and the qth user, $q = 1, 2, \ldots, Q$. The data vector $\boldsymbol{d}_k[n] \in \mathbb{R}^d$ with the total number of spatial streams $d = \sum_{q=1}^{Q} d_q$ is expressed as

$$\boldsymbol{d}_k[n] = \begin{bmatrix} \boldsymbol{d}_{1,k}^{\mathrm{T}}[n] & \boldsymbol{d}_{2,k}^{\mathrm{T}}[n] & \cdots & \boldsymbol{d}_{Q,k}^{\mathrm{T}}[n] \end{bmatrix}^{\mathrm{T}}, \tag{3}$$

where $\boldsymbol{d}_{q,k}[n] \in \mathbb{R}^{d_q}$ denotes the desired signal for the qth user on the kth subcarrier and at the nth time instant when $(k+n)$ is even[2], and d_q denotes the number of spatial streams sent to the qth user. The coefficients $c_{i\ell}$ (cf. Table I) represent the system impulse response determined by the synthesis and analysis filters. The PHYDYAS prototype filter [13] is used, and the overlapping factor is chosen to be $K = 4$. For more details about FBMC/OQAM systems, the reader is referred to [5]. Moreover, $\boldsymbol{n}_k[n]$ denotes the combined additive white Gaussian noise vector with variance σ_n^2.

The operation of precoding the signals on the kth subcarrier at the nth time instant is symbolized by $\boldsymbol{f}\left(\boldsymbol{d}_k[n]\right) \in \mathbb{C}^{M_{\mathrm{T}}^{(\mathrm{BS})}}$. It can correspond to linear precoding, non-linear precoding, as well as the combination of linear and non-linear precoding. For instance, in case of linear precoding, we have

$$\boldsymbol{f}\left(\boldsymbol{d}_k[n]\right) = \boldsymbol{F}_k[n]\boldsymbol{d}_k[n], \tag{4}$$

where $\boldsymbol{F}_k[n] \in \mathbb{C}^{M_{\mathrm{T}}^{(\mathrm{BS})} \times d}$ contains the precoding matrices for all users. In this work, our goal is to design $\boldsymbol{f}(\cdot)$ to accomplish the mitigation of the multi-user interference and the intrinsic interference.

[1]Here we only provide the formulas of the channel matrices, precoding matrices, and data vectors on the kth subcarrier and at the nth time instant explicitly due to limited space. In case of the ℓth subcarrier and the ith time instant, the corresponding expressions can be obtained by replacing k and n with ℓ and i, respectively.

[2]For the case where $(k+n)$ is odd, the desired signal on the kth subcarrier and at the nth time instant is pure imaginary, while the intrinsic interference is real [5]. As the two cases are essentially equivalent to each other, we only take the case where $(k+n)$ is even to describe the proposed algorithm in this paper.

III. TOMLINSON HARASHIMA PRECODING (THP) BASED NON-LINEAR PRECODING SCHEMES

A. Generalizing THP for FBMC/OQAM to per-user multi-antenna multi-stream transmissions

First, let us focus on the settings where $M_{\mathrm{T}}^{(\mathrm{BS})} \geq M_{\mathrm{R}}^{(\mathrm{tot})} = d$. We decompose the precoding operation into two steps, i.e.,

$$\boldsymbol{f}(\boldsymbol{d}_k[n]) = \boldsymbol{F}_k^{(\mathrm{L})}[n]\boldsymbol{f}^{(\mathrm{NL})}(\boldsymbol{d}_k[n]) \tag{5}$$

for the signals on the kth subcarrier at the nth time instant. Similarly as in [8], $\boldsymbol{F}_k^{(\mathrm{L})}[n]$, as a linear precoding matrix, is computed such that the intrinsic interference is eliminated after taking the real part of the received signal. Real-valued THP represented by $\boldsymbol{f}^{(\mathrm{NL})}(\boldsymbol{d}_k[n])$ serves to combat the multi-user interference. Then, the second term on the right-hand side of (1) corresponding to the intrinsic interference can be expressed as

$$\sum_{i=n-3}^{n+3}\sum_{\ell=k-1}^{k+1} \boldsymbol{H}_\ell[i]c_{i\ell}\boldsymbol{f}\left(\boldsymbol{d}_\ell[i]\right) = \sum_{i=n-3}^{n+3}\sum_{\ell=k-1}^{k+1} \boldsymbol{H}_\ell[i]\boldsymbol{F}_\ell^{(\mathrm{L})}[i]c_{i\ell}\boldsymbol{f}^{(\mathrm{NL})}\left(\boldsymbol{d}_\ell[i]\right), \quad (\ell,i) \neq (k,n). \tag{6}$$

Note that since $c_{i\ell}\boldsymbol{d}_\ell[i]$ ($\ell = k-1, k, k+1$, $i = n-3, \ldots, n+3$, and $(\ell,i) \neq (k,n)$) are pure imaginary [5] and real-valued THP is employed, the terms $c_{i\ell}\boldsymbol{f}^{(\mathrm{NL})}\left(\boldsymbol{d}_\ell[i]\right)$ in (6) are pure imaginary, i.e.,

$$\mathrm{Re}\left\{c_{i\ell}\boldsymbol{f}^{(\mathrm{NL})}\left(\boldsymbol{d}_\ell[i]\right)\right\} = \boldsymbol{0}, \tag{7}$$

where $\mathrm{Re}\{\cdot\}$ symbolizes the real part of the input argument, and $\mathrm{Im}\{\cdot\}$ is used later to represent the imaginary part. Consequently, the real part of the combined receive vector on the kth subcarrier at the nth time instant can be written as

$$\mathrm{Re}\left\{\boldsymbol{y}_k[n]\right\} = \mathrm{Re}\left\{\boldsymbol{H}_k[n]\boldsymbol{F}_k^{(\mathrm{L})}[n]\right\}\boldsymbol{f}^{(\mathrm{NL})}\left(\boldsymbol{d}_k[n]\right) + (-1)\cdot\sum_{i=n-3}^{n+3}\sum_{\ell=k-1}^{k+1} \mathrm{Im}\left\{\boldsymbol{H}_\ell[i]\boldsymbol{F}_\ell^{(\mathrm{L})}[i]\right\}\mathrm{Im}\left\{c_{i\ell}\boldsymbol{f}^{(\mathrm{NL})}\left(\boldsymbol{d}_\ell[i]\right)\right\} + \mathrm{Re}\left\{\boldsymbol{n}_k[n]\right\}, \quad (\ell,i) \neq (k,n). \tag{8}$$

To cancel the intrinsic interference, $\boldsymbol{F}_\ell^{(\mathrm{L})}[i]$ is calculated such that [8]

$$\mathrm{Im}\left\{\boldsymbol{H}_\ell[i]\boldsymbol{F}_\ell^{(\mathrm{L})}[i]\right\} = \boldsymbol{0}. \tag{9}$$

Therefore, for each subcarrier and each time instant, taking the (ℓ,i) pair as an example, $\begin{bmatrix} \mathrm{Re}\left\{\boldsymbol{F}_\ell^{(\mathrm{L})}[i]\right\}^{\mathrm{T}} & \mathrm{Im}\left\{\boldsymbol{F}_\ell^{(\mathrm{L})}[i]\right\}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ should lie in the nullspace of $\begin{bmatrix} \mathrm{Im}\left\{\boldsymbol{H}_\ell^{(\mathrm{L})}[i]\right\} & \mathrm{Re}\left\{\boldsymbol{H}_\ell^{(\mathrm{L})}[i]\right\} \end{bmatrix}$.

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Hence, the real part of the combined receive vector on the kth subcarrier at the nth time instant can be further expressed as

$$\mathrm{Re}\left\{\boldsymbol{y}_k[n]\right\} = \boldsymbol{H}_k^{(\mathrm{eq})}[n]\boldsymbol{f}^{(\mathrm{NL})}\left(\boldsymbol{d}_k[n]\right) + \mathrm{Re}\left\{\boldsymbol{n}_k[n]\right\} \quad (10)$$

where $\boldsymbol{H}_k^{(\mathrm{eq})}[n] = \mathrm{Re}\left\{\boldsymbol{H}_k[n]\boldsymbol{F}_k^{(\mathrm{L})}[n]\right\} \in \mathbb{R}^{M_{\mathrm{R}}^{(\mathrm{tot})} \times M_{\mathrm{T}}^{(\mathrm{eq})}}$ with $M_{\mathrm{T}}^{(\mathrm{eq})} = 2M_{\mathrm{T}}^{(\mathrm{BS})} - M_{\mathrm{R}}^{(\mathrm{tot})} \geq M_{\mathrm{R}}^{(\mathrm{tot})}$.

Afterwards, the real-valued THP [14] is applied on the equivalent real-valued channel matrix $\boldsymbol{H}_k^{(\mathrm{eq})}[n]$. Decompose $\boldsymbol{H}_k^{(\mathrm{eq})}[n]$ such that

$$\boldsymbol{H}_k^{(\mathrm{eq})}[n] = \left[\begin{array}{cc} \boldsymbol{L}_k[n] & \boldsymbol{0}_{d \times (M_{\mathrm{T}}^{(\mathrm{eq})} - d)} \end{array}\right] \boldsymbol{Q}_k'^{\mathrm{T}}[n], \quad (11)$$

where $\boldsymbol{L}_k[n] \in \mathbb{R}^{d \times d}$ denotes a lower-triangular matrix and $\boldsymbol{Q}'_k[n] \in \mathbb{R}^{M_{\mathrm{T}}^{(\mathrm{eq})} \times M_{\mathrm{T}}^{(\mathrm{eq})}}$ a unitary matrix. Let us also define $\boldsymbol{Q}_k[n] \in \mathbb{R}^{M_{\mathrm{T}}^{(\mathrm{eq})} \times d}$ that contains the first d columns of $\boldsymbol{Q}'_k[n]$. A feedback matrix $\boldsymbol{B}_k[n]$ [14], [8] is computed as

$$\boldsymbol{B}_k[n] = \boldsymbol{\Sigma}_k^{-1}[n]\boldsymbol{L}_k[n] - \boldsymbol{I}_d, \quad (12)$$

where $\boldsymbol{\Sigma}_k[n]$ is a diagonal matrix with the diagonal elements of $\boldsymbol{L}_k[n]$ on its diagonal and can be expressed as

$$\boldsymbol{\Sigma}_k[n] = \begin{bmatrix} \boldsymbol{\Sigma}_{1,k}[n] & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_{2,k}[n] & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Sigma}_{Q,k}[n] \end{bmatrix}. \quad (13)$$

Here $\boldsymbol{\Sigma}_{q,k}[n] \in \mathbb{R}^{d_q \times d_q}$ is a diagonal matrix with respect to the qth user ($q = 1, 2, \ldots, Q$). Then, $\boldsymbol{f}^{(\mathrm{NL})}\left(\boldsymbol{d}_k[n]\right)$ can be expressed as follows to eliminate the multi-user interference and the inter-stream interference for each user

$$\boldsymbol{f}^{(\mathrm{NL})}\left(\boldsymbol{d}_k[n]\right) = \boldsymbol{Q}_k[n]\boldsymbol{s}_k[n], \quad (14)$$

where the rth element of $\boldsymbol{s}_k[n] \in \mathbb{R}^d$, $s_k^r[n]$, takes the following form

$$s_k^r[n] = \mathrm{MOD}\left\{ d_k^r[n] - \sum_{p=1}^{r-1} s_k^p[n] b_k^{(r,p)}[n] \right\}. \quad (15)$$

Here $d_k^r[n]$ denotes the rth element of the data vector $\boldsymbol{d}_k[n]$ for all users on the kth subcarrier at the nth time instant, $b_k^{(r,p)}[n]$ represents the (r,p)th element of the feedback matrix $\boldsymbol{B}_k[n]$, and $\mathrm{MOD}\{\cdot\}$ is a modulo operator defined to limit the constellation size [14].

B. A flexible variant of THP for FBMC/OQAM to alleviate the dimensionality constraint

In this subsection, we turn to the scenarios where $M_{\mathrm{T}}^{(\mathrm{BS})} < M_{\mathrm{R}}^{(\mathrm{tot})}$ and propose an extension of the scheme described above that cannot be employed in such settings. Let us define a real-valued block diagonal matrix $\boldsymbol{D}_k[n] \in \mathbb{R}^{M_{\mathrm{R}}^{(\mathrm{tot})} \times d}$ as the combined decoding matrix for all users on the kth subcarrier at the nth time instant [10], [7]

$$\boldsymbol{D}_k[n] = \begin{bmatrix} \boldsymbol{D}_{1,k}[n] & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D}_{2,k}[n] & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_{Q,k}[n] \end{bmatrix}, \quad (16)$$

where $\boldsymbol{D}_{q,k}[n] \in \mathbb{R}^{M_{\mathrm{R}_q} \times d_q}$ denotes the decoding matrix of the qth user on the kth subcarrier at the nth time instant. Instead of performing the precoding on the combined propagation channel matrix, the following equivalent channel matrix is used

$$\widetilde{\boldsymbol{H}}_k[n] = \boldsymbol{D}_k^{\mathrm{T}}[n]\boldsymbol{H}_k[n] \in \mathbb{C}^{d \times M_{\mathrm{T}}^{(\mathrm{BS})}}. \quad (17)$$

The resulting setting resembles that in Section III-A, and the THP based scheme introduced above is applicable. After performing the precoding, the decoding matrix of each user is further computed based on a certain criterion, e.g., MMSE, and thus the combined decoding matrix $\boldsymbol{D}_k[n]$ is updated. To determine the termination of such an iterative procedure, a metric that measures the residual multi-user interference and the inter-stream interference can be defined and is compared to a certain threshold. Once it falls below the threshold, the convergence is achieved.

In Figure 1, we illustrate this variant of THP and the corresponding receive processing of the qth user ($q = 1, 2, \ldots, Q$) via two block diagrams.

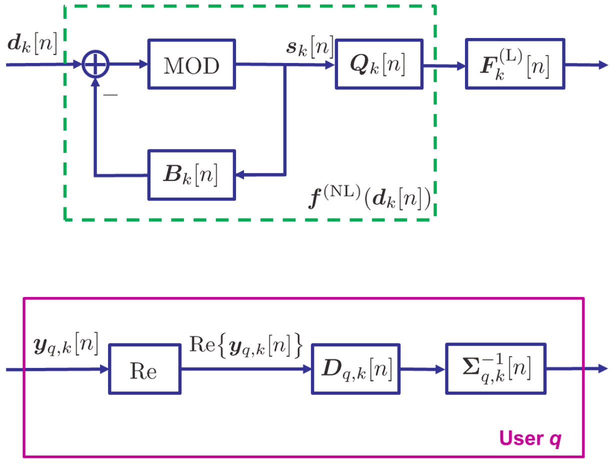

Figure 1. Block diagram of the proposed THP for FBMC/OQAM (top) and the receive processing at the qth user terminal ($q = 1, 2, \ldots, Q$) (bottom)

This coordinated THP scheme is summarized as follows:

- **Step 1**
  The real-valued block diagonal decoding matrix is initialized as $\boldsymbol{D}_k^{(0)}[n] \in \mathbb{R}^{M_{\mathrm{R}}^{(\mathrm{tot})} \times d}$. For the first subcarrier, a random initialization of $\boldsymbol{D}_{q,k}^{(0)}[n]$ ($q = 1, 2, \ldots, Q$) is employed. Otherwise, the decoding matrix that has already been computed for the previous subcarrier is used [7], [15]. In addition, the iteration index denoted by p is set to zero, and a threshold ϵ is set for the stopping criterion.

- **Step 2**
  Set $p \rightarrow p + 1$ and compute the equivalent channel matrix $\widetilde{\boldsymbol{H}}_k^{(p)}[n]$ for the pth iteration such that

  $$\widetilde{\boldsymbol{H}}_k^{(p)}[n] = \boldsymbol{D}_k^{(p-1)^{\mathrm{T}}}[n]\boldsymbol{H}_k[n] \in \mathbb{C}^{d \times M_{\mathrm{T}}^{(\mathrm{BS})}}. \quad (18)$$

- **Step 3**

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Update $\boldsymbol{F}_k^{(\mathrm{L})^{(p)}}[n]$ for the pth iteration. To this end, let us define the following matrix

$$\check{\boldsymbol{H}}_k^{(p)}[n] = \left[\ \mathrm{Im}\left\{\widetilde{\boldsymbol{H}}_k^{(p)}[n]\right\} \quad \mathrm{Re}\left\{\widetilde{\boldsymbol{H}}_k^{(p)}[n]\right\} \ \right]. \tag{19}$$

After performing the singular value decomposition (SVD) of $\check{\boldsymbol{H}}_k^{(p)}[n] \in \mathbb{R}^{d\times 2M_\mathrm{T}^{(\mathrm{BS})}}$, we obtain $\boldsymbol{V}_0^{(p)} \in \mathbb{R}^{2M_\mathrm{T}^{(\mathrm{BS})}\times M_\mathrm{T}^{(\mathrm{eq})}}$ that contains the last $M_\mathrm{T}^{(\mathrm{eq})} = 2M_\mathrm{T}^{(\mathrm{BS})} - r^{(p)}$ right singular vectors forming an orthonormal basis for the null space of $\check{\boldsymbol{H}}_k^{(p)}[n]$. Here $r^{(p)}$ denotes the rank of $\check{\boldsymbol{H}}_k^{(p)}[n]$. Then, $\boldsymbol{F}_k^{(\mathrm{L})^{(p)}}[n]$ is computed as

$$\boldsymbol{V}_0^{(p)} = \begin{bmatrix} \mathrm{Re}\left\{\boldsymbol{F}_k^{(\mathrm{L})^{(p)}}[n]\right\} \\ \mathrm{Im}\left\{\boldsymbol{F}_k^{(\mathrm{L})^{(p)}}[n]\right\} \end{bmatrix} \tag{20}$$

such that

$$\mathrm{Im}\left\{\widetilde{\boldsymbol{H}}_k^{(p)}[n]\boldsymbol{F}_k^{(\mathrm{L})^{(p)}}[n]\right\} = \mathbf{0}. \tag{21}$$

- **Step 4**
  Perform THP on the resulting real-valued equivalent channel $\widetilde{\boldsymbol{H}}_k^{(p)}[n]\boldsymbol{F}_k^{(\mathrm{L})^{(p)}}[n]$ and obtain $\boldsymbol{L}_k^{(p)}[n]$, $\boldsymbol{Q}_k^{(p)}[n]$, $\boldsymbol{\Sigma}_k^{(p)}[n]$, and $\boldsymbol{B}_k^{(p)}[n]$ as described in Section III-A (cf. equations (11) and (12)).
- **Step 5**
  Update the decoding matrix $\boldsymbol{D}_{q,k}^{(p)}[n]$ for each user. Let us first define an equivalent precoding matrix that corresponds the real-valued THP in **Step 4**

$$\boldsymbol{F}_k^{(\mathrm{eq})^{(p)}}[n] = \boldsymbol{Q}_k^{(p)}[n]\left(\boldsymbol{B}_k^{(p)}[n] + \boldsymbol{I}_d\right)^{-1}. \tag{22}$$

Then, we express $\boldsymbol{F}_k^{(\mathrm{eq})^{(p)}}[n] \in \mathbb{R}^{M_\mathrm{T}^{(\mathrm{eq})}\times d}$ as

$$\boldsymbol{F}_k^{(\mathrm{eq})^{(p)}}[n] = \tag{23}$$
$$\left[\ \boldsymbol{F}_{1,k}^{(\mathrm{eq})^{(p)}}[n] \quad \boldsymbol{F}_{2,k}^{(\mathrm{eq})^{(p)}}[n] \quad \cdots \quad \boldsymbol{F}_{Q,k}^{(\mathrm{eq})^{(p)}}[n] \ \right], \tag{24}$$

with $\boldsymbol{F}_{q,k}^{(\mathrm{eq})^{(p)}}[n] \in \mathbb{R}^{M_\mathrm{T}^{(\mathrm{eq})}\times d_q}$ $(q = 1, 2, \ldots, Q)$. Consequently, for the qth user, the following M_{R_q}-by-d_q equivalent channel matrix is defined

$$\widetilde{\boldsymbol{H}}_{q,k}^{(\mathrm{eq})^{(p)}}[n] = \mathrm{Re}\left\{\boldsymbol{H}_{q,k}[n]\boldsymbol{F}_k^{(\mathrm{L})^{(p)}}[n]\right\}\boldsymbol{F}_{q,k}^{(\mathrm{eq})^{(p)}}[n]. \tag{25}$$

Then $\boldsymbol{D}_{q,k}^{(p)}[n]$ is calculated based on the MMSE criterion, where $q = 1, 2, \ldots, Q$.

- **Step 6**
  Compute the term $\xi^{(p)}$ as

$$\xi^{(p)} = \left\|\mathrm{off}\left\{\boldsymbol{D}_k^{(p)^\mathrm{T}}[n]\boldsymbol{H}_k[n]\boldsymbol{F}_k^{(\mathrm{L})^{(p)}}[n]\boldsymbol{F}_k^{(\mathrm{eq})^{(p)}}[n]\right\}\right\|_\mathrm{F}, \tag{26}$$

where $\mathrm{off}\{\cdot\}$ symbolizes the operation of setting the diagonal entries of the input matrix to zero while keeping the off-diagonal entries unchanged. It provides a measure of the residual multi-user interference and inter-stream interference. Once $\xi^{(p)}$ falls below the threshold ϵ set in the initialization stage, the convergence is achieved. Otherwise, we continue with **Step 2** starting another iteration.

It is worth noting that in addition to multi-user MIMO downlink settings where $M_\mathrm{T}^{(\mathrm{BS})} < M_\mathrm{R}^{(\mathrm{tot})}$, the iterative THP algorithm described above can also be treated as a solution for scenarios where $M_\mathrm{R}^{(\mathrm{tot})} > d$ and the original THP approach cannot be directly employed.

IV. Simulation Results

In this section, we evaluate the bit error rate (BER) performance of the proposed THP based algorithms. For all examples, the number of subcarriers is 512, and the total bandwidth is 5 MHz. In case of CP-OFDM, the length of the CP is set to $1/8$ of the symbol period. The ITU Ped-A channel [16] is adopted. Moreover, the PHYDYAS prototype filter [13] with the overlapping factor $K = 4$ is employed. The data symbols are drawn from a 16 QAM constellation. Perfect channel state information is assumed at the transmitter and the receiver.

First, we show the BER performance of the enhanced version of the STHP scheme. Here the base station is equipped with eight antennas, whereas each of the three users has two antennas. Two data streams are sent to each user. The corresponding results are presented in Figure 2. For the purpose

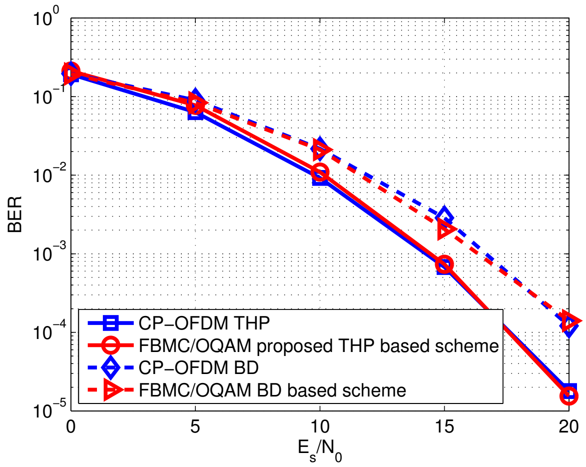

Figure 2. Comparison of the BER performances of different schemes in a multi-user MIMO downlink system where $M_\mathrm{T}^{(\mathrm{BS})} = 8$, $M_\mathrm{R}^{(\mathrm{tot})} = 6$, $d = 6$, and the ITU Ped-A channel is considered

of comparison, the performance of the BD based algorithm [6] is also illustrated. It can be observed that the proposed algorithm achieves a better performance compared to the BD based method in such a scenario where each user is equipped with multiple antennas and receives multiple data streams.

In the sequel, the performance of the iterative THP scheme is assessed. For the second experiment, we consider a symmetric scenario where the number of transmit antennas is equal to the total number of receive antennas, i.e., $M_\mathrm{T}^{(\mathrm{BS})} = M_\mathrm{R}^{(\mathrm{tot})} = 8$, and the total number of data streams is six. The iterative

THP algorithm is employed, and its performance is depicted in Figure 3. We can see that the iterative THP significantly outperforms its linear counterpart IIM-CBF 1 [7] also as an iterative precoding method.

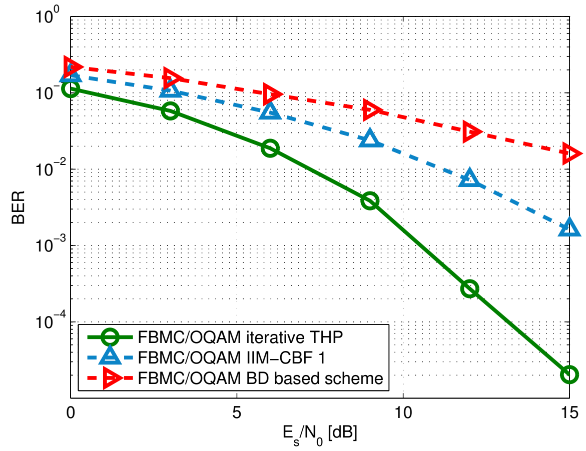

Figure 3. Comparison of the BER performances of different schemes in a multi-user MIMO downlink system where $M_{\mathrm{T}}^{(\mathrm{BS})} = M_{\mathrm{R}}^{(\mathrm{tot})} = 8$, $d = 6$, and the ITU Ped-A channel is considered

We continue with a third example. In this case, the total number of receive antennas exceeds the number of transmit antennas. A comparison of the iterative THP algorithm and the linear coordinated scheme, IIM-CBF 2 [7], has been performed. In Figure 4, we observe that the performance of iterative THP is superior to that of IIM-CBF 2 and is similar to that of the CP-OFDM based multi-user MIMO downlink with the coordinated non-linear precoding approach called dTHP [10]. Note that in the comparison of the linear precoding algorithms IIM-CBF 2 [7] for FBMC/OQAM and LoCCoBF [15] for CP-OFDM, the former achieves a slight gain over the latter. This is due to the fact that a CP is not required in FBMC/OQAM systems. By contrast, such a gain is not observed in case of non-linear precoding. It has been pointed out in [7] that since the propagation channel exhibits frequency selectivity, i.e., $\boldsymbol{H}_k[n] \neq \boldsymbol{H}_{k-1}[n]$ and $\boldsymbol{H}_k[n] \neq \boldsymbol{H}_{k+1}[n]$, schemes based on the concept of coordinated beamforming for FBMC/OQAM systems suffer from residual intrinsic interference. The results presented in Figure 4 imply that the existence of the residual intrinsic interference has a slightly bigger impact on non-linear precoding than on linear precoding for FBMC/OQAM.

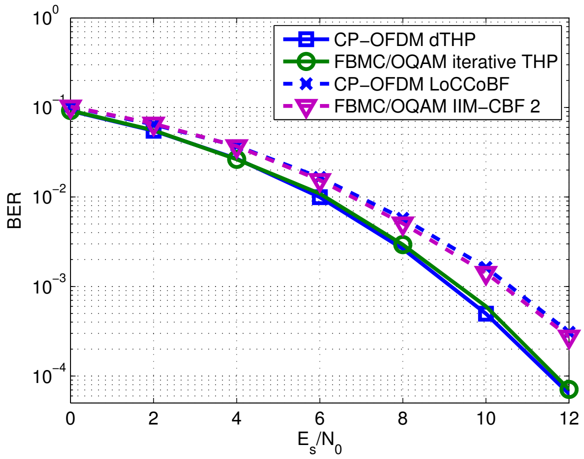

Figure 4. Comparison of the BER performances of different schemes in a multi-user MIMO downlink system where $M_{\mathrm{T}}^{(\mathrm{BS})} = 8$, $M_{\mathrm{R}}^{(\mathrm{tot})} = 10$, $d = 6$, and the ITU Ped-A channel is considered

Finally, we present our investigation on the convergence behavior of the iterative THP technique. The complimentary cumulative distribution function (CCDF) of the number of iterations is presented in Figure 5. The two simulation scenarios for Figure 3 and Figure 4 are considered here. The threshold ϵ for the stopping criterion is set to 10^{-5}, whereas the maximum number of iterations is 50, i.e., the iterative procedure terminates when the number of iterations p reaches 50 even if the term $\xi^{(p)}$ fails to fall below ϵ. We observe in Figure 5 that as the total number of receive antennas increases, a larger number of iterations is required for the convergence of the iterative THP scheme. Compared to the iterative non-linear precoding algorithm dTHP for CP-OFDM based systems, the proposed technique appears to need a bit more iterations. It is very likely due to the fact that the mitigation of the intrinsic interference in FBMC/OQAM systems is taken into account in the iterative THP scheme, and it has an impact on the convergence of the algorithm.

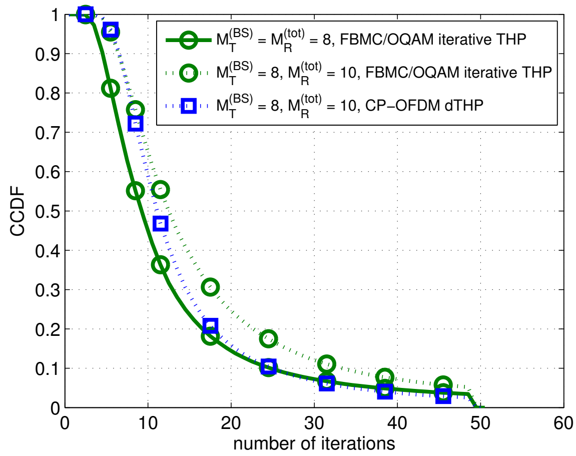

Figure 5. CCDF of the number of iterations for different schemes in two multi-user MIMO downlink settings where $M_{\mathrm{T}}^{(\mathrm{BS})} = 8$, $M_{\mathrm{R}}^{(\mathrm{tot})} = 8$ or 10, $d = 6$

V. Conclusion

We have proposed two THP based non-linear precoding algorithms for the downlink of FBMC/OQAM based multi-user MIMO systems. The first one results from the generalization of the existing STHP approach and thus alleviates the constraint that each user terminal can only have or switch on a single receive antenna. The second scheme belongs to the category of coordinated beamforming techniques and employs an iterative procedure to jointly update the precoding and the decoding. This iterative THP method is applicable to overloaded FBMC/OQAM based multi-user MIMO downlink settings where the total number of receive antennas at the user terminals exceeds the number of transmit antennas at the base station. The simulation results have shown that the proposed non-linear precoding schemes are able to achieve a superior performance compared to a few comparable linear precoding algorithms for FBMC/OQAM. In addition, it has been observed that by employing the proposed schemes, the performance of FBMC/OQAM based multi-user MIMO downlink systems is competitive with that of their CP-OFDM based counterparts.

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