# SEMI-BLIND RECEIVER FOR TWO-HOP MIMO RELAYING SYSTEMS VIA SELECTIVE KRONECKER PRODUCT MODELING

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## ABSTRACT

In this paper we propose a new iterative semi-blind receiver for two-hop MIMO relaying systems using a tensor-based approach. We consider that both the source and the relay use a Khatri-Rao space time code for data transmission. We introduce a Selective Kronecker Product (SKP) modeling that allows to recast the received signal at the destination as a Tucker 3 model with a sparse core. The proposed SKP modeling eliminates the need of the source-destination (SD) link to initialize the receiver, as opposed to the receiver proposed in [1]. Our simulations show that our SKP-ALS receiver achieves a better performance than its state-of-the-art in the literature, that uses an additional SD link, with a lower computational complexity.

***Index Terms***— MIMO relaying systems, space-time coding, semi-blind receiver, selective Kronecker product.

## 1. INTRODUCTION

The use of cooperative communications, e.g., via a relay station, has shown to be a good way to mitigate the wireless channel impairments, such as strong fading and shadowing [2]. Even in single-input multiple-output (SIMO) systems, e.g., mobile station to a base station, the use of a relay station virtually increases the number of antennas, increasing the system diversity [3]. Overall, cooperative communications increases the wireless system diversity, especially when using relays equipped with multiple antennas, while increasing the system coverage and capacity [4–6].

Tensor algebra has been widely used in wireless signal processing, [7–12]. The main reason is that the wireless signal is a multidimensional signal, i.e., we may have multiple antennas (space dimension), different time-slots (time dimension), subcarriers (frequency dimension), polarization, etc. Tensor algebra facilitate the manipulation of these dimensions, also, in general, tensor decompositions have more relaxed uniqueness conditions (e.g., PARAFAC decomposition [13]) than matrices. This means that the estimated system parameters are unique under some permutation and scaling ambiguities. Works related to cooperative two-hop multiple-input multiple-output (MIMO) tensor-based systems can be found in [1, 14–16].

The authors in [14] proposes two semi-blind receivers for two-hop MIMO using a Nested PARAFAC decomposition. The work in [15] propose a new tensor decomposition, called Nested Tucker. It is a generalization of the work in [14], by having full space-time coding tensors. The work in [1] proposes semi-blind receivers for two-hop MIMO systems using Khatri-Rao space time (KRST) coding at the relay and the source. They show that their proposed system model fits a PARATuck 2 model and they derived three semi-blind receivers: PARATuck 2 ALS, Sequential PARAFAC-PARATuck 2 (SPP) and Combined PARAFAC-PARATuck 2 (CPP). These receivers rely on the fact that the source-destination link is available, otherwise, their performance will be degraded. In this paper, we start from a similar model than the one in [1]. However, we do not consider that the source-destination link is available at the receiver since the distance between the source and the destination is so large that a relay is needed. To overcome this issue, we propose a Selective Kronecker Product (SKP) approach in the receiver, where the PARATuck 2 tensor model is reformulated into a Tucker 3 tensor model with a sparse core tensor. We show that using this approach, our proposed SKP-ALS (Selective Kronecker Product - Alternating Least Squares) receiver, obtains a good performance even without the assistance of a source-destination link. In fact, we show that our SKP-ALS receiver has a better performance than the one proposed in [1] with respect to the channel estimation accuracy and the computational complexity.

## 2. NOTATION AND TENSOR PRELIMINARIES OPERATIONS

### 2.1. Notation and Properties

Scalars are denoted by lower-case letters $(a, b, ...)$, vectors by bold lower-case letters $(\mathbf{a}, \mathbf{b}, ..)$, matrices by bold upper-case letters $(\mathbf{A}, \mathbf{B}, ..)$, tensors are defined by calligraphic upper-case letters $(\mathcal{A}, \mathcal{B}, ...)$. $\mathbf{A}^{\mathrm{T}}, \mathbf{A}^{+}, \mathbf{A}^{*}, \mathbf{A}^{\mathrm{H}}$ stand for transpose, Moore-Penrose pseudo-inverse, conjugate and Hermitian of $\mathbf{A}$, respectively. The operators $\otimes$, $\diamond$ and $\circ$ define the Kronecker, Khatri-Rao and the outer product, respectively.

For a matrix $\mathbf{A} \in \mathbb{C}^{I \times R}$, the vec$(\cdot)$ operator vectorizes a matrix by stacking its columns, i.e., vec$(\mathbf{A}) = \mathbf{a} \in \mathbb{C}^{IR \times 1}$, while unvec$(\cdot)$ does the inverse operation, i.e., unvec$(\mathbf{a}) = \mathbf{A} \in \mathbb{C}^{I \times R}$. The frontal slices of a third-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ are matrices denoted by $\mathcal{X}_{..i_3} \in \mathbb{C}^{I_1 \times I_2}$, with $i_3 = \{1, \cdots, I_3\}$. For an $N$th order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times \cdots \times I_N}$ its $n$-mode unfolding is the matrix defined as $[\mathcal{X}]_{(n)} \in \mathbb{C}^{I_n \times I_1, \ldots, I_{n-1} I_{n+1}, \ldots I_N}$. In the case of a fourth-order tensor $\mathcal{G}^{I \times J \times K \times L}$, a generalized unfolding $[\mathcal{G}]_{[(1,3),(2,4)]} \in \mathbb{C}^{IK \times JL}$ is formed by grouping the first and third dimensions ($I$ and $K$) along the rows while grouping the second and fourth dimensions ($J$ and $L$) along the columns, see [17]. The $n$-mode product between a tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times \cdots \times I_n \times \cdots \times I_N}$ and a matrix $\mathbf{A} \in \mathbb{C}^{J_n \times I_n}$ is defined as $\mathcal{Y} = \mathcal{X} \times_n \mathbf{A}$, where $\mathcal{Y} \in \mathbb{C}^{I_1 \times \cdots \times J_n \times \cdots \times I_N}$, so that $[\mathcal{Y}]_{(n)} = \mathbf{A}\,[\mathcal{X}]_{(n)} \in$



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$\mathbb{C}^{J_n \times I_1 \cdots I_{n-1} I_{n+1} \cdots I_N}$. We make use of the following properties

$$\text{vec}(\mathbf{ABC}) = (\mathbf{C}^{\text{T}} \otimes \mathbf{A})\text{vec}(\mathbf{B}) \tag{1}$$

$$\text{vec}(\mathbf{A}\text{Diag}_n(\mathbf{B})\mathbf{C}) = (\mathbf{C}^{\text{T}} \diamond \mathbf{A})\mathbf{b}_n^{\text{T}}, \tag{2}$$

where $\text{Diag}_n(\mathbf{B})$ is a diagonal matrix formed by the $n$th row of $\mathbf{B}$, and $\mathbf{b}_n^{\text{T}}$ is the transposition of the $n$th row vector of $\mathbf{B}$. The Kronecker product between tensors is already very known in literature, in reference [18] the reader can find some useful properties. In fact, the Kronecker product can be considered as the generalized unfolding of the outer product between two or more tensors. For example, consider two third-order tensors $\mathcal{A} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ and $\mathcal{B} \in \mathbb{C}^{J_1 \times J_2 \times J_3}$, the outer product between them results in a sixth-order tensor $\mathcal{C} = \mathcal{A} \circ \mathcal{B} \in \mathbb{C}^{I_1 \times I_2 \times I_3 \times J_1 \times J_2 \times J_3}$.

The Kronecker product between the tensors $\mathcal{A}$ and $\mathcal{B}$ can be viewed as the following generalized unfolding of the tensor $\mathcal{C}$

$$[\mathcal{C}]_{([4,1],[5,2],[6,3])} = \mathcal{A} \otimes \mathcal{B} \in \mathbb{C}^{J_1 I_1 \times J_2 I_2 \times J_3 I_3}. \tag{3}$$

For notation simplicity, let us define $\overline{\mathcal{C}}$ as the third order tensor in Eq. (3), we can view its the frontal slices as

$$\overline{\mathcal{C}}_{..p} = \mathcal{A}_{..i_3} \otimes \mathcal{B}_{..j_3} \in \mathbb{C}^{J_1 I_1 \times J_2 I_2}, \tag{4}$$

where $p = j_3 + (i_3 - 1)J_3$.

### 2.2. Selective Kronecker Product

In the standard Kronecker product operation all the dimensions are combined, i.e., the resulting tensor $\overline{\mathcal{C}}$ has dimensions $\dim\left(\overline{\mathcal{C}}\right) = \max\left[\dim(\mathcal{A}), \dim(\mathcal{B})\right]$, which in this example are three. Let us define a tensor $\overline{\mathcal{D}}$ that combines the first and second mode of $\mathcal{A}$ with the first and second mode of $\mathcal{B}$, respectively. We can write $\overline{\mathcal{D}}$ as

$$\overline{\mathcal{D}} = \mathcal{A} \otimes_{1,2}^{1,2} \mathcal{B} \in \mathbb{C}^{J_1 I_1 \times J_2 I_2 \times J_3 \times I_3}. \tag{5}$$

The lower indices in $\otimes_{1,2}^{1,2}$ indicates which dimensions will be combined in the tensor $\mathcal{A}$ while the upper indices refers to the dimensions of $\mathcal{B}$.

Note that, in the same way as the tensor $\overline{\mathcal{C}}$, the tensor $\overline{\mathcal{D}}$ is a generalized unfolding of tensor $\mathcal{C}$

$$\overline{\mathcal{C}} = [\mathcal{C}]_{([4,1],[5,2],[6,3])}, \quad \overline{\mathcal{D}} = [\mathcal{C}]_{([4,1],[5,2],[6],[3])}. \tag{6}$$

We can write its matrix slices as

$$\overline{\mathcal{D}}_{..j_3 i_3} = \mathcal{A}_{..i_3} \otimes \mathcal{B}_{..j_3} \in \mathbb{C}^{J_1 I_1 \times J_2 I_2} \tag{7}$$

Note that the right-hand side in Eq.(7) is the same as Eq.(4) and Equation (5) represents the SKP which will be useful to reformulate the PARATuck 2 model into a Tucker 3 model.

## 3. SYSTEM MODEL

Consider a two-hop MIMO system, illustrated in Figure 1, where the source does not have a link to the destination and needs the assistance of a relay station in order to forward its data to the destination. For simplicity, perfect timing is assumed in this system. The source is equipped with $M_T$ transmit antennas, the destination has $M_D$ receive antennas, and we assume a half-duplex relay equipped with $M_R$ antennas using an amplify-and-forward (AF) protocol, which means that the transmission is divided into two phases.

**Phase 1**: In Phase 1, the source transmits its data to the relay station. The symbol matrix $\mathbf{S} \in \mathbb{C}^{N \times M_T}$ contains the $N$ symbols

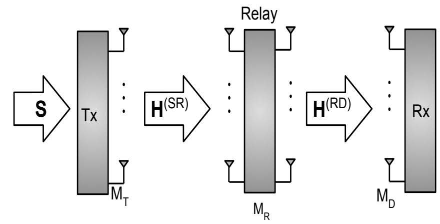


**Fig. 1**: MIMO relaying system.

that are allocated to the $M_T$ transmit antennas. These symbols are encoded using the matrix $\mathbf{V} \in \mathbb{C}^{K \times M_T}$ via a Khatri-Rao Spacte-Time (KRST) code, where $K$ is the time spread, and then, sent to the relay via the source-relay channel $\mathbf{H}^{(\text{SR})} \in \mathbb{C}^{M_R \times M_T}$. The signal received at the relay in the $k$th time slot is given by

$$\mathcal{X}_{..k}^{(\text{SR})} = \mathbf{H}^{(\text{SR})}\text{Diag}_k(\mathbf{V})\,\mathbf{S}^{\text{T}} + \mathcal{N}_{..k}^{(\text{SR})} \in \mathbb{C}^{M_R \times N}, \tag{8}$$

where $\text{Diag}_k(\mathbf{V})$ is a diagonal matrix of size $M_T \times M_T$ formed from the $k$th row of $\mathbf{V}$ and $\mathcal{N}^{(\text{SR})} \in \mathbb{C}^{M_R \times N \times K}$ is the additive white Gaussian noise (AWGN) at the relay station.

**Phase 2**: In Phase 2, the source stays silent while the relay transmits the received signal in Phase 1 with a KRST code using the matrix $\mathbf{U} \in \mathbb{C}^{K \times M_R}$. Then, the relay transmits the encoded signal to the destination via the relay-destination channel $\mathbf{H}^{(\text{RD})} \in \mathbb{C}^{M_D \times M_R}$. The received signal at the destination at the $k$th time slot can be expressed as

$$\mathcal{X}_{..k}^{(\text{SRD})} = \mathbf{H}^{(\text{RD})}\text{Diag}_k(\mathbf{U})\mathcal{X}_{..k}^{(\text{SR})} + \mathcal{N}_{..k}^{(\text{SRD})} \in \mathbb{C}^{M_D \times N} \tag{9}$$

where $\text{Diag}_k(\mathbf{U})$ is a diagonal matrix of size $M_T \times M_T$ formed from the $k$th row of $\mathbf{U}$ and $\mathcal{N}^{(\text{SRD})} \in \mathbb{C}^{M_D \times N \times K}$ is the AWGN at the destination. Note that, neglecting the noise term, the received signal $\mathcal{X}^{(\text{SRD})}$ fits a PARATuck 2 model and is given by

$$\mathcal{X}_{..k}^{(\text{SRD})} = \mathbf{H}^{(\text{RD})}\text{Diag}_k(\mathbf{U})\mathbf{H}^{(\text{SR})}\text{Diag}_k(\mathbf{V})\,\mathbf{S}^{\text{T}} \tag{10}$$

Applying the vec$(\cdot)$ operation to Eq. (10) and defining $\mathbf{x}_k^{(\text{SRD})} = \text{vec}\left(\mathcal{X}_{..k}^{(\text{SRD})}\right) \in \mathbb{C}^{M_D N \times 1}$, we have

$$\begin{aligned}
\mathbf{x}_k^{(\text{SRD})} &= \left(\mathbf{S} \otimes \mathbf{H}^{(\text{RD})}\right)\text{vec}\left(\text{Diag}_k(\mathbf{U})\mathbf{H}^{(\text{SR})}\text{Diag}_k(\mathbf{V})\right) \\
&= \left(\mathbf{S} \otimes \mathbf{H}^{(\text{RD})}\right)\text{Diag}(\mathbf{h}^{(\text{SR})})\left(\text{Diag}_k(\mathbf{V}) \otimes \text{Diag}_k(\mathbf{U})\right),
\end{aligned} \tag{11}$$

where $\mathbf{h}^{(\text{SR})} = \text{vec}(\mathbf{H}^{(\text{SR})}) \in \mathbb{C}^{M_R M_T}$ and $\text{Diag}(\mathbf{h}^{(\text{SR})})$ is a diagonal matrix of size $M_R M_T \times M_R M_T$ formed with the elements of $\mathbf{h}^{(\text{SR})}$. From Eq. (11), by stacking all the $K$ vectors into the columns of a matrix, yields the 3-mode unfolding of the received signal tensor $\mathcal{X}^{(\text{SRD})}$

$$\begin{aligned}
[\mathcal{X}]_{(3)}^{(\text{SRD})\text{T}} &= \left[\mathbf{x}_1^{(\text{SRD})}, \ldots, \mathbf{x}_K^{(\text{SRD})}\right] \in \mathbb{C}^{M_D N \times K} \\
&= \left(\mathbf{S} \otimes \mathbf{H}^{(\text{RD})}\right)\text{Diag}(\mathbf{h}^{(\text{SR})})\left(\mathbf{V}^{\text{T}} \diamond \mathbf{U}^{\text{T}}\right)
\end{aligned} \tag{12}$$

Note that the term $\text{Diag}(\mathbf{h}^{(\text{SR})})$ can be viewed as the 3-mode unfolding of a core tensor $\mathcal{P} \in \mathbb{C}^{M_R \times M_T \times M_R M_T}$ in which can be constructed using the SKP introduced in Section 2.2

$$\mathcal{P} = \underbrace{\left(\mathcal{I}_{3,M_T} \otimes_{2,3}^{2,3} \mathcal{I}_{3,M_R}\right)}_{\in \mathbb{R}^{M_R \times M_T \times M_R M_T \times M_R M_T}} \times_3 \mathbf{h}^{(\text{SR})\text{T}} \in \mathbb{C}^{M_R \times M_T \times 1 \times M_R M_T}, \tag{13}$$



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**Algorithm 1** SKP-ALS receiver

1: Initialize randomly $\hat{\mathbf{H}}_0^{(\mathrm{RD})}$ and $\hat{\mathbf{S}}_0$; it $= 0$;
2: it = it + 1;
3: Calculate an estimate of $\hat{\mathbf{h}}_{it}^{(\mathrm{SR})}$ according to Eq. (21)
4: Reshape Diag($\hat{\mathbf{h}}^{(\mathrm{SR})}$), i.e., $\left[\hat{\mathcal{P}}\right]_{(3)}$, into the tensor $\hat{\mathcal{P}}$
5: Compute an estimate of $\hat{\mathbf{H}}^{(\mathrm{RD})}$ according to Eq. (22)
6: Compute an estimate of $\hat{\mathbf{S}}$ according to Eq. (23)
7: Return to step 2 and repeat until convergence;
8: Remove the scaling ambiguities using Eq. (25).

where $\mathcal{I}_{3,M_T} \in \mathbb{R}^{M_T \times M_T \times M_T}$ and $\mathcal{I}_{3,M_T} \in \mathbb{R}^{M_T \times M_T \times M_T}$ are identity tensors. Equation (13) shows the special structure of the SKP between two identity tensors combined with a vector, yielding into a sparse core tensor, so that $[\mathcal{P}]_{(3)} = \mathrm{Diag}(\mathbf{h}^{(\mathrm{SR})})$. Next, we can express our received signal in tensor notation, neglecting the noise, as

$$\mathcal{X}^{(\mathrm{SRD})} = \mathcal{P} \times_1 \mathbf{H}^{(\mathrm{RD})} \times_2 \mathbf{S} \times_3 \mathbf{L} \in \mathbb{C}^{M_D \times N \times K}, \tag{14}$$

where $\mathbf{L} = \left(\mathbf{V}^{\mathrm{T}} \diamond \mathbf{U}^{\mathrm{T}}\right)^{\mathrm{T}} \in \mathbb{C}^{K \times M_R M_T}$. Note that using the SKP approach, we have expressed our PARATuck 2 model as a special Tucker 3 model, where $\mathcal{P}$ is the sparse core tensor. Since $\mathbf{L}$ is assumed to be known at the receiver, and defining $\mathbf{x}^{(\mathrm{SRD})} = \mathrm{vec}([\mathcal{X}]_{(3)}^{(\mathrm{SRD})\mathrm{T}})$, we can use the following equations to derive our receiver

$$\mathbf{x}^{(\mathrm{SRD})} = \left[\mathbf{L} \diamond \left(\mathbf{S} \otimes \mathbf{H}^{(\mathrm{RD})}\right)\right] \mathbf{h}^{(\mathrm{SR})} \in \mathbb{C}^{M_D N K \times 1} \tag{15}$$

$$[\mathcal{X}]_{(1)}^{(\mathrm{SRD})} = \mathbf{H}^{(\mathrm{RD})} [\mathcal{P}]_{(1)} (\mathbf{L} \otimes \mathbf{S})^{\mathrm{T}} \in \mathbb{C}^{M_D \times NK} \tag{16}$$

$$[\mathcal{X}]_{(2)}^{(\mathrm{SRD})} = \mathbf{S} [\mathcal{P}]_{(2)} \left(\mathbf{L} \otimes \mathbf{H}^{(\mathrm{RD})}\right)^{\mathrm{T}} \in \mathbb{C}^{N \times M_D K}, \tag{17}$$

## 4. SEMI-BLIND SKP-ALS RECEIVER

The proposed SKP-ALS receiver is derived from Eqs. (15) to (17), and consists in solving the following three optimization problems:

$$\hat{\mathbf{h}}^{(\mathrm{SR})} = \underset{\mathbf{h}^{(\mathrm{SR})}}{\mathrm{argmin}} \left\| \mathbf{x}^{(\mathrm{SRD})} - \left[\mathbf{L} \diamond \left(\hat{\mathbf{S}} \otimes \hat{\mathbf{H}}^{(\mathrm{RD})}\right)\right] \mathbf{h}^{(\mathrm{SR})} \right\|_2^2 \tag{18}$$

$$\hat{\mathbf{H}}^{(\mathrm{RD})} = \underset{\mathbf{H}^{(\mathrm{RD})}}{\mathrm{argmin}} \left\| [\mathcal{X}]_{(1)}^{(\mathrm{SRD})} - \mathbf{H}^{(\mathrm{RD})} \left[\hat{\mathcal{P}}\right]_{(1)} \left(\mathbf{L} \otimes \hat{\mathbf{S}}\right)^{\mathrm{T}} \right\|_{\mathrm{F}}^2 \tag{19}$$

$$\hat{\mathbf{S}} = \underset{\mathbf{S}}{\mathrm{argmin}} \left\| [\mathcal{X}]_{(2)}^{(\mathrm{SRD})} - \mathbf{S} \left[\hat{\mathcal{P}}\right]_{(2)} \left(\mathbf{L} \otimes \hat{\mathbf{H}}^{(\mathrm{RD})}\right)^{\mathrm{T}} \right\|_{\mathrm{F}}^2, \tag{20}$$

the solution of which are given, respectively, by

$$\hat{\mathbf{h}}^{(\mathrm{SR})} = \left(\left[\mathbf{L} \diamond \left(\hat{\mathbf{S}} \otimes \hat{\mathbf{H}}^{(\mathrm{RD})}\right)\right]\right)^{+} \mathbf{x}^{(\mathrm{SRD})} \tag{21}$$

$$\hat{\mathbf{H}}^{(\mathrm{RD})} = [\mathcal{X}]_{(1)}^{(\mathrm{SRD})} \left(\left[\hat{\mathcal{P}}\right]_{(1)} \left(\mathbf{L} \otimes \hat{\mathbf{S}}\right)^{\mathrm{T}}\right)^{+} \tag{22}$$

$$\hat{\mathbf{S}} = [\mathcal{X}]_{(2)}^{(\mathrm{SRD})} \left(\left[\hat{\mathcal{P}}\right]_{(2)} \left(\mathbf{L} \otimes \hat{\mathbf{H}}^{(\mathrm{RD})}\right)^{\mathrm{T}}\right)^{+} \tag{23}$$

The proposed SKP-ALS receiver is described in Algorithm 1. To check the convergence, in each step of the SKP-ALS we compute a relative error, given by

$$e_i = \frac{\| [\mathcal{X}]_{(1)}^{(\mathrm{SRD})} - \left[\hat{\mathcal{X}}\right]_{(1)}^{(\mathrm{SRD})} \|_{\mathrm{F}}^2}{\| [\mathcal{X}]_{(1)}^{(\mathrm{SRD})} \|_{\mathrm{F}}^2}, \tag{24}$$

and if $|e_i - e_{i-1}| \leq 10^{-6}$ then convergence is achieved. Otherwise, we set the maximum number of iterations to 100.

### 4.1. Identifiability and Uniqueness

In contrast to the SPP receiver proposed in [1], our proposed receiver does not depend on the symbol estimation via the Source-Destination (SD) link to initialize the SKP-ALS algorithm. In our case, the SD link is not available, which means a more difficult scenario. Therefore, the number $K$ of time slots should be equal or larger than the product $M_R M_T$. For uniqueness, the knowledge of $\mathbf{V}$ and $\mathbf{U}$ must be available at the destination. Moreover, due to the pseudo-inverses in (21) to (23), we need that $N \geq M_T$, $M_D \geq M_R$ and $K \geq M_R M_T$.

As in [1, 14], by assuming the knowledge of $\mathbf{V}$ and $\mathbf{U}$ the permutation ambiguity is avoided, while the knowledge of one row of $\mathbf{S}$ and $\mathbf{H}^{(\mathrm{RD})}$ can be exploited to eliminate the scalar ambiguity. The knowledge of the first row of $\mathbf{H}^{(\mathrm{RD})}$ can be obtained via a simple training procedure [1, 14]. The scaling ambiguity relations are then given by

$$\mathbf{S} = \hat{\mathbf{S}}\mathbf{\Lambda}_s, \;\; \mathbf{H}^{(\mathrm{RD})} = \hat{\mathbf{H}}^{(\mathrm{RD})}\mathbf{\Lambda}_h, \;\; \mathbf{H}^{(\mathrm{SR})} = \mathbf{\Lambda}_h^{-1}\hat{\mathbf{H}}^{(\mathrm{SR})}\mathbf{\Lambda}_s^{-1} \tag{25}$$

where, $\mathbf{\Lambda}_s = \mathrm{Diag}\left(\mathbf{S}_{(1,:)} \oslash \hat{\mathbf{S}}_{(1,:)}\right)$, $\mathbf{\Lambda}_h = \mathrm{Diag}\left(\mathbf{H}_{(1,:)}^{(\mathrm{RD})} \oslash \hat{\mathbf{H}}_{(1,:)}^{(\mathrm{RD})}\right)$ and "$\oslash$" is the element-wise division.

## 5. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed SKP-ALS receiver by comparing it with the state-of-the-art solution in literature [1]. The simulations were averaged using a total of $5 \cdot 10^4$ Monte Carlo trials, wherein for each trial, independent realizations are performed for symbols, channels, and noise. The channel matrices $\mathbf{H}^{(\mathrm{RD})}$ and $\mathbf{H}^{(\mathrm{SR})}$ are assumed to be i.i.d. complex Gaussian matrices, with zero mean and unit variance. The Signal-Noise Ratio (SNR) is controlled by varying the noise power at the relay and the destination, which are assumed to be equal. The symbol matrix $\mathbf{S}$ is normalized such that $\mathbb{E}\left[\mathbf{S}^{\mathrm{H}}\mathbf{S}\right] = N\mathbf{I}_{M_T}$. Since, $K \geq M_R M_T$, we choose truncated Hadamard and truncated DFT matrices for the coding matrices $\mathbf{V} \in \mathbb{C}^{K \times M_T}$ and $\mathbf{U} \in \mathbb{C}^{K \times M_R}$, respectively, and they are normalized by the factors of $\frac{1}{\sqrt{M_T}}$ and $\frac{1}{\sqrt{M_R}}$ to ensure that $\mathbf{L}^{\mathrm{H}}\mathbf{L} = \mathbf{I}_{M_R M_T}$. The simulation parameter are given by: $N = 10$ $K = M_D = 4$, $M_R = M_T = 2$, and we use 4-QAM modulated symbols. For a fair comparison, all the investigated systems have the same spectral efficiency.

In Figure 2, we compare the result from our proposed SKP-ALS receiver with the competing SPP receiver proposed in [1]. Also, as a benchmark, we compare with the zero forcing (ZF) performance with perfect CSI, given by

$$\hat{\mathbf{S}}_{\mathrm{ZF}} = [\mathcal{X}]_{(2)}^{(\mathrm{SRD})} \left([\mathcal{P}]_{(2)} \left(\mathbf{L} \otimes \mathbf{H}^{(\mathrm{RD})}\right)^{\mathrm{T}}\right)^{+}. \tag{26}$$

As it can be observed, our SKP-ALS receiver achieves the same performance as the SPP one. However, the SPP receiver in [1]



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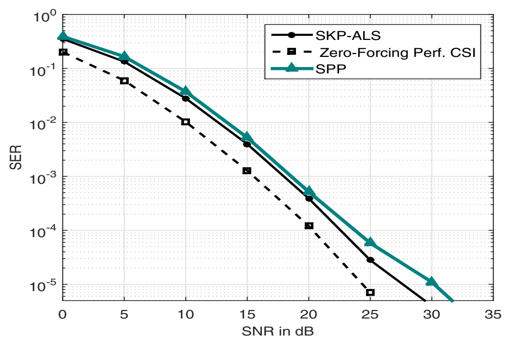

**Fig. 2**: SER $vs$ SNR

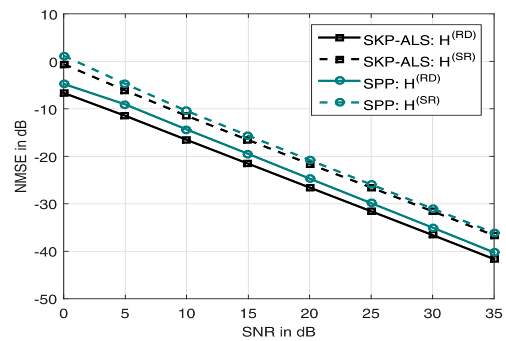

**Fig. 3**: NMSE of the estimated channels.

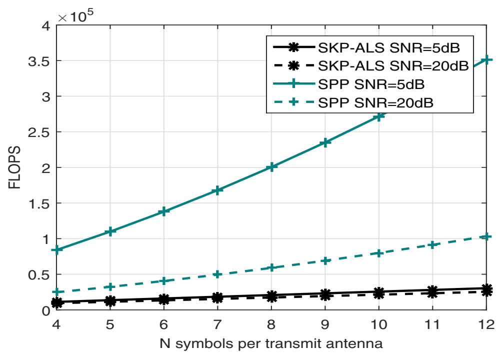

**Fig. 4**: Number of FLOPS for different scenarios.

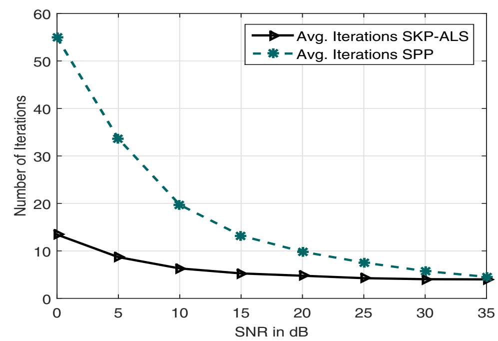

**Fig. 5**: ALS iterations needed to convergence.

needs a priori information of the symbol matrix, which is obtained by considering a Source-Destination (SD) link. Then, the symbols are estimated in a closed-form fashion and used as an initialization of the ALS algorithm. In our case, by recasting our PARATuck 2 model as a special Tucker 3 model, we eliminate this dependence on the SD link, which allows our receiver to cope with more challenging scenarios where the SD link is to weak or not available. In Figure 3, the channel estimation accuracy between the proposed SKP-ALS receiver and the SPP receiver of [1] is compared, using the NMSE as a metric. The NMSE is computed as

$$\text{NMSE} = \frac{1}{\text{MC}} \sum_{r=1}^{\text{MC}} \frac{\left|\left|\mathbf{H}^{(r)} - \hat{\mathbf{H}}^{(r)}\right|\right|_{\text{F}}^{2}}{||\mathbf{H}^{(r)}||_{\text{F}}^{2}}, \tag{27}$$

where MC is the total number of Monte Carlo runs, and $\mathbf{H}$ stands for the source-relay or relay-destination channel matrix. The results depicted in Figure 3 shows that not only we can achieve the same SER without benefiting from SD link initialization, but we also achieve more accurate channel estimates, for the source-relay and the relay-destination channels, using the proposed model.

In Figure 4, we compare the computational complexity in terms of floating-point operations (FLOPS). We assume for matrix multiplication the cost of $\mathcal{O}(4mnp)$ FLOPS and for the Kronecker product a cost of $\mathcal{O}(4mn^2p)$ FLOPS. In Figure 4, we compare two scenarios, low SNR regime (5 dB) and high SNR regime (20 dB). As expected, the proposed SKP-ALS receiver is less complex than the one in [1]. This can be explained by the fact the SPP receiver has a more costly symbol estimation procedure, since the estimation of the full symbol matrix involves $N$ matrix inverses per iteration [1]. In contrast, SKP-ALS allows an all-at-once estimation of the full symbol matrix at every iteration. Also, as can be observed in Figure 5, the SPP needs approximately 34 iterations to converge in the low SNR regime and approximately 10 for the high SNR regime, while for our proposed SKP-ALS the number of iterations varies between 6 and 4 for these SNR regimes.

## 6. CONCLUSIONS

In this paper, we have proposed a semi-blind receiver for a two-hop MIMO system and a new formulation for PARATuck 2 models, that in our application, eliminates the necessity of the additional SD link, unlike the SPP receiver in [1], where the SD link must be available to initialize the ALS algorithm. We have shown that our SKP-ALS achieves the same SER performance as the competing SPP receiver, while providing more accurate channel estimates with a significant computational effort reduction.

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