

# REDUCED-COMPLEXITY DISTRIBUTED BEAMFORMING ALGORITHM FOR INDIVIDUAL RELAY POWER CONSTRAINTS

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## ABSTRACT

This paper presents a distributed beamforming algorithm with a reduced computational complexity for a wireless relay network, where a source-destination pair is assisted by cooperating amplify-and-forward (AF) relays but suffers from additional interference. The relay weights are obtained by maximizing the signal-to-interference-plus-noise ratio (SINR) at the destination subject to individual relay power constraints. Mathematically, this problem is a quasi-convex constrained maximization of a fractional quadratic function, which is typically solved via semidefinite relaxation (SDR) along with a bisection search that requires several iterations. We, however, propose a method based on the SDR approach followed by a change of the optimization variables to convert the original quasi-convex problem into a convex problem. The resulting optimization problem can then be solved in one step without a bisection search, thereby significantly reducing the computational complexity. The effectiveness of the developed procedure is shown by simulations and a complexity analysis. It may also be applicable to similar problems in other applications.

**Index Terms**— Distributed beamforming, quasi-convex problem, semidefinite relaxation, bisection method.

## 1. INTRODUCTION

Establishing reliable and energy-efficient transmissions in distributed ad-hoc networks of nodes has recently been a main concern in the field of wireless communications. It is known that the performance of a wireless network, i.e., its coverage, capacity, and reliability, can be substantially improved by exploiting the cooperation of nodes, termed cooperation diversity [1], [2]. In such schemes, non-transmitting nodes assist each other by relaying source signals through multiple independent paths in the network, which are constructively combined at the destination. These relays create a virtual array of transmit antennas, and thereby provide spatial diversity in order to combat signal fading effects and interference without the need of multiple antennas at the users.

A very effective approach to benefit from cooperative diversity capabilities is distributed relay beamforming [3]-[5], where the relay nodes form a beam towards the destination. Among various relaying protocols, the amplify-and-forward (AF) protocol [2] is of particular interest due to its simplicity and low complexity. References [3]-[5] consider a single source-destination pair and compute the beamforming weights based on the assumption that instantaneous or statistical channel state information (CSI) is available at the receiver. Thus, the weights are computed at the receiver and fed back to the relays. One

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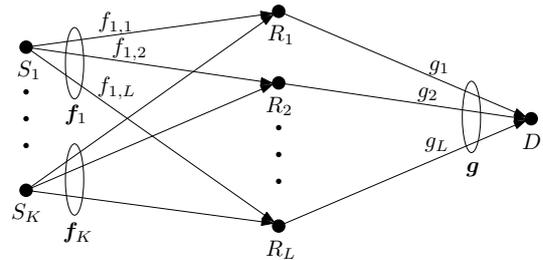


Fig. 1. A network of  $K$  sources,  $L$  relays, and one destination.

of the studied design criteria to obtain the beamforming weights is the problem of maximizing the receiver signal-to-noise ratio (SNR) subject to individual relay power constraints. This type of constraint is highly desirable for practical implementations. Mathematically, the optimization problem is a quasi-convex constrained maximization of a ratio containing quadratic forms. As it cannot be solved as a generalized eigenvector problem due to the individual power constraints, a solution via semidefinite relaxation (SDR) [7] followed by a bisection search was proposed in [4] and [5]. However, the bisection method [6] requires several iterations and solves a convex feasibility semidefinite program (SDP) problem at each step, which is of high computational complexity and inapplicable in practice.

In this paper, we consider a distributed network consisting of a source-destination pair assisted by multiple AF relays that is subject to interference. For this system and given the instantaneous CSI at the receiver, we develop a reduced-complexity distributed beamforming algorithm based on the signal-to-interference-plus-noise ratio (SINR) maximization at the destination under individual relay power constraints. After applying the SDR approach to the optimization problem, a substitution of the optimization variable is proposed, by which the quasi-convex problem is reformulated into a convex SDP optimization problem. This resulting problem can then be solved in one step without the need of a bisection search, which substantially reduces the computational complexity. Its effectiveness is supported by simulations and a complexity analysis. The developed procedure may also be applicable to other quasi-convex fractional quadratic problems in different applications. It should be mentioned that a similar idea was proposed in [8] and [9] for different applications.

## 2. SYSTEM MODEL

Consider a wireless ad-hoc relay network with one source-destination user pair,  $L$  relay nodes and  $K - 1$  interfering source nodes, as shown in Fig. 1. All the nodes are equipped with a single antenna and operate on the same frequency. Furthermore, they are assumed to work in a half-duplex mode, i.e., they cannot receive and transmit at the

same time. As there is no direct link between the  $K$  sources and the destination, the communication between the user pair is carried out via the relays. Moreover, we have flat-fading channels and the distributed network is perfectly synchronized. Each transmission from the sources to the destination is implemented in a two-step scheme, where in the first time-slot, all the sources simultaneously broadcast their signals to the relays. Then, in the second time-slot, the received signals at the relays are processed by a complex beamforming weight and retransmitted to the destination.

In the first transmission stage, the mixture of source signals received at the relays can be modeled as

$$\mathbf{x} = \mathbf{F}\mathbf{P}^{1/2}\mathbf{s} + \boldsymbol{\mu} \in \mathbb{C}^{L \times 1}, \quad (1)$$

where  $\mathbf{F} = [\mathbf{f}_1, \dots, \mathbf{f}_K] \in \mathbb{C}^{L \times K}$  is the channel matrix between the  $K$  sources and the  $L$  relays, and  $\mathbf{f}_i = [f_{i,1}, \dots, f_{i,L}]^T$ ,  $i = 1, \dots, K$ , contains the channel coefficients from the  $i$ -th source to all the relays. The vector  $\mathbf{s} = [s_1, \dots, s_K]^T \in \mathbb{C}^{K \times 1}$  represents the uncorrelated source signals with  $\mathbb{E}\{|s_i|^2\} = 1$ ,  $\mathbf{P} \in \mathbb{R}^{K \times K}$  is the diagonal matrix with the source powers  $P_i$  on its diagonal, and  $\boldsymbol{\mu} \in \mathbb{C}^{L \times 1}$  is the zero-mean circularly symmetric complex Gaussian noise at the relays with variance  $\sigma_\mu^2$ .

In the second stage, the retransmitted signal from the relays can be expressed as

$$\mathbf{r} = \mathbf{W}^H \mathbf{x} \in \mathbb{C}^{L \times 1}, \quad (2)$$

where  $\mathbf{W} = \text{diag}\{\mathbf{w}\}$  and  $\text{diag}\{\cdot\}$  places the elements of  $\mathbf{w}$  on the diagonal of  $\mathbf{W}$ . The vector  $\mathbf{w} = [w_1, \dots, w_L]^T \in \mathbb{C}^{L \times 1}$  contains the complex beamforming weights to be designed. Let us denote  $\mathbf{g} = [g_1, \dots, g_L]^T$  as the channel coefficient vector between the  $L$  relays and the destination. Combining (1) and (2), the received signal at the destination is given by

$$\begin{aligned} y &= \mathbf{g}^T \mathbf{r} + n \\ &= \underbrace{\sqrt{P_d} \mathbf{g}^T \mathbf{W}^H \mathbf{f}_{d s_d}}_{\text{desired signal}} + \underbrace{\mathbf{g}^T \mathbf{W}^H \sum_{k=1, k \neq d}^K \sqrt{P_k} \mathbf{f}_k s_k}_{\text{interference}} \\ &\quad + \underbrace{\mathbf{g}^T \mathbf{W}^H \boldsymbol{\mu} + n}_{\text{effective noise}}, \end{aligned} \quad (3)$$

$$(4)$$

where  $n$  is the zero-mean noise at the destination with variance  $\sigma_n^2$  and the subscript  $d$  denotes the desired signal component. Furthermore, we assume that the channel coefficients, the source symbols, and the noise at the relays and the destination are statistically independent.

The SINR at the destination is given by

$$\text{SINR} = \frac{P_s}{P_i + P_n}, \quad (5)$$

where  $P_s$ ,  $P_i$ , and  $P_n$  represent the power of the desired signal, the interference power, and the noise power at the receiver, respectively. The expressions of these power are derived similarly to [4], but for instantaneous channel realizations. For the power of the desired signal component, we have

$$P_s = \mathbb{E}\{|\sqrt{P_d} \mathbf{g}^T \mathbf{W}^H \mathbf{f}_{d s_d}|^2\} = \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (6)$$

where  $\mathbf{w} = \text{diag}\{\mathbf{W}\} \in \mathbb{C}^{L \times 1}$  and  $\text{diag}\{\cdot\}$  extracts the diagonal of  $\mathbf{W}$ . Moreover,  $\mathbf{R} = P_d \mathbf{h}_d \mathbf{h}_d^H \in \mathbb{C}^{L \times L}$  with  $\mathbf{h}_d = \mathbf{g} \odot \mathbf{f}_d$ , where  $\odot$  denotes the Schur-Hadamard (element-wise) matrix product. The interference power can be written as

$$P_i = \mathbb{E}\left\{\left|\mathbf{g}^T \mathbf{W}^H \sum_{k=1, k \neq d}^K \sqrt{P_k} \mathbf{f}_k s_k\right|^2\right\} = \mathbf{w}^H \mathbf{Q}_i \mathbf{w}, \quad (7)$$

where  $\mathbf{Q}_i = \sum_{k=1, k \neq d}^K P_k \mathbf{h}_k \mathbf{h}_k^H \in \mathbb{C}^{L \times L}$  with  $\mathbf{h}_k = \mathbf{g} \odot \mathbf{f}_k$ . The expression for the noise power at the receiver is given by

$$P_n = \mathbb{E}\{|\mathbf{g}^T \mathbf{W}^H \boldsymbol{\mu} + n|^2\} = \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_n^2, \quad (8)$$

where  $\mathbf{Q}_n = \sigma_\mu^2 ((\mathbf{g}\mathbf{g}^H) \odot \mathbf{I}_L) \in \mathbb{R}^{L \times L}$ . Using (6)-(8), the SINR is written as

$$\text{SINR} = \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\mathbf{w}^H (\mathbf{Q}_i + \mathbf{Q}_n) \mathbf{w} + \sigma_n^2}. \quad (9)$$

Let  $P_r$  be the total relay transmit power expressed as

$$P_r = \mathbb{E}\{\|\mathbf{r}\|^2\} = \mathbf{w}^H \mathbf{D} \mathbf{w}, \quad (10)$$

where  $\mathbf{D} \in \mathbb{R}^{L \times L}$  is a diagonal matrix with  $\mathbf{D} = \mathbf{T} \odot \mathbf{I}_L$ . The covariance matrix  $\mathbf{T}$  can be written as

$$\mathbf{T} = \mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{F}\mathbf{P}\mathbf{F}^H + \sigma_\mu^2 \mathbf{I}_L. \quad (11)$$

On the other hand, the individually transmitted power at the  $l$ -th relay is

$$P_{r_l} = \mathbb{E}\{|r_l|^2\} = |w_l|^2 [\mathbf{D}]_{ll}, \quad (12)$$

where  $[\mathbf{D}]_{ll}$  is the  $(l, l)$ -element of  $\mathbf{D}$  with  $l = 1, \dots, L$  and  $\sum_{l=1}^L P_{r_l} = P_r$ .

### 3. SINR MAXIMIZATION WITH INDIVIDUAL POWER CONSTRAINTS

In this section, we consider the distributed beamforming criterion for the weight computation that maximizes the SINR at the receiver subject to individual relay power constraints. This type of power constraint is of particular interest as in practical relay communications, each relay is restricted in its battery life time and equipped with its own power amplifier. Thus, it is desirable to impose strict power constraints at the relays to consider their remaining battery power and to have the power amplifiers operate in their linear range.

The optimization problem is stated as

$$\begin{aligned} \max_{\mathbf{w}} \quad & \text{SINR} \\ \text{subject to} \quad & P_{r_l} \leq P_{r_l}^{\max} \quad \forall l, l = 1, \dots, L, \end{aligned} \quad (13)$$

where  $P_{r_l}^{\max}$  is the maximum allowable individual relay transmit power at the  $l$ -th relay. Note that because of (12), a total power constraint can be ignored as it only infringes on the allowable individual power limits when their sum exceeds  $P_r$ , which is not the case here.

Due to the fractional quadratic objective function, the problem in (13) is quasi-convex and cannot be solved as a generalized eigenvector problem as a result of the individual power constraints. However, as proposed for the non-interference case [4], the optimization problem can be reformulated by applying the SDR approach [7] and the epigraph form [6] so that it can be written as a convex feasibility SDP problem for any fixed value of the introduced auxiliary variable  $t$ . Thus, a solution via the bisection method [6] can be obtained, which is an iterative procedure that requires solving the same convex feasibility SDP problem in each iteration for refined values of  $t$ . Owing to the iterative nature of the bisection search, the computational complexity is very high and therefore, a novel approach with a lower complexity is developed in the next section. Note that in contrast to [4] and [5], we also consider an interference scenario as shown in Fig. 1.

#### 4. REDUCED-COMPLEXITY ALGORITHM

This section presents the proposed distributed beamforming algorithm based on the SINR maximization at the destination under individual power constraints. It has a significantly reduced computational complexity as it does not require an iterative bisection search.

We start our development by expanding (13), using (9) and (12), to obtain

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\mathbf{w}^H \mathbf{Q} \mathbf{w} + \sigma_n^2} \\ \text{subject to} \quad & |w_l|^2 [\mathbf{D}]_{ll} \leq P_{r_l}^{\max} \quad \forall l, l = 1, \dots, L, \end{aligned} \quad (14)$$

where  $\mathbf{Q} = \mathbf{Q}_i + \mathbf{Q}_n$ . Next, we apply the concept of SDR [7] to the problem in (14) and define the optimization variable  $\mathbf{X} \triangleq \mathbf{w} \mathbf{w}^H \in \mathbb{C}^{L \times L}$ . Using the fact that for any arbitrary matrix  $\mathbf{A}$ , the equation  $\mathbf{w}^H \mathbf{A} \mathbf{w} = \text{Tr}\{\mathbf{A} \mathbf{w} \mathbf{w}^H\}$  holds, (14) is equivalent to

$$\begin{aligned} \max_{\mathbf{X}} \quad & \frac{\text{Tr}\{\mathbf{R} \mathbf{X}\}}{\text{Tr}\{\mathbf{Q} \mathbf{X}\} + \sigma_n^2} \\ \text{subject to} \quad & [\mathbf{D}]_{ll} [\mathbf{X}]_{ll} \leq P_{r_l}^{\max} \quad \forall l, l = 1, \dots, L \\ & \text{rank}\{\mathbf{X}\} = 1, \quad \mathbf{X} \succeq \mathbf{0}, \end{aligned} \quad (15)$$

where  $\text{Tr}\{\cdot\}$  represents the trace-operator and  $\mathbf{X} \succeq \mathbf{0}$  means that  $\mathbf{X}$  is constrained to be positive semidefinite. Note that the optimization problem in (15) is not convex due to the non-convex rank constraint. Therefore, we relax the problem by dropping the rank constraint and solve instead the optimization problem

$$\begin{aligned} \max_{\mathbf{X}} \quad & \frac{\text{Tr}\{\mathbf{R} \mathbf{X}\}}{\text{Tr}\{\mathbf{Q} \mathbf{X}\} + \sigma_n^2} \\ \text{subject to} \quad & [\mathbf{D}]_{ll} [\mathbf{X}]_{ll} \leq P_{r_l}^{\max} \quad \forall l, l = 1, \dots, L \\ & \mathbf{X} \succeq \mathbf{0}. \end{aligned} \quad (16)$$

The relaxed optimization problem in (16) is still a quasi-convex program in the optimization variable  $\mathbf{X}$ . Hence, we propose a substitution of the current optimization variable  $\mathbf{X} \in \mathbb{C}^{L \times L}$  by the new variables  $\mathbf{Y} \in \mathbb{C}^{L \times L}$  and  $z$  according to

$$\mathbf{X} = \mathbf{Y}/z \quad \text{with} \quad \mathbf{Y} \succeq \mathbf{0}, \quad z > 0. \quad (17)$$

Incorporating the new optimization variables into the problem in (16), yields

$$\begin{aligned} \max_{\mathbf{Y}, z} \quad & \frac{\text{Tr}\{\mathbf{R} \mathbf{Y}\}}{\text{Tr}\{\mathbf{Q} \mathbf{Y}\} + z \sigma_n^2} \\ \text{subject to} \quad & [\mathbf{D}]_{ll} [\mathbf{Y}]_{ll} \leq z P_{r_l}^{\max} \quad \forall l, l = 1, \dots, L \\ & \mathbf{Y} \succeq \mathbf{0}, \quad z > 0. \end{aligned} \quad (18)$$

Due to the introduction of the variable  $z$ , the objective function of (18) becomes homogeneous. Thus, without loss of generality, we can fix the term in the denominator of the fractional objective function to be equal to one at the optimal point. By doing so, the problem (18) can be equivalently recast as

$$\max_{\mathbf{Y}, z} \quad \text{Tr}\{\mathbf{R} \mathbf{Y}\} \quad (19a)$$

$$\text{subject to} \quad \text{Tr}\{\mathbf{Q} \mathbf{Y}\} + z \sigma_n^2 = 1 \quad (19b)$$

$$[\mathbf{D}]_{ll} [\mathbf{Y}]_{ll} \leq z P_{r_l}^{\max} \quad \forall l, l = 1, \dots, L \quad (19c)$$

$$\mathbf{Y} \succeq \mathbf{0}, \quad z \geq 0, \quad (19d)$$

which is now a convex semidefinite program (SDP) in the variables  $\mathbf{Y}$  and  $z$  that can be efficiently solved in one step using interior-point

methods [10]. For the sake of the efficiency of convex optimization solvers, we have replaced the strict inequality constraint  $z > 0$  in (18) by  $z \geq 0$  in (19d). This is, however, uncritical as  $z = 0$  is infeasible due to the fact that in this case, we have  $\mathbf{Y} = \mathbf{0}$  from the constraints (19c) and (19d), which then violates the constraint (19b).

Note that due to the semidefinite relaxation, i.e., dropping the non-convex rank constraint, the optimal value  $\mathbf{Y}^*$  of the problem (19) is not necessarily of rank one in general. It represents an upper bound on the maximum value of the problem (14). If  $\mathbf{Y}^*$  happens to be of rank one, the relaxation is tight, i.e., the optimal values of (19) and (14) are equal. In this case, the desired beamforming vector  $\mathbf{w}$  can be extracted as

$$\mathbf{w} = \alpha \mathcal{P}\{\mathbf{X}^*\}, \quad (20)$$

where  $\mathbf{X}^*$  is obtained by resubstituting  $\mathbf{X}^* = \mathbf{Y}^*/z^*$  to fulfill the individual power constraints in (14). Moreover,  $\mathcal{P}\{\cdot\}$  is the normalized principal eigenvector operator and  $\alpha = \sqrt{\lambda_{\max}\{\mathbf{X}^*\}}$  with  $\lambda_{\max}\{\cdot\}$  being the maximum eigenvalue of a matrix. However, if the rank of  $\mathbf{Y}^*$  is greater than one, we only obtain an approximate solution as the feasible set of (19) is a subset of that of problem (15). In this case, several randomization techniques to obtain a suboptimal rank-one solution from  $\mathbf{Y}^*$  have been proposed in [11]. Interestingly, throughout our extensive simulations, we never encountered a case, where the relaxed SDP problem provided a solution with a rank higher than one.

It is worth mentioning that the proposed problem reformulation is also applicable to the case where only statistical channel information, i.e., the second-order statistics of the channels instead of the instantaneous channel realizations, is available [4], [5].

#### 5. COMPLEXITY ANALYSIS

If the problem (14) is solved using the SDR approach combined with the bisection method as in [4], the worst-case computational complexity is the overall cost of solving the same feasibility SDP problem in each of the  $\mathcal{O}(\log_2((u-l)/\epsilon))$  bisection search iterations, where  $u$  and  $l$  are the upper and lower values of the search interval and  $\epsilon$  is the accuracy. The proposed algorithm is also based on the SDR technique but does not require the iterative bisection search as it solves the reformulated problem in one step. Therefore, the reduction in computations of the presented approach is at least  $\mathcal{O}(\log_2((u-l)/\epsilon))$ .

#### 6. SIMULATIONS AND NUMERICAL RESULTS

In this section, we show simulation results that demonstrate the performance of the proposed distributed beamforming algorithm based on the SINR maximization under individual relay power constraints. We also conduct an assessment of the required computational complexity, where we measure the run time of the algorithm. For comparison purposes, we include the SDR approach together with the bisection method developed in [4] to solve the underlying optimization problem and point out the benefits of the proposed algorithm.

In the first experiment, we have  $L = 10$  relays and  $K = 5$  sources in the network, and assume Rayleigh flat-fading channels with unit-variance channel coefficients. The variances of the noise at the relays and the destination are equal and the SNR is 0 dB. Moreover, the desired user transmits with a power of 0 dBW and the signal-to-interference ratio (SIR) for the  $K - 1$  interferers is also fixed at 0 dB. In order to consider the case of individual relay powers, we define the maximum allowable total transmit power

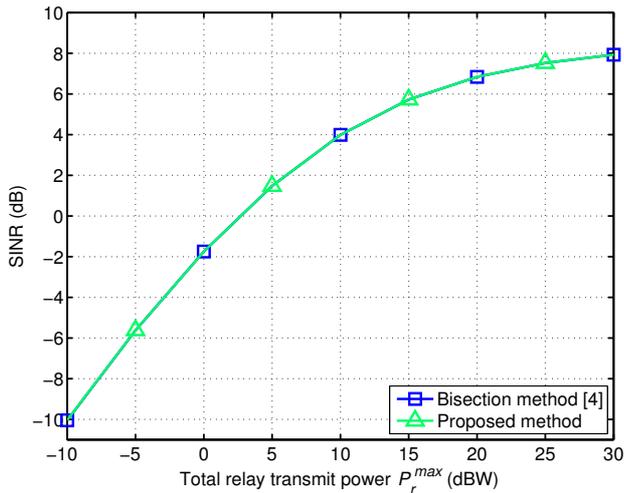


Fig. 2. Maximum SINR versus the maximum total relay power  $P_r^{\max}$  for  $L = 10$ ,  $K = 5$ , SNR = 0 dB, and SIR = 0 dB.

$P_r^{\max}$  and divide the  $L$  relays into two equal groups of  $L/2$  relays, where the individual relay powers of the first group is twice that of the second group. The accuracy  $\epsilon$  for the bisection method required in [4] is set to  $\epsilon = 10^{-4}$ . All the curves are obtained by averaging over 1000 Monte Carlo trials. In Fig. 2, we illustrate the maximum achievable SINR as a function of the total relay transmit power  $P_r^{\max} = \sum_{l=1}^L P_r^{\max}$ , where the individual relay nodes have the aforementioned power limits. In our intensive simulations, we have observed that the solution matrix  $\mathbf{Y}^*$  is always rank one and hence, no randomization technique is required. It can be seen that the performance of the proposed method is identical to the one including the bisection search.

In the second experiment, we evaluate the computational complexity of the proposed distributed beamforming algorithm and compare it to the method of [4]. Specifically, we measure the run time of the algorithms on a laptop with an Intel Core i5 M430 2.27 GHz CPU. We have performed 100 runs of both algorithms and display the average run time for one run in Fig. 3 as a function of the number of relays. The maximum total relay transmit power is  $P_r^{\max} = 10$  dBW, whereas all the other parameters are kept the same. It is apparent that the run time slightly increases for both methods as the number of relays grows. However, due to the non-iterative nature of the proposed algorithm, its run time is significantly lower than that based on the bisection method. In fact, a substantial run time reduction of up to 90 % can be achieved while providing the same performance. Thus, the proposed algorithm should be preferred in practical implementations.

## 7. CONCLUSION

In this paper, we have presented a reduced-complexity distributed beamforming algorithm for the relay weight design based on the SINR maximization at the destination subject to individual relay transmit power constraints. We have considered a distributed network, which consists of a single-antenna source-destination user pair, multiple cooperating relays, and multiple interferers. The resulting optimization problem is a quasi-convex fractional quadratic problem typically solved using the SDR method followed by an iterative bisection search, which is computationally expensive. There-

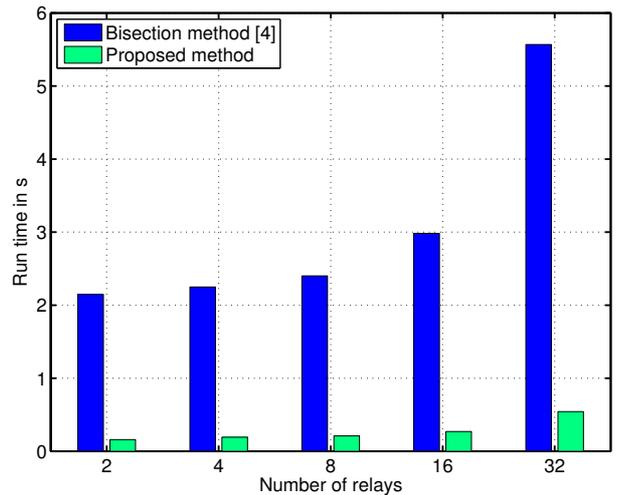


Fig. 3. Run time versus the number of relays for  $K = 5$ ,  $P_r^{\max} = 10$  dBW, SNR = 0 dB, and SIR = 0 dB.

fore, we have developed an alternative approach based on a change of optimization variables with a reduced computational complexity that converts the quasi-convex problem into a convex problem that can be solved in one step. Simulation results and a complexity analysis have shown that the proposed method achieves the same SINR performance while requiring a significantly lower computational complexity.

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