ML-GSVD-based MIMO-NOMA Networks

Liana Khamidullina[1], André L. F. de Almeida[2], and Martin Haardt[1]
[1]*Communication Research Laboratory, Ilmenau University of Technology*, Ilmenau, Germany
[2]*Department of Teleinformatics Engineering, Federal University of Ceará*, Fortaleza, Brazil
e-mail: liana.khamidullina@tu-ilmenau.de, andre@gtel.ufc.br, martin.haardt@tu-ilmenau.de

***Abstract*—Future generations of wireless networks become more and more demanding due to the exponential growth of mobile traffic with billions of connected smart devices. In this paper, we focus on non-orthogonal multiple access (NOMA), which has been considered as a promising technique for 5G and beyond wireless networks. We consider a power-domain downlink MIMO-NOMA system with an arbitrary number of users and propose to design the precoding and decoding matrices based on the Multilinear Generalized Singular Value Decomposition (ML-GSVD) that we have recently proposed. Moreover, we demonstrate how the generalized singular values of the ML-GSVD can be used for the power allocation. We also compare the proposed MIMO-NOMA scheme with orthogonal multiple access (OMA) techniques and provide various numerical results.**

***Index Terms*—MIMO, GSVD, ML-GSVD, tensor decomposition, NOMA.**

I. Introduction

The substantial growth of the global data traffic puts higher demands on 5G and beyond networks. The extensive use of smart devices and the increasing interest in the Internet of things (IoT) require reliable low-latency communication, massive connectivity, and increased data rates. Due to limited bandwidth and limited power resources, providing a sufficient quality of service (QoS) becomes more and more challenging.

In conventional orthogonal multiple access (OMA) techniques, the users exploit the orthogonal resources in time, frequency, code, or in the joint time-frequency domain to avoid multiple access interference. However, since each resource block can only support one user equipment (UE), the number of served users is limited. As an alternative, non-orthogonal multiple access (NOMA) has been proposed to be employed in future wireless communication systems [1]. NOMA allows to use the same non-orthogonal resources to transmit the data of different users simultaneously. It has gained a significant attention due to its potential to considerably improve the spectral efficiency, enhance the connectivity, and reduce latency [2]. In general, NOMA solutions can be separated into two main groups, power-domain and code-domain multiplexing [3]. However, the majority of the NOMA contributions focuses on the power-domain NOMA due to its relatively simple implementation, tolerable user fairness, and spectral efficiency trade-off [4].

The authors gratefully acknowledge the support of the German Research Foundation (DFG) under grant no. HA 2239/14-1 (AdAMMM) and grant no. ZH 640/2-1 (AdAMMM).

Several overviews and surveys on NOMA techniques have been published in recent years. They present the basic principles of the NOMA technique, discuss the open issues, and compare it with the performance of OMA-based transmission [2]–[9]. The authors in [2] additionally highlight the key challenges and the future research trends in NOMA exploration, which includes a performance analysis, beamforming and receiver design, channel estimation, resource allocation, and an extension to MIMO. The study in [10] investigates efficient user clustering and power allocation algorithms for MIMO-NOMA systems. The authors derive the sum rate capacity for the uplink system with a massive antenna array at the base station (BS) and single antenna UEs. A MIMO-NOMA downlink scenario with multiple antenna users has been considered in [11]. The authors propose to use user pairing to enhance the NOMA performance and consider the identity matrix as a precoding matrix to reduce the system overhead. More sophisticated precoding and decoding based on QR and GSVD decompositions have been proposed in [12]–[18]. However, the aforementioned schemes are restricted to systems with two users in one resource block due to the limitation that the GSVD uses only two matrices.

In [19] we have proposed the ML-GSVD as an extension of the GSVD to decompose more than two matrices with one common dimension. Compared to the other multidimensional extensions of the GSVD, the ML-GSVD inherits the properties of the original decomposition, such as orthogonality and the structure of the diagonal matrices. This paper focuses on power-domain NOMA, which employs superposition coding (SC) at the transmitter and successive interference cancellation (SIC) at the receiver. We propose a new ML-GSVD based NOMA scheme for downlink MIMO systems with more than two users in one resource block. We show how the ML-GSVD can be applied to MIMO-NOMA systems with an arbitrary number of users. Additionally, we propose a new power allocation scheme based on the generalized singular values of the ML-GSVD and compare it to the schemes from the literature.

The remainder of the paper is organized as follows. We first provide the system model of the considered power-domain downlink MIMO-NOMA system. Then, we present the design of the precoding and decoding matrices based on the ML-GSVD and consider different scenarios exploiting the properties of this recently proposed decomposition. Next, we demonstrate how the generalized singular values of the ML-

ISBN 978-3-8007-5686-5

GSVD can be used for the power-allocation. We also show the numerical results to evaluate the performance of the ML-GSVD-based NOMA scheme in terms of the achievable rate and compare it to the OMA.

Notation. Matrices and vectors are denoted by upper-case and lower-case bold-faced letters, respectively. The superscripts $\{\cdot\}^{\mathrm{T}}$ and $\{\cdot\}^{\mathrm{H}}$ denote the transpose and Hermitian transpose, respectively, whereas $\operatorname{diag}\{\cdot\}$ is the operation of constructing a diagonal matrix with diagonal elements being the entries of the input vector or constracting a vector from the diagonal elements of the input matrix. The matrix $\boldsymbol{I}_d$ represents a $d \times d$ identity matrix. The i-th row and the j-th column of a matrix $\boldsymbol{A} \in \mathbb{C}^{I \times J}$ is represented by $\boldsymbol{A}(i,:) \in \mathbb{C}^{J}$ and $\boldsymbol{A}(:,j) \in \mathbb{C}^{I}$, respectively, where $i = 1, \ldots, I$ and $j = 1, \ldots, J$.

II. System model

Let us consider a downlink MIMO-NOMA communication system with one base station (BS) and K users, where the BS and the kth user are equipped with M_T and M_{R_k} antennas, respectively (Figure 1). Assuming flat fading, the $M_{R_k} \times M_T$ channel matrix from the BS to the kth user is denoted as $\boldsymbol{H}_k$. In a downlink NOMA network, the BS transmits a superposition of the desired signals of K users with different allocated power to all K users. The power allocation coefficients α_k are inversely proportional to the channel conditions of the users (e.g., path loss). The precoding matrix at the BS is denoted as $\boldsymbol{P} \in \mathbb{C}^{M_T \times M_T}$. Then, the transmitted signal can be written as $\boldsymbol{x} = \boldsymbol{P}\boldsymbol{s}$, and $\boldsymbol{s} = [s_1 \ldots s_{M_T}]^{\mathrm{T}} \in \mathbb{C}^{M_T \times 1}$ is a combined signal for all users, given by

$$\boldsymbol{s} = \sum_{k=1}^{K} \boldsymbol{\Lambda}_k \tilde{\boldsymbol{s}}_k, \tag{1}$$

where $\boldsymbol{\Lambda}_k = \operatorname{diag}[\alpha_{k,1} \ldots \alpha_{k,M_T}] \in \mathbb{R}^{M_T \times M_T}$ and $\tilde{\boldsymbol{s}}_k = [\tilde{s}_{k,1} \ldots \tilde{s}_{k,M_T}]^{\mathrm{T}} \in \mathbb{C}^{M_T \times 1}$ contain the power allocation coefficients and the data for user k, respectively. The power allocation coefficients satisfy $\sum_{k=1}^{K} \boldsymbol{\Lambda}_k^2 = \boldsymbol{I}_{M_T}$. Consequently, the received signal at the kth user can be expressed as

$$\boldsymbol{y}_k = \frac{1}{\sqrt{d_k^{\eta}}} \boldsymbol{H}_k \boldsymbol{P} \boldsymbol{s} + \boldsymbol{n}_k \in \mathbb{C}^{M_{R_k} \times 1}, \tag{2}$$

where $\boldsymbol{n}_k$ denotes the additive Gaussian noise vector of user k, whose elements are i.i.d. complex Gaussian random variables with zero mean and unit variance. The quantity $\frac{1}{\sqrt{d_k^{\eta}}}$ denotes the large-scale fading, where d_k is the distance between the BS and the kth user, and η is the path-loss exponent [13]. We assume that $\mathbb{E}\left\{\tilde{\boldsymbol{s}}\tilde{\boldsymbol{s}}^{\mathrm{H}}\right\} = \boldsymbol{I}_{M_T}$. After applying the decoding matrix $\boldsymbol{D}_k$ at the kth user we get

$$\boldsymbol{D}_k \boldsymbol{y}_k = \frac{1}{\sqrt{d_k^{\eta}}} \boldsymbol{D}_k \boldsymbol{H}_k \boldsymbol{P} \boldsymbol{s} + \boldsymbol{D}_k \boldsymbol{n}_k. \tag{3}$$

In the following section we discuss the design of the precoding and the decoding matrices.

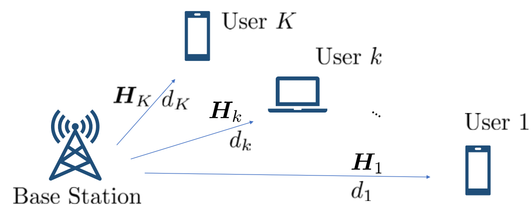

Fig. 1: Downlink MIMO-NOMA system with K users.

III. ML-GSVD-based NOMA scheme

In this paper, we propose to use the ML-GSVD [19] to define the precoding and decoding matrices $\boldsymbol{P}$ and $\boldsymbol{D}_k$ that will jointly diagonalize the channel matrices $\boldsymbol{H}_k$. The ML-GSVD of K matrices $\boldsymbol{H}_k \in \mathbb{C}^{M_k \times N}$ with the common column dimension N and a varying row dimension M_k is defined as [19]

$$\boldsymbol{H}_k = \boldsymbol{B}_k \cdot \boldsymbol{C}_k \cdot \boldsymbol{A}^{\mathrm{H}} \in \mathbb{C}^{M_k \times N}, \tag{4}$$

where $\boldsymbol{A} \in \mathbb{C}^{N \times N}$ is a square, nonsingular matrix, common for all factorizations, and the matrix $\boldsymbol{B}_k \in \mathbb{C}^{M_k \times N}$ has orthonormal columns such that $\boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{B}_k = \boldsymbol{I}_Q$. The matrices $\boldsymbol{C}_k$ are non-negative diagonal matrices, satisfying $\sum_{k=1}^{K} \boldsymbol{C}_k^2 = \boldsymbol{I}_N$. The ML-GSVD is a natural generalization of the GSVD decomposition [20], [21]. In contrast to other extensions of the GSVD, the ML-GSVD preserves the properties of the original decomposition, and for $K = 2$, the ML-GSVD of two matrices corresponds to the GSVD. For the notational simplicity, we assume in this paper that the channel matrices $\boldsymbol{H}_k$ have full rank. Depending on the choices of M_k and N, we can distinguish three different cases:

- *Case I.* $N \leq M_k$, $\forall k$. Then, the matrix $\boldsymbol{C}_k$ has the following structure

$$\boldsymbol{C}_k = \boldsymbol{\Sigma}_k \in \mathbb{R}^{N \times N}, \tag{5}$$

where $\boldsymbol{\Sigma}_k = \operatorname{diag}[\sigma_{k,1}, \cdots, \sigma_{k,N}] \in \mathbb{R}^{N \times N}$ contains the generalized singular values of the ML-GSVD, such that $0 < \sigma_{k,n} < 1$ for $n \in \{1, \cdots, N\}$. The matrices $\boldsymbol{\Sigma}_k$ are full-rank diagonal matrices, and $\sum_{k=1}^{K} \boldsymbol{\Sigma}_k^2 = \boldsymbol{I}_N$. All columns of $\boldsymbol{A}$ are shared for all factorizations, and the decomposition provides only the *common* subspace of size N for all $\boldsymbol{H}_k$s. Therefore, in this paper, we mostly focus on this case. We use the aforementioned common subspace for the MIMO-NOMA transmission of the combined signals to all K users.
- *Case II.* $N > M_k$, for some k, $N < \sum_{k=1}^{K} M_k$. This configuration provides both *private* and *common* subspaces. The matrix $\boldsymbol{C}_k$ has the following structure

$$\boldsymbol{C}_k = \operatorname{diag}\left\{\boldsymbol{0}_{p_l}^{\mathrm{T}} \quad \boldsymbol{\sigma}_k^{\mathrm{T}} \quad \boldsymbol{1}_{p_k}^{\mathrm{T}}\right\} \in \mathbb{R}^{N \times N}, \tag{6}$$

where $\boldsymbol{1}_{p_k}^{\mathrm{T}}$ is a row vector of ones, which corresponds to the private subspace of the matrix $\boldsymbol{H}_k$, while the vector of zeros $\boldsymbol{0}_{p_l}^{\mathrm{T}}$ corresponds to the private subspace of

ISBN 978-3-8007-5686-5

$\boldsymbol{H}_l, l \neq k$, and $\boldsymbol{\sigma}_k$ represents the common subspace. The dimensions of the subspaces depend on M_k, N, as well as on the realization of the tensor. Such a configuration of the tensor can provide a common subspace for all K matrices as well as for groups of matrices. In this case, in general, the decomposition can be expressed as

$$\boldsymbol{H}_k = \Big[\underbrace{\boldsymbol{O}_l}_{N_{l_k}} \; \underbrace{\hat{\boldsymbol{B}}_k^{(c)}}_{N_{c_k}} \; \underbrace{\hat{\boldsymbol{B}}_k^{(p)}}_{N_{p_k}}\Big] \cdot \boldsymbol{C}_k \cdot \begin{bmatrix} \hat{\boldsymbol{A}}_l^{\mathrm{H}(p)} \\ \hat{\boldsymbol{A}}_k^{\mathrm{H}(c)} \\ \hat{\boldsymbol{A}}_k^{\mathrm{H}(p)} \end{bmatrix} \begin{matrix} \} \, N_{l_k} \\ \} \, N_{c_k}, \\ \} \, N_{p_k} \end{matrix} \quad (7)$$

where the superscripts of the matrices denote that they belong to a common (c) or a private (p) subspace. Moreover, N_{c_k} and N_{p_k} are the dimensions of the common and private subspaces of the kth matrix, respectively. The matrix $\boldsymbol{O}_l$ is a matrix of zeros of size $M_k \times N_{l_k}$ which corresponds to the private subspaces of the matrices different from $\boldsymbol{H}_k$ such that $N_{p_k} + N_{c_k} + N_{l_k} = N$. Note that depending on the dimensions of the individual matrices, for some factorizations, the ML-GSVD provides common and private subspaces, whereas for the other factorizations only common subspaces. For example, if for some ks, $M_k < N$, the private subspace of these $\boldsymbol{H}_k$ can be empty. The columns of $\boldsymbol{A}$ which correspond to the common subspace, are shared between the corresponding matrices, while the other columns are specific for each factorization and ensure a private subspace. The common subspace can be used for the NOMA transmission of the combined signals, and the private subspace might be used for confidential messages to ensure security.

- *Case III.* $N \geq \sum_{k=1}^{K} M_k$. In this case, the entries of $\boldsymbol{C}_k$ only contain ones and zeros, i.e., only a private subspace, and the decomposition in (4) can be rewritten as

$$\begin{aligned}\boldsymbol{H}_k &= \begin{bmatrix}\boldsymbol{O}_l & \hat{\boldsymbol{B}}_k^{(p)} & \boldsymbol{O}_j\end{bmatrix} \cdot \begin{bmatrix}\boldsymbol{O}_l & & \\ & \boldsymbol{I}_{M_k} & \\ & & \boldsymbol{O}_j\end{bmatrix} \cdot \begin{bmatrix}\hat{\boldsymbol{A}}_1^{\mathrm{H}(p)} \\ \vdots \\ \hat{\boldsymbol{A}}_k^{\mathrm{H}(p)} \\ \vdots \\ \hat{\boldsymbol{A}}_K^{(p)}\end{bmatrix} \\ &= \hat{\boldsymbol{B}}_k \hat{\boldsymbol{A}}_k^{\mathrm{H}(p)}\end{aligned} \quad (8)$$

  As can be observed from (8), the matrices, in this case, do not share any common factors. All $\boldsymbol{H}_k$s are decomposed separately, and the submatrices in $\boldsymbol{A}$ correspond to a private subspace of each matrix. Thus, we do not consider Case III of the ML-GSVD for the NOMA transmission. However, since it allows to isolate the private channels to users, Case III can be utilized in conventional OMA systems without superposition coding.

Using the ML-GSVD of the channel matrices $\boldsymbol{H}_k$ in (4), the precoding matrix $\boldsymbol{P}$ can be defined as $\boldsymbol{P} = \frac{\sqrt{P}}{\beta}\{\boldsymbol{A}^{\mathrm{H}}\}^{-1}$, where P is the total transmit power at the BS, and β is a power normalization coefficient. The decoding matrix of the kth user is defined as $\boldsymbol{D}_k = \boldsymbol{B}_k^{\mathrm{H}}$. Consequently, the received data at the kth user can be written as

$$\boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{y}_k = \frac{\sqrt{P}}{\beta\sqrt{d_k^{\eta}}} \boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{H}_k \{\boldsymbol{A}^{\mathrm{H}}\}^{-1} \boldsymbol{s} + \boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{n}_k \quad (9)$$

$$= \frac{\sqrt{P}}{\beta\sqrt{d_k^{\eta}}} \boldsymbol{C}_k \boldsymbol{s} + \tilde{\boldsymbol{n}}_k, \quad (10)$$

where $\tilde{\boldsymbol{n}}_k = \boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{n}_k$. After applying the ML-GSVD, the MIMO channels $\boldsymbol{H}_k$ of K users can be considered as independent parallel sub-channels

$$y_{k,n} = \frac{\sqrt{P}}{\beta\sqrt{d_k^{\eta}}} c_{k,n} s_n + n_{k,n}, \quad (11)$$

where $y_{k,n}$, $c_{k,n}$, s_n, and $n_{k,n}$ are the nth elements of $\boldsymbol{y}_k$, $\mathrm{diag}\{\boldsymbol{C}_k\}$, $\boldsymbol{s}$, and $\tilde{\boldsymbol{n}}_k$, respectively. For notational simplicity, we assume that the first user is the farthest, and the Kth user is the closest (strongest). In NOMA, the highest power is assigned to the user with the weakest channel. Therefore, the power allocation coefficients satisfy $\alpha_{1,n}^2 > \cdots > \alpha_{K,n}^2$, $n \in \{1, \ldots, M_T\}$.

Depending on the ratio between the number of transmit and receive antennas, according to the three cases of the ML-GSVD described above, the value of $c_{k,n}$ in (11) can be equal to 0, 1, or $\sigma_{k,n}$, corresponding to private or common subspaces. Notice that signals that are transmitted on the sub-channels that correspond to private subspaces of the kth user will not be detected by other users. Therefore, if $c_{k,n} = 1$, on the nth sub-channel the user k receives

$$y_{k,n} = \frac{\sqrt{P}}{\beta\sqrt{d_k^{\eta}}} s_n + n_{k,n}, \quad (12)$$

Thus, the individual information rate of user k is given as

$$R_n^{(k)} = \log_2\left(1 + \frac{P}{\beta^2 d_k^{\eta} N_0}\right), \quad (13)$$

whereas the rate of other users $l \neq k$ is zero. In this case, the signal s_n can be designed in a way that it only contains the data of user k, which will correspond to a conventional OMA signal. For the NOMA transmission, we use the sub-channels that are shared between several users, i.e., we utilize the common subspace of the ML-GSVD to transmit the superimposed signals of multiple users. For simplicity, we assume that $M_T \leq M_{R_k}$ for all k. This scenario corresponds to the Case I of the ML-GSVD, where the common subspace is shared among all users, and no private subspace exists for any user. In general, Case II of the ML-GSVD can also be used in NOMA systems. Since this case combines both private and common subspaces, only the channels corresponding to the common subspace should be used for the NOMA communication. Assuming transmission via the common channel, we can rewrite (11) as

$$y_{k,n} = \frac{\sqrt{P}}{\beta\sqrt{d_k^{\eta}}} \sigma_{k,n} s_n + n_{k,n}, \quad (14)$$

where $\sigma_{k,n}^2$ is the generalized singular value of the ML-GSVD corresponding to the nth common sub-channel, and the signal

ISBN 978-3-8007-5686-5

s_n contains the combined data of all users. To recover the individual signals, users have to perform the SIC process. Therefore, the information rate of $\tilde{s}_{k,n}$ for the kth user is given by

$$R_n^{(k)} = \log_2\left(1 + \frac{P\alpha_{k,n}^2\sigma_{k,n}^2}{\sum_{j=k+1}^{K} P\alpha_{j,n}^2\sigma_{k,n}^2 + \beta^2 d_k^\eta N_0}\right), \quad (15)$$

where N_0 is the noise power. The closest user, user K, needs to decode the signals of all users before decoding his own. Consequently, assuming the perfect SIC, the achievable rate of the Kth user is given by

$$R_n^{(K)} = \log_2\left(1 + \frac{P\alpha_{K,n}^2\sigma_{K,n}^2}{\beta^2 d_K^\eta N_0}\right). \quad (16)$$

Next we consider the choice of the power allocation coefficients $\alpha_{k,n}$. In general, the power allocation problem is considered as one of the main problems in NOMA systems [4]. In this paper, we propose to allocate the power based on the ML-GSVD generalized singular values.

It has been mentioned before that the $\sigma_{k,n}$s from the ML-GSVD satisfy $0 < \sigma_{k,n} < 1$, and the stronger channel gets a bigger generalized singular value, i.e., $\sigma_{K,n} > \sigma_{1,n}$. This relation can be used to calculate the power allocation coefficients $\boldsymbol{\alpha}_n = [\alpha_{1,n} \dots \alpha_{K,n}]$ that account for the channel conditions of each user. Let us define the matrix $\boldsymbol{C}$ as

$$\boldsymbol{C} = 1 \oslash \begin{bmatrix} \mathrm{diag}\{\boldsymbol{C}_1\}^{\mathrm{T}} \\ \vdots \\ \mathrm{diag}\{\boldsymbol{C}_K\}^{\mathrm{T}} \end{bmatrix}, \quad (17)$$

where the matrices $\boldsymbol{C}_k$ are defined in (4), and $\oslash$ denotes the element-wise division. The values $\boldsymbol{C}(:,n) = [\frac{1}{\sigma_{1,n}} \dots \frac{1}{\sigma_{K,n}}]^{\mathrm{T}}$ are inversely proportional to the generalized singular values of the channel matrices $\boldsymbol{H}_k$. Thus, we can express the power allocation as $\boldsymbol{\alpha}_n = \hat{\boldsymbol{C}}(:,n)$, where $\hat{\boldsymbol{C}}$ denotes the matrix $\boldsymbol{C}$ with normalized columns. Such choice of $\alpha_{k,n}$s means that the power will be allocated according to the channel conditions that are represented by the generalized singular values of the channel.

IV. Simulation Results

In this section, we present the numerical results to assess the performance of the proposed ML-GSVD-based MIMO-NOMA scheme. For the simulations, we consider a downlink MIMO-NOMA scenario with one BS and three users. The noise power is calculated as $N_0 = -174 + \log_{10} B$, where the bandwidth $B = 1\,\mathrm{MHz}$. The sum rate is expressed as $R_{\mathrm{sum}} = \sum_{k=1}^{K}\sum_{n=1}^{M_T} R_n^{(k)}$, while the individual user rates are calculated as $R^{(k)} = \sum_{n=1}^{M_T} R_n^{(k)}$, where $R_n^{(k)}$ is defined in (15). All results are averaged over 1000 Monte Carlo trials.

In the first simulation, each user is equipped with $M_{R_k} = 4$ antennas, and the BS has $M_T = 3$ antennas. This scenario corresponds to the case where the ML-GSVD decomposition provides only the common subspace (*Case I*), i.e., the users receive superimposed data of all users. We set the distances between the BS and the kth user to $d_1 = 200\,\mathrm{m}$, $d_2 = 15\,\mathrm{m}$, and $d_3 = 10\,\mathrm{m}$, respectively. The path-loss exponent is equal to $\eta = 3$. Figure 2 shows the sum rate and the individual rates of each user with different power allocation coefficients. For the fixed power allocation, the coefficients are calculated according to the distances, while in the case of the ML-GSVD-based power allocation, the coefficients are calculated dynamically, based on the generalized singular values. In both cases, the precoding and decoding matrices are determined based on the proposed ML-GSVD-NOMA scheme. As is can be observed, the ML-GSVD-based dynamic power allocation shows a better performance, especially for the sum rate and the individual rates of the second and third users, which is explained by the fact that the fixed power allocation does not adapt to the small changes in the channels.

In the second simulation, we compare the performances of the proposed ML-GSVD-based scheme with the technique

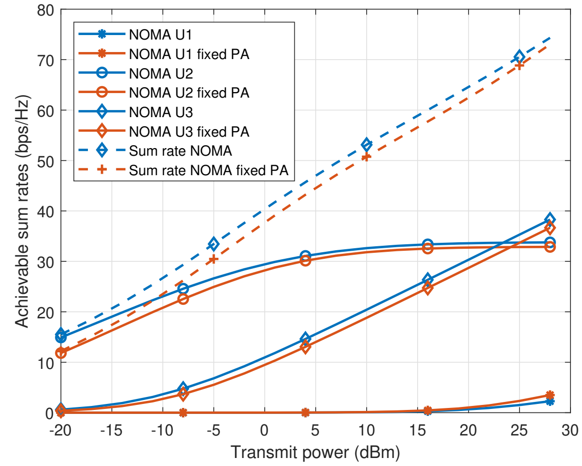

Fig. 2: Rates vs. Transmit power. Comparison of the fixed and ML-GSVD-based power allocation schemes. Parameters: $d_1 = 200\,\mathrm{m}$, $d_2 = 15\,\mathrm{m}$, and $d_3 = 10\,\mathrm{m}$, $\eta = 3$, $M_T = 3$, $M_{R_k} = 4$.

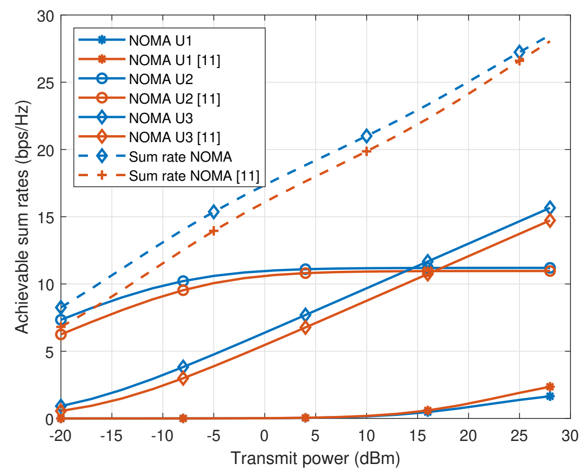

Fig. 3: Rates vs. Transmit power. Parameters: $d_1 = 200\,\mathrm{m}$, $d_2 = 15\,\mathrm{m}$, and $d_3 = 10\,\mathrm{m}$, $\eta = 3$, $M_T = 1$, $M_{R_k} = 4$.

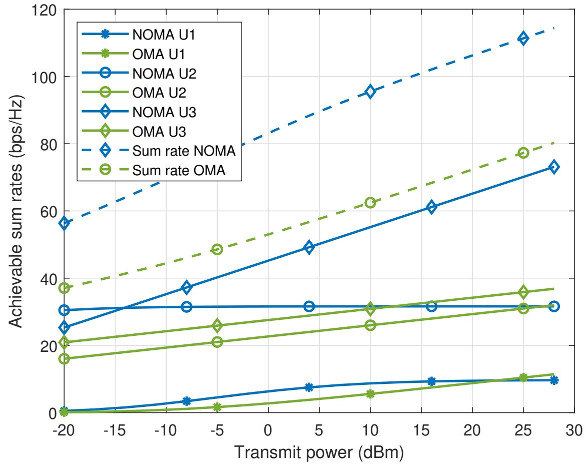

Fig. 4: Rates vs. Transmit power. Comparison of the rates achieved by OMA and NOMA. Parameters: $d_1 = 300\,\text{m}, d_2 = 10\,\text{m}$, and $d_3 = 5\,\text{m}$, $\eta = 2$, $M_T = 3, M_{R_k} = [3, 4, 6]$.

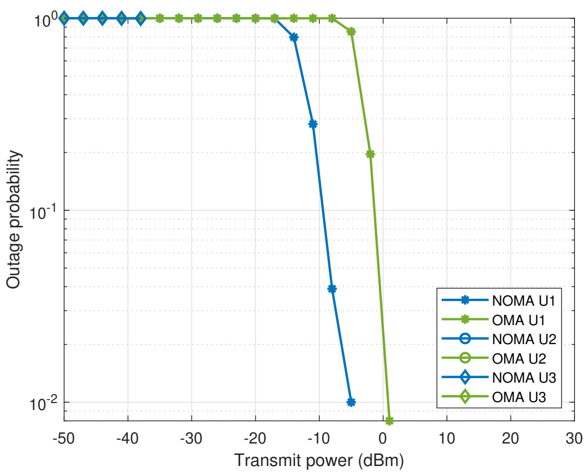

Fig. 5: Outage probability vs. Transmit power. Comparison between OMA and NOMA. Parameters: $d_1 = 300\,\text{m}, d_2 = 10\,\text{m}$, and $d_3 = 5\,\text{m}$, $\eta = 2$, $M_T = 3, M_{R_k} = [3, 4, 6]$.

proposed in [11]. We consider the scenario, where the BS has one antenna[1], and the users have four antennas each. The distances are $d_1 = 200\,\text{m}$, $d_2 = 15\,\text{m}$, and $d_3 = 10\,\text{m}$, respectively. The path-loss exponent is set to $\eta = 3$. The results are shown in Figure 3. As it can be seen, the proposed scheme outperforms the reference scheme in terms of the achievable sum rate and the rates of the second and third users. It should be noted, that the scheme in [11] is limited to the scenario where $M_T < M_R$ and assumes that all users have an equal number of antennas.

In the next simulation, we consider an asymmetric scenario, where the BS has $M_T = 3$ antennas, and three users are equipped with 3, 4, and, 6 antennas, respectively, and compare the performance of the NOMA and OMA schemes. The simulation results are depicted in Figure 4. One can see that the proposed ML-GSVD outperforms the OMA scheme in terms of the achievable rates of all three users. As it can be observed, the achievable rate of the third (strongest) user is much higher in case of NOMA, especially with the increased power, due to the absence of the interference from the weaker users. The rates of the first and the second users are approximately constant at high powers because they consider the signals of the stronger users as a noise. Figure 5 depicts the outage probabilities for NOMA and OMA. The target rates are set to 2 bps/Hz, 4 bps/Hz, and 5 bps/Hz, respectively. As it can be observed, the outage probability of the weak NOMA user is smaller than for the OMA user which makes the NOMA transmission more reliable. Whereas with a given transmit power, the strong users maintain the target rates without outages.

[1]The authors in [11] assume that the number of transmit antennas is equal to the number of clusters, which means that each cluster with K users is maintained by one antenna. Therefore, for the fair comparison, we assume a single antenna BS in this simulation.

V. Conclusions

In this paper, we have presented a new ML-GSVD-based NOMA transmission technique that can be applied in power-domain MIMO-NOMA downlink communication systems with multiple users. It utilizes the recently proposed ML-GSVD decomposition to design the precoding and decoding matrices that jointly diagonalize the channels between the BS and the users. Compared to the GSVD-based technique proposed in [13], the ML-GSVD can support more than two users on one frequency resource, and the common subspace of the ML-GSVD can be employed to transmit the combined signals of the K users. Additionally, we have presented a simple power allocation technique based on the generalized singular values of the ML-GSVD that outperforms the conventional fixed power allocation. The performance of the proposed scheme has been evaluated in terms the achievable rate and compared to traditional OMA and to the state of the art NOMA techniques.

References

[1] R. Kizilirmak, *Non-Orthogonal Multiple Access (NOMA) for 5G Networks*, 12 2016.

[2] L. Dai, B. Wang, Z. Ding, Z. Wang, S. Chen, and L. Hanzo, “A Survey of Non-Orthogonal Multiple Access for 5G,” *IEEE Communications Surveys Tutorials*, vol. 20, no. 3, pp. 2294–2323, 2018.

[3] Z. Ding, X. Lei, G. K. Karagiannidis, R. Schober, J. Yuan, and V. K. Bhargava, “A Survey on Non-Orthogonal Multiple Access for 5G Networks: Research Challenges and Future Trends,” *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 10, pp. 2181–2195, 2017.

[4] M. Aldababsa, M. Toka, S. Gökçeli, G. K. Kurt, and O. Kucur, “A tutorial on nonorthogonal multiple access for 5G and beyond,” *Wireless communications and mobile computing*, vol. 2018, 2018.

[5] S. M. R. Islam, N. Avazov, O. A. Dobre, and K.-s. Kwak, “Power-Domain Non-Orthogonal Multiple Access (NOMA) in 5G Systems: Potentials and Challenges,” *IEEE Communications Surveys Tutorials*, vol. 19, no. 2, pp. 721–742, 2017.

[6] J. Zeng, T. Lv, R. P. Liu, X. Su, M. Peng, C. Wang, and J. Mei, “Investigation on Evolving Single-Carrier NOMA Into Multi-Carrier NOMA in 5G,” *IEEE Access*, vol. 6, pp. 48 268–48 288, 2018.

[7] B. Makki, K. Chitti, A. Behravan, and M.-S. Alouini, “A Survey of NOMA: Current Status and Open Research Challenges,” *IEEE Open Journal of the Communications Society*, vol. 1, pp. 179–189, 2020.

[8] A. Benjebbour, K. Saito, A. Li, Y. Kishiyama, and T. Nakamura, “Non-orthogonal multiple access (NOMA): Concept, performance evaluation and experimental trials,” 10 2015, pp. 1–6.

[9] Y. Cai, Z. Qin, F. Cui, G. Y. Li, and J. A. McCann, “Modulation and Multiple Access for 5G Networks,” *IEEE Communications Surveys Tutorials*, vol. 20, no. 1, pp. 629–646, 2018.

[10] W. A. Al-Hussaibi and F. H. Ali, “Efficient User Clustering, Receive Antenna Selection, and Power Allocation Algorithms for Massive MIMO-NOMA Systems,” *IEEE Access*, vol. 7, pp. 31 865–31 882, 2019.

[11] Z. Ding, F. Adachi, and H. V. Poor, “The Application of MIMO to Non-Orthogonal Multiple Access,” *IEEE Transactions on Wireless Communications*, vol. 15, no. 1, pp. 537–552, 2016.

[12] Z. Ding, L. Dai, and H. V. Poor, “MIMO-NOMA Design for Small Packet Transmission in the Internet of Things,” *IEEE Access*, vol. 4, pp. 1393–1405, 2016.

[13] Z. Chen, Z. Ding, X. Dai, and R. Schober, “Asymptotic Performance Analysis of GSVD-NOMA Systems With a Large-Scale Antenna Array,” *IEEE Transactions on Wireless Communications*, vol. 18, no. 1, pp. 575–590, 2019.

[14] C. Rao, Z. Ding, and X. Dai, “GSVD-Based MIMO-NOMA Security Transmission,” *IEEE Wireless Communications Letters*, pp. 1–1, 2021.

[15] ——, “The Distribution Characteristics of Ordered GSVD Singular Values and its Applications in MIMO-NOMA,” *IEEE Communications Letters*, vol. 24, no. 12, pp. 2719–2722, 2020.

[16] Y. Qi and M. Vaezi, “Secure Transmission in MIMO-NOMA Networks,” *IEEE Communications Letters*, vol. 24, no. 12, pp. 2696–2700, 2020.

[17] M. F. Hanif and Z. Ding, “Robust Power Allocation in MIMO-NOMA Systems,” *IEEE Wireless Communications Letters*, vol. 8, no. 6, pp. 1541–1545, 2019.

[18] Z. Ma, Z. Ding, P. Fan, and S. Tang, “A General Framework for MIMO Uplink and Downlink Transmissions in 5G Multiple Access,” in *2016 IEEE 83rd Vehicular Technology Conference (VTC Spring)*, 2016, pp. 1–4.

[19] L. Khamidullina, A. L. F. de Almeida, and M. Haardt, “Multilinear Generalized Singular Value Decomposition (ML-GSVD) with Application to Coordinated Beamforming in Multi-user MIMO Systems,” in *ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2020, pp. 4587–4591.

[20] C. F. Van Loan, “Generalizing the Singular Value Decomposition,” *SIAM Journal on Numerical Analysis*, vol. 13, pp. 76–83, Mar. 1976.

[21] C. Paige and M. Saunders, “Towards a Generalized Singular Value Decomposition,” *SIAM Journal on Numerical Analysis*, vol. 18, no. 3, pp. 398–405, 1981. [Online]. Available: https://doi.org/10.1137/0718026