

# Distributed Coordinated Beamforming for RIS-Aided Dynamic TDD Systems

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**Abstract**—We consider the joint design of the active and passive beamforming for a reconfigurable intelligent surface (RIS) aided dynamic time-division-duplexing (DTDD) wireless network. An alternating optimization (AO) method is proposed. To reduce the high signaling overhead involved in the centralized solution for the active beamforming design, a distributed coordinated beamforming based on the alternating direction method of multipliers (ADMM) is proposed. The Semidefinite Programming (SDP) technique is adopted for the design of the passive reflection matrix of the RIS. Our design objective is to maximize the minimum signal-to-interference-plus-noise ratio (SINR) of the downlink users while satisfying the total power constraint of the downlink base stations and guaranteeing that the maximum interference seen by the uplink users due to the transmission of the downlink cells is below a pre-defined level. Our numerical results demonstrate that the proposed algorithm converges to the centralized solution in a reasonable number of iterations.

**Index Terms**—Dynamic TDD, MIMO communications, small cells, RIS, ADMM, distributed algorithm.

## I. INTRODUCTION

Due to the rapid advancement in technology and the emergence of diverse mobile applications, there is a massive increase in mobile data traffic generation and demand for higher data rates for the beyond fifth-generation (B5G) wireless communication systems. The deployment of small cell networks has emerged as an effective way to meet the exponential growth of traffic demands for 5G and future wireless networks [1]. Dynamic TDD (DTDD) has been proposed as a solution to satisfy the asymmetric and dynamic traffic demand of small cells [2]. In a DTDD enabled wireless communication system, each cell individually decides its schedule for the uplink and downlink mode operation across different time slots, based on its instantaneous traffic demand and/or interference status [3].

However, the main challenge brought by DTDD is the cross-link interference issue, because adjacent cells may use at a given time different TDD frame configurations according to traffic needs, thereby giving rise to opposite transmission directions among neighboring cells. There are two kinds of cross-link interference: base station-to-base station (BS-to-BS) and user equipment-to-user equipment (UE-to-UE) interference, which may degrade the system performance significantly. Between the two, the BS-to-BS interference is extremely detrimental due to the large transmit power and line-of-sight (LOS) propagation characteristics. Interference management strategies in DTDD systems are more complicated than those in static TDD systems due to the cross-link interference.

Therefore, there is a need to develop an efficient cross-link interference management scheme.

Reconfigurable intelligent surfaces (RISs) have gained significant attention recently, as a low-cost and compact transformational technology for future wireless systems [4]-[6]. In a passive RIS-aided communication system, where the RIS has no radio-frequency chains, the phase shifts of the RIS elements can be adjusted to meet a certain cost function, e.g., the reflected signals add constructively at the intended users and/or destructively at the unintended users. In [7], we demonstrated that the intelligent spectrum control of the RIS can be exploited for interference management in an RIS-aided DTDD system. However, such an implementation may be impractical as a result of the high signaling overhead involved in the collection of all the channel state information at the central processing unit (CPU). To address this issue, we propose a distributed algorithm using the Alternating Direction Method of Multipliers (ADMM) [8] technique for RIS-aided DTDD wireless networks. ADMM is a powerful decomposition technique that blends the superior convergence properties of dual decomposition and the numerical robustness of augmented Lagrangian methods [8].

In this paper, we propose an alternating-optimization (AO) based algorithm for the design of the transmit beamforming vectors and passive reflection coefficients for an RIS-aided DTDD system. Our design objective is to maximize the minimum signal-to-interference-plus-noise ratio (SINR) of the downlink (DL) users while satisfying the total power constraint of the DL BSs and guaranteeing that the maximum interference seen by the uplink (UL) users due to the transmission of the DL cells is below a pre-defined level. We propose a distributed algorithm for the design of the transmit beamforming vectors based on ADMM that can be implemented in a distributed manner at each downlink DL BSs with reasonable amount of information exchange between the coupled DL BSs. We adopt the semidefinite programming (SDP) technique for the design of the passive reflection coefficients of the RIS. We consider the discrete phase-shift model for the RIS. Moreover, we assume that the directions of the small cells have been optimized *a priori*, e.g., using the proposed cell reconfiguration method in [9], and are known at the CPU.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we consider an RIS-aided mmWave DTDD system consisting of  $Q$  small cells, where each cell has a BS with a uniform linear array (ULA) of  $N$  antennas serving  $K$  UEs, each equipped with a single antenna<sup>1</sup>. We assume that the communication is aided by an RIS with  $M$  passive reflecting elements, where the BSs and the RIS are controlled by a CPU via backhaul connections as illustrated in Fig. 1. Let  $\mathcal{Q} \triangleq \{1, \dots, Q\}$  and  $\mathcal{K} \triangleq \{\mathcal{K}_1, \dots, \mathcal{K}_Q\}$  denote the set of all BSs (cells) and UEs, respectively, whereas  $\mathcal{K}_q$  denotes the set of UEs served by BS <sub>$q$</sub> . At the considered time instant, we assume that there are  $|\mathcal{Q}^{\text{ul}}|$  cells operating in the UL direction and  $|\mathcal{Q}^{\text{dl}}|$  cells operating in the DL direction, such that  $|\mathcal{Q}^{\text{ul}}| + |\mathcal{Q}^{\text{dl}}| = Q$  and  $\mathcal{Q}^{\text{ul}} \cap \mathcal{Q}^{\text{dl}} = \emptyset$ . Equally, we assume that there are  $|\mathcal{K}^{\text{ul}}|$  and  $|\mathcal{K}^{\text{dl}}|$  UEs in the UL and DL direction, respectively, such that  $|\mathcal{K}^{\text{ul}}| + |\mathcal{K}^{\text{dl}}| = QK$  and  $\mathcal{K}^{\text{ul}} \cap \mathcal{K}^{\text{dl}} = \emptyset$ .

Let  $\mathbf{H}_q \in \mathbb{C}^{M \times N}$  be the channel matrix from the  $q$ th BS to the RIS,  $\mathbf{h}_{qk} \in \mathbb{C}^M$  be the channel vector from the RIS to the  $k$ th UE in the  $q$ th BS,  $\mathbf{G}_{r,q} \in \mathbb{C}^{N \times N}$  be the channel matrix from the  $q$ th BS to the  $r$ th BS,  $\mathbf{g}_{qk,q} \in \mathbb{C}^N$  be the channel vector from the  $q$ th BS to the  $k$ th UE, and  $g_{r,j,qk} \in \mathbb{C}$  be the channel scalar from  $k$ th UE to the  $j$ th UE. Denote  $b$  as the number of quantization bits of the RIS, such that the number of phase levels is  $2^b$ . The resulting discrete phase shifts of the RIS are expressed as

$$\mathcal{F} \triangleq \left\{ 0, \frac{2\pi}{2^b}, 2\frac{2\pi}{2^b}, \dots, (2^b - 1)\frac{2\pi}{2^b} \right\} \quad (1)$$

The SINR of the  $k$ th UE in the  $q$ th DL cell, i.e.,  $q \in \mathcal{Q}^{\text{dl}}$ , can be expressed as

$$\Gamma_{qk}^{(\text{dl})} = \frac{|\mathbf{h}_{qk}^{\text{BS-UE}} \mathbf{H}_q \mathbf{f}_{qk}|^2}{\sum_{\substack{j \in \mathcal{K}_q^{\text{dl}} \\ j \neq k}} |(\mathbf{h}_{qk}^{\text{BS-UE}})^H \mathbf{f}_{qj}|^2 + \sum_{\substack{n \in \mathcal{Q}^{\text{dl}} \\ n \neq q}} \sum_{i \in \mathcal{K}_n^{\text{dl}}} |(\mathbf{h}_{qk}^{\text{BS-UE}})^H \mathbf{f}_{ni}|^2 + \beta_{qk}} \quad (2)$$

where  $\mathbf{h}_{qk}^{\text{BS-UE}} = (\mathbf{h}_{qk}^H \mathbf{\Theta} \mathbf{H}_q + \mathbf{g}_{qk,q}^H)^H$ ,  $\mathbf{\Theta} = \text{diag}(\boldsymbol{\theta})$  is the RIS reflection diagonal matrix,  $\boldsymbol{\theta} = [e^{j\phi_1}, \dots, e^{j\phi_M}]^T \in \mathbb{C}^M$  with  $\phi_m \in \mathcal{F}$ , and  $\mathbf{f}_{qk} \in \mathbb{C}^N$  is the transmit precoding vector with  $\|\mathbf{f}_{qk}\|_2^2 = p_{qk}^{\text{dl}}$ . Additionally,  $\beta_{qk} = \alpha_{qk} + \sigma_{qk}^2$ , where  $\alpha_{qk} = \sum_{r \in \mathcal{Q}^{\text{ul}}} \sum_{j \in \mathcal{K}_r^{\text{ul}}} |h_{qk,rj}^{\text{UE-UE}}|^2 p_{rj}^{\text{ul}}$  is the interference from the UL UEs, whereas,  $h_{qk,rj}^{\text{UE-UE}} = \mathbf{h}_{qk}^H \mathbf{\Theta} \mathbf{h}_{rj} + g_{qk,rj}$  and  $\sigma_{qk}^2$  is noise variance. We assume that  $\alpha_{qk}$  is known at the  $q$ th DL BS.

Furthermore, the total received BS-BS interference power at the  $r$ th UL BS, i.e.,  $r \in \mathcal{Q}^{\text{ul}}$ , from the DL BSs, can be expressed as

$$I_{pr} = \sum_{q \in \mathcal{Q}^{\text{dl}}} \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r,q}^{\text{BS-BS}} \mathbf{f}_{qk}\|_2^2, \quad (3)$$

where  $\mathbf{H}_{r,q}^{\text{BS-BS}} = \mathbf{H}_r^H \mathbf{\Theta} \mathbf{H}_q + \mathbf{G}_{r,q}$ .

In this paper, we assume that the above defined channels are estimated in advance, e.g., by using the approach in [10].

<sup>1</sup>Notation: Vectors and matrices are written as lowercase and uppercase boldface letters, respectively. The notation  $\diamond$  is used to denote the Khatri-Rao product. The transpose and the conjugate transpose (Hermitian) of  $\mathbf{X}$  are represented by  $\mathbf{X}^T$  and  $\mathbf{X}^H$ , respectively. The  $\text{diag}(\mathbf{x})$  forms a matrix by placing  $\mathbf{x}$  on its main diagonal, and  $\text{vec}(\mathbf{X})$  vectorizes  $\mathbf{X}$  by stacking its columns on top of each other.

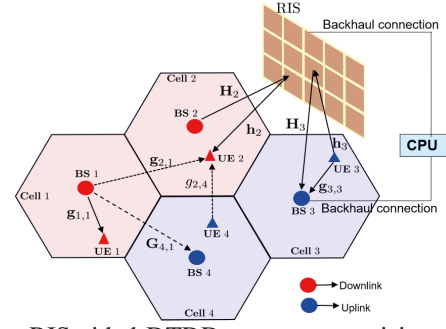


Fig. 1: An RIS-aided DTDD system comprising  $Q = 4$  cells

### A. Problem Formulation

In this paper, we jointly optimize the passive reflection vector of the RIS and the active transmit beamforming vectors of the DL BSs to improve the communication performance. To provide fairness among the DL UEs, we consider the maximization of the minimum SINR of the DL UEs subject to per BS power constraints and the condition that the maximum interference received by each UL cell being less than a QoS imposed threshold. Moreover, we assume the constant modulus constraints (CMCs) for the RIS reflection coefficients. Hence, the system-wide optimization problem is given as

$$\begin{aligned} \max_{\mathbf{f}, \boldsymbol{\theta}} \quad & \min_{q_k} \Gamma_{qk}^{(\text{dl})} \\ \text{s.t.} \quad & \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{f}_{qk}\|_2^2 \leq p_q^{\text{dl}}, \forall q \in \mathcal{Q}^{\text{dl}} \\ & \sum_{q \in \mathcal{Q}^{\text{dl}}} \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r,q}^{\text{BS-BS}} \mathbf{f}_{qk}\|_2^2 \leq \tau_r, \forall r \in \mathcal{Q}^{\text{ul}} \\ & |\boldsymbol{\theta}[m]|^2 = 1, m = 1, \dots, M, \\ & \phi_m \in \mathcal{F} \end{aligned} \quad (4)$$

where  $p_q^{\text{dl}}$  is the maximum transmit power allowed for  $q$ th DL BS and  $\tau_r$  is the predefined interference threshold in order to guarantee a certain QoS of the UL cells. Problem (4) is non-convex due to its joint optimization and the CMCs. Therefore, we adopt the AO approach to update the active transmit beamforming vectors and the passive RIS reflection vector sequentially.

## III. OPTIMIZATION METHOD

In this section, we utilize the AO framework to decouple and simplify the design of the transmit beamforming vectors and the passive RIS reflection vector.

### A. Transmit Beamforming Design

For a given  $\boldsymbol{\theta}$ , we design the transmit beamforming vectors  $\mathbf{f}_q, \forall q \in \mathcal{Q}^{\text{dl}}$ , as the solution to the problem given as

$$\begin{aligned} \max_{\mathbf{f}} \quad & \min_{q_k} \Gamma_{qk}^{(\text{dl})} \\ \text{s.t.} \quad & \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{f}_{qk}\|_2^2 \leq p_q^{\text{dl}}, \forall q \in \mathcal{Q}^{\text{dl}} \\ & \sum_{q \in \mathcal{Q}^{\text{dl}}} \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r,q}^{\text{BS-BS}} \mathbf{f}_{qk}\|_2^2 \leq \tau_r, \forall r \in \mathcal{Q}^{\text{ul}} \end{aligned} \quad (5)$$

Problem (5) can be recast in the epigraph form [11] as

$$\begin{aligned}
 & \max_{\mathbf{f}, \varphi} \quad \varphi \\
 & \text{s.t.} \quad \Gamma_{q_k}^{(\text{dl})} \geq \varphi \\
 & \quad \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{f}_{q_k}\|_2^2 \leq p_q^{\text{dl}}, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \sum_{\forall q \in \mathcal{Q}^{\text{dl}}} \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r,q}^{\text{BS-BS}} \mathbf{f}_{q_k}\|_2^2 \leq \tau_r, \forall r \in \mathcal{Q}^{\text{ul}}
 \end{aligned} \quad (6)$$

A centralized solution to (6) can be obtained using the bisection method [12], [13]. However, the centralized solution is impractical as a result of the high signaling overhead involved in the collection of all the channel state information at the CPU. Hence, to reduce the signaling overhead and workload on the CPU, in the following, we propose a distributed algorithm based on the ADMM approach [8].

To begin, we introduce slack variables which represent the local copies of the variables that couple all the DL BSs. The resulting equivalent problem is expressed as

$$\begin{aligned}
 & \min \quad -\varphi \\
 & \text{s.t.} \quad \tilde{\Gamma}_{q_k}^{(\text{dl})} \geq \gamma_q \\
 & \quad \lambda_{n_j,q}^2 \geq \sum_{k \in \mathcal{K}_q^{\text{dl}}} |(\mathbf{h}_{n_j,q}^{\text{BS-UE}})^H \mathbf{f}_{q_k}|^2, \forall j \in \mathcal{K}_n^{\text{dl}}, \forall n, q \in \mathcal{Q}^{\text{dl}}, n \neq q \\
 & \quad \phi_{r_j,q}^2 \geq \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r_j,q}^{\text{BS-BS}} \mathbf{f}_{q_k}\|_2^2, \forall j \in \mathcal{K}_r^{\text{ul}}, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r_j,q}^{\text{BS-BS}} \mathbf{f}_{q_k}\|_2^2 + \sum_{\forall n \in \mathcal{Q}^{\text{dl}}, n \neq q} \bar{\phi}_{r_j,n}^2 \leq \tau_r, \forall r \in \mathcal{Q}^{\text{ul}} \\
 & \quad \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{f}_{q_k}\|_2^2 \leq p_q^{\text{dl}}, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \bar{\lambda}_{q_k,n} = \mu_{q_k,n}, \\
 & \quad \lambda_{n_j,q} = \mu_{n_j,q}, \forall j \in \mathcal{K}_n^{\text{dl}}, \forall n, q \in \mathcal{Q}^{\text{dl}}, n \neq q \\
 & \quad \phi_{r_j,q} = \psi_{r_j,q}, \forall j \in \mathcal{K}_r^{\text{ul}}, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \bar{\phi}_{r_j,n} = \psi_{r_j,n}, \forall j \in \mathcal{K}_r^{\text{ul}}, \forall n \in \mathcal{Q}^{\text{dl}} \\
 & \quad \gamma_q = \varphi
 \end{aligned} \quad (7)$$

where  $\tilde{\Gamma}_{q_k}^{(\text{dl})} = \frac{|(\mathbf{h}_{q_k,q}^{\text{BS-UE}})^H \mathbf{f}_{q_k}|^2}{\sum_{j \in \mathcal{K}_q^{\text{dl}}, j \neq k} |(\mathbf{h}_{q_k,q}^{\text{BS-UE}})^H \mathbf{f}_{q_j}|^2 + \sum_{n \in \mathcal{Q}^{\text{dl}}, n \neq q} \bar{\lambda}_{q_k,n}^2 + \beta_{q_k}}$ , and  $\gamma_q, \lambda_{n_j,q}, \bar{\lambda}_{q_k,n}, \phi_{r_j,q}$ , and  $\bar{\phi}_{r_j,n}$  are the newly introduced slack variables representing the local copies of the SINR variable, the interference generated by BS  $q$  to user  $n_j$ , the interference at user  $q_k$  generated by neighboring DL BS  $n$  ( $n \neq q$ ), and the UL interference variable from DL BS  $q$  ( $n$ ) respectively. Furthermore,  $\varphi, \mu_{q_k,n}, \mu_{n_j,q}, \psi_{r_j,q}$ , and  $\psi_{r_j,n}$  are the respective global copies to satisfy equivalence between (6) and (7). Next, for convenience, we include all the constraints that can be handled locally at BS  $q$  in the set  $\mathcal{W}_q$  which is defined as

$$\begin{aligned}
 \mathcal{W}_q = \{ & \mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q \mid \tilde{\Gamma}_{q_k}^{(\text{dl})} \geq \gamma_q, \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{f}_{q_k}\|_2^2 \leq p_q^{\text{dl}}, \\
 & \phi_{r_j,q}^2 \geq \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r_j,q}^{\text{BS-BS}} \mathbf{f}_{q_k}\|_2^2, \lambda_{n_j,q}^2 \geq \sum_{k \in \mathcal{K}_q^{\text{dl}}} |(\mathbf{h}_{q,n_j}^{\text{BS-UE}})^H \mathbf{f}_{q_k}|^2 \\
 & \left. \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r_j,q}^{\text{BS-BS}} \mathbf{f}_{q_k}\|_2^2 + \sum_{\forall n \in \mathcal{Q}^{\text{dl}}, n \neq q} \bar{\phi}_{r_j,n}^2 \leq \tau_r \right\}
 \end{aligned} \quad (8)$$

Therefore, problem (7) can be compactly written as

$$\begin{aligned}
 & \min_{\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q, \varphi} \quad -\varphi \\
 & \text{s.t.} \quad \{\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q\} \in \mathcal{W}_q, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \boldsymbol{\lambda}_q = \boldsymbol{\mu}_q, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \boldsymbol{\phi}_q = \boldsymbol{\psi}_q, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \gamma_q = \varphi, \forall q \in \mathcal{Q}^{\text{dl}}
 \end{aligned} \quad (9)$$

where

$$\begin{aligned}
 \boldsymbol{\lambda}_q &= [\{\lambda_{n_j,q}\}_{j \in \mathcal{K}_n^{\text{dl}}, n \in \mathcal{Q}^{\text{dl}}}, \{\bar{\lambda}_{q_k,n}\}_{k \in \mathcal{K}_q^{\text{dl}}, n \in \mathcal{Q}^{\text{dl}}}]^T \\
 \boldsymbol{\mu}_q &= [\{\mu_{n_j,q}\}_{j \in \mathcal{K}_n^{\text{dl}}, n \in \mathcal{Q}^{\text{dl}}}, \{\bar{\mu}_{q_k,n}\}_{k \in \mathcal{K}_q^{\text{dl}}, n \in \mathcal{Q}^{\text{dl}}}]^T \\
 \boldsymbol{\phi}_q &= [\{\phi_{r_j,q}\}_{j \in \mathcal{K}_r^{\text{ul}}, r \in \mathcal{Q}^{\text{ul}}, q \in \mathcal{Q}^{\text{dl}}}]^T \\
 \boldsymbol{\psi}_q &= [\{\psi_{r_j,q}\}_{j \in \mathcal{K}_r^{\text{ul}}, r \in \mathcal{Q}^{\text{ul}}, q \in \mathcal{Q}^{\text{dl}}}]^T
 \end{aligned}$$

$\boldsymbol{\lambda}_q$  and  $\boldsymbol{\mu}_q$  are the transmitter specific interference and consistency vectors respectively; while the vectors  $\boldsymbol{\phi}_q$ , and  $\boldsymbol{\psi}_q$  are the transmitter specific UL interference and consistency vectors, and  $\mathcal{Q}_q^{\text{dl}} \triangleq \mathcal{Q}^{\text{dl}} \setminus q$ , i.e.,  $\mathcal{Q}_q^{\text{dl}}$  is the set including indices of all other DL BSs. Furthermore, we define the following indicator function  $I_q(\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q)$

$$I_q(\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q) = \begin{cases} 0 & (\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q) \in \mathcal{W}_q \\ \infty & \text{otherwise} \end{cases} \quad (10)$$

It is routine to verify that  $\sum_{q \in \mathcal{Q}^{\text{dl}}} \gamma_q = |\mathcal{Q}^{\text{dl}}| \varphi$ , therefore, problem (9) can be expressed equivalently as

$$\begin{aligned}
 & \min_{\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q, \varphi} \quad \sum_{q \in \mathcal{Q}^{\text{dl}}} \left( -\frac{\gamma_q}{|\mathcal{Q}^{\text{dl}}|} + I_q(\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q) \right) \\
 & \text{s.t.} \quad \boldsymbol{\lambda}_q = \boldsymbol{\mu}_q, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \boldsymbol{\phi}_q = \boldsymbol{\psi}_q, \forall q \in \mathcal{Q}^{\text{dl}} \\
 & \quad \gamma_q = \varphi, \forall q \in \mathcal{Q}^{\text{dl}}
 \end{aligned} \quad (11)$$

According to the ADMM concept, we first form the augmented Lagrangian of (11), which is given as

$$\begin{aligned}
 \mathcal{L}_\rho(\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q, \boldsymbol{\chi}_q, \boldsymbol{\xi}_q, \eta_q, \boldsymbol{\mu}_q, \boldsymbol{\psi}_q, \varphi) \\
 &= \sum_{q \in \mathcal{Q}^{\text{dl}}} \left( -\frac{\gamma_q}{|\mathcal{Q}^{\text{dl}}|} + I_q(\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q) + \frac{\rho}{2} \left| \gamma_q - \varphi + \frac{1}{\rho} \eta_q \right|^2 \right. \\
 & \quad \left. + \frac{\rho}{2} \left\| \boldsymbol{\lambda}_q - \boldsymbol{\mu}_q + \frac{1}{\rho} \boldsymbol{\chi}_q \right\|_2^2 + \frac{\rho}{2} \left\| \boldsymbol{\phi}_q - \boldsymbol{\psi}_q + \frac{1}{\rho} \boldsymbol{\xi}_q \right\|_2^2 \right),
 \end{aligned} \quad (12)$$

where  $\rho \geq 0$  is the penalty parameter which helps to achieve numerical stability and faster convergence for the ADMM algorithm [8]. Note that the Lagrangian function (12) is completely separable between the DL BSs. Therefore, in the first step of each iteration of the ADMM algorithm, each BS $_q, \forall q \in \mathcal{Q}^{\text{dl}}$  updates the local variables  $(\mathbf{F}_q^{(t+1)}, \boldsymbol{\lambda}_q^{(t+1)}, \boldsymbol{\phi}_q^{(t+1)}, \gamma_q^{(t+1)})$  by solving the following optimization problem

$$\begin{aligned}
 & \min_{\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q} \quad -\frac{\gamma_q}{|\mathcal{Q}^{\text{dl}}|} + \frac{\rho}{2} |\gamma_q - \varphi^{(t)} + \frac{1}{\rho} \eta_q^{(t)}|^2 + \frac{\rho}{2} \varkappa_q(\boldsymbol{\lambda}_q, \boldsymbol{\phi}_q) \\
 & \text{s.t.} \quad \{\mathbf{F}_q, \boldsymbol{\lambda}_q, \boldsymbol{\phi}_q, \gamma_q\} \in \mathcal{W}_q
 \end{aligned} \quad (13)$$

where

$$\varkappa_q(\boldsymbol{\lambda}_q, \boldsymbol{\phi}_q) = \frac{\rho}{2} \left\| \boldsymbol{\lambda}_q - \boldsymbol{\mu}_q^{(t)} + \frac{1}{\rho} \boldsymbol{\chi}_q^{(t)} \right\|_2^2 + \frac{\rho}{2} \left\| \boldsymbol{\phi}_q - \boldsymbol{\psi}_q^{(t)} + \frac{1}{\rho} \boldsymbol{\xi}_q^{(t)} \right\|_2^2$$

Problem (13) is non-convex due to the non-convex SINR

constraint in the set  $\mathcal{W}_q$ . However, for a fixed  $\gamma_q$ , the problem can be recast as a second-order cone programming (SOCP) problem, which can easily be solved by the CVX tool [14]. According to [15, Algorithm 2], we obtain the optimal  $\gamma_q^*$  used in solving for the other local variables of (13).

Next, the augmented Lagrangian function (12) is minimized over the global variables  $\boldsymbol{\mu}_q, \boldsymbol{\psi}_q$ , and  $\varphi$  while all the other variables are fixed at the current values. This can be expressed as

$$\min_{\boldsymbol{\mu}_q, \boldsymbol{\psi}_q, \varphi} \sum_{q \in \mathcal{Q}^{\text{dl}}} \left( \frac{\rho}{2} \left| \gamma_q^{(t+1)} - \varphi + \frac{1}{\rho} \eta_q^{(t)} \right|^2 + \frac{\rho}{2} \left\| \boldsymbol{\lambda}_q^{(t+1)} - \boldsymbol{\mu}_q + \frac{1}{\rho} \boldsymbol{\chi}_q^{(t)} \right\|_2^2 + \frac{\rho}{2} \left\| \boldsymbol{\phi}_q^{(t+1)} - \boldsymbol{\psi}_q + \frac{1}{\rho} \boldsymbol{\xi}_q^{(t)} \right\|_2^2 \right) \quad (14)$$

The problem in (14) is convex and separable in variables  $\{\boldsymbol{\mu}_q, \boldsymbol{\psi}_q\}_{q \in \mathcal{Q}^{\text{dl}}}$  and  $\varphi$ . Therefore, by setting the gradient of the cost function in (14) with respect to the optimization variables equal to zero, the global variables are updated as follows

$$\boldsymbol{\mu}_{n_j, q}^{(t+1)} = \frac{1}{2} \left( \lambda_{n_j, q}^{(t+1)} + \bar{\lambda}_{q_k, n}^{(t+1)} \right) \quad (15a)$$

$$\boldsymbol{\psi}_{r_j, q}^{(t+1)} = \frac{1}{|\mathcal{Q}^{\text{dl}}|} \left( \sum_{q \in \mathcal{Q}^{\text{dl}}} \left( \phi_{r_j, q}^{(t+1)} + \frac{1}{\rho} \xi_q^{(t)} \right) \right) \quad (15b)$$

$$\varphi^{(t+1)} = \frac{1}{|\mathcal{Q}^{\text{dl}}|} \left( \sum_{q \in \mathcal{Q}^{\text{dl}}} \left( \gamma_q^{(t+1)} + \frac{1}{\rho} \eta_q^{(t)} \right) \right) \quad (15c)$$

Note that (15) is updated after the DL BSs have exchanged their respective local variables. Finally, each BS $_q, \forall q \in \mathcal{Q}^{\text{dl}}$  updates the dual variables associated with the equality constraints as

$$\boldsymbol{\chi}_q^{(t+1)} = \boldsymbol{\chi}_q^{(t)} + \rho(\boldsymbol{\lambda}_q^{(t+1)} - \boldsymbol{\mu}_q^{(t+1)}) \quad (16)$$

$$\boldsymbol{\xi}_q^{(t+1)} = \boldsymbol{\xi}_q^{(t)} + \rho(\boldsymbol{\phi}_q^{(t+1)} - \boldsymbol{\psi}_q^{(t+1)}) \quad (17)$$

$$\boldsymbol{\eta}_q^{(t+1)} = \boldsymbol{\eta}_q^{(t)} + \rho(\gamma_q^{(t+1)} - \varphi^{(t+1)}) \quad (18)$$

We remark that the convergence of the ADMM scheme for convex problems has been established in [8], [16] and in [17] for non-convex problems.

## B. RIS Reflection Matrix Design

With the obtained solution of problem (7), the second sub-problem becomes a feasibility check problem for finding the phase shift matrix  $\boldsymbol{\Theta}$ , with fixed  $\mathbf{F}$  and  $\varphi$ . The problem can be written as

Find  $\boldsymbol{\Theta}$

$$\begin{aligned} \text{s.t. } & \Gamma_{q_k}^{(\text{dl})} \geq \varphi, \forall q \in \mathcal{Q}^{\text{dl}} \\ & \sum_{\forall q \in \mathcal{Q}^{\text{dl}}} \sum_{k \in \mathcal{K}_q^{\text{dl}}} \|\mathbf{H}_{r, q}^{\text{BS-BS}} \mathbf{f}_{q_k}\|^2 \leq \tau_r, \forall r \in \mathcal{Q}^{\text{ul}} \\ & |[\boldsymbol{\theta}]_{[m]}|^2 = 1, m = 1, \dots, M, \\ & \phi_m \in \mathcal{F} \end{aligned} \quad (19)$$

By applying a change of variable,  $\Gamma_{q_k}^{(\text{dl})}$  can be expressed as

$$\Gamma_{q_k}^{(\text{dl})} = \frac{|\mathbf{a}_{q_k}^{\text{H}} \boldsymbol{\theta} + b_{q_k}|^2}{\|\mathbf{C}_{q_k} \boldsymbol{\theta} + \mathbf{d}_{q_k}\|^2 + \sigma_{q_k}^2} \quad (20)$$

where

$$\mathbf{a}_{q_k} = \text{diag}(\mathbf{h}_{q_k, q}) \mathbf{H}_q \mathbf{f}_{q_k} \quad (21)$$

$$b_{q_k} = \mathbf{g}_{q_k, q}^{\text{H}} \mathbf{f}_{q_k} \quad (22)$$

$$\mathbf{d}_{q_k} = \text{stack}\{u_{q_k}^{\text{INT}}, u_{q_k}^{\text{ICI}}, u_{q_k}^{\text{CL}}\} \quad (23)$$

$$\mathbf{C}_{q_k} = \text{stack}\{\mathbf{e}_{q_k}^{\text{INT}}, \mathbf{e}_{q_k}^{\text{ICI}}, \mathbf{e}_{q_k}^{\text{CL}}\}, \quad (24)$$

and

$$u_{q_k}^{\text{INT}} = \sum_{j \in \mathcal{K}_q^{\text{dl}}, j \neq k} \mathbf{g}_{q_k, q}^{\text{H}} \mathbf{f}_{q_j} \quad (25a)$$

$$u_{q_k}^{\text{ICI}} = \sum_{n \in \mathcal{Q}^{\text{dl}}} \sum_{\substack{i \in \mathcal{K}_n^{\text{dl}} \\ n \neq q}} \mathbf{g}_{q_k, n}^{\text{H}} \mathbf{f}_{n_i} \quad (25b)$$

$$u_{q_k}^{\text{CL}} = \sum_{r \in \mathcal{Q}^{\text{ul}}} \sum_{j \in \mathcal{K}_r^{\text{ul}}} g_{q_k, r_j} \quad (25c)$$

$$\mathbf{e}_{q_k}^{\text{INT}} = \sum_{j \in \mathcal{K}_q^{\text{dl}}, j \neq k} \text{diag}(\mathbf{h}_{q_k, q}) \mathbf{H}_q \mathbf{f}_{q_j} \quad (25d)$$

$$\mathbf{e}_{q_k}^{\text{ICI}} = \sum_{n \in \mathcal{Q}^{\text{dl}}} \sum_{\substack{i \in \mathcal{K}_n^{\text{dl}} \\ n \neq q}} \text{diag}(\mathbf{h}_{q_k, n}) \mathbf{H}_n \mathbf{f}_{n_i} \quad (25e)$$

$$\mathbf{e}_{q_k}^{\text{CL}} = \sum_{r \in \mathcal{Q}^{\text{ul}}} \sum_{j \in \mathcal{K}_r^{\text{ul}}} \text{diag}(\mathbf{h}_{q_k, q}) \mathbf{h}_{r_j, q}, \quad (25f)$$

where we define  $\mathbf{X} = \text{stack}\{\mathbf{X}_i\}_{\forall i \in \mathcal{I}} = [\mathbf{X}_1^{\text{T}}, \dots, \mathbf{X}_{|\mathcal{I}|}^{\text{T}}]^{\text{T}}$ , i.e., a function that stacks the input matrices over each other. From the above, problem (19) can be written as

Find  $\boldsymbol{\theta}$

$$\begin{aligned} \text{s.t. } & \frac{|\mathbf{a}_{q_k}^{\text{H}} \boldsymbol{\theta} + b_{q_k}|^2}{\|\mathbf{C}_{q_k} \boldsymbol{\theta} + \mathbf{d}_{q_k}\|^2 + \sigma_{q_k}^2} \geq \varphi, \forall q \in \mathcal{Q}^{\text{dl}} \\ & \|\mathbf{T}_{r,q} \boldsymbol{\theta} + \boldsymbol{\omega}_{r,q}\|^2 \leq \tau_r, \forall r \in \mathcal{Q}^{\text{ul}} \\ & |[\boldsymbol{\theta}]_{[m]}|^2 = 1, m = 1, \dots, M, \\ & \phi_m \in \mathcal{F} \end{aligned} \quad (26)$$

where

$$\mathbf{T}_{r,q} = \sum_{\forall q \in \mathcal{Q}^{\text{dl}}} \sum_{k \in \mathcal{K}_q^{\text{dl}}} (\mathbf{f}_{q_k}^{\text{T}} \mathbf{H}_q^{\text{T}} \diamond \mathbf{H}_r^{\text{H}}), \quad (27a)$$

$$\boldsymbol{\omega}_{r,q} = \mathbf{G}_{r,q} \mathbf{f}_{q_k}. \quad (27b)$$

Next, we exclude the limitation of the discrete phase shifts, and thereafter map the obtained continuous phase shifts to the nearest discrete phase value in  $\mathcal{F}$ . Furthermore, we adopt the SDR technique to recast problem (26) into a convex SDP which can be solved using the CVX tools.

Find  $\boldsymbol{\Omega}$

$$\begin{aligned} \text{s.t. } & \text{tr}(\boldsymbol{\Sigma}_{q_k} \boldsymbol{\Omega}) + |b_{q_k}|^2 \geq \varphi (\text{tr}(\boldsymbol{\Xi}_{q_k} \boldsymbol{\Omega}) + \|\mathbf{d}_{q_k}\|^2), \\ & \text{tr}(\boldsymbol{\Pi}_{r,q} \boldsymbol{\Omega}) + \|\boldsymbol{\omega}_{r,q}\|^2 \leq \tau_r, \forall r \in \mathcal{Q}^{\text{ul}} \\ & \boldsymbol{\Omega}_{m,m} = 1, m = 1, \dots, M+1, \\ & \boldsymbol{\Omega} \succeq 0. \end{aligned} \quad (28)$$

Due to space limitation, we omit the detailed expression of the semidefinite matrices  $\boldsymbol{\Omega}, \boldsymbol{\Sigma}_{q_k}, \boldsymbol{\Xi}_{q_k}$ , and  $\boldsymbol{\Pi}_{r,q}$ , which are directly derived from (26). It is noted that the considered SDP problem has relaxed the rank-one constraint; however, the rank of the obtained solution may not be 1. If the rank is 1, then  $\boldsymbol{\theta}$  can be obtained by using an eigenvalue decomposition.

Otherwise, we can obtain a high-quality rank-one solution by applying Gaussian randomization [18]. The obtained continuous phase shifts given as  $\phi_m = \angle \boldsymbol{\theta}(m)$  are mapped to the discrete ones. This operation can be expressed as

$$\tilde{\phi}_m = \arg \min_{\phi \in \mathcal{F}} |\phi - \phi_m|, \quad (29)$$

where the periodicity with  $2\pi$  is taken into account. Next, the transmit beamforming vectors are updated using the  $\boldsymbol{\theta}$  designed in the previous step. The alternating optimization process continues till the difference between the consecutive value of the cost function becomes smaller than the stopping criteria value,  $\epsilon$ .

### C. Backhaul Signaling and Per-BS complexity Analysis

The ADMM-based solution for the design of the transmit beamforming vectors requires BS  $q$  to broadcast its local variables  $(\boldsymbol{\lambda}_q, \phi_q, \gamma_q)$  to the coupled BSs, with a total of  $2|\mathcal{Q}^{\text{dl}}|(|\mathcal{Q}^{\text{dl}}| - 1)|\mathcal{K}_q^{\text{dl}}| + |\mathcal{Q}^{\text{dl}}|(|\mathcal{Q}^{\text{dl}}| - 1)|\mathcal{K}^{\text{ul}}| + 1$  real valued scalars per iteration. This leads to a reduced signaling/computational load on the CPU, when compared with the centralized solution that requires each BS to exchange its local channel state information with all other BSs. The total number of real valued scalars required to be sent in the centralized solution is  $2|\mathcal{K}^{\text{dl}}|N(|\mathcal{Q}^{\text{dl}}| - 1)|\mathcal{Q}^{\text{dl}}| + 2|\mathcal{Q}^{\text{dl}}|N^2(|\mathcal{Q}^{\text{dl}}| - 1)|\mathcal{Q}^{\text{ul}}|$ , for the exchange of  $\mathbf{h}^{\text{BS-UE}}$  and  $\mathbf{H}^{\text{BS-BS}}$  channels.

Furthermore, the per-iteration complexity of the ADMM-based solution is dominated by the complexity of solving (13) by each BS, which has 1 SOC of size  $(4|\mathcal{K}_q^{\text{dl}}| + 1)$ ,  $|\mathcal{K}_q^{\text{dl}}|$  SOC of size  $(2|\mathcal{K}_q^{\text{dl}}| + 2)$ ,  $|\mathcal{K}_q^{\text{dl}}|$  SOC of size  $(2|\mathcal{K}_q^{\text{dl}}| + 1)$ ,  $|\mathcal{Q}^{\text{dl}}|$  SOC of size  $(N|\mathcal{K}_q^{\text{dl}}| + 1)$ ,  $|\mathcal{Q}^{\text{dl}}|$  SOC of size  $(N|\mathcal{K}_q^{\text{dl}}| + |\mathcal{K}_q^{\text{dl}}| + 1)$ , and 1 SOC of size  $(N|\mathcal{K}_q^{\text{dl}}| + 1)$ . The per-BS computational complexity can be computed using the complexity of a generic interior point method as given in [19], [20].

## IV. NUMERICAL RESULTS

In this section, we show simulation results to evaluate the performance of our proposed method as compared to the centralized baseline scheme [12]. We consider a system with  $Q = 4$  cells, as shown in Fig. 1, where  $|\mathcal{Q}^{\text{dl}}| = 2$  and  $|\mathcal{Q}^{\text{ul}}| = 2$ . We assume  $p_q^{\text{dl}} = P$ ,  $\forall q \in \mathcal{Q}^{\text{dl}}$ ,  $\sigma_1^2 = \dots = \sigma_K^2 = 1$ ,  $p_r^{\text{ul}} = 23$  dBm,  $\tau_r = 4$  dB,  $\forall r \in \mathcal{Q}^{\text{ul}}$ ,  $\epsilon = 10^{-3}$ , and  $[M, N, K] = [256, 16, 2]$ . We assume a Rician flat fading channel model such that, for example, the channel between the  $q$ th BS and the RIS is represented as

$$\mathbf{H}_q = \sqrt{\frac{\kappa}{\kappa + 1}} \mathbf{H}_q^{\text{LOS}} + \sqrt{\frac{1}{\kappa + 1}} \mathbf{H}_q^{\text{NLOS}} \quad (30)$$

where  $\kappa \geq 0$  denotes the Rician  $K$ -factor and  $\mathbf{H}_q^{\text{LOS}}$  denotes the channel associated with the line of sight (LOS) component, while  $\mathbf{H}_q^{\text{NLOS}}$  represents the non-line of sight component, whose entries are independent and identically distributed (i.i.d) and follow the complex Gaussian distribution with zero mean and unit variance. The other channels are modeled in a similar manner.

We study the achievable spectral efficiency using the maximization of the minimum SINR, expressed as  $\log_2(1 + \varphi)$ . Note that from  $\sum_{q \in \mathcal{Q}^{\text{dl}}} \gamma_q = |\mathcal{Q}^{\text{dl}}| \varphi$ , the global variable  $\varphi$  is

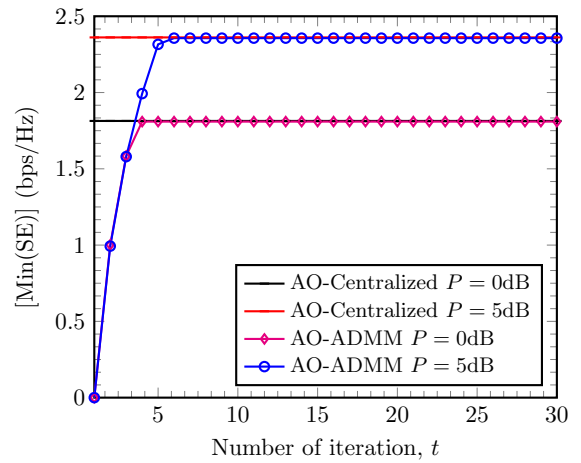


Fig. 2: Convergence behaviour of the AO-ADMM algorithm for different maximum total transmit power,  $\rho = 0.5$ .

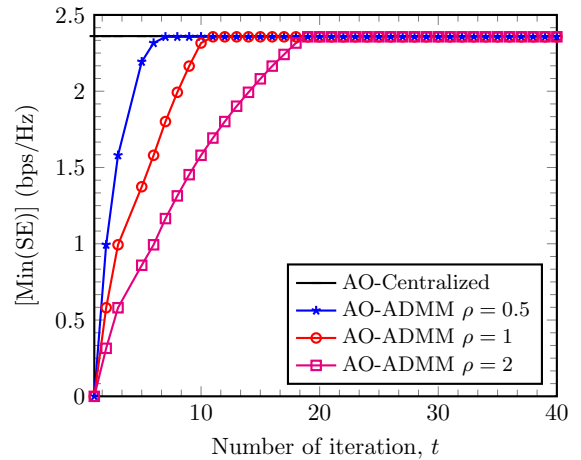


Fig. 3: Convergence behaviour of the AO-ADMM algorithm for different  $\rho$ , assuming that  $P = 5$  dB.

the average SINR that is obtained independently by all the DL BSs. For performance comparison purposes, we include results for the case where problem (4) is solved by adopting an AO approach and problem (6) is solved according to the centralized approach in [12] while the SDP method is used to solve problem (19).

Fig. 2 shows the convergence behaviour of the proposed AO-ADMM solution for different maximum total transmit power values while assuming that  $\rho = 0.5$ . It can be seen that the AO-ADMM solution converges to the AO-centralized solution within 10 iterations for the different maximum transmit power values considered. This confirms the practicality of the proposed design.

In Fig. 3 we investigate the convergence behaviour of the proposed solution for different values of the penalty parameter  $\rho$ . The results show that for all considered values of  $\rho$ , the AO-ADMM solution converges to the AO-centralized solution. However, the rate of the convergence depends on the value of  $\rho$ .

Next, we compare the optimal minimum spectral efficiency obtained using the AO-centralized solution and the AO-ADMM solution for different maximum total transmit power

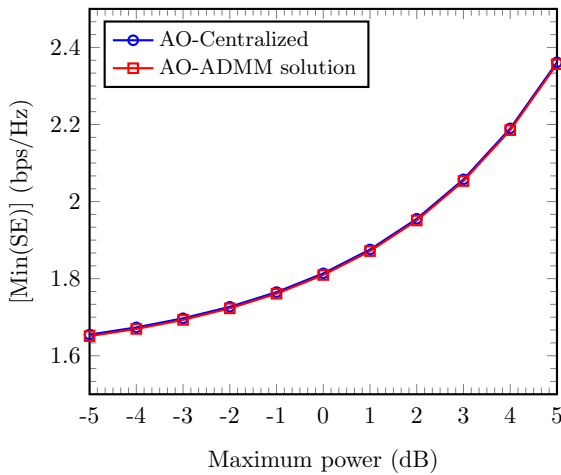


Fig. 4: Minimum SE versus the maximum transmit power,  $\rho = 0.5$ .

values. Fig. 4 clearly shows that the AO-ADMM solution achieves the same performance as the AO-centralized solution for all values of the maximum total transmit power.

## V. CONCLUSIONS

In this paper, we have considered an RIS-aided DTDD wireless network and utilized the Alternating Direction Method of Multipliers (ADMM) based distributed coordinated transmit beamforming to reduce the high signaling overhead involved in collecting all the channel state information at the CPU and SDP for the design of the passive reflection matrix. To this end, AO-ADMM algorithm was proposed. The results demonstrate that the AO-ADMM algorithm converges to the centralized solution after a few iterations.

## ACKNOWLEDGMENTS

This work was supported by Nnamdi Azikiwe University Nigeria, under NEEDS Assessment Fund and the Communication Research Laboratory, Ilmenau University of Technology, Germany.

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