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# RATE SPLITTING AND PRECODING STRATEGIES FOR MULTI-USER MIMO BROADCAST CHANNELS WITH COMMON AND PRIVATE STREAMS

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## ABSTRACT

In this paper, we present a precoder design for multi-user multiple-input multiple-output (MU-MIMO) broadcast systems with rate splitting at the transmitter. The proposed scheme applies to both underloaded and overloaded communication systems and supports the transmission of multiple common and private streams. We show how the generalized singular value (GSVD) and multilinear generalized singular value (ML-GSVD) decompositions can be used to define the number of common and private streams and adjust the message split. Additionally, we present transmit precoding and receive combining designs that allow the simultaneous transmission of common and private streams but do not require successive interference cancellation (SIC) at the receivers and can be used in cases where the total number of streams does not exceed the number of transmit antennas.

***Index Terms*—** Rate splitting, MU-MIMO, GSVD, ML-GSVD.

## 1. INTRODUCTION

An increasing number of smart devices connected to the global network puts higher demands on every successive generation of mobile communications. The sixth generation (6G) networks are anticipated to offer massive connectivity with ultra-reliability and high quality of service, which in turn, requires more sophisticated technologies that will provide more efficient use of the bandwidth and power resources. To this end, the authors in [1, 2] propose the rate splitting multiple access (RSMA) framework, which encompasses the advantages of two extreme interference management strategies, namely, space division multiple access (SDMA) and non-orthogonal multiple access (NOMA), while enabling their joint and flexible use.

In recent years, the RSMA has attracted particular attention from researchers and continues to gain popularity. However, the technology has not yet reached its maturity, and there are still various challenges to be addressed and underdeveloped research directions to be investigated [2]. Most of the publications that overview and explore rate splitting are dedicated to the multiple-input single-output (MISO) systems with single-antenna users [1–4]. Yet, several papers discuss the extension of RSMA to multiple-input multiple-output (MIMO) systems. The authors in [5] characterize the achievable degree-of-freedom (DoF) regions of a general two-receiver MIMO broadcast (BC) and MIMO interference (IC) channels with imperfect channel state information at the transmitter (CSIT). The achievable rates of linearly precoded BC RSMA schemes with $K$ users have been investigated in [6, 7]. Moreover, rate splitting has found an application in massive MIMO systems in [8, 9]. The precoder design and optimization for the multi-user (MU) MIMO downlink with rate splitting has been studied in [10]. The authors additionally validate their results via link-level simulations. However, the model in [10] is limited to a scenario where all receivers have the same number of antennas. The linear and non-linear precoding and stream combining techniques for the RSMA in MIMO systems with imperfect CSIT have been developed in [11, 12]. Moreover, in [13] the authors consider another essential RSMA problem and present adaptive power allocation schemes to distribute the transmit power between the common and private streams with a reduced computational cost. The precoder design for the underloaded and critically loaded[1] downlink MIMO system with rate splitting has been presented in [14, 15].

Similarly, in this contribution, our focus falls on the precoder design for MIMO broadcast systems with common and private streams where both the base station (BS) and the receivers are equipped with multiple antennas. However, in contrast to [11–13], we do not limit the number of common streams to one and allow the transmission of multiple common and private streams. We propose a precoding design for the common and private streams based on the multicast precoder design in [16] and a FlexCoBF scheme in [17]. In contrast to the schemes in [14, 15], the proposed techniques are applicable in both under- and overloaded communication scenarios. Moreover, we show how the generalized singular value (GSVD) [18, 19] and the multilinear generalized singular value (ML-GSVD) [20, 21] decompositions can be used to define the number of common and private streams and adjust the message split. The ML-GSVD extends the GSVD to the multiple matrices case, enabling a joint decomposition of more than two matrices. Therefore, it applies to systems with more than two users where the common streams can also be transmitted to selected groups of users (multi-layer hierarchical RS (HRS)). Additionally, we present the transmit precoding and receive combining designs that enable the simultaneous transmission of common and private streams but do not require successive interference cancellation (SIC) at the receivers and can be used in cases where the total number of streams does not exceed the number of transmit antennas.

*Notation*. Matrices and vectors are denoted by upper-case and lower-case bold-faced letters, respectively. The matrix $\boldsymbol{I}_d$ represents a $d \times d$ identity matrix. The superscripts $\{\cdot\}^{\mathrm{T}}$ and $\{\cdot\}^{\mathrm{H}}$ denote the transpose and Hermitian transpose, respectively. Moreover, $\mathrm{diag}\{\cdot\}$ is the operation of constructing a diagonal matrix with diagonal elements being the entries of the input vector. Additionally, we denote the Frobenius norm of a matrix $\boldsymbol{A}$ by $\|\boldsymbol{A}\|_{\mathrm{F}}$ and the determinant of a matrix $\boldsymbol{A}$ by $|\boldsymbol{A}|$.

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[1]In the under- and critically loaded MU-MIMO systems, the total number of receive antennas is less or equal to the number of antennas at the BS.

## 2. SYSTEM MODEL

We consider a multi-user MIMO downlink system where the base station (BS) equipped with $M_T$ transmit antennas serves $K$ users, each equipped with $M_{R_k}$ receive antennas. The $M_{R_k} \times M_T$ channel matrix from the BS to the $k$th user is represented by $\boldsymbol{H}_k$, and the transmitted data is denoted as $\boldsymbol{x} \in \mathbb{C}^{M_T \times 1}$. The transmitted signal is subject to a total power constraint $\mathbb{E}\left\{\|\boldsymbol{x}\|^2\right\} \leq P_t$. Consequently, the signal received at the $k$th user is given by

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x} + \boldsymbol{n}_k \in \mathbb{C}^{M_{R_k} \times 1}, \tag{1}$$

where $\boldsymbol{n}_k \sim \mathcal{CN}\left(\boldsymbol{0}, \sigma_k^2 \boldsymbol{I}_{M_{R_k}}\right)$ is an Additive White Gaussian Noise (AWGN) vector whose elements are i.i.d. complex Gaussian random variables with zero mean and variance $\sigma_k^2$. Assuming all users' noise variances are equal to one, the transmit SNR equals the total power consumption $P_t$. In this paper, we consider the rate splitting technique to transmit the data to multiple users. In one-layer RSMA every users' message set $\mathcal{W}_k$ is split into common and private parts denoted as $\mathcal{W}_k^c$ and $\mathcal{W}_k^p$, respectively. The common parts are then combined and encoded together into a common stream $\boldsymbol{s}_c$ of size $l_c$, whereas the private message sets $\mathcal{W}_k^p$ are encoded independently into $K$ private streams $\boldsymbol{s}_k$ of size $l_k$, $k = \{1, \ldots, K\}$. Consequently, the resulting data stream vector to be transmitted is equal to $\boldsymbol{s} = \left[\boldsymbol{s}_c^{\mathrm{T}}, \boldsymbol{s}_1^{\mathrm{T}}, \ldots, \boldsymbol{s}_K^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{l \times 1}$, where $l = l_c + \sum_{k=1}^{K} l_k$ denotes the total number of data streams. We assume that the total number of common streams $l_c$ cannot exceed $\min\{M_T, M_{R_i}\}_{i=1}^{K}$ and the total number of private streams satisfies $\sum_{k=1}^{K} l_k \leq M_T$. Subsequently, the common stream $\boldsymbol{s}_c$ and the $K$ private streams $\boldsymbol{s}_k$ are precoded via the precoding matrices $\boldsymbol{P}_c \in \mathbb{C}^{M_T \times l_c}$ and $\boldsymbol{P}_k \in \mathbb{C}^{M_T \times l_k}$, respectively. Thus, the combined transmit precoding matrix can be written as $\boldsymbol{P} = [\boldsymbol{P}_c, \boldsymbol{P}_1, \ldots, \boldsymbol{P}_K] \in \mathbb{C}^{M_T \times l}$. Then, the resulting transmitted signal is expressed as

$$\boldsymbol{x} = \boldsymbol{P}_c \boldsymbol{s}_c + \sum_{k=1}^{K} \boldsymbol{P}_k \boldsymbol{s}_k = \boldsymbol{P}\boldsymbol{s} \in \mathbb{C}^{M_T \times 1}. \tag{2}$$

The signal received at the $k$th user can be rewritten as

$$\begin{aligned} \boldsymbol{y}_k &= \boldsymbol{H}_k \boldsymbol{x} + \boldsymbol{n}_k & (3)\\ &= \boldsymbol{H}_k (\boldsymbol{P}_c \boldsymbol{s}_c + \sum_{k=1}^{K} \boldsymbol{P}_k \boldsymbol{s}_k) + \boldsymbol{n}_k & (4)\\ &= \boldsymbol{H}_k \boldsymbol{P}\boldsymbol{s} + \boldsymbol{n}_k & (5) \end{aligned}$$

with $\mathbb{E}\left\{\boldsymbol{s}\boldsymbol{s}^H\right\} = \boldsymbol{I}_l$. As a result, the total power is subject to the constraint $\|\boldsymbol{P}\|_{\mathrm{F}}^2 \leq P_t$. In the following sections, we present the precoder designs and a message splitting approach based on the GSVD (for two users) and the ML-GSVD (for more than two users). For completeness, in the next section we will briefly review these decompositions.

## 3. GSVD AND ML-GSVD

The ML-GSVD is defined for a set of $K \geq 2$ complex (or real) valued matrices $\boldsymbol{H}_k \in \mathbb{C}^{M_k \times N}$ with the same column dimension $N$ and possibly different row dimensions $M_k$ as follows [20,21]

$$\boldsymbol{H}_k = \boldsymbol{B}_k \cdot \boldsymbol{C}_k \cdot \boldsymbol{A}^{\mathrm{H}} \in \mathbb{C}^{M_k \times N}, \tag{6}$$

where the matrix $\boldsymbol{A} \in \mathbb{C}^{N \times Q}$, $Q = \min\{\sum_{k=1}^{K} M_k, N\}$, is nonsingular and common for all $\boldsymbol{H}_k$s, and the matrix $\boldsymbol{B}_k \in \mathbb{C}^{M_k \times Q}$ corresponding to the $k$-th matrix $\boldsymbol{H}_k$ has unitary columns such that $\boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{B}_k = \boldsymbol{I}_Q$. The matrices $\boldsymbol{C}_k \in \mathbb{R}^{Q \times Q}$ are diagonal with nonnegative entries satisfying $\sum_{k=1}^{K} \boldsymbol{C}_k^2 = \boldsymbol{I}_Q$. We assume that the number of matrices $K$ does not exceed the number of columns $N$. Let $r_k = \text{rank}\left(\boldsymbol{H}_k\right)$. Then, if $r_k < N$ for some $k$, and $\sum_{k=1}^{K} r_k > N$ (Case 2 in [21]), the decomposition has the following structure

$$\boldsymbol{H}_k = \underbrace{\begin{bmatrix}\boldsymbol{O}_{M_k \times Q - r_k} & \hat{\boldsymbol{B}}_k\end{bmatrix}}_{\boldsymbol{B}_k} \cdot \underbrace{\begin{bmatrix}\boldsymbol{O}_{Q-r_k} & & \\ & \boldsymbol{\Lambda}_k & \\ & & \boldsymbol{I}_{p_k}\end{bmatrix}}_{\boldsymbol{C}_k} \cdot \underbrace{\begin{bmatrix}\boldsymbol{A}_{o_k}^{\mathrm{H}} \\ \boldsymbol{A}_{c_k}^{\mathrm{H}} \\ \boldsymbol{A}_{p_k}^{\mathrm{H}}\end{bmatrix}}_{\boldsymbol{A}^{\mathrm{H}}}, \tag{7}$$

where $\boldsymbol{\Lambda}_k = \text{diag}\{\boldsymbol{\lambda}_k\} = \text{diag}\{\lambda_{k,1}, \ldots, \lambda_{k,c_k}\}$ and $\hat{\boldsymbol{B}}_k \in \mathbb{C}^{M_k \times r_k}$. The values of $\boldsymbol{\lambda}_k \in \mathbb{R}^{c_k}$ are in the range $(0,1)$, $c_k$ and $p_k$ ($c_k + p_k = r_k$) are the dimensions of the *common* and *private* subspaces, respectively. The identity matrix $\boldsymbol{I}_{p_k}$ represents the private subspace of the $k$th matrix, whereas $\boldsymbol{\lambda}_k$ corresponds to the common subspace shared between all matrices or groups of matrices. The matrix of zeros $\boldsymbol{O}_{Q-r_k} \in \mathbb{R}^{(Q-r_k)\times(Q-r_k)}$ corresponds to the private subspaces of the matrices different from $\boldsymbol{H}_k$. Note that for $k = 1$, the values on the main diagonal of $\boldsymbol{C}_k$ are sorted in an ascending order, and since $\sum_{k=1}^{K} \boldsymbol{C}_k^2 = \boldsymbol{I}_Q$, the private subspace of the matrix $\boldsymbol{H}_k$ always coincides with the zeros in the remaining matrices. Therefore, for $k \neq 1$, the values on the diagonal of $\boldsymbol{C}_k$ are permuted accordingly. The submatrix $\boldsymbol{A}_{c_k}$ is associated with the common subspace of the $k$th and some (or all) other matrices, and the submatrix $\boldsymbol{A}_{p_k}$ corresponds to the private subspace of $\boldsymbol{H}_k$. The matrix $\boldsymbol{A}_{o_k}$ is associated with the private and common subspaces of other matrices than the matrix $\boldsymbol{H}_k$.

Depending on the dimensions and the individual ranks of the $\boldsymbol{H}_k$s, the private and common subspaces can be empty for some or all $K$ matrices. For instance, if $r_k = N, \forall k$ (Case 1 in [21]), then, $\boldsymbol{C}_k = \boldsymbol{\Lambda}_k \in \mathbb{R}^{r_k \times r_k}$. In this case, the columns of $\boldsymbol{A}$ are shared for all factorizations, and the decomposition provides *only* the *common* subspace of size $N$ for all matrices $\boldsymbol{H}_k$, $k = \{1, \ldots, K\}$. If $\sum_{k=1}^{K} r_k \leq N$ (Case 3 in [21]), the entries of $\boldsymbol{C}_k$ only contain ones and zeros, i.e., the matrix $\boldsymbol{\Lambda}_k$ in $\boldsymbol{C}_k$ in (7) is not present, and *only private* subspaces exist for every $\boldsymbol{H}_k$:

$$\boldsymbol{C}_k = \begin{bmatrix}\boldsymbol{Q}_{Q-r_k} & \\ & \boldsymbol{I}_{r_k}\end{bmatrix}. \tag{8}$$

In this case, the matrices $\boldsymbol{H}_k$ do not share any common factors.

The ML-GSVD is a natural generalization of the GSVD [18,19]. In contrast to other extensions of the GSVD (for instance, HO GSVD in [22]), the ML-GSVD preserves properties of the original decomposition, such as orthogonality as well as common and private subspace structures. In general, the ML-GSVD is an approximation in the least squares sense. However, for $K = 2$, the decomposition is exact and corresponds to the GSVD of two matrices. The dimensionality of the common and private subspaces in such a case can be defined explicitly as follows. Let $q = \text{rank}([\boldsymbol{H}_1^{\mathrm{H}} \;\; \boldsymbol{H}_2^{\mathrm{H}}])$. Then, $p_1 = q - r_2, p_2 = q - r_1, c_1 = c_2 = r_1 + r_2 - q$, and the matrices $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are given by

$$\boldsymbol{C}_1 = \begin{bmatrix}\boldsymbol{O}_{p_2} & & \\ & \boldsymbol{\Lambda}_1 & \\ & & \boldsymbol{I}_{p_1}\end{bmatrix}, \;\; \boldsymbol{C}_2 = \begin{bmatrix}\boldsymbol{I}_{p_2} & & \\ & \boldsymbol{\Lambda}_2 & \\ & & \boldsymbol{O}_{p_1}\end{bmatrix}. \tag{9}$$

The ratios between the corresponding entries of $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$ are called generalized singular values. For more details on the ML-GSVD, GSVD, and their applications, we refer the reader to [18–20], and [21].

## 4. PRECODER DESIGN FOR COMMON AND PRIVATE STREAMS

### 4.1. Channel assignment and precoder design

Considering the MIMO scenario where users can receive multiple data streams, the number of common and private streams transmitted in parallel has to be determined. Moreover, it has been shown in [2] (for the MISO case) that RSMA can be reduced to SDMA or NOMA depending on the strength or orthogonality of the users' channels. In this regard, the ML-GSVD and the GSVD provide an elegant and straightforward way to determine the channels' strengths and alignment. As an example, let us consider the GSVD of two channel matrices $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ with the diagonal terms as in (9). The GSVD separates the channels into three independent subspaces (or parallel virtual channels (VCs)): a common subspace ($\mathcal{S}_{12}$) shared between the two users and two private subspaces ($\mathcal{S}_1$ and $\mathcal{S}_2$) that are orthogonal to each other. The presence or absence of the private and common subspaces depends on the channel conditions. Therefore, we use the GSVD to determine the existence of a common subspace to transmit the common messages and, consequently, the selected number of common streams, which is equal to the size $c_k$ of the common subspace. For scenarios with more than two users, the ML-GSVD can be applied. It estimates the common subspace shared between all $K$ users and between groups of users, which can be further used in multi-layer HRS.

Subsequently, the defined number of common and private streams can be used for the design of the precoding matrices $\boldsymbol{P}_c$ and $\boldsymbol{P}_k$ in (4). The authors in [16] propose a multicast precoding scheme for MIMO orthogonal frequency division multiplexing (OFDM) systems to transmit common messages to a selected group of users. It can also be generalized for MIMO RSMA to transmit the common streams to all or selected groups of users as

$$\boldsymbol{P}_c = \frac{1}{\beta_c} \sum_{k \in \mathcal{S}_{\mathcal{K}}} g_k \boldsymbol{V}_k, \tag{10}$$

where the matrix $\boldsymbol{V}_k$ is the right singular matrix corresponding to the first $l_c = c_k$ singular values of the channel matrix $\boldsymbol{H}_k$, $\mathcal{S}_{\mathcal{K}}$ is the set of users $\mathcal{K}$ that decode the common message, $g_k$ is a power normalization according to the effective channel gain, and $\beta_c$ is a normalization constant to fulfill the transmit power constraint. All users will first decode the common streams $\boldsymbol{s}_c$ by treating the interference from the private streams as noise. Thus, the achievable rate for the common streams at user $k$ can be expressed as

$$R_k^c = \log_2 \left| \boldsymbol{I}_{l_c} + \boldsymbol{P}_c^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} (\boldsymbol{G}_k^c)^{-1} \boldsymbol{H}_k \boldsymbol{P}_c \right|, \tag{11}$$

where the term $\boldsymbol{G}_k^c = \sigma_k^2 \boldsymbol{I}_{M_{R_k}} + \sum_{i=1}^{K} \boldsymbol{H}_k \boldsymbol{P}_i \boldsymbol{P}_i^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}}$ denotes the noise plus interference covariance matrices. To ensure that all $K$ users successfully decode the common message, the resulting common rate $R^c$ should not exceed $\min\{R_i^c\}_{i=1}^{K}$.

The private streams $\boldsymbol{s}_k$ are decoded independently by the corresponding users after performing SIC by subtracting the successfully decoded common signal $\boldsymbol{H}_k \boldsymbol{P}_c \boldsymbol{s}_c$ from the received signal $\boldsymbol{y}_k$. The private precoding matrices $\boldsymbol{P}_k$ can be designed based on the FlexCoBF scheme proposed in [17], which provides freedom in the choice of the linear transmit and receive beamforming strategies and can be applied in both under- and overloaded MU-MIMO systems. Thus, the achievable rates for the private streams can be expressed as

$$R_k^p = \log_2 \left| \boldsymbol{I}_{l_k} + \boldsymbol{P}_k^{\mathrm{H}} \tilde{\boldsymbol{H}}_k^{\mathrm{H}} (\boldsymbol{G}_k^p)^{-1} \tilde{\boldsymbol{H}}_k \boldsymbol{P}_k \right|, \tag{12}$$

where the term $\boldsymbol{G}_k^p = (\sigma_k^2 \boldsymbol{I}_{M_{R_k}} + \sum_{i=1, i \neq k}^{K} \tilde{\boldsymbol{H}}_k \boldsymbol{P}_i \boldsymbol{P}_i^{\mathrm{H}} \tilde{\boldsymbol{H}}_k^{\mathrm{H}})$ corresponds to the noise plus interference from the private streams of the users other than $k$, and $\tilde{\boldsymbol{H}}_k = \boldsymbol{W}_k^{\mathrm{H}} \boldsymbol{H}_k$ is an effective channel matrix, where $\boldsymbol{W}_k$ is a decoding matrix. The resulting rate achieved at user $k$ is written as $R_k = R_k^p + C_k$, where $C_k$ is a portion of the common rate assigned to the $k$th user, such that $\sum_{k=1}^{K} C_k = R^c$. The precoding schemes presented above assume that the number of common streams $l_c$ does not exceed $\min\{M_T, M_{R_i}\}_{i=1}^{K}$. The total number of private streams satisfies $\sum_{k=1}^{K} l_k \leq M_T$. Therefore, the described schemes are applicable in a wide range of scenarios, including cases where the total number of streams $l > M_T$.

### 4.2. Special case: ML-GSVD-based transmission

Next, let us introduce the precoding and decoding schemes for the scenario where the total number of streams transmitted in parallel satisfies $l = l_c + \sum_{k=1}^{K} l_k \leq M_T$. Based on the ML-GSVD of the channel matrices $\boldsymbol{H}_k$, the overall precoding matrix $\boldsymbol{P}$ can be defined as

$$\boldsymbol{P} = \frac{1}{\beta} \{\boldsymbol{A}^{\mathrm{H}}\}^{-1}, \tag{13}$$

where $\beta$ is a normalization coefficient ensuring the total power constraint. The decoding matrix of the $k$th user is defined as $\boldsymbol{W}_k = \boldsymbol{B}_k$. Consequently, the received data at the $k$th user in (5) can be rewritten as

$$\boldsymbol{W}_k^{\mathrm{H}} \boldsymbol{y}_k = \boldsymbol{W}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{P} \boldsymbol{s} + \boldsymbol{W}_k^{\mathrm{H}} \boldsymbol{n}_k \tag{14}$$

$$= \boldsymbol{C}_k \boldsymbol{s} + \tilde{\boldsymbol{n}}_k, \tag{15}$$

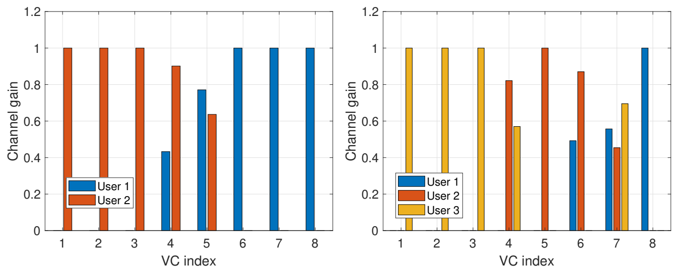
where $\tilde{\boldsymbol{n}}_k = \boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{n}_k$, and $\boldsymbol{C}_k$ is a diagonal matrix as in (6) and (7). As it can be seen from (14), after applying the ML-GSVD, the MIMO channels $\boldsymbol{H}_k$ of $K$ users can be considered as independent parallel virtual channels. Then, common streams can be transmitted on the common VCs (common ML-GSVD subspace) and decoded by the corresponding users. Private VCs (private ML-GSVD subspaces) are used to transmit the private streams that are detected individually by the intended users. It should be noted that by utilizing the ML-GSVD or GSVD for the transmission of private and common streams, we enable the joint detection of private and common streams without performing SIC at receivers, which reduces the complexity of the system and avoids error propagation, especially in higher order RSMA systems where several SIC steps have to be performed. Since the common VC gains are equal to $\lambda_{k,n}$, $n \in \{1, \ldots, c_k\}$, the achievable rate of the $n$th common stream for the $k$th user is given by

$$R_{k,n}^c = \log_2 \left(1 + \frac{P_{c,n} \lambda_{k,n}^2}{\beta^2 \sigma_k^2}\right), \tag{16}$$

where $P_{c,n}$ is the portion of the total transmit power assigned to the common streams. As in (11), the resulting common rate $R_n^c$ for the $n$th stream should not exceed $\min\{R_{i,n}^c\}_{i=1}^{K}$. The private channels have a unit channel gain. Thus, the individual information rate of user $k$ for the private stream $m$, $m \in \{1, \ldots, p_k\}$ can be expressed as

$$R_{k,m}^p = \log_2 \left(1 + \frac{P_{p,m}}{\beta^2 \sigma_k^2}\right), \tag{17}$$

where $P_{p,m}$ is a portion of the total transmit power assigned to the $m$th private stream. The resulting rate achieved at user $k$ is written as $R_k = \sum_{j=1}^{p_k} R_{k,j}^p + \sum_{i=1}^{c_k} C_{k,i}$, where $C_{k,n}$, $n \in \{1, \ldots, c_k\}$, is a portion of the common rate $R_n^c$ assigned to the $k$th user, such that $\sum_{k=1}^{K} C_{k,n} = R_n^c$. We can choose the $C_{k,n}$s to be proportional to $\lambda_{k,n}$ in (7), i.e, the individual VC gains.




(a) $K = 2$. Channels 4 and 5 share a common subspace.

(b) $K = 3$. Channels 4, 6, and 7 share a common subspace.

**Fig. 1**: Virtual channel gains of the users.

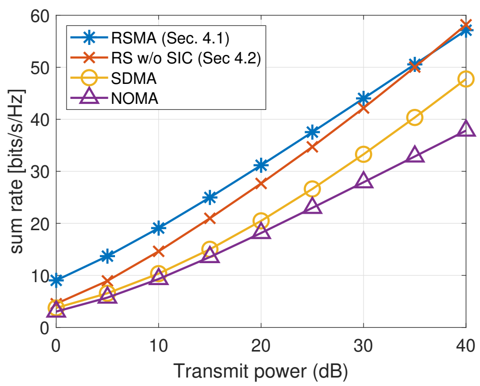



**Fig. 2**: Sum Rates. $K = 2$. Results are averaged over 1000 Monte Carlo trials.

## 5. SIMULATION RESULTS

In this section, we present some numerical results to evaluate the performance of the proposed rate splitting schemes. The first simulation illustrates the private and common subspaces of channel matrices provided by GSVD and ML-GSVD. For the simulations, we consider an uncorrelated Rayleigh fading MIMO channel model. Fig. 1(a) shows the virtual channel gains obtained via the GSVD for an overloaded MIMO downlink system with $M_T = 8$ transmit antennas and two users equipped with $M_{R_k} = 5$ antennas each. As it can be observed, the two users have three private channels (with unit channel gain) and two common channels which can be used to transmit the common message. Since User 2 experiences a higher common channel gain on the VC 4, it can be assigned a larger portion of the common rate on that stream. Fig. 1(b) demonstrates the virtual channel gains obtained with the ML-GSVD of the channels $\boldsymbol{H}_k$ for an asymmetrical scenario where the BS is equipped with $M_T = 8$ antennas, and three users are equipped with $M_{R_1} = 3$, $M_{R_2} = 4$, and $M_{R_3} = 5$ antennas, respectively. The ML-GSVD overcomes the limitation of the GSVD to two matrices and allows a joint decomposition of $K \geq 2$ users' channels. As it can be seen from Fig. 1(b), in such a case the ML-GSVD distinguishes 1, 1, and 3 private VCs for the users 1, 2, and 3, respectively. Moreover, there are two common channels for a group of two users ($\mathcal{S}_{23}$ and $\mathcal{S}_{12}$) and one common channel for all three users ($\mathcal{S}_{123}$).

In the next simulation experiments, we assess the performance of the proposed precoding schemes. First, we consider a two-user downlink MIMO scenario with one BS equipped with $M_T = 5$ transmit antennas and two receivers both equipped with $M_{R_k} = 3$ antennas. The total number of streams is equal to $M_T$. This scenario corresponds to the GSVD providing both common and private subspaces. Fig. 2 shows the sum rate performances for the precoding schemes proposed in Sections 4.1 and 4.2 and their comparison to NOMA (data of the 1st user is transmitted on the private streams, and data of the 2nd user is transmitted on the common streams) and SDMA (zero power is assigned to the common streams). We perform the power allocation based on an exhaustive search, maximizing the sum rate and assuming that the users have equal weights. Fig. 3 shows the achievable sum rates for a scenario with $K = 3$ users, where the BS and every user is equipped with $M_T = 10$ and $M_{R_k} = 4$ antennas, respectively. In this case, the common streams can be sent to all three or to a group of two users (for $K > 2$ we employ the ML-GSVD to define the common/private subspaces and the number of streams). Such a scenario corresponds to the hierarchical RS where the number of SIC layers $L$ can increase up to 4 ($L = 2^{K-1}$, see [2] for details). The scheme described in Section 4.2 allows transmitting and receiving the common and private steams without SIC. As shown in Figs. 2 and 3, it performs better than the RSMA scheme for the higher SNRs. In general, both proposed rate splitting techniques outperform the SDMA and NOMA for the presented two- and three-users scenarios.

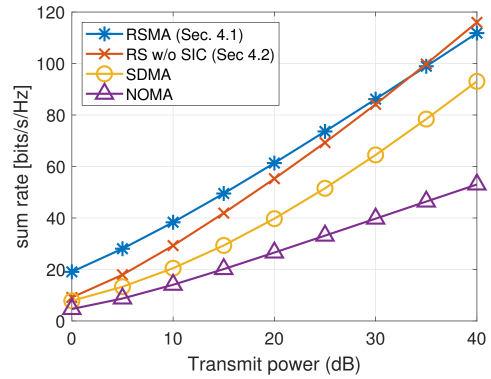



**Fig. 3**: Sum Rates. $K = 3$. Results are averaged over 1000 Monte Carlo trials.

## 6. CONCLUSIONS

In this paper, we have proposed a precoder design for MIMO broadcast systems with rate splitting at the transmitter. We have shown how the GSVD and the ML-GSVD can be employed to define the number of common and private streams as well as to adjust the message split. The proposed scheme applies to both underloaded and overloaded communication systems and supports the transmission of multiple common and private streams. Moreover, for the cases where the total number of streams does not exceed the number of transmit antennas, we have introduced a transmit precoding and receive combining designs based on the ML-GSVD, which allow simultaneous transmission of common and private streams but do not require SIC at the receivers. The use of the ML-GSVD overcomes the two-user limitation of the GSVD, allowing its application to systems with more than two users where the common streams can also be transmitted to selected groups of users (multi-layer hierarchical or generalized RS). Simulation results have shown that both proposed rate splitting schemes outperform SDMA and NOMA.

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