

EMPACt for Adaptive Spatial Filtering of GNSS Signals

Anju Elza Achenkunju, Damir Rakhimov, Joseph K. Chege, and Martin Haardt

Communications Research Laboratory, Ilmenau University of Technology,

P. O. Box 100565, D-98684 Ilmenau, Germany

Email: {anju-elza.achenkunju, damir.rakhimov, joseph.chege, martin.haardt}@tu-ilmenau.de

Abstract—Jamming poses a major challenge to Global Navigation Satellite Systems (GNSS), whose signals arrive at receivers well below the noise floor. This paper presents ESPRIT-MPDR with phase and amplitude compensation (EMPACt), a three-stage jammer mitigation method for a four-element uniform rectangular array (URA) evaluated using real phased-array data. EMPACt combines direction-of-arrival estimation using the ESPRIT algorithm with a constrained minimum power distortionless response (MPDR) beamformer to suppress interference while preserving the desired satellite signals. In addition, amplitude normalization and phase compensation techniques are introduced to compensate for the array imperfections. Experimental results demonstrate that EMPACt performs better than uncompensated zenith-steered MPDR and power-inversion beamforming, both in jammer-free and jamming conditions. These findings highlight the effectiveness of adaptive spatial filtering for improving the robustness of GNSS receivers in jamming environments.

Index Terms—GNSS, jamming, beamforming, adaptive array signal processing, EMPACt

I. INTRODUCTION

Global Navigation Satellite Systems (GNSS) are widely used in many applications that require position, velocity, and time (PVT) information. However, their signals are highly vulnerable to jamming [1] due to their extremely low received power [2]. Jamming refers to the transmission of radio frequency (RF) signals in the frequency band of GNSS signals with the intention of preventing the receiver from correctly acquiring and tracking the GNSS signals. A comprehensive overview of mitigation approaches, including spatial, temporal, and inertial methods, is discussed in [3].

Antenna arrays are commonly used in receivers to mitigate interference through spatial filtering. Techniques such as beamforming, null-steering, and subspace projection [4] are typically employed to reduce the effects of jamming in GNSS signals.

The dataset used in this work is part of the EUSIPCO Phased Array Signal Processing Student Challenge 2025 [5]. Several methods have been proposed to mitigate jamming for

this dataset. In [6], the authors used the multiple signal classification (MUSIC) algorithm to estimate the direction of arrival (DOA) of the jammer. Then, a minimum power distortionless response (MPDR) beamformer pointing towards the zenith was applied to suppress the interference. The results showed that the jammer could be mitigated without prior knowledge of its location using MPDR. In [7], a power inversion (PI) beamformer and a blind steering (BS) approach were applied to remove the interference. However, none of these previous works considered amplitude or phase compensation of the received antenna signals prior to the processing of the received signals from the antennas.

In this paper, we propose ESPRIT-MPDR with phase and amplitude compensation (EMPACt) for jammer mitigation in GNSS signals received by an antenna array. The method consists of three main blocks. The first block is the amplitude and phase compensation to restore the Vandermonde structure of the received signals and reduce the effect of array imperfections. The presence of the phase compensation procedure distinguishes EMPACt from the previously available approaches, such as [6], [7]. This procedure is based on 1-D ESPRIT. The second module performs estimation of the angular coordinates of the jammer via 2-D ESPRIT. The third block calculates the MPDR beamformer with multiple linear constraints to suppress the interference. The signals after beamforming are processed using the FGI-GSRx software receiver [8] from the Finnish Geospatial Research Institute (FGI) to obtain information about the receiver location. The performance of the proposed solution is validated on the dataset for the GNSS signals received by a four-element phased-array under dynamic interference conditions [5]. These experimental results using the real phased-array data demonstrate that the proposed approach effectively suppresses jamming and enables reliable GNSS positioning under time-varying interference.

Notation: Scalars are denoted as italic letters (a, A), column vectors as lower-case bold-face letters ($\mathbf{a}$), and matrices as upper-case bold-face letters ($\mathbf{A}$). We use the superscripts $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$, $(\cdot)^{-1}$, and $(\cdot)^+$ for transposition, Hermitian transposition, complex conjugation, matrix inversion, and matrix pseudoinverse, respectively. The Kronecker product of two matrices $\mathbf{A}$ and $\mathbf{B}$ is denoted as $\mathbf{A} \otimes \mathbf{B}$. The Hadamard (elementwise) product of two vectors $\mathbf{a}$ and $\mathbf{b}$ is denoted as $\mathbf{a} \odot \mathbf{b}$. The operation $\text{vec}\{\mathbf{A}\}$ stacks the columns of $\mathbf{A}$ in a vector.

The work of D. Rakhimov, J. K. Chege, and M. Haardt was supported in part by the German Research Foundation (DFG) under Grant 402834619 (AdAMMM-II, HA 2239/14-3).

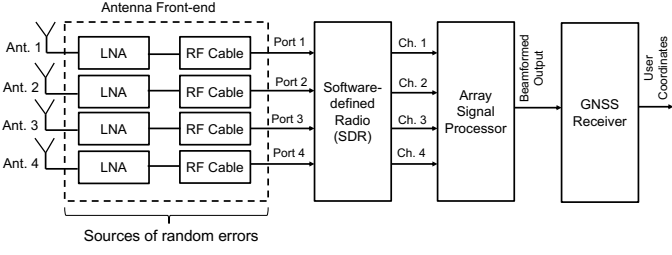


Fig. 1: Multi-antenna GNSS receiver architecture illustrating the sources of random amplitude and phase errors at the receiver.

II. SYSTEM MODEL

This section describes the antenna array configuration, the mathematical signal model, the effects of array imperfections, and the spatial processing steps applied to the received signals.

A. Measurement Setup and Array Configuration

Figure 1 illustrates the complete measurement and processing setup. A four-element active GPS antenna array is connected to a multi-channel software-defined radio (SDR) platform. The SDR provides synchronized multi-channel signal acquisition, and the recorded antenna signals are further processed by the array signal processor before being forwarded to the GNSS receiver. Although the receiver channels of the SDR are phase coherent, the effects of RF front-end components, such as low-cost active GPS antennas and cables of different lengths, are not compensated before data acquisition. Thus, the received signal experiences additional time-invariant random gain and phase variations across different channels.

B. Signal Model

Let us consider a uniform rectangular array (URA). The array lies in the xy -plane and the z -axis points outward, normal to the array. It is of size $M_x \times M_y$, where M_x and M_y are the number of sensors in the x - and y -directions, respectively. The total number of elements is equal to $M = M_x M_y$. The spacing between the antenna elements in the x - and y -directions is equal to $\Delta_x = \Delta_y = \Delta$. The visualization of the array configuration for $M_x = 2$ and $M_y = 2$ is given in Figure 2.

We assume that a mixture of L planar wavefronts is impinging on the array. The received signal at the time instance t can be written as

$$\mathbf{x}(t) = \mathbf{A}_S \mathbf{s}_S(t) + \mathbf{A}_J \mathbf{s}_J(t) + \boldsymbol{\eta}(t) \in \mathbb{C}^M, \quad (1)$$

where $\mathbf{A}_S \in \mathbb{C}^{M \times L_S}$ and $\mathbf{s}_S(t) \in \mathbb{C}^{L_S}$ are the steering matrix and the vector with complex-valued path gains at time t for signals impinging from L_S GPS satellites, $\mathbf{A}_J \in \mathbb{C}^{M \times L_J}$ and $\mathbf{s}_J(t) \in \mathbb{C}^{L_J}$ are the steering matrix and the vector with complex-valued path gains at time t corresponding to L_J jammers. The vector $\boldsymbol{\eta}(t) \in \mathbb{C}^M$ collects the samples of Zero Mean Circularly Symmetric Complex Gaussian (ZMCSCG) noise with the spatial covariance matrix $\mathbf{R}_\eta = \sigma_\eta^2 \mathbf{I}_M \in \mathbb{C}^{M \times M}$. We assume that the total number of sources is $L = L_S + L_J$. All sources are assumed to be narrowband

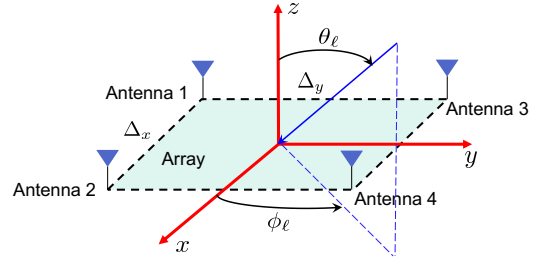


Fig. 2: Configuration of the four-element uniform rectangular array (URA).

and to have the same carrier frequency. We also assume that both $\mathbf{s}_i(t)$, $\forall i \in \{S, J\}$ and $\boldsymbol{\eta}(t)$ are mutually independent.

The array steering matrices for signals from satellites and jammers have an identical structure and can be written as

$$\mathbf{A} = [\mathbf{a}(\mu_1, \nu_1) \ \mathbf{a}(\mu_2, \nu_2) \ \cdots \ \mathbf{a}(\mu_L, \nu_L)] \in \mathbb{C}^{M \times L}, \quad (2)$$

where $\mathbf{a}(\mu_\ell, \nu_\ell)$ is the array steering vector corresponding to the ℓ -th source. Here, μ_ℓ and ν_ℓ are the spatial frequencies in the x - and y -directions, respectively. The spatial frequencies are given by

$$\mu_\ell = -\frac{2\pi}{\lambda_{\text{cr}}} \Delta_x \cos(\phi_\ell) \sin(\theta_\ell) \quad (3)$$

$$\nu_\ell = -\frac{2\pi}{\lambda_{\text{cr}}} \Delta_y \sin(\phi_\ell) \sin(\theta_\ell), \quad (4)$$

where $1 \leq \ell \leq L$ and λ_{cr} is the wavelength of the impinging signal. In the case of a URA, the array steering vector $\mathbf{a}(\mu, \nu)$ is expressed as

$$\mathbf{a}(\mu, \nu) = \mathbf{a}_y(\nu) \otimes \mathbf{a}_x(\mu) \in \mathbb{C}^{M \times 1}, \quad (5)$$

where $\mathbf{a}_x(\mu)$ and $\mathbf{a}_y(\nu)$ are the steering vectors in the x - and y -directions, respectively. They have a Vandermonde structure and are defined as

$$\mathbf{a}_x(\mu) = [1 \ e^{j\mu} \ \cdots \ e^{j(M_x-1)\mu}]^T \in \mathbb{C}^{M_x \times 1}, \quad (6)$$

$$\mathbf{a}_y(\nu) = [1 \ e^{j\nu} \ \cdots \ e^{j(M_y-1)\nu}]^T \in \mathbb{C}^{M_y \times 1}. \quad (7)$$

We assume that the GPS signals received on Earth are below the noise floor [2]. In addition, the studies in [6], [7] have shown that only a single jammer is present in the dataset recordings. Therefore, we consider only a single jammer during our analysis. As a result, the expression (1) can be rewritten as

$$\mathbf{x}(t) \approx \mathbf{a}_J \mathbf{s}_J(t) + \boldsymbol{\eta}(t) \in \mathbb{C}^{M \times 1}. \quad (8)$$

In the case when multiple realizations of $\mathbf{x}(t)$ are available, they can be written in the form of a matrix $\mathbf{X} \in \mathbb{C}^{M \times N}$ as

$$\mathbf{X} = [\mathbf{x}(t_1) \ \mathbf{x}(t_2) \ \cdots \ \mathbf{x}(t_N)], \quad (9)$$

where $\mathbf{x}(t_n)$ is the snapshot for the time instance t_n , $1 \leq n \leq N$.

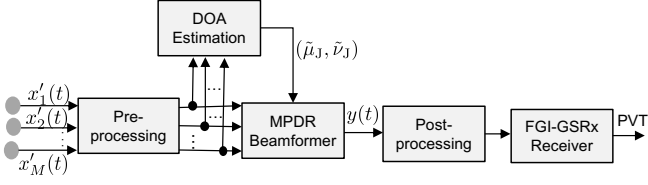


Fig. 3: Overview of EMPACT for spatial filtering.

C. Array Imperfections

Our initial analysis of the dataset brought us to the conclusion that the measured signals in the dataset differ from the assumed model, which is mainly caused by the presence of imperfections inherent to physical arrays. These imperfections introduce random amplitude and phase variations in each channel, which are not considered in the model (1). We assume that these imperfections are mainly caused by parameter deviations in different RF channels, such as different gains of low-noise amplifiers (LNAs) and different cable lengths. In addition, they may affect the overall system performance. For example, phase errors across different channels lead to the loss of the Vandermonde structure (5).

To account for additional amplitude and phase errors, the expression (8) can be rewritten as

$$\mathbf{x}'(t) = (\mathbf{a}_J \odot \boldsymbol{\phi}_{\text{err}} \odot \boldsymbol{\alpha}_{\text{err}}) s_J(t) + \boldsymbol{\eta} = \mathbf{a}'_J s_J(t) + \boldsymbol{\eta}, \quad (10)$$

where $\boldsymbol{\phi}_{\text{err}} = [e^{j\varphi_1}, e^{j\varphi_2}, \dots, e^{j\varphi_M}]^T$ and $\boldsymbol{\alpha}_{\text{err}} = [\alpha_1, \alpha_2, \dots, \alpha_M]^T$ are the vectors comprising random phase offsets and the vector with random amplitude gains in each RF channel. The resulting array steering vector with imperfections is denoted as $\mathbf{a}'_J = \mathbf{a}_J \odot \boldsymbol{\phi}_{\text{err}} \odot \boldsymbol{\alpha}_{\text{err}} \in \mathbb{C}^M$. We denote the matrix consisting of N imperfect snapshots as $\mathbf{X}'$.

D. Spatial Processing of the Received Signal

To mitigate the impact of the jammer in the spatial domain, beamforming is applied to form a null in the jammer direction while preserving the signals arriving from the satellites. In this case, the received signal after applying a beamforming vector $\mathbf{w} \in \mathbb{C}^{M \times 1}$ can be written as

$$\mathbf{y}^T = \mathbf{w}^H \mathbf{X}' \in \mathbb{C}^{1 \times N}. \quad (11)$$

Initially, the beamforming vector $\mathbf{w} \in \mathbb{C}^{M \times 1}$ is unavailable, and it has to be calculated based on the received signals. Our goal is to find such a beamforming vector $\mathbf{w} \in \mathbb{C}^{M \times 1}$ to ensure a stable GPS signal demodulation throughout the entire period of the jammer activity.

III. PROPOSED METHOD

The EMPACT algorithm for adaptive spatial filtering comprises several stages, which are shown in Figure 3. During the first stage, the amplitude normalization and phase compensation are applied to the received signals to restore the Vandermonde structure. Then, the spatial frequencies of the jamming signal are estimated using the 2-D ESPRIT algorithm. Since the GPS signal is much weaker than the interference, the estimation algorithm provides the estimates

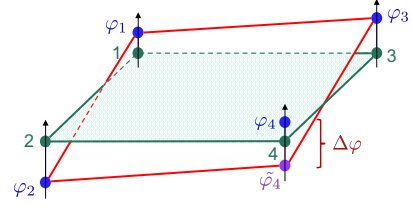


Fig. 4: Illustration of the phase compensation procedure.

only for the jammer. Afterwards, the beamforming vector is calculated using the MPDR algorithm. The estimated frequencies are used to build constraints for the MPDR beamformer to form a null in the direction of the jammer. Finally, the signal after beamforming is scaled and converted into a 16-bit integer for subsequent processing by the FGI-GSRx receiver. A pseudocode of the proposed algorithms is presented in Algorithm 1.

A. Amplitude Normalization and Phase Compensation

As it is stated in Section II-C, the signals in different channels have a mismatch in power levels. However, many direction-of-arrival (DOA) estimation algorithms assume uniform amplitudes across all array elements. To satisfy this assumption, amplitude normalization is applied to equalize the received signal magnitudes across the channels. We normalize it with the inverse of the RMS value of the power in order to get unit power. The matrix with N snapshots for signals after normalization can be written as

$$\mathbf{X}_{\text{norm}} = \text{diag}(\boldsymbol{\beta})^{-1} \cdot \mathbf{X}', \quad (12)$$

where $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_M]$ is the vector with the square root of the power averaged over a window of N snapshots for different channels. The m -th element of the vector $\boldsymbol{\beta}$ can be found by

$$\beta_m = \sqrt{\frac{1}{N} \sum_{n=1}^N |x_m(t_n)|^2}. \quad (13)$$

In order to compensate for the phase mismatch between the antennas, we propose a phase compensation method based on 1-D ESPRIT to restore the Vandermonde structure prior to DOA estimation. The proposed compensation method is applicable only to 2×2 URAs when a single jammer is present. In addition, it is specifically designed only to restore the Vandermonde structure of the received signals. This procedure consists of two steps. During the first step, the spatial frequencies are estimated for parallel subarrays by applying 1-D ESPRIT to each of them. For two identical subarrays, the direction estimates of a far-field source are expected to be identical. If these estimates differ, we assume that this is caused by random phase offsets in different channels. For example, the initial analysis of the dataset shows that the estimates $\hat{\mu}_{12}$ and $\hat{\mu}_{34}$ obtained for different antenna pairs, such as 1 & 2 and 3 & 4 in Figure 4, are not equal. We calculate the correction vector $\Delta \mathbf{a}_{34} = [1 \ e^{j(\hat{\mu}_{12} - \hat{\mu}_{34})}]^T$ to make the estimated angular frequency of the 3 & 4 pair to be equal to the

angular frequency of the 1 & 2 pair, which is parallel to it. The corresponding steering vector for the 3 & 4 pair of antennas after the correction can be written as $\tilde{\mathbf{a}}_{34} = \mathbf{a}'_{34} \odot \Delta \mathbf{a}_{34}$. Even though each antenna element has a random phase error, the resulting phase error vector after correction $\tilde{\phi}_{\text{err}} \in \mathbb{C}^{4 \times 1}$ has a Vandermonde structure that allows us to rewrite it as

$$\tilde{\phi}_{\text{err}} = \phi_{\text{err}} \odot \Delta \phi = \mathbf{a}_y(\Delta \nu) \otimes \mathbf{a}_x(\Delta \mu), \quad (14)$$

where $\Delta \mu = \varphi_2 - \varphi_1$ and $\Delta \nu = \varphi_3 - \varphi_1$, $\Delta \phi = [1, 1, 1, e^{j(\hat{\mu}_{12} - \hat{\mu}_{34})}]^T$ is the vector with phase corrections. As a result, the phase-compensated array steering vector $\tilde{\mathbf{a}}$ with restored Vandermonde structure can be represented as

$$\tilde{\mathbf{a}}_J = \mathbf{a}'_J \odot \Delta \phi = \underbrace{\mathbf{a}_y(\nu_J + \Delta \nu)}_{\tilde{\nu}_J} \otimes \underbrace{\mathbf{a}_x(\mu_J + \Delta \mu)}_{\tilde{\mu}_J}, \quad (15)$$

where $\Delta \mu$ and $\Delta \nu$ are random offsets of spatial frequencies in x - and y - directions that depend on the random phases in different channels. The matrix for the received signal after phase correction can be written as

$$\tilde{\mathbf{X}} = \text{diag}(\Delta \phi) \mathbf{X}_{\text{norm}}. \quad (16)$$

One can notice that for the proposed procedure, the phase correction $e^{j(\hat{\mu}_{12} - \hat{\mu}_{34})}$ is applied only to the last antenna element to restore the Vandermonde structure of the array. This procedure is based on the fact that the array steering vector $\mathbf{a}(\mu, \nu)$ of the ideal URA is Kronecker-separable, as shown in (5). Equivalently, $\mathbf{a}(\mu, \nu)$ can be reshaped into a matrix $\mathbf{A}_{\text{mat}} \in \mathbb{C}^{2 \times 2}$, which can be expressed as an outer product, i.e., $\mathbf{A}_{\text{mat}} = \mathbf{a}_x(\mu) \mathbf{a}_y(\nu)^T$, since $\text{vec}\{\mathbf{A}_{\text{mat}}\} = \mathbf{a}_y(\nu) \otimes \mathbf{a}_x(\mu) \in \mathbb{C}^4$. Therefore, for an ideal 2×2 URA, $\mathbf{A}_{\text{mat}}$ is a rank-one matrix and it satisfies $\det(\mathbf{A}_{\text{mat}}) = 0$ [9]. Similarly, if we reshape the uncompensated steering vector $\mathbf{a}'(\mu, \nu) = \mathbf{a}(\mu, \nu) \odot \phi_{\text{err}}$ into a matrix, we have

$$\mathbf{A}'_{\text{mat}} = \begin{bmatrix} e^{j\varphi_1} & e^{j(\nu+\varphi_3)} \\ e^{j(\mu+\varphi_2)} & e^{j(\mu+\nu+\varphi_4)} \end{bmatrix}. \quad (17)$$

Imposing the zero-determinant constraint on (17) results in a phase-consistency condition given by

$$\varphi_1 + \varphi_4 = \varphi_2 + \varphi_3, \quad (18)$$

which should be satisfied after compensation. The correction vector $\Delta \phi$ provides an estimate of the error in the phase-consistency condition, since $\Delta \hat{\varphi} = \hat{\mu}_{12} - \hat{\mu}_{34} \approx (\varphi_2 + \varphi_3) - (\varphi_1 + \varphi_4)$. Applying this correction to the last antenna element¹ results in a modified phase $\tilde{\varphi}_4 = \varphi_4 + \Delta \hat{\varphi} = \varphi_2 + \varphi_3 - \varphi_1$ which is exactly the condition required in (18).

An illustrative example of an array with random phase errors and the corresponding phase compensation procedure is provided in Figure 4. There, the green dots represent the ideal phase values in different channels, which all lie on a plane. The blue dots represent random phase offsets in different channels. In general, it is not possible to draw a plane through the blue

¹In fact, for a 2×2 array, the phase correction can be applied to any antenna element since there is only one constraint on the phases. Here, we apply it to the last element without loss of generality.

Algorithm 1 EMPACT

Input: A batch of N snapshots $\mathbf{X} \in \mathbb{C}^{M \times N}$

Output: User location

1. Preprocessing:
 - $\mathbf{X}_{\text{norm}} \leftarrow$ Apply amplitude normalization according to (12)
 - $\Delta \hat{\phi} \leftarrow$ Estimate the phase adjustment via 1-D ESPRIT
 - $\tilde{\mathbf{X}} \leftarrow$ Apply phase compensation according to (16).
 2. DOA Estimation:
 - $\mathbf{U}_s \leftarrow$ Estimate the signal subspace of $\tilde{\mathbf{X}}$
 - $(\tilde{\mu}_J, \tilde{\nu}_J) \leftarrow$ 2-D ESPRIT($\mathbf{U}_s$)
 3. Beamformer Design:
 - $\hat{\mathbf{R}} \leftarrow \frac{1}{N} \tilde{\mathbf{X}} \tilde{\mathbf{X}}^H$
 - $\hat{\mathbf{C}} \leftarrow [\mathbf{a}(\tilde{\mu}_1, \tilde{\nu}_1), \mathbf{a}(\tilde{\mu}_2, \tilde{\nu}_2), \mathbf{a}(\tilde{\mu}_J, \tilde{\nu}_J)]$
 - $\mathbf{g} \leftarrow [1, 1, 0]^T$
 - $\mathbf{w} \leftarrow \hat{\mathbf{R}}^{-1} \hat{\mathbf{C}} (\hat{\mathbf{C}}^H \hat{\mathbf{R}}^{-1} \hat{\mathbf{C}})^{-1} \mathbf{g}$
 4. Beamforming:
 - $\mathbf{Y} \leftarrow \mathbf{w}^H \tilde{\mathbf{X}}$
 5. Postprocessing:
 - Output signal normalization
 6. FGI-GSRx receiver[8]:
 - GPS signal decoding
 - User localization
-

dots. As a result, the corresponding Vandermonde structure is lost. The goal is to find a phase adjustment $\Delta \varphi$ such that all resulting phases lie on a plane, which means that the Vandermonde structure is restored.

B. Estimation of subspaces

2-D ESPRIT relies on the signal subspace for parameter estimation. For the estimation of the signal subspace spanned by the columns of $\mathbf{U}_s \in \mathbb{C}^{M \times L}$, the singular value decomposition (SVD) of $\tilde{\mathbf{X}} \in \mathbb{C}^{M \times N}$ is computed as

$$\tilde{\mathbf{X}} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \Sigma_s & \mathbf{0} \\ \mathbf{0} & \Sigma_n \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix}, \quad (19)$$

where $\mathbf{U}_s$ contains the left-singular vectors corresponding to the L dominant singular values, while $\mathbf{U}_n$ contains the remaining $M-L$ left-singular vectors. In this work, we assume $M = 4$, $L = 1$, $N = 10^8$.

C. Standard ESPRIT in element space

Standard ESPRIT is a subspace-based DOA estimation algorithm that exploits the rotational invariance property of two identical subarrays within an antenna array to estimate the directions of arrival [10].

Selection matrices are defined to select the two subarrays in each direction. Here, the two selection matrices in the x -direction can be written as [10]

$$\mathbf{J}_{1,\mu} = \mathbf{I}_{M_y} \otimes [\mathbf{I}_{M_x-1} \quad \mathbf{0}_{(M_x-1) \times 1}] \quad (20)$$

$$\mathbf{J}_{2,\mu} = \mathbf{I}_{M_y} \otimes [\mathbf{0}_{(M_x-1) \times 1} \quad \mathbf{I}_{M_x-1}], \quad (21)$$

and the selection matrices in the y -direction can be written as

$$\mathbf{J}_{1,\nu} = [\mathbf{I}_{M_y-1} \quad \mathbf{0}_{(M_y-1) \times 1}] \otimes \mathbf{I}_{M_x} \quad (22)$$

$$\mathbf{J}_{2,\nu} = [\mathbf{0}_{(M_y-1) \times 1} \quad \mathbf{I}_{M_y-1}] \otimes \mathbf{I}_{M_x}. \quad (23)$$

In the case of a URA with only one source, the shift invariance property can be expressed as [10]

$$\mathbf{J}_{1,\mu} \tilde{\mathbf{a}} \omega_\mu = \mathbf{J}_{2,\mu} \tilde{\mathbf{a}} \quad \text{and} \quad \mathbf{J}_{1,\nu} \tilde{\mathbf{a}} \omega_\nu = \mathbf{J}_{2,\nu} \tilde{\mathbf{a}}, \quad (24)$$

where $\omega_\mu = e^{j\tilde{\mu}}$ and $\omega_\nu = e^{j\tilde{\nu}}$. The steering vector $\tilde{\mathbf{a}}$ is unknown during the estimation. However, it can be replaced by an estimate of the signal subspace $\mathbf{u}_s$, since they span the same signal subspace in the noiseless case or in the case of an infinite number of snapshots. Therefore, the shift invariance equation can be rewritten as

$$\mathbf{J}_{1,\mu} \mathbf{u}_s \omega_\mu = \mathbf{J}_{2,\mu} \mathbf{u}_s \quad \text{and} \quad \mathbf{J}_{1,\nu} \mathbf{u}_s \omega_\nu = \mathbf{J}_{2,\nu} \mathbf{u}_s. \quad (25)$$

The quantities ω_μ and ω_ν can be estimated using a least-squares solution as

$$\hat{\omega}_\mu = (\mathbf{J}_{1,\mu} \mathbf{u}_s)^\dagger \mathbf{J}_{2,\mu} \mathbf{u}_s \quad \text{and} \quad \hat{\omega}_\nu = (\mathbf{J}_{1,\nu} \mathbf{u}_s)^\dagger \mathbf{J}_{2,\nu} \mathbf{u}_s. \quad (26)$$

Then, the estimates of the spatial frequencies for the x - and y - directions can be calculated as

$$\hat{\mu} = \arg(\hat{\omega}_\mu) \quad \text{and} \quad \hat{\nu} = \arg(\hat{\omega}_\nu). \quad (27)$$

D. MPDR Beamformer with multiple linear constraints

The MPDR beamformer [11] minimizes the array output power subject to linear constraints that preserve the desired signal while suppressing interference. The MPDR weights are obtained by solving the following constrained optimization problem

$$\min_{\mathbf{w}} \mathbf{J}(\mathbf{w}) = \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{C} = \mathbf{g}^H, \quad (28)$$

where $\mathbf{R} \in \mathbb{C}^{M \times M}$ is the spatial covariance matrix, $\mathbf{C} \in \mathbb{C}^{M \times k}$ is the constraint matrix, $\mathbf{g} \in \mathbb{C}^k$ is the gain vector, and k is the number of constraints. A solution of the optimization problem (28) can be written as [11]

$$\mathbf{w}_{\text{mpdr}} = \mathbf{R}^{-1} \mathbf{C} (\mathbf{C}^H \mathbf{R}^{-1} \mathbf{C})^{-1} \mathbf{g}. \quad (29)$$

However, in practice, the matrix $\mathbf{R}$ is not known a priori, and it has to be estimated from the received signal. Thus, for our solution, we substitute $\mathbf{R}$ in (29) by the sample covariance matrix $\hat{\mathbf{R}} \in \mathbb{C}^{M \times M}$ that is estimated as

$$\hat{\mathbf{R}} = \frac{1}{N} \tilde{\mathbf{X}} \tilde{\mathbf{X}}^H. \quad (30)$$

For the matrix $\mathbf{C}$, three linear constraints are implemented, i.e., $k = 3$. One constraint corresponds to the estimated jammer direction, while the other two correspond to the directions of the desired GNSS signals. Since the exact DOAs of the GNSS signals are unknown, two representative directions are selected within the visibility region in spatial frequency space (radius π), sufficiently separated from the jammer direction. These directions are generated using Vogel's spiral algorithm [12], which distributes points uniformly within a circular region using a golden-angle increment.

In this case, the constraint matrix, $\mathbf{C}$ and the gain vector, $\mathbf{g}$ in (28) can be expressed as

$$\mathbf{C} = [\mathbf{a}(\tilde{\mu}_1, \tilde{\nu}_1) \quad \mathbf{a}(\tilde{\mu}_2, \tilde{\nu}_2) \quad \mathbf{a}(\tilde{\mu}_J, \tilde{\nu}_J)] \quad (31)$$

$$\mathbf{g} = [1 \quad 1 \quad 0], \quad (32)$$

where $\mathbf{a}(\tilde{\mu}_1, \tilde{\nu}_1)$ and $\mathbf{a}(\tilde{\mu}_2, \tilde{\nu}_2)$ are the array steering vectors of the desired signal, and $\mathbf{a}(\tilde{\mu}_J, \tilde{\nu}_J)$ is the array steering vector of the jammer. This places a gain of 1 in the desired signal directions and a null in the direction of the jammer. The beamformer output is scaled and formatted to be processed by the GNSS receiver as described in Section IV.

IV. EXPERIMENTAL VALIDATION

A. Methodology

Due to the large number of samples, the data are processed in chunks. A sliding-window approach is used with a window length of 10 s and a step size of 1 s between successive chunks. In the case of the MPDR beamformer, for each chunk, signal directions are generated using Vogel's spiral within the spatial frequency region of radius π . From this sequence, the first two points that satisfy a minimum Euclidean separation of 0.8 in the spatial frequency domain from the jammer direction are selected as the desired directions. Here, the FGI-GSRx receiver [8] is used to verify the performance of the beamformer in removing the interference from the received signal. For the receiver, the true location is computed using the first 100 s of interference-free data by averaging the position estimates obtained when the received signal from Antenna 1 is processed by the receiver.

B. Description of Dataset

The dataset [5] consists of four files, each corresponding to one antenna in the array. Each file contains complex baseband in-phase and quadrature (IQ) samples of the GNSS signal received at each antenna over a period of 300 s. These files contain the GPS L1 coarse/acquisition (C/A) signal with carrier frequency 1575.42 MHz. The sampling frequency is 10 Msps. In the array, the antenna element spacing $\Delta_x = \Delta_y = 10$ cm. Initially, the signals are interference-free, and after approximately 100 s, a jammer is introduced. The power of the jammer increases gradually over time. The data were measured in the experimental campaign held in Jammertest 2024 in Norway [13]. These data were collected under several scenarios where malicious platforms try to disrupt the GNSS receiver functionalities [5].

C. Synthetic Simulations

To evaluate the proposed phase compensation approach, simulations were carried out using synthetic data for a 2×2 array. We considered 1000 snapshots and carried out 1000 independent trials during which random phase errors and amplitude gains were applied to all antenna elements. In each trial, the source spatial frequencies were randomly generated, while independent antenna phase errors were generated uniformly over $[0, 2\pi)$. To account for gain mismatch, independent amplitude errors were applied to the antenna elements.

Figure 5 shows the normalized mean squared error (NMSE) of the reconstructed signal as a function of SNR for four cases: phase error combined with amplitude mismatch, amplitude mismatch only, phase error only, and the proposed

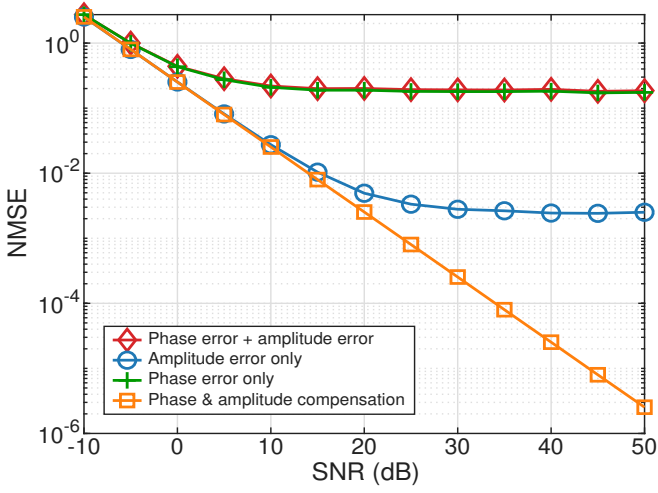


Fig. 5: Reconstruction accuracy (NMSE) versus SNR for a 2×2 array showing the impact of errors and compensation techniques (1000 trials, $N = 1000$ snapshots).

compensation approach applied to the combined phase-and-amplitude-error case. For the latter case, antenna-wise amplitude normalization is first performed, followed by phase-error compensation based on the proposed compensation method. The noiseless received signal is modeled as

$$\mathbf{z}_0 = \mathbf{a}\mathbf{s}, \quad (33)$$

where $\mathbf{a}$ denotes the array steering vector and $\mathbf{s}$ denotes the source signal. The corresponding noisy received signal is given by

$$\mathbf{z} = \mathbf{z}_0 + \boldsymbol{\eta}. \quad (34)$$

The spatial frequencies of the received signal in each case, $(\hat{\mu}, \hat{\nu})$, are estimated using 2-D ESPRIT. Then the array steering vector $\mathbf{a}_{\text{rec}}$ can be reconstructed using (5). The source signal can then be estimated as

$$\hat{\mathbf{s}} = \mathbf{a}_{\text{rec}}^+ \mathbf{z} \quad (35)$$

and the reconstructed received signal is obtained as

$$\hat{\mathbf{z}}_{\text{rec}} = \mathbf{a}_{\text{rec}} \hat{\mathbf{s}}. \quad (36)$$

The NMSE is calculated with respect to the corresponding noiseless received signal and averaged over the independent trials for each SNR value. The NMSE is defined as

$$\text{NMSE} = \frac{1}{T} \sum_{t=1}^T \frac{\|\mathbf{z}_0(t) - \hat{\mathbf{z}}_{\text{rec}}(t)\|_F^2}{\|\mathbf{z}_0(t)\|_F^2}, \quad (37)$$

where T is the number of trials.

D. Numerical experiments with the dataset

From Figure 6, it can be seen that a jammer is present as the power at Antenna 1 increases rapidly after 150 s. The preliminary analysis based on the Akaike Information Criterion (AIC) and Minimum Description Length (MDL)

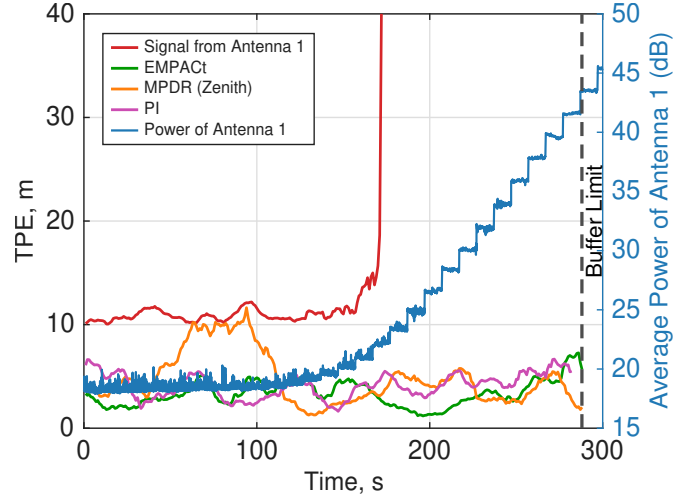


Fig. 6: 3-D total position error from FGI-GSRx receiver. For the presented data, a moving average of 10 s is applied.

confirmed that only one jammer is present in the dataset. The two signal constraints obtained for the MPDR beamformer are $(\mu_1, \nu_1) = (-0.46, 0.42)$ and $(\mu_2, \nu_2) = (0.08, -0.89)$.

Figure 6 presents the 10 s moving average of the total position error (TPE) in meters with respect to the true position over the duration of the entire dataset, along with the average received power at Antenna 1. The position error is calculated as

$$\text{TPE} = \sqrt{(E_{\text{ref}} - E)^2 + (N_{\text{ref}} - N)^2 + (U_{\text{ref}} - U)^2}, \quad (38)$$

where E , N , U denote the estimated receiver location in the local East-North-Up (ENU) coordinate system, and E_{ref} , N_{ref} , and U_{ref} denote the corresponding reference location. The reference location is estimated as an average location during the first 100 s, when the jammer is inactive.

Looking at these results, we can notice that the red line, representing the received signal at Antenna 1, breaks at 174 s as the jammer power increases sufficiently to prevent the receiver from processing the signal. All the other beamforming methods considered here successfully suppress the jamming signal throughout the duration of 288 s, which is limited by the 12 s buffer of the FGI-GSRx receiver. For comparison with the EMPACT algorithm, we consider two other algorithms. Firstly, the MPDR beamformer steered towards the zenith direction of the array [6] is considered. The next algorithm considered is the Power Inversion (PI) algorithm [7]. Neither of these methods requires the estimation of the DOA of the jammer.

The RMSE of the TPE for each algorithm is calculated as

$$\text{RMSE} = \sqrt{\frac{1}{N_s} \sum_{t=t_1}^{t_2} \text{TPE}(t)^2}, \quad (39)$$

where N_s denotes the number of samples in the considered time interval. The overall RMSE is evaluated over the complete dataset, i.e., $t \in [0, 300]$ s. Additionally, the RMSE is separately evaluated for the jammer-free interval ($t \in [0, 105]$ s)

and the interval where the jammer is present ($t \in [106, 300]$ s). The RMSE values obtained for the different algorithms are summarized in Table I.

TABLE I: RMSE comparison of different algorithms

Method	Overall RMSE	Jammer-free	Jammer present
Antenna 1	20.90 m	10.90 m	30.12 m
EMPACT	3.80 m	3.53 m	3.95 m
MPDR (Zenith)	5.64 m	7.67 m	4.04 m
PI	4.55 m	4.29 m	4.69 m

EMPACT demonstrates a more consistent positioning accuracy throughout the duration of the dataset, both in the pre-jamming phase (1–105 s) and in the interval where the jammer is present (from 106 s). Over the entire dataset, EMPACT achieves a root mean square error (RMSE) of 3.80 m, whereas the MPDR-based approach attains an RMSE of 5.64 m and the PI approach attains an RMSE of 4.55 m, demonstrating the effectiveness of the proposed method in improving the positioning accuracy.

During the pre-jamming phase, EMPACT maintains an RMSE of 3.53 m and an RMSE of 3.95 m for the duration where the jammer is present. This improvement, observed even in the absence of jamming, highlights the benefit of compensating for array imperfections, including phase and amplitude mismatch across antenna elements. EMPACT consistently achieves a lower positioning error throughout the dataset. These results validate the effectiveness of incorporating array imperfection compensation into the beamforming framework for GNSS anti-jamming applications using a 4-element antenna array.

V. CONCLUSIONS

This paper presents a three-stage jammer mitigation method for suppressing a time-varying GNSS jammer using a four-element antenna array and real phased-array GNSS data. In the pre-processing stage, amplitude normalization and phase compensation are applied to compensate for array imperfections and restore the Vandermonde structure of the received signals. In the second stage, the direction of arrival (DOA) of the jammer is estimated using the ESPRIT algorithm. The estimated DOA is then incorporated as a null constraint in an MPDR beamformer to suppress the interference in the final stage. Experimental results demonstrate that the proposed approach effectively suppresses the jammer and enables reliable

GNSS positioning even with increasing interference power. The method maintains stable positioning accuracy throughout the duration of the entire dataset. EMPACT is limited to a 2×2 URA and a single jammer scenario, and future work will investigate larger arrays and multiple interference sources. Overall, the study shows that low-complexity array processing with a small antenna array can provide an effective anti-jamming capability for GNSS receivers operating in challenging interference environments.

REFERENCES

- [1] D. Borio, F. Dovis, H. Kuusniemi, and L. Lo Presti, "Impact and Detection of GNSS Jammers on Consumer Grade Satellite Navigation Receivers," *Proceedings of the IEEE*, vol. 104, no. 6, pp. 1233–1245, Jun. 2016.
- [2] E. D. Kaplan and C. J. Hegarty, *Understanding GPS/GNSS: Principles and Applications*, 3rd ed. Boston, MA, USA: Artech House, 2017.
- [3] G. X. Gao, M. Sgammini, M. Lu, and N. Kubo, "Protecting GNSS Receivers From Jamming and Interference," *Proceedings of the IEEE*, vol. 104, no. 6, pp. 1327–1338, Jun. 2016.
- [4] W. Sun and M. G. Amin, "Maximum Signal-to-Noise Ratio GPS Anti-jam Receiver With Subspace Tracking," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP 2005)*, vol. 4, Philadelphia, PA, USA, Mar. 2005, pp. 1085–1088.
- [5] European Signal Processing Conference (EUSIPCO 2025), "Phased Array Signal Processing Student Challenge," <https://eusipco2025.org/phased-array-signal-processing-student-challenge/>, Palermo, Italy, Sep. 2025.
- [6] A. Brochard, J. Lesouple, L. Ortega, E. L. Rojano Fernández, and P. Thevenon, "Zenith beamforming with antenna array for gnss interference mitigation: The winning solution of the eusipco 2025 phased array signal processing student challenge," *Signal Processing*, vol. 246, pp. 1–11, Sep. 2026.
- [7] M. Mikkonen, M. Z. H. Bhuiyan, D. Borio, and V. Hytönen, "Detection, Localization and Mitigation of Noise-like Jamming via Phased Array Signal Processing," 2025. [Online]. Available: <http://hdl.handle.net/10138/600590>
- [8] M. Liaquat, M. Z. H. Bhuiyan, S. Islam, I. Pääkkönen, and S. Kaasalainen, "An Enhanced FGI-GSRx Software-Defined Receiver for the Execution of Long Datasets," *Sensors*, vol. 24, no. 12, pp. 1–18, 2024.
- [9] G. Strang, *Linear Algebra and Its Applications*, 4th ed. Belmont, CA, USA: Thomson, Brooks/Cole, 2006.
- [10] M. Haardt, M. Zoltowski, C. Mathews, and J. Nossék, "2D Unitary ESPRIT for Efficient 2D Parameter Estimation," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP 1995)*, vol. 3, Michigan, USA, May 1995, pp. 2096–2099.
- [11] H. L. Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*. New York, NY, USA: Wiley-Interscience, 2002.
- [12] H. Vogel, "A Better Way to Construct the Sunflower Head," *Mathematical Biosciences*, vol. 44, no. 3-4, pp. 179–189, Jun. 1979.
- [13] N. Gerrard, T. A. Solend, A. Rødningsby, Ø. Karlsen, T. Levin, H. Hauglin, K. Svartveit, C. B. Skjetne, A. M. Solberg, T. Rødningen, and Ø. Borlaug, "Jammertest: An Open GNSS Interference Test Arena to Accelerate the Development of Resilient GNSS Applications," *Engineering Proceedings*, vol. 126, no. 1, pp. 180–191, 2026.