Enhanced Tensor Based Semi-blind Estimation Algorithm for Relay-Assisted MIMO Systems

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Abstract. In this paper we study tensor based semi-blind estimation algorithms for one-way amplify-and-forward relaying systems. By exploiting the tensor structure of the equivalent channel, a bilinear alternating least squares (ALS) algorithm is proposed to jointly estimate the data and the channels at the destination. Sufficient conditions for the unique estimation are derived. The simulation results show that the proposed algorithms outperforms the state of the art algorithms.

1 Introduction

Multiple-input and multiple-output (MIMO) techniques can enhance the performance of relaying networks, e.g., [1]. To fully exploit the MIMO gain, channel knowledge is required at the transmitter and the receiver. Traditionally matrix based solutions are used to estimate MIMO channels, e.g., [5]. Recently, research results in [8,9] have shown that a tensor-based channel estimation method for relaying networks can provide a better estimation accuracy and requires less training overhead compared to the matrix based solution. Due to the use of training sequences, training based estimation methods provide accurate estimation results [8]. But they sacrifice physical resources which can be used for data transmission. Hence, blind or semi-blind estimation techniques, which do not need the transmission of training sequences, become more attractive, e.g., [10,12]. More specifically, in [12] a semi-blind estimation algorithm, which jointly estimates the data and the channels based on the PARAFAC-PARATUCK tensor decomposition of the received signal, is proposed for a two-hop one-way amplify-and-forward (AF) relaying assisted MIMO system. It is shown in [12] that the received signal via the source-to-destination (SD) link satisfies a PARAFAC tensor model, and the received signal via the source-to-relay-to-destination (SRD) link satisfies the PARATUCK2 tensor model [2]. The general PARATUCK2 decomposition requires a diagonal relay amplification matrix and is computed using an alternating least squares (ALS) method. More precisely, a trilinear ALS is proposed in [12]. Moreover, Kruskal's identifiability condition [4] must be satisfied for uniqueness. These stringent conditions have restricted the applicability of the proposed tensor based algorithms in [12].

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In this paper we introduce enhanced tensor based semi-blind algorithms for a joint data and channel estimation in the two-hop AF relaying system. In contrast to [12], our design is applicable even if a full relay amplification matrix is used. Our contribution can be summarized as: (1) A demodulation step is introduced during the estimation of the data symbols to provide well-conditioned data estimates, which can accelerate the converge of the ALS algorithm as compared to [12]. (2) In contrast to the trilinear ALS, bilinear ALS algorithms for jointly estimating the source-to-relay (SR) channel, the relay-to-destination (RD) channel, and the data are proposed. In each iteration, algebraic solutions for computing the SR channel and the RD channel are introduced based on the PARAFAC tensor model. Sufficient conditions for a unique estimation up to scaling ambiguities are derived. The derived conditions are more relaxed compared to Kruskal's condition in [12]. Numerical results show that significant improvements are achieved in terms of channel estimation accuracy and system BER performance using only a few number of iterations.

2 System Model

We consider an AF relaying scenario where one source node communicates with one destination node via a relay assisted network as in [12]. The source, the relay, and the destination have M_{S}, M_{R}, and M_{D} antennas, respectively. We assume that the channel is i.i.d. frequency flat and quasi-static block fading. The matrices $\boldsymbol{H}^{(\mathrm{SD})} \in \mathbb{C}^{M_{\mathrm{D}} \times M_{\mathrm{S}}}$, $\boldsymbol{H}^{(\mathrm{SR})} \in \mathbb{C}^{M_{\mathrm{R}} \times M_{\mathrm{S}}}$, and $\boldsymbol{H}^{(\mathrm{RD})} \in \mathbb{C}^{M_{\mathrm{D}} \times M_{\mathrm{R}}}$ denote the SD channel, the SR channel, and the RD channel, respectively. All the nodes operate in half-duplex modes and thus a complete transmission takes two phases. In the first phase, the source transmits to the relay and the destination. In the second phase, the source is silent and the relay transmits to the destination. To perform a blind estimation, the source transmits data using the Khatri-Rao space-time (KRST) coding scheme proposed in [10]. Assume that N blocks of KRST coded symbols are transmitted and during the n-th block ($n \in \{1, \cdots, N\}$) the transmitted signal is denoted as $\boldsymbol{X}_n = \mathrm{D}_n\{\boldsymbol{S}\}\boldsymbol{C}^{\mathrm{T}} \in \mathbb{C}^{M_{\mathrm{S}} \times K}$, where $\boldsymbol{S} \in \mathbb{C}^{N \times M_{\mathrm{S}}}$ denotes the overall data to be transmitted in N blocks, and $\boldsymbol{C} \in \mathbb{C}^{K \times M_{\mathrm{S}}}$ is the known KRST coding matrix, which maps M_{S} symbols to K time slots, i.e., the spatial code rate is M_{S}/K [10]. The operation $\mathrm{D}_n\{\boldsymbol{A}\}$ creates a diagonal matrix by aligning the elements of the n-th row of $\boldsymbol{A}$ onto its diagonal. Therefore, the n-th block of the received signal via the SD link is given by [12]

$$\boldsymbol{Y}_n^{(\mathrm{SD})} = \boldsymbol{H}^{(\mathrm{SD})}\mathrm{D}_n\{\boldsymbol{S}\}\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{V}_n^{(\mathrm{SD})} \in \mathbb{C}^{M_{\mathrm{D}} \times K}, \tag{1}$$

where $\boldsymbol{V}_n^{(\mathrm{SD})} \in \mathbb{C}^{M_{\mathrm{D}} \times K}$ denotes the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise and each element has a variance of σ_d^2. The received signal after N blocks can be expressed as a three-way tensor with the n-th frontal slice denoted by (1). The resulting PARAFAC model is given by

$$\boldsymbol{\mathcal{Y}}^{(\mathrm{SD})} = \boldsymbol{\mathcal{I}}_{3,M_{\mathrm{S}}} \times_1 \boldsymbol{H}^{(\mathrm{SD})} \times_2 \boldsymbol{C} \times_3 \boldsymbol{S} + \boldsymbol{\mathcal{V}}^{(\mathrm{SD})} \in \mathbb{C}^{M_{\mathrm{D}} \times K \times N} \tag{2}$$

where $\boldsymbol{\mathcal{I}}_{3,M_{\mathrm{S}}}$ is the identity tensor and $\times_i$ denotes the i-mode product [3]. Similarly, the n-th block of the received signal via the SRD link is expressed as [12]

$$\boldsymbol{Y}_n^{(\mathrm{SRD})} = \boldsymbol{H}^{(\mathrm{RD})}\boldsymbol{G}_n\boldsymbol{H}^{(\mathrm{SR})}\mathrm{D}_n\{\boldsymbol{S}\}\boldsymbol{C}^{\mathrm{T}} + \bar{\boldsymbol{V}}_n^{(\mathrm{SRD})} \in \mathbb{C}^{M_{\mathrm{D}}\times K} \tag{3}$$

where $\bar{\boldsymbol{V}}_n^{(\mathrm{SRD})} = \boldsymbol{H}^{(\mathrm{RD})}\boldsymbol{G}_n\boldsymbol{H}^{(\mathrm{SR})}\boldsymbol{V}_n^{(\mathrm{R})} + \boldsymbol{V}_n^{(\mathrm{SRD})}$ denotes the effective noise, where $\boldsymbol{V}_n^{(\mathrm{R})}$ and $\boldsymbol{V}_n^{(\mathrm{SRD})}$ are the ZMCSCG noise at the relay and the destination with a variance of σ_{r}^2 per element of $\boldsymbol{V}_n^{(\mathrm{R})}$, and $\boldsymbol{G}_n \in \mathbb{C}^{M_{\mathrm{R}}\times M_{\mathrm{R}}}$ is the n-th relay amplification matrix.

Our objective is to develop tensor based semi-blind estimation schemes such that the channels $\boldsymbol{H}^{(\mathrm{SD})}$, $\boldsymbol{H}^{(\mathrm{SR})}$, $\boldsymbol{H}^{(\mathrm{RD})}$, and the data matrix $\boldsymbol{S}$ can be uniquely identified.

3 Enhanced Design in the Direct Link

An estimate of $\boldsymbol{S}$ and $\boldsymbol{H}^{(\mathrm{SD})}$ in the direct link can be obtained via the Khatri-Rao factorization as described in [12]. To this end, the 2-mode unfolding of the tensor $\boldsymbol{\mathcal{Y}}^{(\mathrm{SD})}$ is expressed as

$$\left[\boldsymbol{\mathcal{Y}}^{(\mathrm{SD})}\right]_{(2)} \approx \boldsymbol{C}(\boldsymbol{H}^{(\mathrm{SD})} \diamond \boldsymbol{S})^{\mathrm{T}}, \tag{4}$$

where $\approx$ implies that the noise is ignored and $\diamond$ is the Khatri-Rao product. If the coding matrix $\boldsymbol{C}$ has a full column rank, i.e., $K \geq M_{\mathrm{S}}$, by pre-multiplying $\boldsymbol{C}^{+}$ on both sides of (4) an estimate of the Khatri-Rao product is obtained as $\hat{\boldsymbol{Y}}^{(\mathrm{SD})} = \left(\boldsymbol{C}^{+}\left[\boldsymbol{\mathcal{Y}}^{(\mathrm{SD})}\right]_{(2)}\right)^{\mathrm{T}} \approx \boldsymbol{H}^{(\mathrm{SD})} \diamond \boldsymbol{S}$, where $^{+}$ is the Moore-Penrose pseudo inverse. The resulting problem is a Khatri-Rao factorization problem, for which a LS solution can be obtained using the singular value decomposition (SVD) [5,6,8,12] and a detailed implementation is also found in [8]. The drawback of this method is that the Khatri-Rao factorization has inherent scaling ambiguities, i.e., one scaling ambiguity per column. A typical way to resolve these ambiguities is to assume that one row of $\boldsymbol{S}$ is known [9,12]. The described estimation procedure so far follows [12] exactly.

The estimates in the SD link and especially the estimated signal matrix $\hat{\boldsymbol{S}}$ will be used to initialize the trilinear ALS based PARATUCK2 decomposition in the SRD link [12]. It is known that the ALS algorithm is sensitive to ill-conditioned matrices due to the inverse operation. We observe that this happens quite often when the algorithms in [12] are applied. To deal with this phenomenon, we propose to demodulate the entries of the data matrix $\hat{\boldsymbol{S}}$. For this purpose, an element-wise hard-decision demodulation technique [7] is used and the output of the demodulation step is denoted as $\hat{\boldsymbol{S}}_{\mathrm{demod}}$. Numerical results show that the matrix $\hat{\boldsymbol{S}}_{\mathrm{demod}}$ is in general well-conditioned. Hence, this demodulation step is applied in both the SD link and the SRD link. Furthermore, we apply a LS based refinement of the channel estimate $\hat{\boldsymbol{H}}^{(\mathrm{SD})}$. This is simply computed by using the 1-mode unfolding of $\boldsymbol{\mathcal{Y}}^{(\mathrm{SD})}$, i.e., $\hat{\boldsymbol{H}}_{\mathrm{enh}}^{(\mathrm{SD})} \approx \left[\boldsymbol{\mathcal{Y}}^{(\mathrm{SD})}\right]_{(1)} ((\hat{\boldsymbol{S}}_{\mathrm{demod}} \diamond \boldsymbol{C})^{\mathrm{T}})^{+}$.

4 Bilinear ALS Algorithm for the SRD Link

Let us briefly review the trilinear ALS algorithm proposed in [12]. If the relay amplification matrix $\boldsymbol{G}_n$ in (3) is a diagonal matirx, i.e., $\boldsymbol{G}_n = \mathrm{D}_n\{\boldsymbol{G}\}$ and $\boldsymbol{G} \in \mathbb{C}^{N \times M_{\mathrm{R}}}$, the obtained M_{D}-by-K-by-N tensor by stacking N received blocks satisfies a PARATUCK2 model [2], where (3) is the n-th frontal slice of the corresponding tensor. In our case, the matrices $\boldsymbol{G}$ and $\boldsymbol{C}$ can be designed and we need to estimate $\boldsymbol{S}$, $\boldsymbol{H}^{(\mathrm{SR})}$, and $\boldsymbol{H}^{(\mathrm{RD})}$ from (3). When two out of three parameters are fixed, (3) can be rewritten as a linear function of the third parameter. Therefore, a trilinear ALS algorithm can be applied and an exact PARATUCK2 decomposition can be achieved. According to Kruskal's condition [4], to ensure the uniqueness of the PARATUCK2 decomposition, i.e., up to scaling ambiguities, it is required that $K \geq M_{\mathrm{S}}$ and $\min(M_{\mathrm{S}}, M_{\mathrm{D}}) \geq M_{\mathrm{R}}$ [12]. To resolve the scaling ambiguities, one row of $\boldsymbol{H}^{\mathrm{RD}}$ or one column of $\boldsymbol{H}^{\mathrm{SR}}$ needs to be known. Moreover, since $\boldsymbol{S}$ is involved in both the SD link, i.e., (1), and the SRD link, i.e., (3), a better estimation of $\boldsymbol{S}$ and $\boldsymbol{H}^{(\mathrm{SD})}$ might be obtained if (1) and (3) are combined. Depending on whether (1) and (3) are jointly exploited for estimating $\boldsymbol{S}$ or not, the proposed algorithms in [12] are divided into the combined PARAFAC/PARATUCK2 (CPP) method and the sequential PARAFAC/PARATUCK2 (SPP), respectively. In the following, we discuss our proposed approaches based on the CPP method. But the extension to the SPP method is straightforward.

4.1 Bilinear ALS Based Design

If we do not restrict ourselves to the PARATUCK2 model in [2], more flexibilities are obtained, e.g., the use of a full relay amplification matrix. To see this, define $\boldsymbol{H}_n^{(\mathrm{SRD})} = \boldsymbol{H}^{(\mathrm{RD})}\boldsymbol{G}_n\boldsymbol{H}^{(\mathrm{SR})}$, where $\boldsymbol{G}_n$ can be a full matrix in contrast to [12]. Assume that an ALS algorithm is used. Using equation (3), a LS estimate of the effective SRD channel $\boldsymbol{H}_n^{(\mathrm{SRD})}$ at the n-th block in the i-th step is calculated as

$$\hat{\boldsymbol{H}}_{n,i}^{(\mathrm{SRD})} \approx \boldsymbol{Y}_n^{(\mathrm{SRD})}(\mathrm{D}_n\{\hat{\boldsymbol{S}}_{i-1}\}\boldsymbol{C}^{\mathrm{T}})^{+} \in \mathbb{C}^{M_{\mathrm{D}} \times M_{\mathrm{S}}}, \tag{5}$$

$\forall n$, where $\hat{\boldsymbol{S}}_{i-1}$ denotes the estimate of $\boldsymbol{S}$ in the $(i-1)$-th step. To obtain $\hat{\boldsymbol{H}}_{n,i}^{(\mathrm{SRD})}$ uniquely, it is sufficient that $\boldsymbol{C}$ has full column rank, i.e., $K \geq M_{\mathrm{S}}$, because in this case $\mathrm{D}_n\{\hat{\boldsymbol{S}}_{i-1}\}$ is invertible. This coincides with the requirement in the SD link. Afterwards, the channels $\hat{\boldsymbol{H}}_i^{(\mathrm{RD})}$ and $\hat{\boldsymbol{H}}_i^{(\mathrm{SR})}$ are estimated algebraically from the tensor representation of the equivalent channel $\bar{\boldsymbol{\mathcal{H}}}_i^{(\mathrm{SRD})} = \left[\hat{\boldsymbol{H}}_{1,i}^{(\mathrm{SRD})} \sqcup_3 \hat{\boldsymbol{H}}_{2,i}^{(\mathrm{SRD})} \cdots \sqcup_3 \hat{\boldsymbol{H}}_{N,i}^{(\mathrm{SRD})}\right] \in \mathbb{C}^{M_{\mathrm{D}} \times M_{\mathrm{S}} \times N}$, where $\sqcup_3$ denotes the concatenation of matrices along the third dimension [8]. The proposed estimation methods will be introduced in the sequel. After that, the estimated equivalent channel is reconstructed as $\tilde{\boldsymbol{H}}_{n,i}^{(\mathrm{SRD})} = \hat{\boldsymbol{H}}_i^{(\mathrm{RD})}\boldsymbol{G}_n\hat{\boldsymbol{H}}_i^{(\mathrm{SR})}$. Then by combining (1) and (3), i.e., the CPP concept in [12] is used, a LS estimate of the n-th row of $\boldsymbol{S}$ in the i-th step is determined as

$$\hat{\boldsymbol{s}}_{n,i} \approx \begin{bmatrix} \boldsymbol{C} \diamond \hat{\boldsymbol{H}}^{(\mathrm{SD})} \\ \boldsymbol{C} \diamond \tilde{\boldsymbol{H}}_{n,i}^{(\mathrm{SRD})} \end{bmatrix}^{+} \begin{bmatrix} \boldsymbol{y}_n^{(\mathrm{SD})} \\ \boldsymbol{y}_n^{(\mathrm{SRD})} \end{bmatrix}, \tag{6}$$

$\forall n$, where $\boldsymbol{y}_n^{(\mathrm{SD})} = \mathrm{vec}\{\boldsymbol{Y}_n^{(\mathrm{SD})}\}$ and $\boldsymbol{y}_n^{(\mathrm{SRD})} = \mathrm{vec}\{\boldsymbol{Y}_n^{(\mathrm{SRD})}\}$. The conclusion in [12, Theorem 1] can be still applied to (6). That is, the condition $K \geq M_{\mathrm{S}}$ is sufficient to guarantee that a unique pseudo inversion is obtained in (6). Since in the i-th iteration step only two parameters, i.e., $\hat{\boldsymbol{H}}_{n,i}^{(\mathrm{SRD})}$ and $\hat{\boldsymbol{s}}_{n,i}$, are computed using the ALS algorithm, $\forall n$, we name this algorithm the bilinear ALS method. Due to the nature of an ALS method, the uniqueness is guaranteed up to a diagonal scaling matrix. However, since one row of $\boldsymbol{S}$ is assumed to be known in Sect. 3, the scaling ambiguity is resolved.

In the following we introduce algebraic methods for obtaining $\hat{\boldsymbol{H}}_i^{(\mathrm{RD})}$ and $\hat{\boldsymbol{H}}_i^{(\mathrm{SR})}$ in the i-th iteration. The index i will be dropped for notational simplicity. Moreover, let us define $\boldsymbol{\mathcal{G}} = [\boldsymbol{G}_1 \sqcup_3 \boldsymbol{G}_2 \cdots \sqcup_3 \boldsymbol{G}_N] \in \mathbb{C}^{M_{\mathrm{R}} \times M_{\mathrm{R}} \times N}$.

4.2 Khatri-Rao Factorization (KRF) Based Approach

The tensor $\bar{\boldsymbol{\mathcal{H}}}^{(\mathrm{SRD})} \approx \boldsymbol{\mathcal{G}} \times_1 \boldsymbol{H}^{(\mathrm{RD})} \times_2 \boldsymbol{H}^{(\mathrm{SR})^{\mathrm{T}}}$ satisfies a Tucker2 tensor model. Let $r_{\boldsymbol{\mathcal{G}}}$ be the rank of the tensor $\boldsymbol{\mathcal{G}}$ [8]. Then the PARAFAC decomposition of $\boldsymbol{\mathcal{G}}$ is computed as $\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{I}}_{r_{\boldsymbol{\mathcal{G}}}} \times_1 \boldsymbol{F}_1 \times_2 \boldsymbol{F}_2 \times_3 \boldsymbol{F}_3$, where $\boldsymbol{F}_1 \in \mathbb{C}^{M_{\mathrm{R}} \times r_{\boldsymbol{\mathcal{G}}}}$, $\boldsymbol{F}_2 \in \mathbb{C}^{M_{\mathrm{R}} \times r_{\boldsymbol{\mathcal{G}}}}$, and $\boldsymbol{F}_3 \in \mathbb{C}^{N \times r_{\boldsymbol{\mathcal{G}}}}$ are the corresponding factor matrices. Inserting this PARAFAC decomposition of $\boldsymbol{\mathcal{G}}$ into the Tucker2 model, we get

$$\bar{\boldsymbol{\mathcal{H}}}^{(\mathrm{SRD})} \approx \boldsymbol{\mathcal{I}}_{3,r_{\boldsymbol{\mathcal{G}}}} \times_1 (\boldsymbol{H}^{(\mathrm{RD})}\boldsymbol{F}_1) \times_2 (\boldsymbol{H}^{(\mathrm{SR})^{\mathrm{T}}}\boldsymbol{F}_2) \times_3 \boldsymbol{F}_3. \tag{7}$$

Equation (7) is also a PARAFAC decomposition. Note that we can construct $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, and $\boldsymbol{F}_3$ such that $\boldsymbol{H}^{(\mathrm{RD})}$ and $\boldsymbol{H}^{(\mathrm{SR})}$ can be uniquely estimated. By applying the 3-mode unfolding of (7), we obtain

$$\left[\bar{\boldsymbol{\mathcal{H}}}^{(\mathrm{SRD})}\right]_{(3)} \approx \boldsymbol{F}_3 \cdot ((\boldsymbol{H}^{(\mathrm{RD})}\boldsymbol{F}_1) \diamond (\boldsymbol{H}^{(\mathrm{SR})^{\mathrm{T}}}\boldsymbol{F}_2))^{\mathrm{T}}. \tag{8}$$

It is straightforward to see that (8) yields a similar Khatri-Rao structure as (4). To estimate $\boldsymbol{H}^{(\mathrm{RD})}$ and $\boldsymbol{H}^{(\mathrm{SR})}$, we first isolate the Khatri-Rao product from (8). For this purpose, it is sufficient that $\boldsymbol{F}_3$ has a full column rank, i.e., $N \geq r_{\boldsymbol{\mathcal{G}}}$. Then the remaining problem is a Khatri-Rao factorization problem, which can be solved up to a diagonal scaling matrix $\boldsymbol{\Lambda}_{\mathrm{f}} \in \mathbb{C}^{r_{\boldsymbol{\mathcal{G}}} \times r_{\boldsymbol{\mathcal{G}}}}$. That is, the obtained two factor matrices $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ of the Khatri-Rao factorization can be written as $\boldsymbol{B}_1 = \boldsymbol{H}^{(\mathrm{RD})}\boldsymbol{F}_1\boldsymbol{\Lambda}_{\mathrm{f}}$ and $\boldsymbol{B}_2 = \boldsymbol{H}^{(\mathrm{SR})^{\mathrm{T}}}\boldsymbol{F}_2\boldsymbol{\Lambda}_{\mathrm{f}}^{-1}$. To resolve this scaling ambiguity, we still assume that one column of $\boldsymbol{H}^{(\mathrm{RD})}$ or $\boldsymbol{H}^{(\mathrm{SR})}$ is known, e.g., it could be obtained via a training based initial estimate between the relay and the destination [12]. After the scaling ambiguity is resolved, i.e., $\boldsymbol{\Lambda}_{\mathrm{f}}$ is estimated, the following relationships are valid, i.e., $\boldsymbol{B}_1\boldsymbol{\Lambda}_{\mathrm{f}}^{(-1)} = \boldsymbol{H}^{(\mathrm{RD})}\boldsymbol{F}_1$ and $\boldsymbol{B}_2\boldsymbol{\Lambda}_{\mathrm{f}} = \boldsymbol{H}^{(\mathrm{SR})^{\mathrm{T}}}\boldsymbol{F}_2$.

To obtain $\boldsymbol{H}^{(\mathrm{RD})}$ and $\boldsymbol{H}^{(\mathrm{SR})}$ uniquely, we require that $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ have full row rank, i.e., $r_{\mathcal{G}} \geq M_{\mathrm{R}}$. Also, to render the pseudo inversions numerically stable, $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ should have orthogonal rows. Overall, the Khatri-Rao factorization (KRF) based approach can be applied if $N \geq r_{\mathcal{G}} \geq M_{\mathrm{R}}$. To reduce the channel estimation overhead, $r_{\mathcal{G}}$ should be as small as possible. Therefore, we choose $r_{\mathcal{G}} = M_{\mathrm{R}}$. The advantage of this KRF approach is that an algebraic solution is obtained. The disadvantage of it is that the resulting scaling ambiguity is the same as in [12].

Finally, the proposed bilinear algorithm is described in Algorithm 1 and the corresponding sufficient conditions to estimate the channels $\boldsymbol{H}^{(\mathrm{RD})}$, $\boldsymbol{H}^{(\mathrm{SR})}$, and the symbol matrix $\boldsymbol{S}$ in the SRD link without ambiguities are summarized as follows: the identifiablity is guaranteed if $\boldsymbol{C}$ and $\boldsymbol{F}_3$ have full column rank, and $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ have full row rank, i.e., $K \geq M_{\mathrm{S}}$ and $N \geq M_{\mathrm{R}}$. Note that in contrast to [12] the proposed bilinear ALS algorithm does not impose any restrictions on the number of antennas at the source, the relay, or the destination.

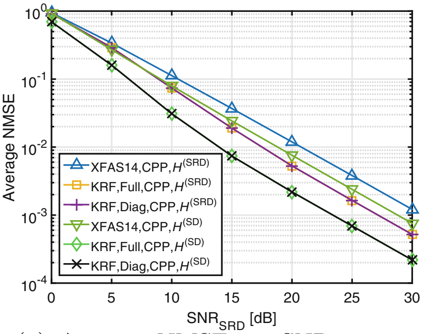
Remark. When $\boldsymbol{G}_n$ is a diagonal matrix, i.e., $\boldsymbol{G}_n = \mathrm{D}_n\{\boldsymbol{G}\}$, it is a special case of the PARAFAC based design, i.e., $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are identity matrices. In such a case the equivalent channel is expressed as $\boldsymbol{H}_n^{(\mathrm{SRD})} \approx \boldsymbol{H}^{(\mathrm{RD})}\mathrm{D}_n\{\boldsymbol{G}\}\boldsymbol{H}^{(\mathrm{SR})}$. It has the same structure as (1) and thus an equivalent PARAFAC model is obtained as $\bar{\boldsymbol{\mathcal{H}}}^{(\mathrm{SRD})} \approx \boldsymbol{\mathcal{I}}_{3,M_{\mathrm{S}}} \times_1 \boldsymbol{H}^{(\mathrm{RD})} \times_2 \boldsymbol{H}^{(\mathrm{SR})^{\mathrm{T}}} \times_3 \boldsymbol{G}$. The channels $\boldsymbol{H}^{(\mathrm{RD})}$ and $\boldsymbol{H}^{(\mathrm{RD})}$ can then be obtained by using its 3-mode unfolding $\left[\bar{\boldsymbol{\mathcal{H}}}^{(\mathrm{SRD})}\right]_{(3)} \approx \boldsymbol{G}(\boldsymbol{H}^{(\mathrm{RD})} \diamond \boldsymbol{H}^{(\mathrm{SR})^{\mathrm{T}}})^{\mathrm{T}}$ and the LS Khatri-Rao factorization.

Algorithm 1. A bilinear ALS algorithm for estimating $\boldsymbol{H}^{(\mathrm{RD})}$, $\boldsymbol{H}^{(\mathrm{SR})}$, and $\boldsymbol{S}$ in the SRD link using the CPP method

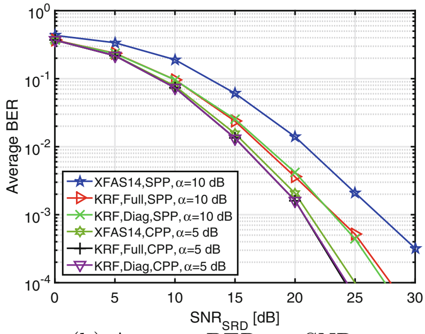
1: **Initialize:** set initial value of $\hat{\boldsymbol{S}}_0$ using the estimate from the SD link, $\hat{\boldsymbol{H}}^{(\mathrm{SD})} = \hat{\boldsymbol{H}}^{(\mathrm{SD})}_{\mathrm{enh}}$, $i = 1$, and the threshold value ϵ.
2: **Main step:**
3: **repeat**
4: Calculate $\hat{\boldsymbol{H}}^{(\mathrm{SRD})}_{n,i}$ using (5).
5: Estimate $\hat{\boldsymbol{H}}^{(\mathrm{RD})}_i$ and $\hat{\boldsymbol{H}}^{(\mathrm{SR})}_i$ using the KRF approach and obtain $\tilde{\boldsymbol{H}}^{(\mathrm{SRD})}_{n,i} = \hat{\boldsymbol{H}}^{(\mathrm{RD})}_i \boldsymbol{G}_n \hat{\boldsymbol{H}}^{(\mathrm{SR})}_i$.
6: Compute $\hat{\boldsymbol{s}}_{n,i}$ using (6) and then demodulate $\hat{\boldsymbol{s}}_{n,i}$ using the hard-decision demodulation method [7].
7: **until** $\sum_{n=1}^{N} \|\boldsymbol{Y}_n^{(\mathrm{SRD})} - \hat{\boldsymbol{H}}^{(\mathrm{RD})}_i \boldsymbol{G}_n \hat{\boldsymbol{H}}^{(\mathrm{SR})}_i \mathrm{D}_n\{\hat{\boldsymbol{S}}_i\}\boldsymbol{C}^{\mathrm{T}}\| \leq \epsilon$
8: **Output:** $\boldsymbol{H}^{(\mathrm{RD})}$, $\boldsymbol{H}^{(\mathrm{SR})}$, $\boldsymbol{S}$, and $\hat{\boldsymbol{H}}^{(\mathrm{SD})}$.

5 Simulation Results and Concluding Remarks

The proposed bilinear ALS algorithms are evaluated using Monte Carlo simulations. The simulated channels $\boldsymbol{H}^{(\mathrm{SD})}$, $\boldsymbol{H}^{(\mathrm{SR})}$ and $\boldsymbol{H}^{(\mathrm{RD})}$ are uncorrelated Rayleigh fading channels. The transmit power at the source and the relay are



(a) Average NMSE vs. SNR, $\alpha = 0$ dB.



(b) Average BER vs. SNR.

Fig. 1. Comparison of different algorithms. $M_{\mathrm{S}} = M_{\mathrm{D}} = M_{\mathrm{R}} = K = 2$, $N = 4$, $N_f = 40$, and QPSK modulation.

set to unity. The noise power at the relay and the destination are identical, i.e., $\sigma_{\mathrm{d}}^2 = \sigma_{\mathrm{r}}^2 = \sigma_{\mathrm{n}}^2$. The SNR of the relay-assisted link and the direct link are denoted by $\mathrm{SNR}_{\mathrm{SRD}}$ and $\mathrm{SNR}_{\mathrm{SD}}$, respectively. We have $\mathrm{SNR}_{\mathrm{SRD}} = \mathrm{SNR}_{\mathrm{SD}} + \alpha$ [dB] and $\alpha \geq 0$, which is a realistic assumption [12]. The coding matrix $\boldsymbol{C}$ is chosen as a Vandermonde matrix, i.e., $C_{k,m} = e^{\frac{\jmath 2\pi(k-1)(m-1)}{M_{\mathrm{S}}}}$, $k \in \{1, \cdots K\}$, and $m \in \{1, \cdots M_{\mathrm{S}}\}$.

The channel estimation accuracy is measured by using the normalized mean squared error (NMSE) criterion. Let $\boldsymbol{H}^{(X)}$ and $\hat{\boldsymbol{H}}^{(X)}$ denote the true channel and the estimated channel, where $X \in \{\mathrm{SD}, \mathrm{SRD}\}$. It implies that for the SRD link, the equivalent channel is evaluated [12]. For each simulation, the NMSE is defined as

$$\mathrm{e}(\boldsymbol{H}^{(X)}, \hat{\boldsymbol{H}}^{(X)}) = \frac{\|\boldsymbol{H}^{(X)} - \hat{\boldsymbol{H}}^{(X)}\|_{\mathrm{F}}^2}{\|\boldsymbol{H}^{(X)}\|_{\mathrm{F}}^2}. \tag{9}$$

The sphere decoder described in [11] is used to decode the modulated signal at the end. The BER performance is determined by transmitting a total number of N_f blocks of KRST coded symbols, where the first N blocks are also used for blind channel estimation. Since the proposed algorithms in [12] outperform the training based solutions in [5,9], we only compare our proposed algorithms to the algorithms in [12], which are denoted as "XFAS14". Note that the XFAS14 algorithms are applicable only if a diagonal relay amplification matrix is used. Moreover, "full" and "diag" stand for whether a full relay amplification matrix or a diagonal relay amplification matrix is used. All the simulation results are averaged over 2000 channel realizations.

Figure 1a and 1b demonstrate the performance of the bilinear ALS algorithms in terms of the NMSE and the BER performance when a CPP or a SPP method is used. When a full relay amplification matrix, $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are set to M_{R}-by-M_{R} DFT matrices while $\boldsymbol{F}_3$ is a Vandermonde matrix. When a diagonal relay amplification matrix is used as explained in Remark 1, the matrix $\boldsymbol{G} \in \mathbb{C}^{M_{\mathrm{R}} \times M_{\mathrm{R}}}$

is set to a Vandermonde matrix. As depicted in Fig. 1a, the KRF approach always provides the best channel estimates compared to the XFAS14 algorithms regardless whether a CPP or a SPP procedure is applied. It also provides a better BER performance especially when the SPP method is applied. When the CPP based method is used, the performance gain over the XFAS14 algorithm is not significant due to the fact that the estimate of $\boldsymbol{S}$ from the direct link dominates the performance. Moreover, all the proposed algorithms require only a few iterations compared to the XFAS14 algorithm as shown in Table 1. Finally, we conclude that the proposed bilinear ALS algorithms are computationally more efficient and can provide a better performance compared to the state of the art algorithms in [12]. Moreover, although a full relay amplification matrix contains more free parameters than a diagonal one, to fully exploit this freedom an appropriate design of the relay amplification matrix is desired.

Table 1. Comparison of the average number of required iterations when the SPP method is used, $M_{\mathrm{S}} = M_{\mathrm{D}} = M_{\mathrm{R}} = K = 2$, $N = 4$, and $\alpha = 10$ dB under different values of $\mathrm{SNR}_{\mathrm{SRD}}$.

Algorithm	$\mathrm{SNR}_{\mathrm{SRD}} = 0$ dB	$\mathrm{SNR}_{\mathrm{SRD}} = 15$ dB	$\mathrm{SNR}_{\mathrm{SRD}} = 30$ dB
XFAS14	107.2	99.2	55.1
KRF (full)	3.8	2.6	2.0
KRF (diag)	3.8	2.7	2.0

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