Multi-Linear Encoding and Decoding for MIMO Systems

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***Abstract*—The objective of future wireless communication systems is to provide a reliable and high quality of service. We propose multi-linear encoding and decoding strategies by exploiting Kronecker-structured constant modulus constellations for providing a low bit error ratio (BER) in multiple-input-multiple-output (MIMO) systems. The encoding schemes are based on the one-layer Khatri-Rao, two-layer Khatri-Rao and hybrid Kronecker-Khatri-Rao encoding processes. The corresponding multi-linear decoders consist of closed-form algorithms based on rank-one approximations of matrices and/or tensors. Compared with the convolutional codes with hard and soft Viterbi decoders, the proposed multi-linear encoding and decoding strategies outperform the latter in terms of BER for the same spectral efficiency.**

***Index Terms*—Multi-linear encoding and decoding, multidimensional least squares Kronecker factorization, least squares Khatri-Rao factorization.**

I. Introduction

The fifth-generation (5G) promises the fulfilling of the new communication needs, which is low-latency, high speed besides with high-reliability communication for end-to-end service [1]. A prominent feature of error-correcting codes for data and control channels are low-density-parity-check (LDPC) codes and polar codes [1]. These traditional codes have different performances and computational complexities depending on code rate and block lengths [2], [3].

A new class of linear numerical codes based on tensor codes formed as the Kronecker product, is proposed in [4]. A folded-tree decoder is proposed in [5], for efficient maximum-likelihood (ML) decoding strategy for Kronecker product-based (KPB) for Reed-Muller (RM) and polar codes. The authors in [6], proposed a receiver strategy which exploits cross-coding using tensor space-time coding (TSTC) based on Kronecker product.

This paper is based on the follow-up of [7], where we have proposed coding and decoding strategy. The encoding is based on Kronecker-structured constant modulus constellations property of M-PSK (phase shift keying) and the decoder is based on Kronecker-RoD (Rank-one detector), using single-input-single-output (SISO) scenario. We propose three different multi-linear encoding strategies for multiple-input-multiple-output (MIMO) system. The proposed schemes are one-layer Khatri-Rao, two-layer Khatri-Rao and hybrid Kronecker-Khatri-Rao multi-linear encoding schemes. At the receiver, we derive the corresponding decoding algorithms by exploiting the algebraic structure of the different multi-linear encoding schemes. The decoding schemes are based on rank-one approximations and have good noise rejection capabilities. As shown in our numerical results, the newly proposed schemes have superior performance in terms of bit error ratio (BER) metric as compared to the hard and soft Viterbi decoding at the same spectral efficiency.

***Notation*:** Scalars are denoted by lower-case italic letters $(a, b, \dots)$, vectors by bold lower-case italic letters $(\boldsymbol{a}, \boldsymbol{b}, \dots)$, matrices by bold upper-case italic letters $(\boldsymbol{A}, \boldsymbol{B}, \dots)$, tensors are defined by calligraphic upper-case letters $(\mathcal{A}, \mathcal{B}, \dots)$, $\boldsymbol{A}^{\mathrm{T}}$,$\boldsymbol{A}^{*}$,$\boldsymbol{A}^{\mathrm{H}}$ stand for transpose, conjugate and Hermitian of $\boldsymbol{A}$, respectively. The operators $\otimes$, $\diamond$ and $\circ$ define the Kronecker, Khatri-Rao and the outer product, respectively.

II. System Model

We consider a multiple-input-multiple-output (MIMO) wireless communication system, where the base station (BS) is equipped with N_t transmit antennas and the user equipment (UE) is equipped with M_r receive antennas. The received signal matrix $\boldsymbol{Y} \in \mathbb{C}^{M_r \times L}$ can be written as

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{N}, \tag{1}$$

where $\boldsymbol{H} \in \mathbb{C}^{M_r \times N_t}$ denotes the MIMO channel matrix, assumed to be known at the receiver. The matrix $\boldsymbol{X} \in \mathbb{C}^{N_t \times L}$ represents the coded data block where L represents the length of the coded block. $\boldsymbol{N} \in \mathbb{C}^{M_r \times L}$ is the additive white Gaussian noise matrix having distribution $\mathcal{CN}\left(\boldsymbol{0}_{M_r \times L}, \sigma_n^2 \boldsymbol{I}_{M_r \times L}\right)$. Assuming classical linear receivers and the estimate of the coded block $\hat{\boldsymbol{X}}$ by using minimum mean square error (MMSE) or zero forcing (ZF) equalizer is given by

$$\hat{\boldsymbol{X}} = \boldsymbol{W}\boldsymbol{Y}, \tag{2}$$

where $\boldsymbol{W}$ is defined for MMSE as

$$\boldsymbol{W} = \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma_n^2 \boldsymbol{I}_{N_t}\right)^{-1} \boldsymbol{H}^{\mathrm{H}}, \tag{3}$$

and for ZF as

$$\boldsymbol{W} = \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\right)^{-1} \boldsymbol{H}^{\mathrm{H}}. \tag{4}$$

III. MIMO Transmission Schemes

In this section, the different proposed transmission strategies are used to design the coded matrix $\boldsymbol{X}$. The multi-linear encoding is used to spread the modulated data in both temporal and spatial domain. The principle behind the multi-linear modulation is explained in [7] and is defined as, the Kronecker

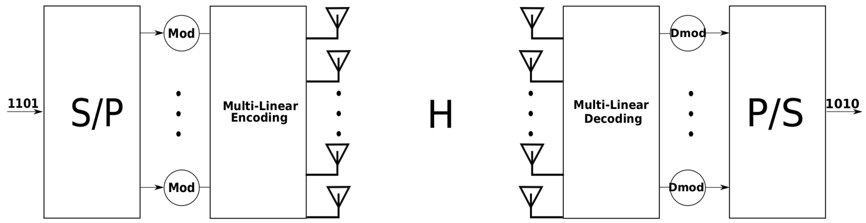

Fig. 1. Multi-linear encoding and decoding in MIMO systems.

product of two or more constant modulus symbols (M-PSK) can result in a known M-PSK modulated symbols. The system model is shown in Fig. 1.

A. Scheme 1 (One-layer Khatri-Rao encoding)

In scheme 1, the modulated data is spread along the temporal domain only. Every transmit antenna transmits a different data stream as

$$\boldsymbol{x}^{(n)} = \boldsymbol{s}_M^{(n)} \otimes \cdots \otimes \boldsymbol{s}_1^{(n)} \in \mathbb{C}^{L\times 1}, \tag{5}$$

where $n = 1, \ldots, N_t$ and $\boldsymbol{s}_m^{(n)} \in \mathbb{C}^{Q\times 1}, m = 1, \ldots, M$, with $L = Q^M$, where L is length of the nth data stream. According to (5), nth data stream is the result of Kronecker product of M symbol vectors composed of constant modulus constellations. We refer the reader to [7]. Let us define the following symbol matrix $\boldsymbol{S}_m$ as

$$\boldsymbol{S}_m = \left[\boldsymbol{s}_m^{(1)} \ldots \boldsymbol{s}_m^{(N_t)}\right] \in \mathbb{C}^{Q\times N_t}, \tag{6}$$

where $m = 1, \ldots, M$. By collecting N_t symbol vectors in matrix $\boldsymbol{X}$ using (5) as

$$\boldsymbol{X} = \left[\boldsymbol{x}^{(1)} \ldots \boldsymbol{x}^{(N_t)}\right]^{\mathrm{T}} \in \mathbb{C}^{N_t\times L} \tag{7}$$

This matrix $\boldsymbol{X}$ can be represented as,

$$\boldsymbol{X} = \begin{bmatrix} [\boldsymbol{s}_M^{(1)} \otimes \cdots \otimes \boldsymbol{s}_1^{(1)}]^{\mathrm{T}} \\ \vdots \\ [\boldsymbol{s}_M^{(N_t)} \otimes \cdots \otimes \boldsymbol{s}_1^{(N_t)}]^{\mathrm{T}} \end{bmatrix} \in \mathbb{C}^{N_t\times L}. \tag{8}$$

where $n = 1, \ldots, N_t$, which can be compactly written in terms of Khatri-Rao product as

$$\boldsymbol{X} = [\boldsymbol{S}_M \diamond \cdots \diamond \boldsymbol{S}_1]^{\mathrm{T}} \in \mathbb{C}^{N_t\times L}. \tag{9}$$

At the receiver after applying linear (ZF/MMSE) filtering to get the estimate of $\hat{\boldsymbol{X}}$ as shown in (2). The transmitted matrices $\{\boldsymbol{S}_M, \ldots, \boldsymbol{S}_1\}$ are decoded by applying the Kronecker-RoD approach explained in [7]. For this scheme, the receiver processing consists of applying N_t rank-one tensor approximations using tensor-power-method detector (TPMD) [7]. Therefore, scheme 1 can be seen as straightforward generalization of [7] to the MIMO case. Note that, N_t TPMD's steps can be implemented in parallel. The one-layer Khatri-Rao decoder is shown in Algorithm 1.

In the following, we present two additional multi-linear encoding and decoding schemes that exploit the spatial dimension at the transmitter by combining both temporal and spatial spreading of data streams at different degrees. This is achieved by means of Khatri-Rao and Kronecker product-structured encoding.

Algorithm 1 One-layer Khatri-Rao decoder

```
procedure INITIALIZATION(X̂)
    for n = 1 : N_t do
        [ŝ_M^(n), ..., ŝ_1^(n)] ← TPMD(x̂^(n))
    end for
end procedure
```

B. Scheme 2 (Two-layer Khatri-Rao encoding)

In scheme 2, as the name suggests, the coded matrix $\boldsymbol{X}$ is designed using a two-step of spreading process. The transmit antennas are partitioned into N_1 groups of N_2 antennas each, so that $N_t = N_1 N_2$. Let us define the following symbol matrices $\boldsymbol{S}_m^{(1)}$ and $\boldsymbol{S}_m^{(2)}$ as

$$\boldsymbol{S}_m^{(1)} = \left[\boldsymbol{s}_m^{(1,1)} \ldots \boldsymbol{s}_m^{(1,N_1)}\right] \in \mathbb{C}^{Q\times N_1}, \tag{10}$$

$$\boldsymbol{S}_m^{(2)} = \left[\boldsymbol{s}_m^{(2,1)} \ldots \boldsymbol{s}_m^{(2,N_2)}\right] \in \mathbb{C}^{Q\times N_2}, \tag{11}$$

where $m = 1, \ldots, M$. Following the Khatri-Rao encoding structure of scheme 1 in (9), we define two temporally spread data blocks as,

$$\boldsymbol{X}_1 = \left[\boldsymbol{S}_M^{(1)} \diamond \cdots \diamond \boldsymbol{S}_1^{(1)}\right]^{\mathrm{T}} \in \mathbb{C}^{N_1\times L}, \tag{12}$$

$$\boldsymbol{X}_2 = \left[\boldsymbol{S}_M^{(2)} \diamond \cdots \diamond \boldsymbol{S}_1^{(2)}\right]^{\mathrm{T}} \in \mathbb{C}^{N_2\times L}, \tag{13}$$

spatial spreading is achieved by taking the Khatri-Rao product between the temporally-coded block matrices $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ to get

$$\begin{aligned} &\boldsymbol{X} = \boldsymbol{X}_1 \diamond \boldsymbol{X}_2 \in \mathbb{C}^{N_t\times L} \\ &= \begin{bmatrix} [\boldsymbol{s}_M^{(1,1)} \otimes \cdots \otimes \boldsymbol{s}_1^{(1,1)}]^{\mathrm{T}} \\ \vdots \\ [\boldsymbol{s}_M^{(1,N_1)} \otimes \cdots \otimes \boldsymbol{s}_1^{(1,N_1)}]^{\mathrm{T}} \end{bmatrix} \diamond \begin{bmatrix} [\boldsymbol{s}_M^{(2,1)} \otimes \cdots \otimes \boldsymbol{s}_1^{(2,1)}]^{\mathrm{T}} \\ \vdots \\ [\boldsymbol{s}_M^{(2,N_2)} \otimes \cdots \otimes \boldsymbol{s}_1^{(2,N_2)}]^{\mathrm{T}} \end{bmatrix} \end{aligned} \tag{14}$$

Such a Khatri-Rao encoding implements a *spatial cross-spreading* of $\{\boldsymbol{S}_m^{(1)}\}$ and $\{\boldsymbol{S}_m^{(2)}\}$ in the spatial domain $m = 1, \ldots, M$. In other words, the columns of $\boldsymbol{X}_1$ are spatially spread across N_2 antennas, while the columns of $\boldsymbol{X}_2$ are spatially spread across N_1 antennas. After ZF/MMSE filtering, the decoding processes consist of two layers. In the first layer, least squares Khatri-Rao factorization (LS-KRF) [8], [9] is applied to estimate $\hat{\boldsymbol{X}}_1, \hat{\boldsymbol{X}}_2$. The second layer consists of applying N_1 TPMD's [7] to decode $\boldsymbol{S}_m^{(1)}$ and N_2 TPMD's to decode $\boldsymbol{S}_m^{(2)}$, where $m = 1, \ldots, M$. Note that the $(N_1 + N_2)$

TPMD's can operate in parallel. A summary of the two-layer Khatri-Rao decoder is shown in Algorithm 2.

Algorithm 2 Two-layer Khatri-Rao decoder

```
procedure INITIALIZATION(X̂)
    [X̂_1, X̂_2] ← LS-KRF(X̂)
    for n = 1 : N_1 do
        [ŝ_M^(1,n), ..., ŝ_1^(1,n)] ← TPMD(x̂_1^(n))
    end for
    for n = 1 : N_2 do
        [ŝ_M^(2,n), ..., ŝ_1^(2,n)] ← TPMD(x̂_2^(n))
    end for
end procedure
```

C. Scheme 3 (Hybrid Kronecker-Khatri-Rao encoding)

Scheme 3 is a hybrid combination of Kronecker and Khatri-Rao encodings. In the first layer, the data blocks are generated by a joint spatial-temporal Kronecker-product encoding of multiple symbol matrices as

$$\boldsymbol{X}_1 = \boldsymbol{S}_M^{(1)} \otimes \cdots \otimes \boldsymbol{S}_1^{(1)} \in \mathbb{C}^{N_1 \times L}, \tag{15}$$

$$\boldsymbol{X}_2 = \boldsymbol{S}_M^{(2)} \otimes \cdots \otimes \boldsymbol{S}_1^{(2)} \in \mathbb{C}^{N_2 \times L}, \tag{16}$$

where $L = Q^M$, $N_1 = P_1^M$, $N_2 = P_2^M$, $N_t = N_1 N_2$ and $\boldsymbol{S}_m^{(1)} \in \mathbb{C}^{P_1 \times Q}, \boldsymbol{S}_m^{(2)} \in \mathbb{C}^{P_2 \times Q}, m = 1, \ldots, M$. For notational convenience, we have fixed the size of the matrices $\boldsymbol{S}_m^{(1)}$, $\boldsymbol{S}_m^{(2)}$, although matrices with arbitrary dimensions could also be considered. In the second layer, following the same approach of scheme 2, the resultant data blocks (15) and (16) are cross-spread along the spatial domain via the Khatri-Rao product which gives $\boldsymbol{X} = \boldsymbol{X}_1 \diamond \boldsymbol{X}_2 \in \mathbb{C}^{N_t \times L}$. After ZF/MMSE equalization, the decoding of $\hat{\boldsymbol{X}}$ also follows a two-layer approach. The first layer decoding is exactly the same as that of scheme 2, where LS-KRF is applied. As for the second layer, the transmitted symbol matrices are estimated by solving multidimensional least-squares-Kronecker factorization (MLS-KronF) [10], [11] that consists of optimizing the following problem

$$\left[\hat{\boldsymbol{S}}_M^{(i)}, \ldots, \hat{\boldsymbol{S}}_1^{(i)}\right] = \arg \min_{\boldsymbol{S}_M^{(i)}, \ldots, \boldsymbol{S}_1^{(i)}} \left\| \hat{\boldsymbol{X}}_i - \boldsymbol{S}_M^{(i)} \otimes \cdots \otimes \boldsymbol{S}_1^{(i)} \right\|_{\mathrm{F}}, \tag{17}$$

it can be shown, that problem (17) can be translated into the following rank-one tensor approximation problem [12]

$$\left[\hat{\boldsymbol{s}}_M^{(i)}, \ldots, \hat{\boldsymbol{s}}_1^{(i)}\right] = \arg \min_{\boldsymbol{s}_M^{(i)}, \ldots, \boldsymbol{s}_1^{(i)}} \left\| \hat{\mathcal{X}}_i - \boldsymbol{s}_M^{(i)} \circ \cdots \circ \boldsymbol{s}_1^{(i)} \right\|_2, \tag{18}$$

where the Mth order tensor $\mathcal{X}_i \in \mathbb{C}^{P_i Q \times \cdots \times P_i Q}$ is obtained by reshaping $\hat{\boldsymbol{X}}_i, i = 1, 2$ according to [12]. The so-called hybrid Khatri-Rao-Kronecker decoder is shown in Algorithm 3. Note that lines 3 and 4 of Algorithm 3 can be executed in parallel.

Algorithm 3 Hybrid Khatri-Rao-Kronecker decoder

```
1: procedure INITIALIZATION(X̂)
2:     [X̂_1, X̂_2] ← LS-KRF(X̂)
3:     [Ŝ_M^(1), ..., Ŝ_1^(1)] ← MLS-KronF(X̂_1)
4:     [Ŝ_M^(2), ..., Ŝ_1^(2)] ← MLS-KronF(X̂_2)
5: end procedure
```

For scheme 1, $N_t QM$ useful symbols (9) are transmitted via $N_t L$ channel uses, which gives the code rate

$$R_1 = \frac{N_t QM}{N_t Q^M} = \frac{QM}{Q^M}. \tag{19}$$

For the second scheme, $(N_1 + N_2)MQ$ information symbols are transmitted as shown in (12), (13), yielding the following code rate

$$R_2 = \frac{(N_1 + N_2)QM}{N_t Q^M}. \tag{20}$$

As for scheme 3, $(P_1 + P_2)QM$ useful symbols (15), (16) are transmitted during the same $N_t L$ channel uses, which gives

$$R_3 = \frac{(P_1 + P_2)QM}{N_t Q^M}, \tag{21}$$

Note that, since $(P_1 + P_2) \leq (N_1 + N_2) \leq N_t$, the code rates of the three proposed schemes are related by $R_3 \leq R_2 \leq R_1$. It is worth noting that the estimates of the multiple symbol matrices as outputs of Algorithms 1,2 and 3 are affected by complex scaling ambiguities. For coherent modulation/demodulation schemes, these ambiguities can be removed by resorting to pilot symbols appended to the transmitted symbol matrices. A discussion on strategies to compensate the scaling ambiguities is omitted due to limited space. Note, however that, for differential modulation schemes, these ambiguities are irrelevant.

D. Computational complexity of the proposed decoders

The computational complexity of the one-layer Khatri-Rao decoder (scheme 1) is N_t times the complexity of a single TPMD [7], which gives $\mathcal{O}\left(8N_t MJQ^4\right)$, where J is the number of iterations of a single TPMD. For the two-layer Khatri-Rao decoder (scheme 2), the LS-KRF decoding layer requires L rank-one matrix approximations, which gives $\mathcal{O}\left(LN_2^2 N_1\right)$. Since the second layer requires $(N_1 + N_2)$ TPMDs, the overall complexity of the two-layer Khatri-Rao decoder is $\mathcal{O}\left(LN_2^2 N_1 + \left((N_1 + N_2)\left(8MJQ^4\right)\right)\right)$. Finally, for the hybrid Khatri-Rao-Kronecker decoder (scheme 3), the overall complexity is $\mathcal{O}\left(LN_2^2 N_1 + MQ^4\left(P_1^4 + P_2^4\right)\right)$, where the second term denotes the complexity of the two MLS-KronF procedures (steps 3 and 4 of Algorithm 3). Recall that the proposed multi-linear decoders consist of closed-form algorithms based on rank-one approximations of matrices and/or tensors, for which several efficient implementations exist in the literature. In addition, the routines of the three decoding algorithms can be executed in parallel, which results in fast processing with short delays.

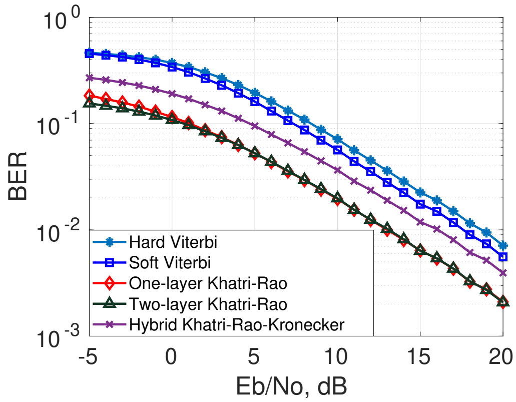

Fig. 2. BER performance with ZF receiver (4×4 MIMO).

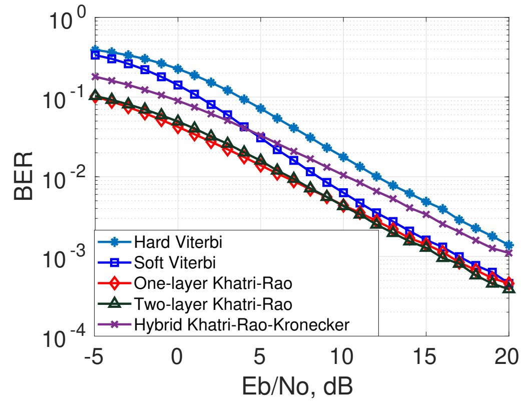

Fig. 3. BER performance with MMSE receiver (4×4 MIMO).

TABLE I
CODE RATES OF THE SIMULATED SCHEMES

Scheme	MIMO	Code rate
1	4×4	1/2
2	4×4	1/2
3	4×4	3/8
1	16×16	1/2
2	16×16	1/4
3	16×16	1/8

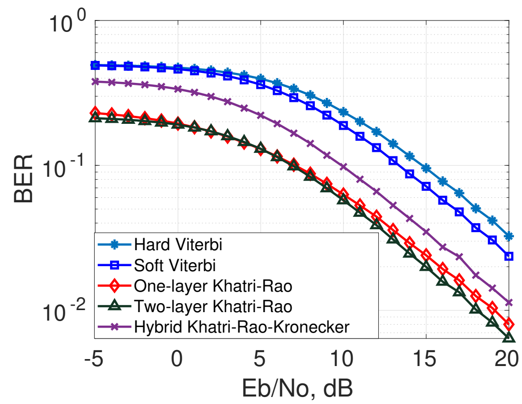

Fig. 4. BER performance with ZF receiver (16×16 MIMO).

IV. Numerical Results

We evaluate the performance of the proposed multi-linear encoding and decoding schemes in terms of their BER for 4-PSK modulated symbols. The MIMO channel coefficients follow a zero mean and unit variance complex Gaussian distribution. To have a fair comparison, a half-rate convolutional encoder (with hard and soft Viterbi decoding) is used. The multi-linear encoding parameters (common to all schemes) are $M = 4, Q = 2, L = Q^M = 16$. The code rates of the simulated schemes are shown in Table I.

In Fig. 2, different transmission schemes are compared for the 4×4 MIMO case with ZF receiver. All of them perform satisfactorily and offer improved performance compared to the half-rate conventional codes with Viterbi decoding. The performance gap is more pronounced at the low Eb/No range. The one-layer and two-layer Khatri-Rao decoders (schemes 1 and 2, respectively) exhibit similar performances, which are better than that of the hybrid Khatri-Rao-Kronecker decoder. The gains of the proposed multi-linear encoding/decoding schemes over the reference solution comes from the noise rejection capability of the tensor rank-1 approximation (second decoding layer of Algorithms 1, 2 and 3), which increases with the number M of encoded symbol matrices. For a target of BER of 10^{-2}, the one-layer Khatri-Rao and two-layer Khatri-Rao decoders provide an approximate gain of 6 dB over the soft Viterbi decoder. Fig. 3, shows the performance of the same schemes but now using the MMSE receiver. As the MMSE receiver takes into account the noise variance, the noise rejection capability of the three schemes is not the dominant factor for the performance (in contrast to the ZF receiver case). Nevertheless, the one-layer and two-layer Khatri-Rao decoders outperform the reference solutions in the low Eb/No range. Fig. 4, we compared the different schemes for a 16×16 MIMO system using the ZF receiver. The BER curves behave similarly to those of Fig. 2 but the performance gains of the multi-linear encoding and decoding schemes over the baseline solutions increase. For a target BER of 10^{-1}, the two-layer Khatri-Rao decoder has a 8 dB gain as compared to the soft Viterbi decoder. However, note that this gain comes at the expense of a reduced code rate, as can be seen in Table I. In this scenario, the one-layer Khatri-Rao decoder is a good choice because it has the same code rate as the Viterbi decoder while offering lower error rates. The slope of BER curves of the different schemes are almost the same, showing the same diversity order at higher Eb/No, but different coding gains.

V. Conclusion

We proposed multi-layer encoding and decoding strategies for Kronecker-structured constant modulus constellations. The decoding algorithms are based on closed-form rank-one approximations of matrices and/or tensors and showed improved BER performance of the proposed schemes compared to reference Viterbi decoders assuming a perfect channel knowledge. A perspective of this work is its extension to the case of a joint semi-blind channel estimation and symbol decoding, which would avoid the prior ZF/MMSE filtering stage and would relax the channel knowledge assumption.

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