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Effects of Imperfect Channel State Information on Achievable Rates of Precoded Multi-user MIMO Broadcast Channels with Limited Feedback

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***Abstract*—We consider multi-user MIMO broadcast channels with limited feedback. A recently proposed linear precoding technique, regularized block diagonalization (RBD), is used to mitigate multi-user interference. We assume that each receiver has estimated channel state information (CSI) via downlink training and independently quantizes its channel by using an efficient channel quantization scheme that we propose in this paper. The transmitter acquires the quantized CSI from each receiver through a noiseless and delayed feedback channel. The achievable rates are studied under these assumptions. We derive an upper bound for the rate loss compared to the case that the transmitter has perfect CSI and quantify the impact of channel estimation errors, quantization errors, and outdated quantized CSI on the rate loss. Furthermore, we provide an expression of the number of feedback bits needed per user to maitain that bound. It is found that the delay of the feedback is the predominant cause of performance degradation in the case of rapidly changing channel impluse response.**

I. Introduction

Multi-user MIMO broadcast channels attract much attention due to a very significant increase in capacity and spectral efficiency compared to point-to-point MIMO channels. The base station (BS) can use space division multiple access (SDMA) to simultaneously transmit data streams to a group of users by appropriate utilization of spatial dimensions. Obviously, the multi-user MIMO benefit comes from being able to know the instantaneous channel state information (CSI) at the BS. Otherwise the multi-user MIMO benefit is completely lost and point-to-point MIMO should be preferred.

In frequency division duplex (FDD) systems, a feedback channel is needed for the BS to acquire the knowledge of CSI from each receiver. In this work we focus on FDD systems and in order to further reduce the feedback overhead we consider the feedback of quantized channel state information instead of analog feedback.

Among the different SDMA strategies we choose regularized block diagonalization (RBD) due to its low complexity compared to dirty paper coding (DPC) and a significantly improved sum rate and diversity order compared to other linear precoding techniques [1]. But at the same time RBD increases the challenge of channel quantization at the receiver side in contrast to zero-forcing (ZF) or block diagonalization (BD) precoding [2], where the transmitter only requires the channel direction information, i.e., knowledge of the subspaces spanned by the users' channel matrices. For RBD precoding the transmitter additionally requires the channel magnitude information, which defines the strength of the eigenmodes of the users' channel matrices. One of the key contributions in this work is that instead of the channel quantization scheme proposed previously [3], [4], which only provides channel direction information, we present a new efficient channel quantization scheme to provide both channel direction and magnitude information.

In this paper we consider that each receiver has imperfect CSI, which is the basis of the channel quantization. We further consider that each receiver sends the quantized CSI to the BS through a noiseless delayed feedback channel. The channel estimation error, quantization error, and feedback delay cause a degradation of the quality of the CSI provided at the BS and have a non-negligible effect. The aim of this work is to assess this effect on the achievable data rates of a multi-user MIMO system employing RBD precoding. We derive an upper bound for the rate loss and quantify the number of feedback bits needed per user to maintain the bound.

II. System Model

We consider a multi-user MIMO system with a single BS and K users, where the BS has an antenna array of M_{T} elements and the ith user has M_{R_i} receive antennas. The propagation channel between the BS and each user is assumed to obey a block fading model, which means that the channel is constant over one block of L symbol intervals and possibly varying from block to block. During one block the entries of the channel are uncorrelated Gaussian i.i.d. and have zero mean and unit variance. The channel estimation (via downlink training) and data transmission are performed in one block. After that new training allows us to estimate the channel for the next block.

The received signal of the ith user within one block is expressed as

$$\boldsymbol{Y}_i = \boldsymbol{H}_i \boldsymbol{X} + \boldsymbol{Z}_i \tag{1}$$

where the matrix $\boldsymbol{Z}_i \in \mathbb{C}^{M_{\mathrm{R}_i} \times L}$ denotes Additive White Gaussian Noise (AWGN) with unit variance. The matrix $\boldsymbol{H}_i \in \mathbb{C}^{M_{\mathrm{R}_i} \times M_{\mathrm{T}}}$ represents the channel matrix from the BS to the ith user. We assume spatially uncorrelated Rayleigh fading for all the channels. The matrix $\boldsymbol{X}$ is composed of the following two phases

- **Training Phase:**
$$\boldsymbol{X}^p = A_p \boldsymbol{S}^p \tag{2}$$
where $\boldsymbol{S}^p \in \mathbb{C}^{M_{\mathrm{T}} \times L_1}$ is the matrix of pilot symbols shared by all users. L_1 is the number of pilot symbols per transmit antenna.
- **Data Transmission Phase:**
$$\boldsymbol{X}^d = \boldsymbol{F}\boldsymbol{S}^d = \textstyle\sum_{i=1}^{K} \boldsymbol{F}_i \boldsymbol{S}_i^d \tag{3}$$
where the matrix $\boldsymbol{S}_i^d \in \mathbb{C}^{M_{\mathrm{T}} \times (L - L_1)}$ contains the transmitted data symbols for the ith user. The matrix $\boldsymbol{F}_i \in \mathbb{C}^{M_{\mathrm{T}} \times M_{\mathrm{T}}}$ denotes the RBD precoding matrix for user i.

The total transmit power is equal to P, where $P = P^d + P^p$. The fraction $P^p = \alpha P$ is used for channel estimation and $P^d = (1-\alpha)P$ is for data transmission. We further assume that the power P^d is equally distributed between K users. We have

$$A_p = \sqrt{\frac{\alpha P}{M_{\mathrm{T}}}}, \quad \mathrm{E}\left\{ \boldsymbol{S}^p \boldsymbol{S}^{pH} \right\} = \boldsymbol{I}_{M_{\mathrm{T}}} \tag{4}$$

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and

$$\mathrm{E}\left\{\boldsymbol{S}_i^d \boldsymbol{S}_i^{dH}\right\} = \frac{(1-\alpha)P}{K \cdot M_{\mathrm{T}}} \boldsymbol{I}_{M_{\mathrm{T}}} = \frac{P_s}{K} \boldsymbol{I}_{M_{\mathrm{T}}}, \quad \mathrm{tr}(\boldsymbol{F}_i \boldsymbol{F}_i^H) = 1 \ . \tag{5}$$

The signal-to-noise ratio (SNR) is defined as the ratio between P_s and the noise variance, i.e., $\mathrm{SNR} = P_s$.

A. Regularization Block Diagonalization Precoding

RBD precoding is designed to extract the full diversity gain and a high sum-rate capacity with an arbitrary number of antennas at the user terminals. The precoding matrix of user i is described as [1]

$$\boldsymbol{F}_i = \gamma \boldsymbol{F}_{a_i} \boldsymbol{F}_{b_i} \ , \tag{6}$$

where

$$\boldsymbol{F}_{a_i} = \widetilde{\boldsymbol{V}}_i (\widetilde{\boldsymbol{\Sigma}}_i^T \widetilde{\boldsymbol{\Sigma}}_i + \mu \boldsymbol{I}_{M_{\mathrm{T}}})^{-1/2} \ . \tag{7}$$

Here the matrix $\boldsymbol{F}_{a_i}$ is used to suppress multi-user interference (MUI) while balancing it with noise enhancement. The matrix $\boldsymbol{F}_{b_i}$ can further optimize the system performance by optimal power loading. In this work we assume that $\boldsymbol{F}_{b_i}$ is unitary. The parameter γ is chosen to set the total transmit power to the power constraint P^d, and $\mu = \left(\frac{P^d}{M_{\mathrm{R}} \sigma_n^2}\right)^{-1}$ where M_{R} denotes the total number of receive antennas for all K users. The matrices $\widetilde{\boldsymbol{V}}_i \in \mathbb{C}^{M_{\mathrm{T}} \times M_{\mathrm{T}}}$ and $\widetilde{\boldsymbol{\Sigma}}_i \in \mathbb{C}^{(M_{\mathrm{R}} - M_{\mathrm{R}_i}) \times M_{\mathrm{T}}}$ are the right singular vectors and the diagonal matrix of the singular values of the combined channel matrix of all other users, respectively.

In [1] equation (7) is derived under the condition $\mathrm{E}\left\{\boldsymbol{S}_i^d \boldsymbol{S}_i^{dH}\right\} = \boldsymbol{I}_{M_{\mathrm{T}}}$. Taking into account $\mathrm{E}\left\{\boldsymbol{S}_i^d \boldsymbol{S}_i^{dH}\right\} = \frac{P_s}{K} \boldsymbol{I}_{M_{\mathrm{T}}}$ in our system model, we rewrite the expressions in (22) and (26) of [1] as

$$\boldsymbol{F}_a = \min_{\boldsymbol{F}_a} \ \mathrm{E}\left\{ \sum_{i=1}^K \left\| \widetilde{\boldsymbol{H}}_i \boldsymbol{F}_{a_i} \boldsymbol{S}_i^d \right\|_{\mathrm{F}}^2 + \frac{\|\boldsymbol{Z}_i\|_{\mathrm{F}}^2}{\beta^2} \right\} \tag{8}$$

$$\begin{aligned} \beta^2 &= \frac{P^d}{\sum_{i=1}^K \mathrm{tr}(\boldsymbol{F}_{a_i} \boldsymbol{S}_i^d \boldsymbol{S}_i^{dH} \boldsymbol{F}_{a_i}^H)} \\ &= \frac{K \cdot M_{\mathrm{T}}}{\sum_{i=1}^K \mathrm{tr}(\boldsymbol{F}_{a_i} \boldsymbol{F}_{a_i}^H)} \ . \end{aligned} \tag{9}$$

Then using a similar derivation as in [1] we get a new expression for $\boldsymbol{F}_{a_i}$ under the condition $\mathrm{E}\left\{\boldsymbol{S}_i^d \boldsymbol{S}_i^{dH}\right\} = \frac{P_s}{K} \boldsymbol{I}_{M_{\mathrm{T}}}$ as

$$\boldsymbol{F}_{a_i} = \widetilde{\boldsymbol{V}}_i \left(\frac{P_s}{K} \widetilde{\boldsymbol{\Sigma}}_i^T \widetilde{\boldsymbol{\Sigma}}_i + \frac{M_{\mathrm{R}} \sigma_n^2}{K \cdot M_{\mathrm{T}}} \boldsymbol{I}_{M_{\mathrm{T}}}\right)^{-1/2} \ . \tag{10}$$

Furthermore, we model the MUI caused by all other users to the user i as Gaussian distributed with variance σ_{MUI}^2. Then we have

$$\begin{aligned} & M_{\mathrm{R}_i} \sigma_{\mathrm{MUI}}^2 \\ &= \mathrm{E}\left\{ \sum_{j \neq i, j=1}^K \left\| \boldsymbol{H}_i \boldsymbol{F}_j \boldsymbol{S}_j^d \right\|_{\mathrm{F}}^2 \right\} \\ &= \sum_{j \neq i, j=1}^K \mathrm{E}\left\{ \left\| \boldsymbol{H}_i \boldsymbol{F}_j \boldsymbol{S}_j^d \right\|_{\mathrm{F}}^2 \right\} \\ &= \sum_{j \neq i, j=1}^K \mathrm{tr}(\boldsymbol{H}_i \boldsymbol{F}_j \mathrm{E}\left\{ \boldsymbol{S}_j^d \boldsymbol{S}_j^{dH} \right\} \boldsymbol{F}_j^H \boldsymbol{H}_i^H) \\ &= \sum_{j \neq i, j=1}^K \mathrm{tr}(\boldsymbol{H}_i \widetilde{\boldsymbol{V}}_j (\tfrac{P_s}{K} \widetilde{\boldsymbol{\Sigma}}_j^T \widetilde{\boldsymbol{\Sigma}}_j + \tfrac{M_{\mathrm{R}} \sigma_n^2}{K \cdot M_{\mathrm{T}}} \boldsymbol{I}_{M_{\mathrm{T}}})^{-1} \tfrac{P_s}{K} \widetilde{\boldsymbol{V}}_j^H \boldsymbol{H}_i^H) \\ &= \sum_{j \neq i, j=1}^K \mathrm{tr}(\boldsymbol{H}_i \widetilde{\boldsymbol{V}}_j (\widetilde{\boldsymbol{\Sigma}}_j^T \widetilde{\boldsymbol{\Sigma}}_j + \tfrac{M_{\mathrm{R}} \sigma_n^2}{P_s \cdot M_{\mathrm{T}}} \boldsymbol{I}_{M_{\mathrm{T}}})^{-1} \widetilde{\boldsymbol{V}}_j^H \boldsymbol{H}_i^H) \\ &\overset{(1)}{=} \sum_{j \neq i, j=1}^K \mathrm{tr}(\boldsymbol{H}_i \widetilde{\boldsymbol{V}}_j (\widetilde{\boldsymbol{\Sigma}}_j^T \widetilde{\boldsymbol{\Sigma}}_j + \mu \boldsymbol{I}_{M_{\mathrm{T}}})^{-1} \widetilde{\boldsymbol{V}}_j^H \boldsymbol{H}_i^H) \\ &\overset{(2)}{=} \sum_{j \neq i, j=1}^K \mathrm{tr}(\boldsymbol{H}_i [\widetilde{\boldsymbol{V}}_j^{(1)} \ \widetilde{\boldsymbol{V}}_j^{(0)}] (\widetilde{\boldsymbol{\Sigma}}_j^T \widetilde{\boldsymbol{\Sigma}}_j + \mu \boldsymbol{I}_{M_{\mathrm{T}}})^{-1} [\widetilde{\boldsymbol{V}}_j^{(1)} \ \widetilde{\boldsymbol{V}}_j^{(0)}]^H \boldsymbol{H}_i^H) \\ &\overset{(3)}{\approx} \sum_{j \neq i, j=1}^K \mathrm{tr}(\boldsymbol{H}_i \widetilde{\boldsymbol{V}}_j^{(1)} \widetilde{\boldsymbol{\Lambda}}_j^{-2} \widetilde{\boldsymbol{V}}_j^{(1)H} \boldsymbol{H}_i^H) \ . \end{aligned} \tag{11}$$

Note that at step (1) we got the same expression as in the case when we calculate $\left\| \boldsymbol{H}_i \boldsymbol{F}_j \boldsymbol{S}_j^d \right\|_{\mathrm{F}}^2$ with equation (7) and $\mathrm{E}\left\{\boldsymbol{S}_j^d \boldsymbol{S}_j^{dH}\right\} = \boldsymbol{I}_{M_{\mathrm{T}}}$. At step (2) we separate $\widetilde{\boldsymbol{V}}_j$ into $\widetilde{\boldsymbol{V}}_j^{(1)} \in \mathbb{C}^{M_{\mathrm{T}} \times (M_{\mathrm{R}} - M_{\mathrm{R}_j})}$ and $\widetilde{\boldsymbol{V}}_j^{(0)} \in \mathbb{C}^{M_{\mathrm{T}} \times (M_{\mathrm{T}} - M_{\mathrm{R}} + M_{\mathrm{R}_j})}$, which refer to the right singular vectors corresponding to non-zero singular values and the right singular vectors corresponding to zero singular values, respectively. In contrast to $\widetilde{\boldsymbol{\Sigma}}_j$ which increases with the SNR, the value of μ converges to zero with increasing SNR. In this case we neglect $\mu \boldsymbol{I}_{M_{\mathrm{T}}}$ to reach step (3), where we use $\widetilde{\boldsymbol{\Lambda}}_j \in \mathbb{C}^{(M_{\mathrm{R}} - M_{\mathrm{R}_j}) \times (M_{\mathrm{R}} - M_{\mathrm{R}_j})}$ to represent the economy-size version of $\widetilde{\boldsymbol{\Sigma}}_j$.

From equation (11) we can get

$$\sigma_{\mathrm{MUI}}^2 = \frac{1}{M_{\mathrm{R}_i}} \sum_{j \neq i, j=1}^K \mathrm{tr}(\boldsymbol{H}_i \widetilde{\boldsymbol{V}}_j^{(1)} \widetilde{\boldsymbol{\Lambda}}_j^{-2} \widetilde{\boldsymbol{V}}_j^{(1)H} \boldsymbol{H}_i^H). \tag{12}$$

The values of $\widetilde{\boldsymbol{\Lambda}}_j$ are directly proportional to the increasing SNR. As a result, at high SNR under the condition that $M_{\mathrm{T}} \geq M_R$, the MUI converges to zero and each user transmits only in the null space of all other users as in BD precoding.

Let us consider the data transmission phase. Then we can rewrite equation (1) as

$$\begin{aligned} \boldsymbol{Y}_i^d &= \boldsymbol{H}_i \boldsymbol{F}_i \boldsymbol{S}_i^d + \sum_{j=1, j \neq i}^K \boldsymbol{H}_i \boldsymbol{F}_j \boldsymbol{S}_j^d + \boldsymbol{Z}_i \\ &= \boldsymbol{H}_i \boldsymbol{F}_i \boldsymbol{S}_i^d + \boldsymbol{V}_i \ . \end{aligned} \tag{13}$$

Here we combine the Gaussian noise $\boldsymbol{Z}_i$ and the MUI for user i as a new noise $\boldsymbol{V}_i \sim \mathcal{CN}(0, (1 + \sigma_{\mathrm{MUI}}^2) \boldsymbol{I}_{M_{\mathrm{R}_i}})$.

III. Channel Quantization Scheme

Recently some research has been performed for limited feedback systems with channel direction quantization [3], [4]. The channel quantization model proposed in these references is suitable to perform ZF and BD precoding which only require channel direction information at the transmitter and cannot be naturally extended to the RBD case. In this section we propose a new efficient channel quantization scheme in order to provide the transmitter with both channel direction and magnitude information.

Our channel quantization scheme is based on the equation [5]

$$\mathrm{vec}\{\hat{\boldsymbol{H}}_i\} = \arg\min_{\boldsymbol{w}_j \in \mathcal{C}} \ d^2(\mathrm{vec}\left\{\boldsymbol{H}_i\right\}, \boldsymbol{w}_j) \tag{14}$$

where $\hat{\boldsymbol{H}}_i \in \mathbb{C}^{M_{\mathrm{R}_i} \times M_{\mathrm{T}}}$ is the quantized version of the channel matrix $\boldsymbol{H}_i$. The vector $\boldsymbol{w}_j \in \mathbb{C}^{M_{\mathrm{R}_i} \cdot M_{\mathrm{T}} \times 1}$ is one codeword of the quantization codebook $\mathcal{C}$ used at the ith user, which is fixed beforehand and known to the transmitter and user i. The vector $\mathrm{vec}\{\boldsymbol{H}_i\}$ represents the stacked form of the channel matrix $\boldsymbol{H}_i$ and $d(\mathrm{vec}\left\{\boldsymbol{H}_i\right\}, \boldsymbol{w}_j)$ is the distance metric. We consider the chordal distance [6] which leads to an efficient vector quantization algorithm for designing the codebook. Then we have

$$d(\mathrm{vec}\left\{\boldsymbol{H}_i\right\}, \boldsymbol{w}_j) = \sqrt{\sin^2 \Theta} \ . \tag{15}$$

Here, Θ is the principal angle between the two vectors.

Instead of directly quantizing the channel matrix, we first quantize the stacked vector of the channel matrix $\boldsymbol{H}_i$ according to equation (14), i.e., the codeword which is closest to $\mathrm{vec}\left\{\boldsymbol{H}_i\right\}$ is chosen as $\mathrm{vec}\{\hat{\boldsymbol{H}}_i\}$. Then we reshape $\mathrm{vec}\{\hat{\boldsymbol{H}}_i\}$ to get $\hat{\boldsymbol{H}}_i$. The advantages of our proposed channel quantization scheme are:

- To perform quantization, instead of calculating the minimum value of the sum of the principal angles spanned by columns of the channel matrices, we calculate the minimum value of the angle spanned by the two vectors.
- By quantizing the stacked vector of the channel matrix, we keep the relative magnitude information for the columns of the channel matrix and avoid the loss of the channel magnitude information which is caused by quantizing the channel matrix as a unitary codeword matrix directly.

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A. Codebook Construction and Quantization Distortion

In this work we consider random vector quantization (RVQ) and analyze the performance averaged over all possible random codebooks.

We assume that each codebook $\mathcal{C}$ consists of 2^B unit norm codewords ($\mathcal{C} = \{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{2^B}\}$). Each of the codewords has the dimension $M_{\mathrm{R}_i} \cdot M_{\mathrm{T}} \times 1$ and is independently and uniformly chosen from $\mathbb{G}_{n,m}(\mathbb{C})$. Here $\mathbb{G}_{n,m}(\mathbb{C})$ is the set of all m-dimensional planes in the n-dimensional Euclidean space. In this work $m = 1$, $n = M_{\mathrm{R}_i} \cdot M_{\mathrm{T}}$ and B is the number of feedback bits per user. One random codebook is generated for each user.

The distortion associated with a given random codebook $\mathcal{C}$ for the quantization of $\boldsymbol{H}_i$ is defined as

$$D(\mathcal{C}) = \mathrm{E}\Big\{ \min_{\boldsymbol{w}\in\mathcal{C}} \; d^2(\mathrm{vec}\,\{\boldsymbol{H}_i\}, \boldsymbol{w}) \Big\} \tag{16}$$

From [7] an upper bound to the quantization distortion is given by

$$\begin{aligned} D(\mathcal{C}) &\leq \bar{D}(\mathcal{C}) \\ &= \frac{\Gamma(\frac{1}{n-1})}{n-1} \cdot (C_{n,m,\xi})^{-\frac{1}{n-1}} \cdot 2^{-\frac{B}{n-1}} \end{aligned} \tag{17}$$

where $C_{n,m,\xi} = \frac{1}{(n-1)!} \prod_{i=1}^{m} \frac{\Gamma(\frac{\xi}{2}(n-i+1))}{\Gamma(\frac{\xi}{2}(m-i+1))} = 1$, if $\xi = 2$, $m = 1$. Here, $\Gamma(\cdot)$ represents the Gamma function. Therefore the upper bound can be simplified to

$$\bar{D}(\mathcal{C}) = \frac{\Gamma(\frac{1}{n-1})}{n-1} \cdot 2^{-\frac{B}{n-1}} \tag{18}$$

IV. System Performance Analysis

The transmitter calculates the precoding matrices by using the acquired CSI, which in general does not exactly match the actual channels because of the channel estimation error, quantization error, and feedback delay. Compared to the case where the BS has perfect CSI, a degradation of the system performance with respect to the achievable data rates will be introduced. In this section we analyze the effect of the imperfect CSI on the degradation in detail.

A. Effect of Channel Quantization Error

First, we assume that during one block each user estimates its channel perfectly and feeds back the quantized CSI to the BS without time delay. The mismatch between the available CSI at the BS and the actual channels is only due to the channel quantization error. In order to quantify the introduced rate loss, we derive an upper bound for it and define the number of feedback bits per user needed to maintain the bound. In this subsection we derive this upper bound by using a more precise method than the one used in [5].

Theorem 1. The rate loss per user incurred due to the channel quantization error relative to RBD with perfect CSI can be upper bounded by

$$\begin{aligned} \Delta R_i(P) &= \big[R_i(P) - R_{i,\mathrm{LF}}(P)\big] \\ &\leq M_{\mathrm{R}_i} \log_2(1 + \sigma^2_{\mathrm{MUI}} + \frac{P_s}{M_{\mathrm{T}}} \cdot D(\mathcal{C})) \end{aligned} \tag{19}$$

where $R_i(P)$ is the ergodic rate achieved by perfect CSI based RBD and $R_{i,\mathrm{LF}}(P)$ refers to the ergodic rate achieved by the limited feedback CSI based RBD.

Proof: The received signal in equation (13) is modified as

$$\boldsymbol{Y}_i^d(k) = \boldsymbol{H}_i(k)\hat{\boldsymbol{F}}_i(k)\boldsymbol{S}_i^d(k) + \sum_{j=1, j\neq i}^{K} \boldsymbol{H}_i(k)\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}_j^d(k) + \boldsymbol{Z}_i(k)\ . \tag{20}$$

Here $\hat{\boldsymbol{F}}_i(k)$ refers to the ith user's RBD precoding matrix which is calculated from the feedback of the quantized CSI and k is the index of the current block. We assume

$$\boldsymbol{H}_i(k) = \hat{\boldsymbol{H}}_i(k) + \hat{\boldsymbol{H}}_i^e(k) \tag{21}$$

where $\hat{\boldsymbol{H}}_i(k)$ is the quantized version of the channel matrix $\boldsymbol{H}_i(k)$ and $\hat{\boldsymbol{H}}_i^e(k)$ denotes the quantization error matrix where each elements has zero mean and variance $\hat{\sigma}_e^2$. Substituting (21) into equation (20), we get

$$\begin{aligned} \boldsymbol{Y}_i^d(k) \;=\; & \hat{\boldsymbol{H}}_i(k)\hat{\boldsymbol{F}}_i(k)\boldsymbol{S}_i^d(k) + \textstyle\sum_{j=1}^{K} \hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}_j^d(k) + \\ & \underbrace{\textstyle\sum_{j=1, j\neq i}^{K} \hat{\boldsymbol{H}}_i(k)\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}_j^d(k) + \boldsymbol{Z}_i(k)}_{\boldsymbol{V}_i(k)}\ . \end{aligned} \tag{22}$$

The rate loss $\Delta R_i(P)$ for user i can be expressed as

$$\begin{aligned} & R_i(P) - R_{i,\mathrm{LF}}(P) \\ \overset{(1)}{=} & \mathrm{E}\Big\{ \log_2 \det\Big(\boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i} + \frac{P_s}{K}\boldsymbol{H}_i(k)\boldsymbol{F}_i(k)\boldsymbol{F}_i^H(k)\boldsymbol{H}_i^H(k)\Big) - \\ & \underbrace{\log_2 \det(\boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i})}_{a1}\Big\} - \mathrm{E}\Big\{ \log_2 \det\Big(\boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i} + \\ & \frac{P_s}{K}\hat{\boldsymbol{H}}_i(k)\hat{\boldsymbol{F}}_i(k)\hat{\boldsymbol{F}}_i^H(k)\hat{\boldsymbol{H}}_i^H(k) + \\ & \frac{P_s}{K}\textstyle\sum_{j=1}^{K}\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big)\Big\} + \mathrm{E}\Big\{ \log_2 \det\Big(\boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i} + \\ & \frac{P_s}{K}\textstyle\sum_{j=1}^{K}\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big)\Big\} \\ \overset{(2)}{\leq} & \mathrm{E}\Big\{ \log_2 \det\Big(\boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i} + \frac{P_s}{K}\textstyle\sum_{j=1}^{K}\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big)\Big\} \\ \overset{(3)}{\leq} & \log_2 \det(\boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i} + \frac{P_s}{K}\textstyle\sum_{j=1}^{K}\mathrm{E}\Big\{\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big\}) \\ = & M_{\mathrm{R}_i}\log_2(1 + \sigma^2_{\mathrm{MUI}} + \frac{P_s}{M_{\mathrm{T}}M_{\mathrm{R}_i}} \cdot D(\mathcal{C})) \\ \leq & M_{\mathrm{R}_i}\log_2(1 + \sigma^2_{\mathrm{MUI}} + \frac{P_s}{M_{\mathrm{T}}} \cdot D(\mathcal{C}))\ . \end{aligned}$$

Here, at step (1) we neglect the positive term (a1), which leads to the bound (2). Then we use Jensen's inequality to get bound (3). We know

$$\begin{aligned} D(\mathcal{C}) &= \mathrm{E}\Big\{ \min\; d^2(\mathrm{vec}\,\{\boldsymbol{H}_i(k)\}, \mathrm{vec}\{\hat{\boldsymbol{H}}_i(k)\})\Big\} \\ &= \mathrm{E}\Big\{ \big\|\mathrm{vec}\{\boldsymbol{H}_i(k)\} - \mathrm{vec}\{\hat{\boldsymbol{H}}_i(k)\}\big\|_{\mathrm{F}}^2 \Big\} \\ &= \mathrm{tr}(\mathrm{E}\left\{\mathrm{vec}(\hat{\boldsymbol{H}}_i^e(k))\mathrm{vec}(\hat{\boldsymbol{H}}_i^e(k))^H\right\}) \\ &= M_{\mathrm{T}}M_{\mathrm{R}_i}\hat{\sigma}_e^2 \end{aligned} \tag{23}$$

which leads to $\hat{\sigma}_e^2 = \frac{D(\mathcal{C})}{M_{\mathrm{T}}M_{\mathrm{R}_i}}$. Then we have

$$\begin{aligned} & \mathrm{E}\Big\{\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big\} \\ & = \mathrm{E}\Big\{\hat{\boldsymbol{H}}_i^e(k)\widetilde{\boldsymbol{V}}_j(k)\Big(\frac{P_s}{K}\widetilde{\boldsymbol{\Sigma}}_j^T\widetilde{\boldsymbol{\Sigma}}_j + \frac{M_{\mathrm{R}}\sigma_n^2}{K\cdot M_{\mathrm{T}}}\boldsymbol{I}_{M_{\mathrm{T}}}\Big)^{-1}\widetilde{\boldsymbol{V}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big\} \\ & = \mathrm{E}\Big\{\hat{\boldsymbol{H}}_i^e(k)\Big(\frac{P_s}{K}\widetilde{\boldsymbol{\Sigma}}_j^T\widetilde{\boldsymbol{\Sigma}}_j + \frac{M_{\mathrm{R}}\sigma_n^2}{K\cdot M_{\mathrm{T}}}\boldsymbol{I}_{M_{\mathrm{T}}}\Big)^{-1}\hat{\boldsymbol{H}}_i^{eH}(k)\Big\} \\ & = \hat{\sigma}_e^2\mathrm{tr}\Big((\frac{P_s}{K}\widetilde{\boldsymbol{\Sigma}}_j^T\widetilde{\boldsymbol{\Sigma}}_j + \frac{M_{\mathrm{R}}\sigma_n^2}{K\cdot M_{\mathrm{T}}}\boldsymbol{I}_{M_{\mathrm{T}}})^{-1}\Big)\boldsymbol{I}_{M_{\mathrm{R}_i}} \\ & = \hat{\sigma}_e^2\mathrm{tr}(\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k))\boldsymbol{I}_{M_{\mathrm{R}_i}} \\ & = \hat{\sigma}_e^2\boldsymbol{I}_{M_{\mathrm{R}_i}} \\ & = \frac{D(\mathcal{C})}{M_{\mathrm{T}}M_{\mathrm{R}_i}}\boldsymbol{I}_{M_{\mathrm{R}_i}} \end{aligned} \tag{24}$$

which provides us the rate loss upper bound proposed in equation (19).

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Theorem 2. In order to maintain the above rate loss that is not larger than a given bound $\log_2 b$, the number of feedback bits per user B is calculated as:

$$\begin{aligned} B &= (n-1)\cdot\log_2 P_s - (n-1)\cdot\log_2\left(b^{1/M_{\mathrm{R}_i}} - 1 - \sigma^2_{\mathrm{MUI}}\right) - \\ &\quad (n-1)\log_2 M_{\mathrm{T}} + (n-1)\cdot\log_2\left(\Gamma(\frac{1}{n-1})/(n-1)\right) \\ &\approx \frac{n-1}{3}\mathrm{SNR} - (n-1)\log_2 M_{\mathrm{T}} \\ &\quad - (n-1)\cdot\log_2(b^{1/M_{\mathrm{R}_i}} - 1 - \sigma^2_{\mathrm{MUI}}) \end{aligned} \quad (25)$$

This expression can be proved by replacing the $\Delta R_i(P)$ in equation (19) by $\log_2 b$. For $D(\mathcal{C})$, we use the expression of the quantization distortion upper bound $\bar{D}(\mathcal{C})$ from equation (18), then we solve for B as a function of P_s and b.

The simulation result is shown in Figure 1(a), where we consider a two users MIMO limited feedback system with $M_{\mathrm{T}} = 4$, $M_{\mathrm{R}_i} = 2$ and $\sigma^2_{\mathrm{MUI}} = 0.25$. The bound of the rate loss is adjusted by the value of b which is set to 2, i.e., we should have a 1 bps/Hz rate offset which refers to a 3 dB gap. In Figure 1(a) we compare the performance of the system using the channel quantization scheme we propose with the channel quantization scheme proposed in [3], [4]. To ensure a fair comparison, we keep the same number of feedback bits for both channel quantization schemes. It can be seen that the 3 dB rate loss gap can only be maintained by our channel quantization scheme. In the absence of channel magnitude information at the BS, such a required rate loss bound cannot be satisfied.

B. Effect of Channel Estimation Error and Feedback Delay

Considering realistic channel propagation conditions, we first assume that each user has imperfect CSIR which will further degrade the CSI available at the BS.

With orthogonal pilots symbols, each user can estimate its channel independently. We denote the estimation of the matrix $\boldsymbol{H}_i(k)$ and the channel estimation error by $\breve{\boldsymbol{H}}_i(k)$ and $\breve{\boldsymbol{H}}^e_i(k)$, respectively. Thus,

$$\boldsymbol{H}_i(k) = \breve{\boldsymbol{H}}_i(k) + \breve{\boldsymbol{H}}^e_i(k)\ . \quad (26)$$

Here we consider an MMSE estimator, then $\breve{\boldsymbol{H}}^e_i(k)$ and $\breve{\boldsymbol{H}}_i(k)$ are independent and have a zero-mean complex Gaussian distribution. We can show that the covariance matrix of the channel estimation error is [8]

$$\mathrm{E}\left\{\breve{\boldsymbol{H}}^e_i(k)\breve{\boldsymbol{H}}^{eH}_i(k)\right\} = \sigma^2_{\mathrm{err}}\boldsymbol{I}_{M_{\mathrm{R}_i}} \quad (27)$$

with $\sigma^2_{\mathrm{err}} = \frac{1}{1+A_p^2}$.

Second, we assume that the feedback channel is noiseless and delayed by one block duration. This means that the feedback CSI of the channel block k will be used to optimize the transmission over the channel realization of the block $(k+1)$. In order to investigate the influence of the feedback delay, it is necessary to build a channel state evolution model. In this work we use a Markov chain model described in [9] to track the channel state given the knowledge of the past. Let ρ indicate the temporal correlation coefficient of the channel. The channel matrix for the block $(k+1)$ can be written as the function of the channel for the kth block

$$\boldsymbol{H}_i(k+1) = \rho\boldsymbol{H}_i(k) + \sqrt{1-\rho^2}\boldsymbol{E},\ 0<\rho<1\ . \quad (28)$$

Here $\boldsymbol{E}$ is a $M_{\mathrm{R}_i} \times M_{\mathrm{T}}$ complex Gaussian matrix where each elements has zero mean and unit variance.

Theorem 3. The rate loss per user incurred due to the channel quantization error, estimation error, and feedback delay relative to RBD with perfect CSI can be upper bounded by

$$\begin{aligned} \Delta R_i(P) &= \left[R_i(P) - R_{i,\mathrm{LF}}(P)\right] \\ &\le M_{\mathrm{R}_i}\log_2\Big(\frac{1}{\rho^2} + \sigma^2_{\mathrm{MUI}} + \frac{P_s}{M_{\mathrm{T}}}\sigma^2_{\mathrm{err}} + \\ &\qquad P_s(\frac{1}{\rho^2} - 1) + \frac{P_s}{M\mathrm{T}}\cdot D(\mathcal{C})\Big) \end{aligned} \quad (29)$$

where $R_i(P)$ is the ergodic rate achieved by perfect CSI based RBD and $R_{i,\mathrm{LF}}(P)$ refers to the ergodic rate achieved by the limited feedback CSI based RBD.

Proof: The received signal in equation (20) is modified as

$$\begin{aligned} \boldsymbol{Y}^d_i(k+1) &= \boldsymbol{H}_i(k+1)\hat{\boldsymbol{F}}_i(k)\boldsymbol{S}^d_i(k+1) + \\ &\quad \textstyle\sum_{j=1,j\neq i}^{K}\boldsymbol{H}_i(k+1)\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}^d_j(k+1) + \boldsymbol{Z}_i(k+1)\ . \end{aligned} \quad (30)$$

According to the equations (26), (28), and (21), we rewrite the matrix $\boldsymbol{H}_i(k+1)$ as

$$\boldsymbol{H}_i(k+1) = \rho(\hat{\boldsymbol{H}}_i(k) + \hat{\boldsymbol{H}}^e_i(k) + \breve{\boldsymbol{H}}^e_i(k)) + \sqrt{1-\rho^2}\boldsymbol{E} \quad (31)$$

and substitute it into equation (30) to get

$$\begin{aligned} \boldsymbol{Y}^d_i(k+1) &= \rho\hat{\boldsymbol{H}}_i(k)\hat{\boldsymbol{F}}_i(k)\boldsymbol{S}^d_i(k+1) + \sum_{j=1}^{K}\rho\hat{\boldsymbol{H}}^e_i(k)\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}^d_j(k+1) \\ &+ \underbrace{\sum_{j=1}^{K}\rho\breve{\boldsymbol{H}}^e_i(k)\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}^d_j(k+1)}_{\boldsymbol{N}_1} + \underbrace{\sum_{j=1}^{K}\sqrt{1-\rho^2}\boldsymbol{E}\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}^d_j(k+1)}_{\boldsymbol{N}_2} \\ &+ \underbrace{\sum_{j=1,j\neq i}^{K}\rho\hat{\boldsymbol{H}}_i(k)\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}^d_j(k+1) + \boldsymbol{Z}_i(k+1)}_{\boldsymbol{V}_i(k+1)}\ . \end{aligned} \quad (32)$$

Taking into account the independence between $\breve{\boldsymbol{H}}^e_i(k)$, $\boldsymbol{E}$, $\hat{\boldsymbol{H}}_i(k)$ and $\boldsymbol{Z}_i(k+1)$, we combine the last four terms in above equation as a new Gaussian noise $\boldsymbol{U}_i(k+1)$ and we get $\boldsymbol{R}_{\boldsymbol{U}_i\boldsymbol{U}_i} = \boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i} + \boldsymbol{R}_{\boldsymbol{N}_1\boldsymbol{N}_1} + \boldsymbol{R}_{\boldsymbol{N}_2\boldsymbol{N}_2}$. The covariance $\boldsymbol{R}_{\boldsymbol{N}_2\boldsymbol{N}_2}$ can be calculated as follows

$$\begin{aligned} &\boldsymbol{R}_{\boldsymbol{N}_2\boldsymbol{N}_2} \\ &= \textstyle\sum_{j=1}^{K}\mathrm{E}\left\{(1-\rho^2)\boldsymbol{E}\hat{\boldsymbol{F}}_j(k)\boldsymbol{S}^d_j(k+1)\boldsymbol{S}^{dH}_j(k+1)\hat{\boldsymbol{F}}^H_j(k)\boldsymbol{E}^H\right\} \\ &= (1-\rho^2)\frac{P_s}{K}\textstyle\sum_{j=1}^{K}\mathrm{E}\left\{\boldsymbol{E}\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}^H_j(k)\boldsymbol{E}^H\right\} \\ &= (1-\rho^2)\frac{P_s}{K}\sum_{j=1}^{K}\mathrm{E}\left\{\boldsymbol{E}\widetilde{\boldsymbol{V}}_j(k)(\frac{P_s}{K}\widetilde{\boldsymbol{\Sigma}}^T_j\widetilde{\boldsymbol{\Sigma}}_j + \frac{M_{\mathrm{R}}\sigma^2_n}{K\cdot M_{\mathrm{T}}}\boldsymbol{I}_{M_{\mathrm{T}}})^{-1}\widetilde{\boldsymbol{V}}^H_j(k)\boldsymbol{E}^H\right\} \\ &= (1-\rho^2)\frac{P_s}{K}\textstyle\sum_{j=1}^{K}\mathrm{tr}((\frac{P_s}{K}\widetilde{\boldsymbol{\Sigma}}^T_j\widetilde{\boldsymbol{\Sigma}}_j + \frac{M_{\mathrm{R}}\sigma^2_n}{K\cdot M_{\mathrm{T}}}\boldsymbol{I}_{M_{\mathrm{T}}})^{-1})\boldsymbol{I}_{M_{\mathrm{R}_i}} \\ &= (1-\rho^2)\frac{P_s}{K}\textstyle\sum_{j=1}^{K}\mathrm{tr}(\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}^H_j(k))\boldsymbol{I}_{M_{\mathrm{R}_i}} \\ &= (1-\rho^2)P_s\boldsymbol{I}_{M_{\mathrm{R}_i}}\ . \end{aligned} \quad (33)$$

Calculating the covariance matrix $\boldsymbol{R}_{\boldsymbol{N}_1\boldsymbol{N}_1}$ in the same way, we have $\boldsymbol{R}_{\boldsymbol{N}_1\boldsymbol{N}_1} = \frac{\rho^2}{M_{\mathrm{T}}}P_s\sigma^2_{\mathrm{err}}\boldsymbol{I}_{M_{\mathrm{R}_i}}$. Therefore the new noise $\boldsymbol{U}_i(k+1)$ is a Gaussian noise with variance $(1+\rho^2\sigma^2_{\mathrm{MUI}} + \frac{\rho^2}{M_{\mathrm{T}}}P_s\sigma^2_{\mathrm{err}} + P_s(1-\rho^2))$.

The rate loss $\Delta R_i(P)$ for user i is

$$\begin{aligned} &R_i(P) - R_{i,\mathrm{LF}}(P) \\ &= \mathrm{E}\Big\{\log_2\det\Big(\boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i} + \frac{P_s}{K}\boldsymbol{H}_i(k)\boldsymbol{F}_i(k)\boldsymbol{F}^H_i(k)\boldsymbol{H}^H_i(k)\Big) \\ &\quad \underbrace{-\log_2\det(\boldsymbol{R}_{\boldsymbol{V}_i\boldsymbol{V}_i})}_{a1}\Big\} - \mathrm{E}\Big\{\log_2\det\Big(\frac{1}{\rho^2}\boldsymbol{R}_{\boldsymbol{U}_i\boldsymbol{U}_i} + \end{aligned}$$

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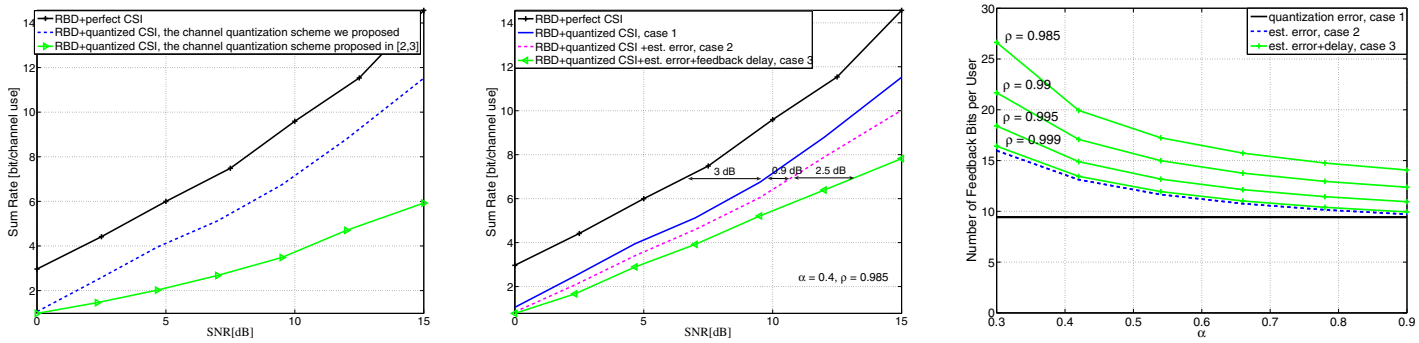

(a) (b) (c)

Fig. 1. Performance comparison for RBD precoded MIMO system with 2 users, $4 \times \{2, 2\}$ Rayleigh fading channels and limited feedback

$$\frac{P_s}{K}\hat{\boldsymbol{H}}_i(k)\hat{\boldsymbol{F}}_i(k)\hat{\boldsymbol{F}}_i^H(k)\hat{\boldsymbol{H}}_i^H(k) + \frac{P_s}{K}\sum_{j=1}^{K}\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big)\Big\} + \mathrm{E}\Big\{\log_2\det\Big(\frac{1}{\rho^2}\boldsymbol{R}_{\boldsymbol{U}_i\boldsymbol{U}_i} + \frac{P_s}{K}\sum_{j=1}^{K}\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big)\Big\}$$

$$\overset{(1)}{\leq}\mathrm{E}\Big\{\log_2\det\Big(\frac{1}{\rho^2}\boldsymbol{R}_{\boldsymbol{U}_i\boldsymbol{U}_i} + \frac{P_s}{K}\sum_{j=1}^{K}\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big)\Big\}$$

$$\overset{(2)}{\leq}\log_2\det\Big(\frac{1}{\rho^2}\boldsymbol{R}_{\boldsymbol{U}_i\boldsymbol{U}_i} + \frac{P_s}{K}\sum_{j=1}^{K}\mathrm{E}\Big\{\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\Big\}\Big)$$

$$\leq M_{\mathrm{R}_i}\log_2\Big(\frac{1}{\rho^2} + \sigma^2_{\mathrm{MUI}} + \frac{1}{M_{\mathrm{T}}}P_s\sigma^2_{\mathrm{err}} + P_s(\frac{1}{\rho^2} - 1) + \frac{P_s}{M_{\mathrm{T}}}D(\mathcal{C})\Big)\ .$$

Similarly to the proof for the theorem 1 we first neglect the positive term (a1) to get the bound (1). Then we apply Jensen's inequality to reach the bound (2). We know $\mathrm{E}\{\hat{\boldsymbol{H}}_i^e(k)\hat{\boldsymbol{F}}_j(k)\hat{\boldsymbol{F}}_j^H(k)\hat{\boldsymbol{H}}_i^{eH}(k)\} = \frac{D(\mathcal{C})}{M_{\mathrm{T}}M_{\mathrm{R}_i}}$, which results in the rate loss upper bound we proposed.

Theorem 4. In order to maintain the above rate loss that is not larger than a given bound $\log_2 b$, the number of feedback bits per user B is calculated as

$$B \approx \frac{n-1}{3}\mathrm{SNR} - (n-1)\log_2 M_{\mathrm{T}} - (n-1)\log_2(b^{1/M_{\mathrm{R}_i}} - \frac{1}{\rho^2} - \sigma^2_{\mathrm{MUI}} - \frac{1}{M_{\mathrm{T}}}P_s\sigma^2_{\mathrm{err}} - P_s(\frac{1}{\rho^2} - 1))\ . \tag{34}$$

This expression can be proved in the same way as Theorem 2 by replacing the $\Delta R_i(P)$ in equation (29) by $\log_2 b$. If we set $\rho = 1$ or $\sigma^2_{\mathrm{err}} = 0$ in equations (29) and (34), we can quantify the influence of the channel estimation error and feedback delay separately.

A Comparison of the simulated results in three different cases is provided in the sequel. We still consider a two users MIMO limited feedback system with $M_{\mathrm{T}} = 4$, $M_{\mathrm{R}_i} = 2$ and $\sigma^2_{\mathrm{MUI}} = 0.25$. The difference between the three cases is that for case 1 we assume that each user has perfect CSIR, for case 2 we assume that each user only knows the estimated CSIR and for case 3 we assume that each user has estimated CSIR and feeds the quantized CSI back to the BS experiencing one block delay. In Figure 1(b) we use the same number of feedback bits per user for the three cases to compare the resulting rate losses. In Figure 1(c) we compare the required number of feedback bits per user for the three cases in order to have the same rate loss bound $\log_2 b = 2.26$ (bps/Hz) at $\mathrm{SNR} = 10$ dB. From these simulation results we can see that the channel estimation error and the feedback delay further increase the rate loss gap and the number of feedback bits needed to maintain the required rate loss bound. Due to the high number of required feedback bits for the delayed feedback, we do not recommend this close-loop strategy in case of a fast-varying system.

V. Conclusion

We have presented an analysis of the achievable data rates of RBD precoded multi-user MIMO broadcast channels by considering pilot-based channel estimation, channel state quantization, and delayed feedback. We first evaluate the performance of the channel quantization scheme we proposed, bound the rate loss and quantify the number of feedback bits needed per user. Furthermore by introducing the channel estimation error and feedback delay, we quantify the effect of them on the achievable rates and compare the different reachable rate loss bounds and the required number of feedback bits. From this study, we see that the performance of limited feedback systems strongly depends on the temporal correlation between successive channel realizations. Accordingly, a high temporal correlation can help us to compress the feedback rate.

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References

[1] V. Stankovic and M. Haardt, "Generalized design of multi-user MIMO precoding matrices," *IEEE Trans. on Wireless Communications*, vol. 7, pp. 953–961, 2007.

[2] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multi-user MIMO channels," *IEEE Trans. Signal Processing*, vol. 52, pp. 461–471, Feb. 2004.

[3] N. Jindal, "MIMO broadcast channels with finite rate feedback," *IEEE Trans. Information Theory*, vol. 52, pp. 5045–5059, Nov. 2006.

[4] N. Ravindran and N. Jindal, "MIMO broadcast channels with block diagonalization and finite rate feedback," in *Proc. IEEE ICASSP*, 2007.

[5] B. Song, F. Roemer, and M. Haardt, "Efficient channel quantization scheme for multi-user MIMO broadcast channels with RBD precoding," in *Proc. IEEE Int. Conf. Acoust., Speech, and Signal Processing (ICASSP)*, Las Vegas, NV, Apr. 2008, pp. 2389–2392.

[6] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing lines, planes, etc.: Packings in Granssmanian space," *Expr.Math*, vol. 5, pp. 139–159, 1996.

[7] W. Dai, Y. Liu, and B. Rider, "Quantization bounds on Grassmann manifolds of arbitary dimensions and MIMO communication with feedback," in *Proc. IEEE GLOBECOM*, 2005.

[8] H. Poor, *An introduction to signal detection and estimation*. NY, USA: Springer-Verlage New York, 1994.

[9] S. Haykin, *Adaptive filter theory*. NJ, Prentice Hall: Englewood Cliff, 1996.

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