# Algebraic Norm-Maximizing (ANOMAX) Transmit Strategy for Two-Way Relaying With MIMO Amplify and Forward Relays

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***Abstract*—Two-way relaying is a promising scheme to achieve the ubiquitous mobile access to a reliable high data rate service, which is targeted for future mobile communication systems. In this contribution, we investigate two-way relaying with an amplify and forward relay, where the relay as well as the terminals are equipped with multiple antennas. Assuming that the terminals possess channel knowledge, the bidirectional two-way relaying channel is decoupled into two parallel effective single-user MIMO channels by subtracting the self-interference at the terminals. Thereby, any single-user MIMO technique can be applied to transmit the data. We derive an algebraic norm-maximizing (ANOMAX) transmit strategy by finding the relay amplification matrix which maximizes the weighted sum of the Frobenius norms of the effective channels and discuss the implications of this solution on the resulting signal to noise ratios. Finally, we compare ANOMAX to other existing transmission strategies via numerical computer simulations.**

***Index Terms*—Amplify and forward, MIMO, two-way relaying.**

## I. Introduction

FUTURE mobile communication systems face challenging targets: the radio access is expected to support very high data rates and to guarantee a certain quality of service. To achieve these goals, a large network node density is desirable since this lowers the path loss and increases the available degrees of freedom for network optimization. Usually, network installation and maintenance cost limit the number of possible network nodes. For this reason, deploying simple and cheap intermediate relay nodes is an attractive solution.

Relays can be used in many different ways. Among the relaying schemes, two-way relaying is known to use the radio resources particularly efficiently. In two-way relaying, a bidirectional transmission between two terminals is achieved in two subsequent transmission phases: First both terminals transmit to the relay, then the relay transmits back to both terminals. The two-way communication channel was already studied by Shannon [10] and has been rediscovered as a means to compensate the spectral efficiency loss in one-way relaying due to the half duplex constraint of the relay [4], [7].

Manuscript received May 26, 2009; revised June 13, 2009. First published June 30, 2009; current version published August 12, 2009. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Eduard A. Jorswieck.

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Digital Object Identifier 10.1109/LSP.2009.2026453

In contrast to decode and forward relays, which decode the transmission from the terminals and reencode them in the second phase, we focus our attention on amplify and forward relays which just amplify the received signal to transmit it back to the terminals. It is known that the bidirectional two-way relaying channel can be transformed into two separate single-user MIMO channels by subtracting the self-interference at the user terminals. In this paper we derive an algebraic norm-maximizing (ANOMAX) transmit strategy by finding the relay amplification matrix that maximizes the weighted sum of the Frobenius norms of the corresponding effective channel matrices since this maximizes the energy of the desired signals. We demonstrate that ANOMAX has a positive effect on the SNR for dominant eigenmode transmission. However, the SNR is not maximized. Instead, ANOMAX can be considered as a sub-optimal solution with the advantage of having a very low computational complexity.

Moreover, we compare ANOMAX to transmission strategies from previous publications. For example, the authors of [2] also propose to subtract the self interference to decouple the channels. However, the authors consider a scaled identity matrix at the relay, since they claim that “the relay pre-decoder matrix which satisfies bidirectional link simultaneously by using SVD may not exist” [*sic*]. The authors of [11] propose to use zero forcing (ZF) or minimum mean square error (MMSE) transceive filters at the relay. In case of the ZF filter, subtracting the self-interference at the terminals is not required since it is already canceled by the multiplication with the relay amplification matrix. As we show in our simulations, this is detrimental to the performance since channel energy is suppressed that can be used to enhance the transmission instead. Finally, [3], [6] and others consider single antenna terminals or single antenna relays only, whereas ANOMAX is applicable to the more general MIMO case.

This paper is organized as follows: The notation is introduced in Section II. In Section III, the system description and the data model are presented. The derivation of ANOMAX is presented in Section IV, followed by numerical computer simulations in Section V. Finally, conclusions are drawn in Section VI.

## II. Notation

The superscripts $^*$, $^{\mathrm{T}}$, $^{\mathrm{H}}$, and $^+$ represent complex conjugation, transposition, Hermitian transposition, and the Moore-Penrose pseudo inverse, respectively. The $\mathrm{vec}\{.\}$ operator aligns all the elements of a matrix into a column vector by stacking the column vectors of the matrix, i.e., $\mathrm{vec}\{\boldsymbol{A}\} \in \mathbb{C}^{M \cdot N \times 1}$ for $\boldsymbol{A} \in \mathbb{C}^{M \times N}$. The inverse operation to reshape a column vector back into a matrix is denoted via $\mathrm{unvec}_{M \times N}\{\boldsymbol{a}\} \in \mathbb{C}^{M \times N}$ for a vector $\boldsymbol{a} \in \mathbb{C}^{M \cdot N \times 1}$.

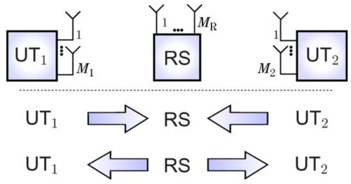


Fig. 1. Two-way relaying system model.

Moreover, the Kronecker product between two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ is symbolized by $\boldsymbol{A} \otimes \boldsymbol{B}$. Note that the Kronecker product satisfies the following well-known property [1]

$$\mathrm{vec}\{\boldsymbol{A} \cdot \boldsymbol{X} \cdot \boldsymbol{B}\} = (\boldsymbol{B}^{\mathrm{T}} \otimes \boldsymbol{A}) \cdot \mathrm{vec}\{\boldsymbol{X}\} \tag{1}$$

for $\boldsymbol{A} \in \mathbb{C}^{M \times N}$, $\boldsymbol{X} \in \mathbb{C}^{N \times P}$, and $\boldsymbol{B} \in \mathbb{C}^{P \times Q}$.

Finally, $||\boldsymbol{a}||_2$ and $||\boldsymbol{A}||_{\mathrm{F}}$ denote the two-norm of the vector $\boldsymbol{a}$ and the Frobenius norm of the matrix $\boldsymbol{A}$, respectively. By definition, the norms fulfills the following identity

$$||\boldsymbol{A}||_{\mathrm{F}} = ||\mathrm{vec}\{\boldsymbol{A}\}||_2 \quad \forall \boldsymbol{A}. \tag{2}$$

## III. System Description

### A. Data Model

The scenario under investigation is depicted in Fig. 1. We consider the communication between two user terminals $\mathsf{UT}_1$ and $\mathsf{UT}_2$ with the help of an intermediate relay station $\mathsf{RS}$. The terminals $\mathsf{UT}_1$ and $\mathsf{UT}_2$ are equipped with $M_1$ and $M_2$ antennas, respectively. The number of antennas at the relay station is denoted by $M_{\mathrm{R}}$.

In two-way relaying, the transmission takes place in two phases. In the first phase, both terminals transmit to the relay using the same resources, so that their transmissions interfere. Assuming frequency-flat fading, the signal received by the relay can be expressed as

$$\boldsymbol{r} = \boldsymbol{H}_1 \cdot \boldsymbol{x}_1 + \boldsymbol{H}_2 \cdot \boldsymbol{x}_2 + \boldsymbol{n}_{\mathrm{R}}$$

where $\boldsymbol{H}_1 \in \mathbb{C}^{M_{\mathrm{R}} \times M_1}$ and $\boldsymbol{H}_2 \in \mathbb{C}^{M_{\mathrm{R}} \times M_2}$ represent the MIMO channels between the terminals and the relay, $\boldsymbol{x}_1 \in \mathbb{C}^{M_1 \times 1}$ and $\boldsymbol{x}_2 \in \mathbb{C}^{M_2 \times 1}$ are the transmitted signals from the terminals, and the vector $\boldsymbol{n}_{\mathrm{R}}$ contains the noise component at the relay.

In the second transmission phase, the relay transmits to both terminals. Since we assume amplify and forward relays, the signal transmitted by the relay can be expressed as

$$\bar{\boldsymbol{r}} = \gamma \cdot \boldsymbol{G} \cdot \boldsymbol{r} = \boldsymbol{G}^{(\gamma)} \cdot \boldsymbol{r} \in \mathbb{C}^{M_{\mathrm{R}} \times 1}$$

where $\boldsymbol{G} \in \mathbb{C}^{M_{\mathrm{R}} \times M_{\mathrm{R}}}$ is the complex relay amplification matrix which is normalized such that $||\boldsymbol{G}||_{\mathrm{F}} = 1$ and $\gamma \in \mathbb{R}$ represents the amplification factor. The relay can compute $\gamma$ via

$$\gamma = \sqrt{\frac{P_{\mathrm{T,R}}}{||\boldsymbol{r}||_2^2}} \tag{3}$$

where $P_{\mathrm{T,R}}$ is the relay power constraint. Note that since $||\bar{\boldsymbol{r}}||_2 \leq ||\boldsymbol{G}^{(\gamma)}||_{\mathrm{F}} \cdot ||\boldsymbol{r}||_2 = \gamma \cdot ||\boldsymbol{r}||_2 = \sqrt{P_{\mathrm{T,R}}}$ this normalization guarantees that the power constraint stays satisfied.

The terminals receive the transmitted vector $\bar{\boldsymbol{r}}$ through their reverse channels. We also assume that reciprocity holds, so that the reverse channels are the transpose of the forward channels. We can then express the received signals $\boldsymbol{y}_1 \in \mathbb{C}^{M_1 \times 1}$ and $\boldsymbol{y}_2 \in \mathbb{C}^{M_2 \times 1}$ in the following way

$$\boldsymbol{y}_1 = \boldsymbol{H}_1^{\mathrm{T}} \cdot \boldsymbol{G}^{(\gamma)} \cdot (\boldsymbol{H}_1 \cdot \boldsymbol{x}_1 + \boldsymbol{H}_2 \cdot \boldsymbol{x}_2 + \boldsymbol{n}_{\mathrm{R}}) + \boldsymbol{n}_1 \tag{4}$$

$$\boldsymbol{y}_2 = \boldsymbol{H}_2^{\mathrm{T}} \cdot \boldsymbol{G}^{(\gamma)} \cdot (\boldsymbol{H}_1 \cdot \boldsymbol{x}_1 + \boldsymbol{H}_2 \cdot \boldsymbol{x}_2 + \boldsymbol{n}_{\mathrm{R}}) + \boldsymbol{n}_2 \tag{5}$$

where $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ represent the noise at the terminals. For notational convenience, we define the effective channels $\boldsymbol{H}_{i,j}^{(\mathrm{e})}$ in the following way

$$\boldsymbol{H}_{i,j}^{(\mathrm{e})} = \boldsymbol{H}_i^{\mathrm{T}} \cdot \boldsymbol{G} \cdot \boldsymbol{H}_j, \quad i,j = 1,2. \tag{6}$$

Expanding (4) and (5) we obtain the following alternative representation by using (6)

$$\boldsymbol{y}_1 = \gamma \cdot \boldsymbol{H}_{1,1}^{(\mathrm{e})} \cdot \boldsymbol{x}_1 + \gamma \cdot \boldsymbol{H}_{1,2}^{(\mathrm{e})} \cdot \boldsymbol{x}_2 + \tilde{\boldsymbol{n}}_1$$
$$\boldsymbol{y}_2 = \gamma \cdot \boldsymbol{H}_{2,2}^{(\mathrm{e})} \cdot \boldsymbol{x}_2 + \gamma \cdot \boldsymbol{H}_{2,1}^{(\mathrm{e})} \cdot \boldsymbol{x}_1 + \tilde{\boldsymbol{n}}_2$$

where $\tilde{\boldsymbol{n}}_1$ and $\tilde{\boldsymbol{n}}_2$ denote the effective noise terms which consist of the terminals' own noise and the forwarded relay noise contribution. Observe that the received signal comprises three terms. The first term represents the self-interference which the user terminal receives from its own transmissions. If channel knowledge is available, this term can be subtracted from the received signal. The second term contains the desired transmissions from the other terminal received via the effective channels $\boldsymbol{H}_{1,2}^{(\mathrm{e})}$ and $\boldsymbol{H}_{2,1}^{(\mathrm{e})}$, and the third term summarizes the noise contributions. Ignoring the impact of channel estimation errors, the effective transmissions can therefore be decoupled into two parallel single user MIMO channels $\boldsymbol{H}_{1,2}^{(\mathrm{e})}$ and $\boldsymbol{H}_{2,1}^{(\mathrm{e})}$ by subtracting the self-interference.

$$\boldsymbol{z}_1 = \boldsymbol{y}_1 - \gamma \cdot \boldsymbol{H}_{1,1}^{(\mathrm{e})} \cdot \boldsymbol{x}_1 = \gamma \cdot \boldsymbol{H}_{1,2}^{(\mathrm{e})} \cdot \boldsymbol{x}_2 + \tilde{\boldsymbol{n}}_1 \tag{7}$$

$$\boldsymbol{z}_2 = \boldsymbol{y}_2 - \gamma \cdot \boldsymbol{H}_{2,2}^{(\mathrm{e})} \cdot \boldsymbol{x}_2 = \gamma \cdot \boldsymbol{H}_{2,1}^{(\mathrm{e})} \cdot \boldsymbol{x}_1 + \tilde{\boldsymbol{n}}_2. \tag{8}$$

Any single-user MIMO technique can be applied to transmit data over these channels, e.g., dominant eigenmode transmission (DET) or spatial multiplexing (SMUX) [5].

Since both $\boldsymbol{H}_{1,2}^{(\mathrm{e})}$ and $\boldsymbol{H}_{2,1}^{(\mathrm{e})}$ are influenced by the relay amplification matrix $\boldsymbol{G}$, we focus on the question how $\boldsymbol{G}$ should be chosen to optimize the transmission via the effective channels in Section IV.

### B. Acquisition of Channel Knowledge

Our system concept depends on the assumption that the self-interference can be subtracted so that the transmission takes place over the decoupled effective single-user MIMO channels. This assumption is only valid if both terminals possess perfect knowledge of their own channels to the relay as well as the channels between the other terminal and the relay. These channels are in general difficult to acquire. In [9] and [8] we have proposed tensor-based channel estimation schemes that provide both terminals with all relevant channel parameters without the need for any feedback. While the TENCE algorithm derived in [9] is purely algebraic, its Structured Least Squares (SLS) based extension introduced in [8] performs an iterative refinement of TENCE. Note that both schemes are applicable to arbitrary antenna configurations.

For both channel estimation schemes, we require the same training phase where the terminals transmit pilot sequences that are forwarded by the relay [9]. As an extension to [9] and [8], we can also obtain estimates for both channel matrices $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ from the received training data at the relay station by solving an overdetermined linear least squares problem. Consequently, we assume that the relay station possesses channel knowledge to compute a suitable amplification matrix for the data transmission phase.

The terminals use the channel knowledge to compute their pre- and decoding vectors. In order to subtract the self-interference from the received data, they need to estimate the equivalent channels including pre- and postprocessing. As we demonstrate in Section IV-B, ANOMAX should be combined with dominant eigenmode transmission (DET). Therefore, the equivalent channels are scalars given by $h_{i,j}^{\rm eq} = \boldsymbol{d}_i^{\rm T} \cdot \boldsymbol{H}_{i,j}^{\rm (e)} \cdot \boldsymbol{p}_j$, $i,j = 1,2$, where $\boldsymbol{p}_j \in \mathbb{C}^{M_j \times 1}$ is the precoding vector used by terminal $j$ and $\boldsymbol{d}_i^{\rm T} \in \mathbb{C}^{1 \times M_i}$ is the decoding vector used by terminal $i$. The vectors $\boldsymbol{d}_i$ and $\boldsymbol{p}_j$ are computed from the left and right dominant singular vectors of the effective channel matrices $\boldsymbol{H}_{i,j}^{\rm (e)}$, respectively. Terminal $i$ receives its desired signal via the channel $h_{i,j}^{\rm eq}$ and its self-interference via the channel $h_{i,i}^{\rm eq}$ and, therefore, needs estimates of both to process the received signal. Even though these two channel taps could be computed directly from the estimated channels, this estimate may not be very reliable since it consists of many estimated terms and may also contain sign ambiguities. We, therefore, propose to transmit additional pilot symbols over the equivalent channels at the beginning of the data transmission phase in order to refine the estimate of the equivalent channels. Note that since only two channel taps need to be estimated, already two pilot symbols are sufficient.

## IV. Algebraic Norm-Maximizing (ANOMAX) Transmit Strategy

### A. Derivation of ANOMAX

As we demonstrate in (7) and (8), the bidirectional two-way relaying transmission can be decoupled into two parallel single-user MIMO transmissions if the self-interference is subtracted at the user terminals. By a proper choice of the relay amplification matrix $\boldsymbol{G}$ we can influence these channels.

We propose to find the relay amplification matrix as a solution to the following cost function

$$\boldsymbol{G}_\beta = \arg\max_{\boldsymbol{G}} J_\beta(\boldsymbol{G}) \text{ s.t. } \|\boldsymbol{G}\|_{\rm F} = 1, \text{ where}$$
$$J_\beta(\boldsymbol{G}) = \beta^2 \cdot \left\|\boldsymbol{H}_{1,2}^{\rm (e)}\right\|_{\rm F}^2 + (1-\beta)^2 \cdot \left\|\boldsymbol{H}_{2,1}^{\rm (e)}\right\|_{\rm F}^2$$

and $\beta \in [0,1]$ is a weighting factor. Therefore, the weighted sum of the Frobenius norms of the effective single-user MIMO channels is maximized. We first present the solution to this cost function, a discussion and motivation are given in Section IV-B.

Inserting the definitions of the effective channels from (6) yields

$$J(\boldsymbol{G}) = \beta^2 \left\|\boldsymbol{H}_1^{\rm T} \cdot \boldsymbol{G} \cdot \boldsymbol{H}_2\right\|_{\rm F}^2 + (1-\beta)^2 \left\|\boldsymbol{H}_2^{\rm T} \cdot \boldsymbol{G} \cdot \boldsymbol{H}_1\right\|_{\rm F}^2.$$

Applying identities (1), (2), and a series of algebraic manipulations, we rewrite this cost function in the following way

$$\begin{aligned}
J(\boldsymbol{G}) &= \left\|\beta \cdot \mathrm{vec}\left\{\boldsymbol{H}_1^{\rm T} \cdot \boldsymbol{G} \cdot \boldsymbol{H}_2\right\}\right\|_2^2 \\
&\quad + \left\|(1-\beta) \cdot \mathrm{vec}\left\{\boldsymbol{H}_2^{\rm T} \cdot \boldsymbol{G} \cdot \boldsymbol{H}_1\right\}\right\|_2^2 \\
&= \left\|\beta\left(\boldsymbol{H}_2^{\rm T} \otimes \boldsymbol{H}_1^{\rm T}\right) \cdot \mathrm{vec}\{\boldsymbol{G}\}\right\|_2^2 \\
&\quad + \left\|(1-\beta)\left(\boldsymbol{H}_1^{\rm T} \otimes \boldsymbol{H}_2^{\rm T}\right) \cdot \mathrm{vec}\{\boldsymbol{G}\}\right\|_2^2 \\
&= \left\|\begin{bmatrix} \beta(\boldsymbol{H}_2 \otimes \boldsymbol{H}_1)^{\rm T} \cdot \mathrm{vec}\{\boldsymbol{G}\} \\ (1-\beta)(\boldsymbol{H}_1 \otimes \boldsymbol{H}_2)^{\rm T} \cdot \mathrm{vec}\{\boldsymbol{G}\} \end{bmatrix}\right\|_2^2 \\
&= \left\|[\beta(\boldsymbol{H}_2 \otimes \boldsymbol{H}_1), (1-\beta)(\boldsymbol{H}_1 \otimes \boldsymbol{H}_2)]^{\rm T} \cdot \mathrm{vec}\{\boldsymbol{G}\}\right\|_2^2 \\
&= \left\|\boldsymbol{K}_\beta^{\rm T} \cdot \boldsymbol{g}\right\|_2^2
\end{aligned}$$

where in the last step we have introduced the definitions $\boldsymbol{g} = \mathrm{vec}\{\boldsymbol{G}\}$ and $\boldsymbol{K}_\beta = [\beta(\boldsymbol{H}_2 \otimes \boldsymbol{H}_1), (1-\beta)(\boldsymbol{H}_1 \otimes \boldsymbol{H}_2)] \in \mathbb{C}^{M_{\rm R}^2 \times 2 \cdot M_1 \cdot M_2}$. Note that the normalization constraint $\|\boldsymbol{G}\|_{\rm F} = 1$ translates to $\|\boldsymbol{g}\|_2 = 1$ for the vector $\boldsymbol{g}$. The solution of the new optimization problem is straightforward. To this end, introduce the SVD of $\boldsymbol{K}_\beta$ as $\boldsymbol{K}_\beta = \boldsymbol{U}_\beta \cdot \boldsymbol{\Sigma}_\beta \cdot \boldsymbol{V}_\beta^{\rm H}$. Then it is easy to see that

$$\begin{aligned}
\max_{\boldsymbol{g}, \|\boldsymbol{g}\|_2 = 1} \left\|\boldsymbol{K}_\beta^{\rm T} \cdot \boldsymbol{g}\right\|_2^2 &= \max_{\boldsymbol{g}, \|\boldsymbol{g}\|_2 = 1} \boldsymbol{g}^{\rm H} \cdot \boldsymbol{K}_\beta^* \cdot \boldsymbol{K}_\beta^{\rm T} \cdot \boldsymbol{g} \\
&= \max_{\boldsymbol{g}, \|\boldsymbol{g}\|_2 = 1} \frac{\boldsymbol{g}^{\rm H} \cdot \boldsymbol{K}_\beta^* \cdot \boldsymbol{K}_\beta^{\rm T} \cdot \boldsymbol{g}}{\boldsymbol{g}^{\rm H} \cdot \boldsymbol{g}} \\
&= \lambda_{\max}\left\{\boldsymbol{K}_\beta^* \cdot \boldsymbol{K}_\beta^{\rm T}\right\} = \sigma_1^2
\end{aligned} \tag{9}$$

where $\sigma_1$ is the largest singular value of $\boldsymbol{K}_\beta$. Moreover, the maximizing $\boldsymbol{g}$ in (9) is given by $\boldsymbol{g} = \boldsymbol{u}_{\beta,1}^*$, where $\boldsymbol{u}_{\beta,1}$ is the first column of $\boldsymbol{U}_\beta$, i.e., the dominant left singular vector of $\boldsymbol{K}_\beta$. Consequently, the final solution for the optimal relay amplification matrix is computed via

$$\boldsymbol{G}_\beta = \mathrm{unvec}_{M_{\rm R} \times M_{\rm R}}\left\{\boldsymbol{u}_{\beta,1}^*\right\}.$$

### B. Discussion

The ANOMAX solution has some interesting properties. First of all, it can be shown that the solution with equal weighting always satisfies $\boldsymbol{G}_{0.5}^{\rm T} = \boldsymbol{G}_{0.5}$. Consequently, the effective channels are also symmetric, i.e., $\boldsymbol{H}_{1,2}^{\rm (e)} = \boldsymbol{H}_{2,1}^{\rm (e)^{\rm T}}$. Obviously, $\boldsymbol{G}_\beta$ is not symmetric anymore for $\beta \neq 0.5$. Moreover, we can show that $\boldsymbol{G}_0 = \boldsymbol{u}_2^* \cdot \boldsymbol{u}_1^{\rm H}$ and $\boldsymbol{G}_1 = \boldsymbol{u}_1^* \cdot \boldsymbol{u}_2^{\rm H}$, where $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ are the dominant left singular vectors of the channel matrices $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$. Consequently, for the limiting cases $\beta = 0$ and $\beta = 1$ ANOMAX yields a rank-1 solution. For any other $\beta$, ANOMAX still directs most of the power onto the first eigenmode, resulting in effective channel matrices which are almost rank one. We have verified numerically that for $M_1, M_2, M_{\rm R} \in [2, 3, \ldots, 10]$ on the average the power of the first eigenmode is between 50 and 500 times larger than the sum of the power in the other eigenmodes. Consequently, as long as the number of antennas is not too large, ANOMAX should be combined with a single-stream transmission technique over the effective channels, e.g., dominant eigenmode transmission (DET). DET results in an SNR gain of $\lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of $\boldsymbol{H}_{i,j}^{\rm (e)} \cdot \boldsymbol{H}_{i,j}^{\rm (e)^{\rm H}}$ [5]. Since the effective channels are almost rank-one we have $\|\boldsymbol{H}_{i,j}^{\rm (e)}\|_{\rm F}^2 \approx \lambda_{\max}$. We therefore conclude that ANOMAX results in an SNR improvement for DET. However, since $\boldsymbol{G}$ also influences the noise power, we cannot formally state that ANOMAX *maximizes* the SNR.

## V. Simulation Results

In this section we present numerical computer simulations to support the results derived in the previous section. We consider uncorrelated Rayleigh fading channels, a transmit power of one and the same noise level $\sigma^2$ at both terminals and the relay. Therefore, the SNR is defined as $\mathrm{SNR} = 1/\sigma^2$. To ensure a fair comparison, the relay amplification matrix $\boldsymbol{G}$ is scaled to Frobenius norm one for each scheme. We compare ANOMAX with the ZF and MMSE approaches introduced in [11] (including the subtraction of duplex interference (SDI)) as well as fixing $\boldsymbol{G}$ to a DFT matrix of size $M_{\rm R} \times M_{\rm R}$.

In Fig. 2 we compare the bit error rate of DET averaged over both terminals for various choices of $\boldsymbol{G}$ using an uncoded QPSK

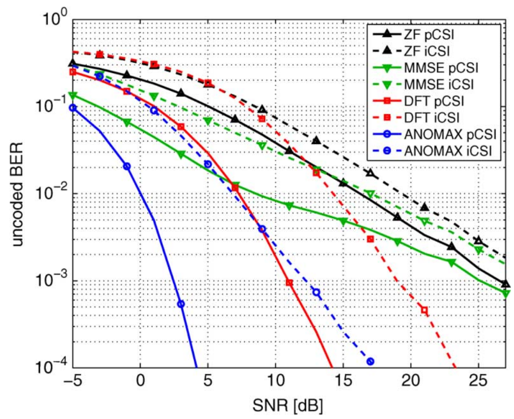


Fig. 2. Mean bit error rate for single stream uncoded QPSK transmission vs. the SNR with perfect CSI (pCSI) and imperfect CSI (iCSI).

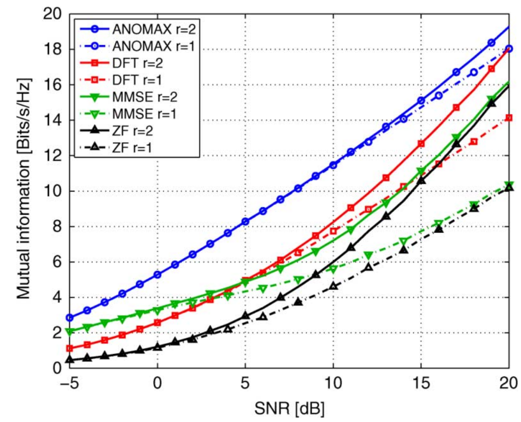


Fig. 3. Maximum mutual information vs. the SNR where $r$ indicates the maximum number of streams allowed.

modulation. The antenna configuration is given by $M_1 = 2$, $M_2 = 2$, $M_{\rm R} = 5$. The solid curves represent the case where the terminals and the relay have perfect channel state information (CSI). For the dashed curves, the channel is estimated at the terminals using TENCE with SLS [8] and at the relay as described in Section III-B. Note that the ZF approach requires no CSI at the terminals and the DFT approach requires no CSI at the relay. We observe that the channel inversion techniques ZF and MMSE suffer from the problem of noise enhancement, since a 5×4 channel matrix is inverted at the relay which can lead to a severe noise enhancement if the spread of the singular values is large. Consequently, in a two-way relaying scenario, where the self-interference can be subtracted at the terminals, it is detrimental to force parts of the channel to zero as in ZF and MMSE, since this also leads to a loss of energy of the desired signal.

In Fig. 3 we display the maximum mutual information of the bidirectional link for the same scenario, computed via

$$I = I_{1,2} + I_{2,1}$$
$$I_{i,j} = \underset{\boldsymbol{Q}_j, \mathrm{tr}\{\boldsymbol{Q}_j\}=1}{\arg\max} \log_2 \det\left\{\boldsymbol{I}_{M_i} + \tilde{\boldsymbol{R}}_{i,i}^{-1}\boldsymbol{H}_{i,j}^{(\mathrm{e})}\boldsymbol{Q}_j\boldsymbol{H}_{i,j}^{(\mathrm{e})^{\mathrm{H}}}\right\}$$

where $\tilde{\boldsymbol{R}}_{i,i}$ is the covariance matrix of the effective noise term $\tilde{\boldsymbol{n}}_i$ at terminal $i$. In this scenario, the number of streams $r$ can be 1 or 2. For the dashed lines only the first stream is used (DET) whereas for the solid lines both streams are active and the power is distributed via water pouring. As before, using the interference constructively instead of forcing it to zero is beneficial, since it increases the mutual information.

## VI. Conclusions

In this paper, we derive an algebraic norm-maximizing (ANOMAX) transmit strategy for the amplification matrix of an amplify and forward relay in a two-way relaying scenario, where both terminals and the relay possess channel knowledge. We first show that by subtracting the self-interference terms at the user terminals the two-way relaying channel is decoupled into two parallel effective single-user MIMO channels, which depend on the choice of the relay amplification matrix. We then optimize this matrix to maximize the Frobenius norms of both effective channels and derive a simple algebraic solution. Moreover, we show that this solution can be generalized to distribute the power unevenly between the two links by introducing weighting in the cost function, if desired. Via numerical computer simulations we demonstrate the superiority of ANOMAX compared to alternative existing transmission strategies.

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