

Iterative Tensor Receiver for MIMO-GFDM systems

Damir Rakhimov*, Sai Pavan Deram*, Bruno Sokal†, Kristina Naskovska*, André de Almeida†, and Martin Haardt*

* Communications Research Laboratory, Ilmenau University of Technology, P. O. Box 100565, D-98684 Ilmenau, Germany

Email: {damir.rakhimov, sai-pavan.deram, martin.haardt}@tu-ilmenau.de

† Department of Teleinformatics Engineering, Federal University of Ceara (UFC), Fortaleza, Brazil

Abstract—In this paper, we present a tensor MIMO-GFDM system model that is based on the double contraction operator. Based on the derived system model, we propose an iterative tensor based MIMO-GFDM receiver, that is initialized with the channel estimation obtained via pilots transmitted in the first data frame. The proposed algorithm exploits the tensor structure by using several unfoldings of the received signal sequentially to obtain estimates of the transmitted symbols and the channel. Simulation results show the tensor gain for the proposed algorithm in addition to the improved channel estimation. Numerical results confirm that the receiver requires the same amount of pilots as the Zero Forcing (ZF) receiver, while having a better symbol error rate (SER) performance and a better channel estimation accuracy.

Index Terms—MIMO-GFDM, Tensor Contraction, Iterative Receiver

I. INTRODUCTION

Generalized Frequency Division Multiplexing (GFDM) is one of the promising next generation modulation schemes to meet the requirements of beyond 5G mobile communication systems [1]. In GFDM, modulation is performed in blocks, where each block contains a few subcarriers and a group of symbols. Each subcarrier is filtered to reduce out of band emissions. On the other hand, the filtering process breaks the orthogonality between subcarriers, leading to additional intersymbol interference (ISI) to the received signal. As a result, the receiver design requires compulsory ISI cancellers.

Due to the filtering and the ISI cancellers, GFDM has a higher complexity compared to OFDM. There are studies presenting algorithms with reduced complexity. For example, in [2] the authors explain how to exploit the structure of the GFDM modulation matrix to decrease the complexity and to improve the performance of the GFDM transceivers using a matrix based approach. However, it does not take into account the multidimensional structure of MIMO-GFDM. We refer the interested readers to the references about the multilinear algebra and tensor decompositions in [3], [4], [5]. Communication systems have been described by using tensors, for example, in [6], [7], and [8]. Moreover, in [9] the authors use the tensor approach to work with GFDM. They modeled the GFDM transmit signal using a PARATUCK2 decomposition. In addition, the authors proposed a simple iterative MIMO-GFDM receiver.

In this paper, we derive the MIMO-GFDM signal model using the tensor contraction operator [4], [7]. The proposed model provides a simpler approach to describe MIMO-GFDM systems and provides more insight into the structure of the signals. Then we derive an iterative semi-blind receiver for frequency selective MIMO channels. The proposed algorithm uses several unfoldings of the received signal tensor sequentially to obtain estimates of the transmitted symbols and the channel.

II. NOTATION AND TENSOR ALGEBRA

In this paper, the following notation is used. Scalars are denoted as capital or lower-case italic letters A, a . A vector, matrix, and tensor are denoted as bold-faced small letters ‘ \mathbf{a} ’, and bold-faced capital letters ‘ \mathbf{A} ’, bold-faced calligraphic letters ‘ \mathcal{A} ’, respectively. The symbols ‘ \otimes ’ and ‘ \diamond ’ denote Kronecker product and Khatri-Rao product, respectively. The following superscripts T , $^{-1}$ and $^+$ denote transposition, matrix inversion, and Moore-Penrose pseudo matrix inversion, respectively.

Consider a 3-way tensor $\mathcal{A} \in \mathbb{C}^{M_1 \times M_2 \times M_3}$ whose r -mode unfolding is denoted as $[\mathcal{A}]_{(r)}$ and a matrix $\mathbf{U}_r \in \mathbb{C}^{J \times M_r}$, $r \in \{1, 2, 3\}$, the r -mode product is denoted as $\mathcal{Y} = \mathcal{A} \times_r \mathbf{U}_r$ and the r -mode unfolding of the resulting tensor is $[\mathcal{Y}]_{(r)} = \mathbf{U}_r [\mathcal{A}]_{(r)}$, $r \in \{1, 2, 3\}$ [3]. Consider a 3-way tensor with a Tucker structure as follows

$$\mathcal{X} = \mathcal{T} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \times_3 \mathbf{A}^{(3)} \in \mathbb{C}^{M_1 \times M_2 \times M_3}, \quad (1)$$

where $\mathcal{T} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ is a core tensor and $\mathbf{A}^{(1)} \in \mathbb{C}^{M_1 \times I_1}$, $\mathbf{A}^{(2)} \in \mathbb{C}^{M_2 \times I_2}$, $\mathbf{A}^{(3)} \in \mathbb{C}^{M_3 \times I_3}$ are the factor matrices. The 2-mode unfolding [3] of such a tensor can be written as

$$[\mathcal{X}]_{(2)} = \mathbf{A}^{(2)} [\mathcal{T}]_{(2)} (\mathbf{A}^{(3)} \otimes \mathbf{A}^{(1)})^T \in \mathbb{C}^{M_2 \times M_1 M_3}. \quad (2)$$

If the core tensor in (1) is an identity tensor $\mathcal{T} = \mathcal{I}_{3,I} \in \mathbb{C}^{I \times I \times I}$, then the tensor and the 2-mode unfolding can be written as

$$\mathcal{X} = \mathcal{I}_{3,I} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \times_3 \mathbf{A}^{(3)} \in \mathbb{C}^{M_1 \times M_2 \times M_3}, \quad (3)$$

$$[\mathcal{X}]_{(2)} = \mathbf{A}^{(2)} (\mathbf{A}^{(3)} \diamond \mathbf{A}^{(1)})^T \in \mathbb{C}^{M_2 \times M_1 M_3}, \quad (4)$$

The contraction is a tensor operator which can be seen as a generalized inner product. The matrix multiplication of two matrices $\mathbf{A} \in \mathbb{C}^{M_1 \times I}$ and $\mathbf{B} \in \mathbb{C}^{I \times M_2}$ can be represented as a contraction operation $\mathbf{A} \bullet_{\frac{1}{2}} \mathbf{B}$. The contraction between the 2-mode of a tensor $\mathcal{A} \in \mathbb{C}^{I_1 \times K \times I_3}$ and the 1-mode of the tensor $\mathcal{B} \in \mathbb{C}^{K \times J_2 \times J_3}$ can be written as

$$\mathcal{Z} = \mathcal{A} \bullet_{\frac{1}{2}} \mathcal{B} = \sum_{k=1}^K \mathcal{A}_{(:,k,:)} \mathcal{B}_{(k,:)} \in \mathbb{C}^{I_1 \times I_3 \times J_2 \times J_3} \quad (5)$$

The generalized unfolding [10] of the tensor \mathcal{Z} is defined as

$$[\mathcal{Z}]_{([1,2],[3,4])} = [\mathcal{A}]_{(2)}^T [\mathcal{B}]_{(1)} \in \mathbb{C}^{I_1 I_3 \times J_2 J_3} \quad (6)$$

The contraction can also be performed along several dimensions. The double contraction of two tensors $\mathcal{A} \in \mathbb{C}^{I_1 \times K \times I_3 \times L}$ and $\mathcal{B} \in \mathbb{C}^{K \times L \times J_3}$ is defined as in [7]

$$(\mathcal{A} \bullet_{2,4} \mathcal{B})_{I_1 \times I_3 \times J_3} = \sum_{l=1}^L \sum_{k=1}^K \mathcal{A}_{(:,k,:l)} \mathcal{B}_{(k,l,:)}. \quad (7)$$

The vectorization operator $\text{vec}\{\mathcal{A}\}$ arranges all the elements in a matrix or a tensor as a column vector. We will use the following properties of Kronecker products

$$\text{vec}\{\mathbf{A}\mathbf{X}\mathbf{B}\} = (\mathbf{B}^T \otimes \mathbf{A}) \text{vec}\{\mathbf{X}\} \quad (8)$$

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \diamond \mathbf{D}) = (\mathbf{A}\mathbf{C} \diamond \mathbf{B}\mathbf{D}). \quad (9)$$

Moreover $\|\cdot\|_{\text{F}}^2$ and $\|\cdot\|_{\text{H}}^2$ denote Frobenius norm and higher order norm, respectively. A matrix ‘ \mathbf{A} ’ in the frequency domain is represented as ‘ $\tilde{\mathbf{A}}$ ’.

III. PROBLEM FORMULATION

A. System model

Consider a MIMO system with M_{T} transmit antennas and M_{R} receive antennas. The modulated GFDM signal is given as in [1]

$$x_n^{(m_{\text{T}})} = \sum_{r=1}^R \sum_{m=1}^M d_{r,m}^{(m_{\text{T}})} p_{r,n} g_{m,n}, \quad (10)$$

where M is the number of symbols transmitted on R subcarriers and $N = RM$ is the block length. Here, $d_{r,m}^{(m_T)}$ are the complex modulated data symbols at each transmit antenna. The data symbols are filtered using the filter coefficients $g_{m,n}$ and shifted to the corresponding subcarrier by $p_{r,n} = \exp(j2\pi\frac{r}{N}n)$.

As mentioned in [1], GFDM is a block modulation scheme, where each block of data is filtered using a prototype filter to reduce out of band (OOB) emissions. The filtered symbols can be represented as

$$t_{r,n}^{(m_T)} = \sum_{m=1}^M d_{r,m}^{(m_T)} g_{m,n}. \quad (11)$$

Let $\mathcal{D} \in \mathbb{C}^{R \times M \times M_T}$ be a tensor with all modulated data symbols for one frame and $\mathbf{G} \in \mathbb{C}^{M \times N}$ be a matrix with the filter coefficients, such that $\mathcal{D}_{(r,m,m_T)} = d_{r,m,m_T}$ and $\mathbf{G}_{(m,n)} = g_{m,n}$. Then the tensor $\mathcal{T} \in \mathbb{C}^{R \times N \times M_T}$ which represents the filtered data blocks for all transmit antennas can be written as

$$\mathcal{T} = \mathcal{D} \times_2 \mathbf{G}^T \in \mathbb{C}^{R \times N \times M_T}. \quad (12)$$

Next, the filtered symbols $t_{r,n}^{(m_T)}$ are shifted to the corresponding subcarriers to get the GFDM signal for m_T -th transmit antennas.

$$x_n^{(m_T)} = \sum_{r=1}^R t_{r,n}^{(m_T)} p_{r,n}. \quad (13)$$

Let $\mathbf{P} \in \mathbb{C}^{R \times N}$ be a matrix where r -th row contains N samples of the r -th subcarrier, i.e., $\mathbf{P}_{(r,n)} = p_{r,n} = \exp(2\pi\frac{r}{N}n)$, $n \in 1, \dots, N$ and $\mathcal{P}_D = \mathcal{I}_{3,N} \times_1 \mathbf{P} \in \mathbb{C}^{R \times N \times N}$. The GFDM transmit signal $\mathbf{X} \in \mathbb{C}^{N \times M_T}$ can be described by using the double contraction operator as follows

$$\mathbf{X} = \mathcal{P}_D \bullet_{1,2} \mathcal{T} \in \mathbb{C}^{N \times M_T}, \quad (14)$$

Then we can rewrite the GFDM transmit signal using the unfoldings of the tensor \mathcal{P}_D and the tensor \mathcal{T} as

$$\mathbf{X} = [\mathcal{P}_D]_{(3)} [\mathcal{T}]_{(3)}^T \in \mathbb{C}^{N \times M_T}. \quad (15)$$

The 3-mode unfoldings of the tensor with the filtered symbols $\mathcal{T} \in \mathbb{C}^{R \times N \times M_T}$ can be written using equation (2) as

$$[\mathcal{T}]_{(3)} = [\mathcal{D}]_{(3)} (\mathbf{G}^T \otimes \mathbf{I}_K)^T \in \mathbb{C}^{M_T \times KN}, \quad (16)$$

where $[\mathcal{D}]_{(3)} \in \mathbb{C}^{M_T \times RM}$ is the 3-mode unfolding of the tensor \mathcal{D} . Similarly, the 3-mode unfolding of \mathcal{P}_D can be written using (4) as

$$[\mathcal{P}_D]_{(3)} = (\mathbf{I}_N \diamond \mathbf{P})^T. \quad (17)$$

Using (16) and (17) in (15) we get

$$\mathbf{X} = (\mathbf{I}_N \diamond \mathbf{P})^T ([\mathcal{D}]_{(3)} (\mathbf{G}^T \otimes \mathbf{I}_K)^T)^T. \quad (18)$$

Using the property (9) for the Kronecker product we can write

$$\begin{aligned} \mathbf{X} &= ((\mathbf{G}^T \otimes \mathbf{I}_K)^T (\mathbf{I}_N \diamond \mathbf{P}))^T [\mathcal{D}]_{(3)}^T \\ &= ((\mathbf{G} \otimes \mathbf{I}_K) (\mathbf{I}_N \diamond \mathbf{P}))^T [\mathcal{D}]_{(3)}^T \\ &= (\mathbf{G} \diamond \mathbf{P})^T [\mathcal{D}]_{(3)}^T \in \mathbb{C}^{N \times M_T}. \end{aligned} \quad (19)$$

Applying the DFT matrix $\mathbf{F} \in \mathbb{C}^{N \times N}$ along the first dimension of the $\mathbf{X} \in \mathbb{C}^{N \times M_T}$, we obtain the frequency domain representation of the transmit signal

$$\tilde{\mathbf{X}} = \mathbf{F} (\mathbf{G} \diamond \mathbf{P})^T [\mathcal{D}]_{(3)}^T \in \mathbb{C}^{N \times M_T}. \quad (20)$$

Let us define the modulation matrix as $\mathbf{Q} = (\mathbf{G} \diamond \mathbf{P}) \in \mathbb{C}^{RM \times N}$. Since $\mathbf{Q} \cdot \mathbf{Q}^H \neq \mathbf{I}_{RM}$ there is additional ISI in the received signal.

B. Channel model

We consider a frequency selective quasi-static MIMO channel. We assume that the maximum propagation delay for all antenna pairs does not exceed $\tau_{\max} = L \cdot T_s$, where L is the number of samples and T_s is the sampling interval. Let $\mathbf{h}^{(m_R, m_T)} \in \mathbb{C}^L$ be the channel impulse response between the receive and transmit antenna pair (m_R, m_T) in the time domain. Let $\mathcal{H} \in \mathbb{C}^{L \times M_R \times M_T}$ be the tensor with the collection of all channel impulse responses for all $M_T \cdot M_R$ antenna pairs, such that $\mathcal{H}_{(:,m_R,m_T)} = \mathbf{h}^{(m_R,m_T)}$. We can represent the channel between the (m_R, m_T) antenna pair in the frequency domain as $\tilde{\mathbf{h}}^{(m_R, m_T)} = \mathbf{F}_L \mathbf{h}^{(m_R, m_T)} \in \mathbb{C}^N$, where $\mathbf{F}_L \in \mathbb{C}^{N \times L}$ contains the first L columns of the DFT matrix \mathbf{F} . Also, let $\tilde{\mathbf{H}}^{(m_T)} \in \mathbb{C}^{N \times M_R}$ be the channel in the frequency domain between the m_T -th transmit antenna and all M_R receive antennas. Then if we concatenate the channel matrices $\tilde{\mathbf{H}}^{(m_T)}$, $m_T \in \{1, \dots, M_T\}$ along the third dimension, we obtain the channel tensor for all antenna pairs in the frequency domain $\tilde{\mathcal{H}} \in \mathbb{C}^{N \times M_R \times M_T}$,

$$\tilde{\mathcal{H}} = [\tilde{\mathbf{H}}^{(1)} \text{ } \text{ } \text{ } \tilde{\mathbf{H}}^{(2)} \text{ } \tilde{\mathbf{H}}^{(M_T)}] \in \mathbb{C}^{N \times M_R \times M_T}. \quad (21)$$

On the other side, the channel tensor in the frequency domain can be obtained via the Fourier transform of the channel tensor in the time domain along the first dimension as $\tilde{\mathcal{H}} = \mathcal{H} \times_1 \mathbf{F}_L$.

After the removal of the cyclic prefix, the channel matrix in the frequency domain for each pair of transmit (m_T) and receive (m_R) antennas is a diagonal matrix, $\tilde{\mathbf{H}}^{(m_R, m_T)} = \text{diag}\{\tilde{\mathbf{h}}^{(m_R, m_T)}\} \in \mathbb{C}^{N \times N}$. Then the diagonalized channel tensor that incorporates information about channel transfer functions for all $M_T \cdot M_R$ pairs of transmit and receive antennas is $\tilde{\mathcal{H}}_D \in \mathbb{C}^{N \times N \times M_R \times M_T}$. It can be represented as

$$\tilde{\mathcal{H}}_{D(:, :, m_R, m_T)} = \text{diag}\{\tilde{\mathcal{H}}_{(:, m_R, m_T)}\} \in \mathbb{C}^{N \times N}. \quad (22)$$

C. Received signal

In multi-carrier communication systems the received signals are usually represented via the slice-wise multiplication of the diagonalized channel tensor $\tilde{\mathcal{H}}_D$ and the transmit signal $\tilde{\mathbf{X}}$ in the frequency domain after the removal of the cyclic prefix. One slice of the signal that corresponds to the m_R -th receive antenna can be written as

$$\tilde{\mathbf{y}}^{(m_R)} = \sum_{m_T=1}^{M_T} \tilde{\mathbf{H}}^{(m_R, m_T)} \tilde{\mathbf{x}}^{(m_T)} + \mathbf{n}^{(m_R)} \in \mathbb{C}^N, \quad (23)$$

where $\tilde{\mathbf{x}}^{(m_T)} = \tilde{\mathbf{X}}_{(:, m_T)} \in \mathbb{C}^N$ is a vector of the transmitted signal for the m_T -th transmit antenna and all N subcarriers, $\mathbf{n}^{(m_R)} \in \mathbb{C}^N$ is a vector with samples of the AWGN at the m_R -th receive antenna and all N subcarriers.

Then, following the assumption of the quasi-static MIMO channel, which means that the channel is constant during K data frames, we can represent the GFDM transmitted signal as a tensor $\tilde{\mathcal{X}} \in \mathbb{C}^{N \times M_T \times K}$ by concatenating the transmitted frames along the third dimension.

$$\tilde{\mathcal{X}} = [\tilde{\mathbf{X}}^{(1)} \text{ } \text{ } \text{ } \tilde{\mathbf{X}}^{(2)} \text{ } \tilde{\mathbf{X}}^{(K)}] \in \mathbb{C}^{N \times M_T \times K}, \quad (24)$$

where $\tilde{\mathbf{X}}^{(k)}$ for the k -th frame is defined in equation (20).

Finally, the entire received signal in the frequency domain after the removal of the cyclic prefix can be represented using the double contraction operator in the following form

$$\tilde{\mathbf{Y}} = \tilde{\mathcal{H}}_D \bullet_{2,4} \tilde{\mathcal{X}} + \tilde{\mathcal{N}} \in \mathbb{C}^{N \times M_R \times K}. \quad (25)$$

where $\tilde{\mathcal{N}} \in \mathbb{C}^{N \times M_R \times K}$ is a tensor with samples of the AWGN. As shown in [7] the generalized unfolding of the channel tensor $\tilde{\mathcal{H}}_D \in$

$\mathbb{C}^{N \times N \times M_R \times M_T}$ has a block diagonal structure and can be expressed as

$$[\tilde{\mathcal{H}}_D]_{([1,3],[2,4])} = \tilde{\mathbf{H}} \diamond (\mathbf{1}_{M_T}^T \otimes \mathbf{I}_N) \in \mathbb{C}^{N M_R \times N M_T}, \quad (26)$$

where $\tilde{\mathbf{H}} = [\tilde{\mathcal{H}}]_{(2)} \in \mathbb{C}^{M_R \times N M_T}$ is the 2-mode unfolding of $\tilde{\mathcal{H}}$. The relationship between the channel in the time and the frequency domain is given by $\tilde{\mathbf{H}} = \mathbf{H} \cdot (\mathbf{I}_{M_T} \otimes \mathbf{F}_L^T)$, where $\mathbf{H} \in \mathbb{C}^{M_R \times L M_T}$ is the channel in the time domain. The unfolding of the noiseless received signal $\tilde{\mathcal{Y}}_0 \in \mathbb{C}^{N \times M_R \times K}$ can be represented using the generalized unfoldings of the tensors $\tilde{\mathcal{H}}_D \in \mathbb{C}^{N \times N \times M_R \times M_T}$ and $\tilde{\mathcal{X}} \in \mathbb{C}^{N \times M_T \times K}$ as

$$\begin{aligned} [\tilde{\mathcal{Y}}_0]_{(3)}^T &= [\tilde{\mathcal{H}}_D]_{([1,3],[2,4])} [\tilde{\mathcal{X}}]_{([1,2],3)} \\ &= (\tilde{\mathbf{H}} \diamond (\mathbf{1}_{M_T}^T \otimes \mathbf{I}_N)) [\tilde{\mathcal{X}}]_{(3)}^T \in \mathbb{C}^{N M_R \times K}. \end{aligned} \quad (27)$$

If we consider $[\tilde{\mathcal{X}}]_{(3)} \in \mathbb{C}^{K \times N M_T}$, the k -th row corresponds to the vectorized GFDM transmitted signal of the k -th frame

$$[\tilde{\mathcal{X}}]_{(3)} = \begin{bmatrix} \text{vec} \{ \tilde{\mathcal{X}}^{(1)} \}^T \\ \vdots \\ \text{vec} \{ \tilde{\mathcal{X}}^{(K)} \}^T \end{bmatrix} \in \mathbb{C}^{K \times N M_T}. \quad (28)$$

We can represent the vectorization of one frame of the GFDM transmitted signal (20) using property (8) in the following form

$$\text{vec} \{ \tilde{\mathcal{X}}^{(k)} \} = (\mathbf{I}_{M_T} \otimes \mathbf{F}(\mathbf{G} \diamond \mathbf{P})^T) \text{vec} \{ \mathcal{D}_{([1,2],[3])}^{(k)} \}, \quad (29)$$

where $\mathcal{D}^{(k)} \in \mathbb{C}^{R \times M \times M_T}$ is the tensor with the modulated data symbols of the k -th frame. Let us define $\Phi = (\mathbf{I}_{M_T} \otimes \mathbf{F}(\mathbf{G} \diamond \mathbf{P})^T) \in \mathbb{C}^{R M M_T \times N M_T}$. Then we can represent the 3-mode unfolding of the tensor $\tilde{\mathcal{X}}$ in the following form

$$\begin{aligned} [\tilde{\mathcal{X}}]_{(3)} &= \begin{bmatrix} \text{vec} \{ \mathcal{D}_{([1,2],[3])}^{(1)} \}^T \Phi \\ \vdots \\ \text{vec} \{ \mathcal{D}_{([1,2],[3])}^{(K)} \}^T \Phi \end{bmatrix} \\ &= \begin{bmatrix} \text{vec} \{ \mathcal{D}_{([1,2],[3])}^{(1)} \}^T \\ \vdots \\ \text{vec} \{ \mathcal{D}_{([1,2],[3])}^{(K)} \}^T \end{bmatrix} \Phi \\ &= [\tilde{\mathcal{D}}]_{(4)} \Phi \in \mathbb{C}^{K \times N M_T}, \end{aligned} \quad (30)$$

where $\tilde{\mathcal{D}} \in \mathbb{C}^{R \times M \times M_T \times K}$ is a 4-D data tensor, which is constructed by concatenation of the data frames $\mathcal{D}^{(k)}$, $k \in \{1, \dots, K\}$ along the fourth dimension. Then, the noiseless received signal tensor $\tilde{\mathcal{Y}}_0 \in \mathbb{C}^{N \times M_R \times K}$ can be expressed as

$$\tilde{\mathcal{Y}}_0 = \mathcal{I}_{3, N M_T} \times_1 (\mathbf{1}_{M_T}^T \otimes \mathbf{I}_N) \times_2 \tilde{\mathbf{H}} \times_3 ([\tilde{\mathcal{D}}]_{(4)} \Phi). \quad (31)$$

IV. ITERATIVE GFDM RECEIVER

In this section, using the PARAFAC representation as in [3] of the noiseless received signal in (31) and assuming that the number of receive antennas is greater than the number of transmit antennas, i.e., $M_R \geq M_T$, we derive algorithms for channel and symbol estimation.

From the 2-mode unfolding of the received signal $\tilde{\mathcal{Y}} \in \mathbb{C}^{N \times M_R \times K}$ we obtain an expression for channel estimation in the time domain.

$$\hat{\mathbf{H}} = [\tilde{\mathcal{Y}}]_{(2)} \left((\mathbf{I}_{M_T} \otimes \mathbf{F}_L^T) ([\tilde{\mathcal{X}}]_{(3)} \diamond (\mathbf{1}_{M_T}^T \otimes \mathbf{I}_N))^T \right)^+, \quad (32)$$

while the 3-mode unfolding of the equation (31) can be used to obtain estimates of the transmitted symbols

$$[\hat{\mathcal{D}}]_{(4)} = [\tilde{\mathcal{Y}}]_{(3)} \left(\Phi \cdot (\tilde{\mathbf{H}} \diamond (\mathbf{1}_{M_T}^T \otimes \mathbf{I}_N))^T \right)^+. \quad (33)$$

We assume the presence of pilots in the first frame, so the first frame at each transmit antenna would be as follows

$$\mathbf{D}^{(m_T)} = \mathbf{D}_p + \mathbf{D}_d \in \mathbb{C}^N, \quad (34)$$

where \mathbf{D}_p are known pilot symbols and \mathbf{D}_d are data symbols. The pilot based channel estimate $\tilde{\mathcal{H}}_p$ is obtained from pilot symbols within the first frame [11].

Algorithm 1: ALTERNATING LEAST-SQUARES ALGORITHM

Initialization: $\tilde{\mathcal{H}}_p$;

while does not exceed the maximum number of iterations, does not reach a predefined minimum, or the error of the cost function changes within two consecutive iterations **do**

Steps

1. Update

$$[\hat{\mathcal{D}}]_{(4)} = [\tilde{\mathcal{Y}}]_{(3)} \left(\Phi \cdot (\tilde{\mathbf{H}} \diamond (\mathbf{1}_{M_T}^T \otimes \mathbf{I}_N))^T \right)^+$$

2. Project the estimated data on to finite alphabet Ω

3. Compute

$$[\hat{\mathcal{X}}]_{(3)} = [\hat{\mathcal{D}}]_{(4)} (\mathbf{I}_{M_T} \otimes \mathbf{F}(\mathbf{G} \diamond \mathbf{P})^T)^T$$

4. Update

$$\hat{\mathbf{H}} = [\tilde{\mathcal{Y}}]_{(2)} \left((\mathbf{I}_{M_T} \otimes \mathbf{F}_L^T) ([\hat{\mathcal{X}}]_{(3)} \diamond (\mathbf{1}_{M_T}^T \otimes \mathbf{I}_N))^T \right)^+$$

5. Compute the channel in frequency domain

$$\hat{\tilde{\mathbf{H}}} = \hat{\mathbf{H}} \cdot (\mathbf{I}_{M_T} \otimes \mathbf{F}_L^T)$$

end

Result: $\hat{\mathcal{D}}$ and $\hat{\mathbf{H}}$

To check the convergence of the proposed algorithm the cost function is defined as $\| \tilde{\mathcal{Y}} - \hat{\mathcal{Y}}_0 \|_{\mathbf{H}}^2$, where $\hat{\mathcal{Y}}_0$ is the reconstructed noiseless tensor of the received signal which can be found as

$$\hat{\mathcal{Y}}_0 = \mathcal{I}_{3, N M_T} \times_1 (\mathbf{1}_{M_T}^T \otimes \mathbf{I}_N) \times_2 \hat{\mathbf{H}} (\mathbf{I}_{M_T} \otimes \mathbf{F}_L^T) \times_3 [\hat{\mathcal{D}}]_{(4)} \Phi. \quad (35)$$

V. SIMULATION RESULTS

We provide some numerical results to show the performance of the proposed algorithm. We compare the performance of the Iterative GFDM receiver with the Zero Forcing algorithm using Monte Carlo simulations. We consider the sampling frequency $f_s = 7.68$ MHz during simulations. Also, we impose the equidistant spacing between the subcarriers with the pilot symbols $\Delta N = 20$. The number of the subcarriers in one block $R = 32$ and the number of the symbols $M = 15$, so that the block length equals to $N = R \cdot M$. The duration of the cyclic prefix is chosen to be 32 samples. For the transmitted data symbols we assume 4-QAM modulation. We use the 3GPP Pedestrian channel (PED A) model. All the simulation results are averaged over 3000 different channel realizations. For each channel realization we run a maximum of 7 iterations of the ALS algorithm. The threshold for the cost error during the simulations is equal to 10^{-7} .

In Figure 1a, Figure 1b, and Figure 1c we depict the SER (Symbol Error Rate) as a function of E_b/N_0 (Energy per bit / Noise spectral density) for different antenna configurations. In all the three cases the proposed algorithm outperforms the ZF receiver. A significant tensor gain can be observed for $K = 1$ and $K > 1$ in Figure 1a and Figure 1b. The gain saturates as K increases, which can be observed in all the three configurations.

We calculate the normalized channel estimation error (NMSE) as

$$\text{NMSE} = \mathbb{E} \left\{ \frac{\| \mathcal{H} - \hat{\mathcal{H}} \|_{\mathbf{H}}^2}{\| \mathcal{H} \|_{\mathbf{H}}^2} \right\}. \quad (36)$$

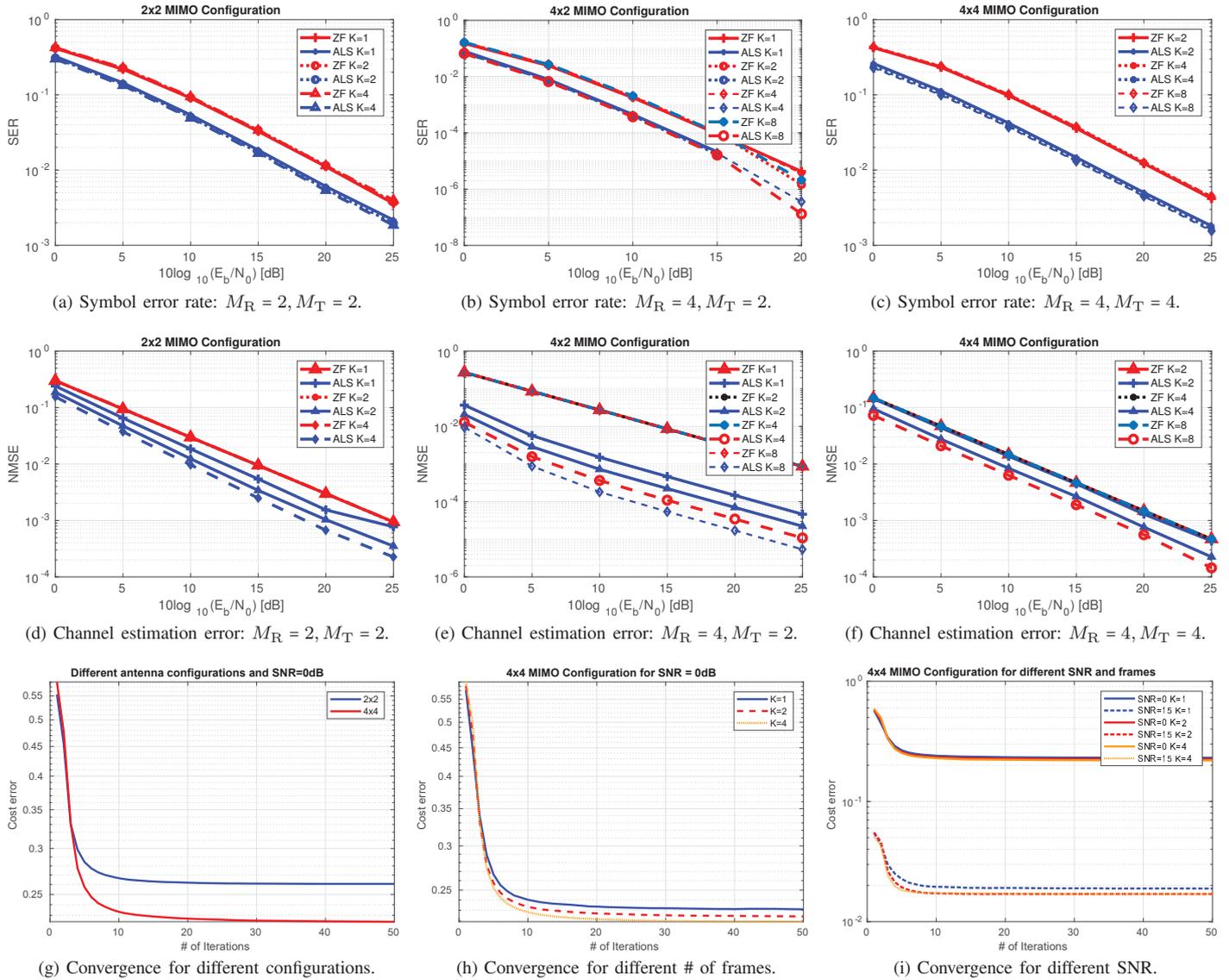


Fig. 1: Numerical results of performance of the Iterative MIMO-GFDM receiver for different scenarios

Figure 1d, Figure 1e, and Figure 1f depicts the NMSE as a function of E_b/N_0 for different antenna configurations. The NMSE for the ZF receiver is the same for all $K \geq 1$, because we obtain the channel estimation based only on information from the first frame with pilots. While, as can be observed in Figures 1d, 1e, and 1f, as the number of frames increases, the channel NMSE for the proposed algorithm decreases.

Finally in Figures 1g, 1h and 1i we depict the value of the cost function versus the number of iterations. The results are averaged over 200 different channel realizations. Figure 1g shows that with the increase of the number of antennas, the slope of convergence curve also increases. Figure 1h shows a faster convergence to the minimum for $K > 1$. Figure 1i shows that in case of high SNR scenarios, the algorithm converges for a fewer number of iterations and the error in the cost function is also low. Furthermore, the proposed receiver has no constraints over the number of frames K . One of the reasons for the fast convergence of the algorithm is the initialization with the pilot based channel estimation and the projection of the estimated symbols onto the finite alphabet of the selected modulation scheme.

VI. CONCLUSION

In this paper, we have proposed an iterative tensor based MIMO-GFDM receiver. For the proposed receiver, several unfoldings of the received signal tensor are used sequentially to obtain estimates of the transmitted symbols and improved estimates of the channel. The algorithm is initialized with the channel estimates obtained via pilots in the first frame. The numerical results confirm that the receiver requires the same amount of pilots as the ZF receiver, while having a better SER performance and a lower channel estimation error. In addition, we present the new tensor MIMO-GFDM system model that is based on the double contraction operator.

This model provides new opportunities for a future work regarding the MIMO-GFDM based communication systems, such as finding the best pilot sequences, investigating more general GFDM systems, when not all subcarriers or subsymbols are used for data transmission, and investigating new solutions that include coding [12].

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