

# MULTI-USER MIMO DOWNLINK BEAMFORMING OVER CORRELATED MIMO CHANNELS

*Veljko Stankovic and Martin Haardt*

Ilmenau University of Technology, Communications Research Laboratory  
P.O. Box 100565, 98684 Ilmenau, Germany  
Phone: +49 (3677) 69-2615, Fax: +49 (3677) 69-1195  
{veljko.stankovic, martin.haardt}@tu-ilmenau.de

## ABSTRACT

**Multi-user (MU) multiple-input multiple-output (MIMO) systems provide a high capacity while offering the benefits of space division multiple access. The channel state information (CSI) at the base station (BS)/access point (AP) can be used to precode the signals and to perform most of the complex processing at the BS/AP which results in a simplification of the users' terminals. If perfect CSI is available at the transmitter it can be used to completely eliminate interference at the BS/AP. In situations when the channel variations are too fast to obtain short-term CSI, the long-term channel state information can be also used to improve the system performance. In this paper we propose a new approach to MU precoding based on long-term CSI. It can be used with previously defined precoding techniques originally requiring perfect CSI at the transmitter. Thereby we exploit the knowledge of the spatial correlation at the transmitter to improve the system throughput. The more antennas there are at the BS/AP, the greater are the benefits of MU MIMO precoding.**

## 1. INTRODUCTION

Multiple-input, multiple-output (MIMO) techniques have been recognized as a key technology that provides the high data rates required for future wireless communication systems. In a multi-user scenario, multiple antennas at both ends of the link allow us to combine the benefits of simultaneous signaling to different users (SDMA) and simultaneous transmission of multiple data streams to a single user (SMUX). It is very important to have CSI at the base station (BS) or access point (AP) since it allows joint processing of all users' signals which results in a significant performance improvement and increased data rates.

---

This work has been partially performed in the framework of the IST project IST-2003-507581 WINNER, which is partly funded by the European Union. The authors would like to acknowledge the contributions of their colleagues.

In [1] the authors propose a linear multi-user (MU) precoding technique called block diagonalization (BD) that can exploit both perfect and long-term channel state information (CSI) to completely eliminate multi-user interference (MUI). The MUI is eliminated by setting the modulation matrix of each user to lie in the null space of all other users' channel matrices. The main disadvantage of this technique is the performance loss due to the zero MUI constraint and the limitation on the total number of receive antennas. The total number of receive antennas has to be less or equal to the total number of transmit antennas.

In order to relax the constraint on the total number of receive antennas we have to allow some MUI. In [2] a technique called successive minimum mean-squared-error (SMMSE) precoding has been introduced. It minimizes the mean-squared-error per receive antenna of each user separately. This technique can provide a higher array and diversity gain with perfect CSI at the transmitter than BD, especially at low SNRs.

It has been shown [3] that the capacity of a MIMO system increases linearly with the minimum out of the number of receive and transmit antennas. Fading correlation reduces the system capacity, especially when there is no channel knowledge available at the transmitter [4]. If it is impossible to acquire perfect CSI at the BS/AP, the spatial channel correlation can nevertheless be used to effectively eliminate or reduce multi-user interference (MUI). In this paper we investigate the performance of MU MIMO downlink precoding techniques when there is either short-term CSI or long-term CSI available at the transmitter. We propose a new approach for exploiting the knowledge of the spatial correlation at the transmitter that allows the use of precoding techniques originally designed for perfect CSI at the transmitter. Thereby, we can use successive minimum mean-square-error (SMMSE) for which the number of receive antennas can be greater than the number of transmit antennas together with long-term CSI.

This paper is organized as follows: In Section 2, we describe the MU downlink channel. In Section 3, we present

the precoding techniques that will be compared and in Section 4, we show the results of the simulations. A short summary follows in the Section 5.

## 2. SYSTEM MODEL

We consider a MU MIMO downlink channel, where  $M_T$  transmit antennas are located at the base station and  $M_{R_i}$  receive antennas are located at the  $i$ -th mobile station (MS),  $i = 1, 2, \dots, K$ . There are  $K$  users (or MSs) in the system. The total number of receive antennas is

$$M_R = \sum_{i=1}^K M_{R_i}.$$

A block diagram of such a system is depicted in Fig. 1. We will use the notation  $\{M_{R_1}, \dots, M_{R_K}\} \times M_T$  to describe the antenna configuration of the system. Let us assume a flat fading channel. The MIMO channel to user  $i$  is denoted as  $\mathbf{H}_i \in \mathbb{C}^{M_{R_i} \times M_T}$ . Moreover, the combined channel matrix is given by

$$\mathbf{H} = [\mathbf{H}_1^T \quad \mathbf{H}_2^T \quad \dots \quad \mathbf{H}_K^T]^T.$$

Consider a cellular communications system where it is not possible to track fast variations of the users' channels but where we are able to obtain information about the transmit correlation at the transmitter. This might happen in a typical wide area scenario. The channel of each user is modeled as

$$\mathbf{H}_i = \mathbf{R}_{r_i}^{1/2} \mathbf{H}_{w_i} \mathbf{R}_{t_i}^{1/2} \quad (1)$$

where  $\mathbf{H}_{w_i}$  is a spatially white unit variance flat fading MIMO channel of dimension  $M_{R_i} \times M_T$ , whereas  $\mathbf{R}_{r_i}$  and  $\mathbf{R}_{t_i}$  are receive and transmit covariance matrices with  $\text{tr}(\mathbf{R}_{r_i}) = M_{R_i}$  and  $\text{tr}(\mathbf{R}_{t_i}) = M_T$ .

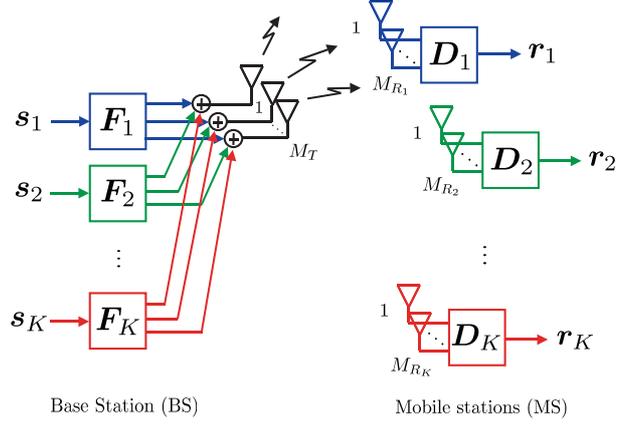
In this paper we assume a scenario where the MS is surrounded by a rich scattering environment and the BS/AP antennas are separated by less than the coherence distance. These propagation conditions correspond to a cellular communication systems typically characterized by a low angular spread at the BS/AP. On the other hand, the angular spread at the mobile is often very large and thus low spatial correlation can be achieved with relatively small antenna separation. Hence, we can write

$$\mathbf{R}_{r_i} = \mathbf{I}_{M_{R_i}}, \quad \mathbf{R}_{t_i} = \frac{M_T}{\text{tr}(\mathbf{A}^* \mathbf{A}^T)} \mathbf{A}^* \mathbf{A}^T \quad (2)$$

and the channel is modelled as

$$\mathbf{H}_i = \sqrt{\frac{M_T}{\text{tr}(\mathbf{A}^* \mathbf{A}^T)}} \mathbf{H}_{w_i} \mathbf{A}^T \quad (3)$$

where  $\mathbf{A} \in \mathbb{C}^{M_T \times N}$  is an array steering matrix containing  $N$  array response vectors of the transmitting antenna array corresponding to  $N$  directions of departure [4].



**Fig. 1.** Block diagram of a multi-user downlink MIMO system.

The  $i$ -th user's transmit correlation matrix is defined as  $E\{\mathbf{H}_i^H \mathbf{H}_i\} = M_{R_i} \mathbf{R}_{t_i}$ , with  $\text{tr}(E\{\mathbf{H}_i^H \mathbf{H}_i\}) = M_{R_i} M_T$ .

## 3. MULTI-USER PRECODING

### 3.1. Perfect CSI at the transmitter

In a TDD system, using the reciprocity principle it is possible to use the estimated uplink channel for downlink transmission. This information can be used to perform joint precoding at the BS/AP of the users' signals. In this paper we compare the performance of two precoding techniques. The first one is block diagonalization (BD) which was first proposed in [1] and is modified here for long-term CSI. It forces all MUI to zero by choosing each user's modulation matrix  $\mathbf{F}_i$  to lie in the null space of all other users' channel matrices. The second one is called SMMSE and it was proposed in [2] for short-term CSI at the transmitter. It balances the MUI by minimizing the mean-square-error per receive antenna for each user separately. Moreover, it is able to eliminate inter-stream interference between different data streams sent to the same user.

### 3.2. Long term CSI at the transmitter

If we assume that the channel is varying too rapidly to track its mean, the information regarding the relative geometry of the propagation paths is captured by a colored spatial correlation matrix. This problem is also addressed in [5] where the authors assume that each user's channel matrix can be represented as  $\mathbf{H}_i = \mathbf{A}_i \mathbf{B}_i$  where the transmitter has the information about the matrix  $\mathbf{B}_i \in \mathbb{C}^{r_i \times M_T}$  but not  $\mathbf{A}_i \in \mathbb{C}^{M_{R_i} \times r_i}$ ,  $r_i \leq M_{R_i}$ . The MUI in the system can be set to zero by performing BD on the matrices  $\mathbf{B}_i$ . This solution corresponds to beamforming based on the long-term beams with the additional constraint that the  $i$ -th user's long-term

beams are in the null space of all other users' long-term beams.

If we define the singular value decomposition (SVD) of the  $i$ -th user's correlation matrix  $\mathbf{R}_{t_i}$  as

$$\mathbf{R}_{t_i} = \mathbf{Q}_i \mathbf{\Lambda}_i \mathbf{Q}_i^H \quad (4)$$

the matrix  $\mathbf{R}_{t_i}^{1/2}$  can be written as  $\mathbf{R}_{t_i}^{1/2} = \mathbf{Q}_i \mathbf{\Lambda}_i^{1/2} \mathbf{Q}_i^H$ . Next, we introduce the matrix

$$\widehat{\mathbf{H}}_i = \mathbf{\Lambda}_i^{1/2} \mathbf{Q}_i^H. \quad (5)$$

It can easily be shown that the solution proposed in [5] is similar to the solution obtained by applying BD on the matrices  $\widehat{\mathbf{H}}_i$ . Unlike in [5] where the authors address only the cancellation of the MUI without the optimization of the isolated single-user performance, the approach based on (5) completely defines the modulation matrices. It was shown that when only the channel correlation matrix is available at the transmitter, the optimum strategy is to transmit on the long-term eigenmodes of matrix  $\mathbf{R}_{t_i}$  [6].

Next, we define an MMSE-like solution based on the long-term CSI at the BS/AP. To this end, we will use the matrix  $\widehat{\mathbf{H}}_i$  defined in (5) as a long-term equivalent channel and perform the precoding on this matrix as if it represented the actual channel. Matrices  $\widehat{\mathbf{H}}_i, i = 1, 2, \dots, K$ , contain all the information about the long-term subspace of each user available at the transmitter, in this case the BS/AP. To illustrate this fact we look at zero-forcing (ZF) precoding that is defined as the pseudo-inverse of the channel.

The pseudo-inverse of the matrix  $\widehat{\mathbf{H}}_i$  is equal to

$$\left( \widehat{\mathbf{H}}_i^H \widehat{\mathbf{H}}_i \right)^{-1} \widehat{\mathbf{H}}_i^H = \left( \mathbf{Q}_i \mathbf{\Lambda}_i \mathbf{Q}_i^H \right)^{-1} \mathbf{Q}_i \mathbf{\Lambda}_i^{1/2} = \mathbf{Q}_i \mathbf{\Lambda}_i^{-1/2}.$$

From the previous equation we can see that the pseudo-inverse of the matrix defined in (5) also results in a transmission on the scaled long-term beams  $\mathbf{Q}_i$  of the channel. This means that we could apply the same linear precoding techniques requiring perfect CSI at the transmitter also if we only have the information on the transmit correlation matrices. But instead of using the exact channel knowledge we will use the matrix in (5). In this paper, we consider a multi-user downlink scenario and we will investigate the performance of BD and SMMSE applied on (5).

### 3.3. Block diagonalization (BD)

Block diagonalization was first proposed in [1]. It is restricted to channels where the number of transmit antennas  $M_T$  is greater or equal to the total number of receive antennas in the network  $M_R$ .

Let us define the precoder matrices as

$$\mathbf{F} = \left[ \mathbf{F}_1 \quad \mathbf{F}_2 \quad \dots \quad \mathbf{F}_K \right] \in \mathbb{C}^{M_T \times r} \quad (6)$$

where  $\mathbf{F}_i \in \mathbb{C}^{M_T \times r_i}$  is the  $i$ -th user's precoder matrix. Moreover,  $r \leq M_R$  is the total number of the transmitted data stream sequences, whereas  $r_i \leq M_{R_i}$  is the number of data stream sequences transmitted to the  $i$ -th user. We can find the optimal precoding matrix  $\mathbf{F}$  such that all MUI is zero by choosing a precoding matrix  $\mathbf{F}_i$  that lies in the null space of the other users' channel matrices. Thereby, a MU MIMO downlink channel is decomposed into multiple parallel independent SU MIMO channels [5], [7].

If we define  $\widetilde{\mathbf{H}}_i$  as

$$\widetilde{\mathbf{H}}_i = \left[ \mathbf{H}_1^T \quad \dots \quad \mathbf{H}_{i-1}^T \quad \mathbf{H}_{i+1}^T \quad \dots \quad \mathbf{H}_K^T \right]^T \quad (7)$$

the zero MUI constraint forces  $\mathbf{F}_i$  to lie in the null space of  $\widetilde{\mathbf{H}}_i$ . From the singular value decomposition (SVD) of  $\widetilde{\mathbf{H}}_i$  whose rank is  $\widetilde{L}_i$

$$\widetilde{\mathbf{H}}_i = \widetilde{\mathbf{U}}_i \widetilde{\mathbf{\Sigma}}_i \left[ \begin{array}{cc} \widetilde{\mathbf{V}}_i^{(1)} & \widetilde{\mathbf{V}}_i^{(0)} \end{array} \right]^H \quad (8)$$

we choose the last right  $M_T - \widetilde{L}_i$  singular vectors  $\widetilde{\mathbf{V}}_i^{(0)} \in \mathbb{C}^{M_T \times M_T - \widetilde{L}_i}$  which form an orthogonal basis for the null space of  $\widetilde{\mathbf{H}}_i$ . The equivalent channel of user  $i$  after eliminating the MUI is identified as  $\mathbf{H}_i \widetilde{\mathbf{V}}_i^{(0)}$ , whose dimension is  $M_{R_i} \times (M_T - \widetilde{L}_i)$  and is equivalent to a system with  $M_T - \widetilde{L}_i$  transmit antennas and  $M_{R_i}$  receive antennas. Each of these equivalent SU MIMO channels has the same properties as a conventional SU MIMO channel. Define the SVD

$$\mathbf{H}_i \widetilde{\mathbf{V}}_i^{(0)} = \mathbf{U}_i \mathbf{\Sigma}_i \left[ \begin{array}{cc} \mathbf{V}_i^{(1)} & \mathbf{V}_i^{(0)} \end{array} \right]^H \quad (9)$$

and let the rank of the  $i$ -th user's equivalent channel matrix be  $L_i$ . The product of the first  $L_i$  singular vectors  $\mathbf{V}_i^{(1)}$  and  $\widetilde{\mathbf{V}}_i^{(0)}$  produces an orthogonal basis of dimension  $L_i$  and represents the transmission vectors that with proper power loading maximize the information rate for user  $i$  subject to the zero MUI constraint.

If there is only long-term CSI available at the transmitter, we use the long-term equivalent channel (5) instead of the exact channel  $\mathbf{H}_i$  in equations (7) and (9).

### 3.4. Successive MMSE transmit precoding (SMMSE)

MMSE precoding can improve the system performance by introducing a certain amount of interference especially for users equipped with a single antenna. However, it suffers a performance loss when it attempts to mitigate the interference between two closely spaced antennas as in the case when the user terminal is equipped with more than one receive antenna. In [2] a new algorithm has been introduced that deals with this problem by successively calculating the columns of the precoding matrix  $\mathbf{F}$  for each of the receive antennas separately.

The precoding matrix in equation (6) is written as

$$\mathbf{F} = \mathbf{F}_a \cdot \mathbf{F}_b, \quad (10)$$

where

$$\mathbf{F}_a = [\mathbf{F}_{a_1} \quad \mathbf{F}_{a_2} \quad \cdots \quad \mathbf{F}_{a_K}] \in \mathbb{C}^{M_T \times M_R},$$

and

$$\mathbf{F}_b = \begin{bmatrix} \mathbf{F}_{b_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{b_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{F}_{b_K} \end{bmatrix} \in \mathbb{C}^{M_R \times r},$$

with  $\mathbf{F}_{a_i} \in \mathbb{C}^{M_T \times M_{R_i}}$  and  $\mathbf{F}_{b_i} \in \mathbb{C}^{M_{R_i} \times r_i}$ .

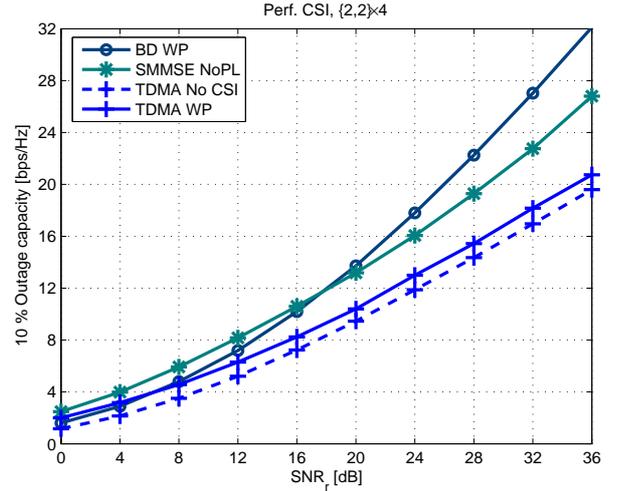
The columns in the precoding matrix  $\mathbf{F}_{a_i}$ , each corresponding to one receive antenna, are calculated successively. For the  $i$ -th user,  $i = 1, \dots, K$ , and  $j$ -th receive antenna  $j = 1, \dots, M_{R_i}$  we define the matrix  $\bar{\mathbf{H}}_i^{(j)}$  as

$$\bar{\mathbf{H}}_i^{(j)} = \begin{bmatrix} \mathbf{h}_{i,j}^T \\ \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{i-1} \\ \mathbf{H}_{i+1} \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \quad (11)$$

where  $\mathbf{h}_{i,j}^T$  is the  $j$ -th row of the  $i$ -th user's channel matrix  $\mathbf{H}_i$ . The corresponding column of the precoding matrix  $\mathbf{F}_{a_i}$  is equal to the first column of the matrix

$$\mathbf{F}_{i,j} = \beta \left( \bar{\mathbf{H}}_i^{(j)H} \bar{\mathbf{H}}_i^{(j)} + \alpha \mathbf{I}_{M_T} \right)^{-1} \bar{\mathbf{H}}_i^{(j)H} \quad (12)$$

where  $\alpha = M_R \sigma_n^2 / P_T$ ,  $P_T$  is the total transmit power, and  $\sigma_n^2$  is the variance of a zero mean additive white Gaussian noise. After calculating the beamforming vectors for all receive antennas in this fashion, the equivalent combined channel matrix of all users is equal to  $\mathbf{H}\mathbf{F}_a \in \mathbb{C}^{M_R \times M_R}$  after the precoding. For high SNR ratios, this matrix is also block diagonal. We can now apply any other previously defined SU MIMO technique on the  $i$ -th user's equivalent channel matrix  $\mathbf{H}_i \mathbf{F}_{a_i}$ . After the precoding using the matrix  $\mathbf{F}_{a_i}$ , we first perform the singular value decomposition (SVD)  $\mathbf{H}_i \mathbf{F}_{a_i} = \mathbf{U}_i \Sigma_i \left[ \mathbf{V}_i^{(1)} \quad \mathbf{V}_i^{(0)} \right]^H$  and define the matrix  $\mathbf{F}_{b_i}$  as  $\mathbf{F}_{b_i} = \mathbf{V}_i^{(1)}$ . Then, if we want to maximize the capacity of the system use water-pouring (WP) on the eigenmodes of all users or if we want to extract the maximum diversity and array gain, we transmit only on the dominant eigenmode. Dominant eigenmode transmission (DET) provides the maximum SNR at the receiver and the best



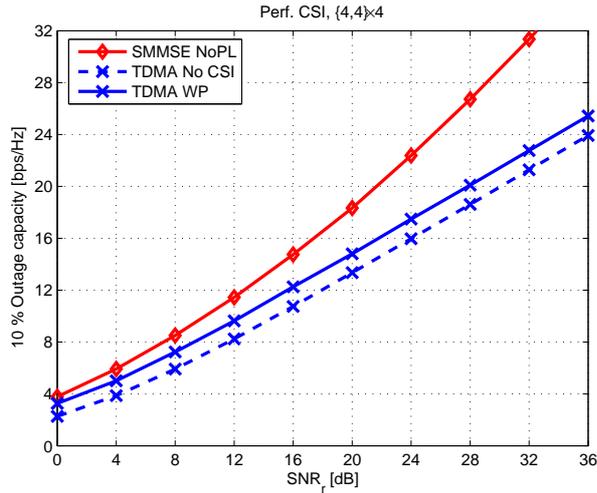
**Fig. 2.** 10 % outage capacity with perfect CSI at the transmitter as a function of the receive SNR<sub>r</sub>. Each user is equipped with two antennas.

BER performance [6]. The complexity of SMMSE is only slightly higher than the one of BD. By using SMMSE we efficiently improve the system performance by introducing MUI and by eliminating inter-stream interference. Again, if the channel is varying too rapidly, we can obtain only long-term CSI at the transmitter and instead of the exact channel matrix  $\mathbf{H}_i$  we will use the long-term equivalent channel as defined in (5).

#### 4. SIMULATION RESULTS

In this section we evaluate the 10 % outage capacity performance of the BD and SMMSE precoding algorithms in a downlink multi-user wide area scenario. We assume that the system is employing OFDM and we calculate the capacity only on one subcarrier. The channel on each subcarrier is modeled according to (1). We assume that the number of users in the system is large and that the scheduling always chooses two users that are well separated in space. The receive signal-to-noise ratio is defined as  $\text{SNR}_r = P_T / \sigma_n^2$ , where  $P_T$  is the total transmit power per subcarrier and  $\sigma_n^2$  is the variance of the zero mean additive white Gaussian noise.

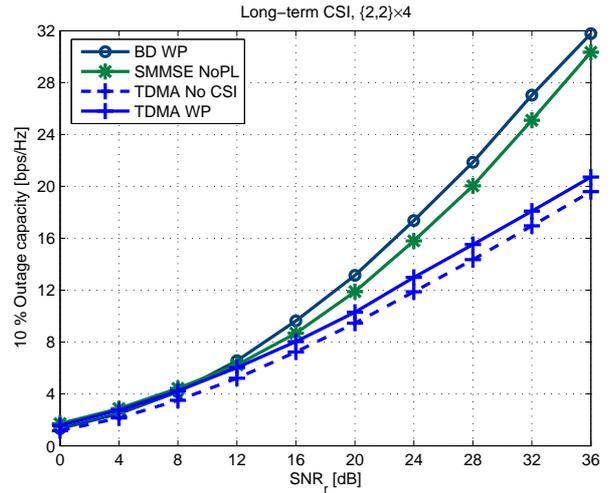
In Fig. 2, we compare the performance of BD and SMMSE when there is perfect CSI available at the transmitter and when the users are equipped with two antennas each. With BD we use water-pouring (WP) and with SMMSE no power loading (NoPL). SMMSE has an advantage over BD at low SNRs while BD can provide higher capacity at high SNRs. However, BD is limited to cases when the total number of receive antennas at the MSs is less or equal to the to-



**Fig. 3.** 10 % outage capacity with perfect CSI at the transmitter as a function of the receive SNR<sub>r</sub>. Each user is equipped with four antennas. The BD solution does not exist in this case.

tal number of transmit antennas [5]. From Figure 3 we can see that when the users are equipped with 4 antennas each, SMMSE improves the capacity at some SNRs by more than 4 bits/sec/Hz. As a comparison we also show the performance of a TDMA system. If there is no CSI we assume no processing at the transmitter. If there is CSI available at the transmitter we assume the transmitter is performing a singular-value decomposition and water-pouring. SMMSE always performs better than TDMA and the SDMA gains are especially high at higher SNRs.

In Fig. 4 we compare the performance of BD and SMMSE when there is only long-term CSI available at the transmitter. Power loading based on the long-term eigenvalues of the channel is used. We can see that both techniques have a similar performance at low SNRs and that at high SNRs BD offers a better performance. Unlike the previous case, when we have perfect CSI available at the transmitter, BD is not limited by the number of receive antennas. This is the consequence of the fact that we perform the precoding on an equivalent channel. When the channel is rank deficient like it is the case in the wide area scenario, the dimension of the equivalent channel is different from the actual and the dimensionality restriction is met if the rank of the transmit correlation matrix is less than the number of receive antennas. The performance of BD and SMMSE in the system with configuration  $\{4, 4\} \times 4$  can be seen in Fig. 5. We should note here that the performance of these precoding techniques does not change significantly compared to the case when there is perfect CSI available at the transmitter, which is a consequence of the smart scheduling assumption [8], [9]. If designed well a scheduling algorithm selects

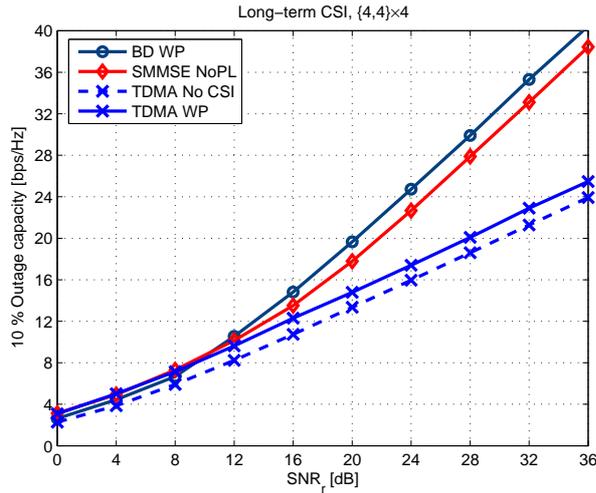


**Fig. 4.** 10 % outage capacity with perfect CSI at the transmitter as a function of the receive SNR<sub>r</sub>. Each user is equipped with two antennas.

the users that are well separated in space. In this case even the knowledge about the transmit correlation of the channel allows us to completely eliminate multi-user interference at high SNRs. The more antennas there are at the BS/AP, the greater are the benefits of MU MIMO precoding. This is shown in Fig. 6 where the performance of BD and SMMSE is compared with a TDMA system with the configuration  $\{2, 2\} \times 8$ . From the last figure, we can also see that as the number of antennas at the BS/AP increases the difference between the BD and SMMSE reduces.

## 5. CONCLUSIONS

In this paper we propose a new precoding approach that allows the use of precoding techniques requiring perfect CSI at the transmitter in cases when only the long-term transmit correlation matrices are known. We compare the performance of two MIMO precoding techniques BD and SMMSE using long-term CSI. SMMSE balances the MUI and has no restrictions considering the number of antennas at the MS. In contrast, BD has zero MUI and the number of antennas has to be less or equal to the number of transmit antennas. With perfect CSI at the transmitter, SMMSE provides a higher capacity than BD at low SNRs. By performing the precoding based on the long-term CSI, BD and SMMSE have a similar performance at low SNRs. When the number of antennas at the BS/AP is increased, the performance of BD or SMMSE further improves compared to the TDMA system. Moreover, by increasing the number of antennas at the BS/AP the difference between BD and SMMSE with long-term CSI reduces further. The perfor-

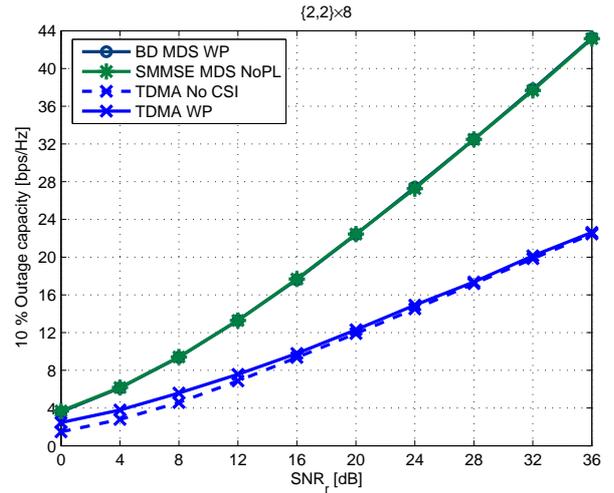


**Fig. 5.** 10 % outage capacity with long-term CSI at the transmitter as a function of the receive SNR<sub>r</sub>. Each user is equipped with four antennas.

mance of BD does not change significantly when the precoding is performed based on long-term CSI. This is mainly the consequence of the scheduling. A good scheduling algorithm can select a group of users that are well separated in space and can be served simultaneously. Then these precoding techniques deliver a much higher performance than TDMA systems. SMMSE cannot provide the same throughput at high SNRs as BD. However, SMMSE has the advantage of having no dimensionality restrictions and can use an increased number of antennas at the MSs to further improve the system capacity. Thereby, we have shown that these two precoding techniques can be used even when there is no perfect CSI available at the transmitter if we use them on the matrices  $\widehat{H}_i$  instead of the exact channel matrices  $H_i$ .

## 6. REFERENCES

- [1] Q. Spencer and M. Haardt, "Capacity and downlink transmission algorithms for a multi-user MIMO channel," in *Proc. 36th Asilomar Conf. on Signals, Systems, and Computers, Pacific Grove, CA, IEEE Computer Society Press*, November 2002.
- [2] V. Stankovic and M. Haardt, "Multi-user MIMO downlink precoding for users with multiple antennas," in *Proc. of the 12-th Meeting of the Wireless World Research Forum (WWRF), Toronto, ON, Canada*, November 2004.
- [3] E. Telatar, "Capacity of multi-antenna Gaussian channels," *ATT Bell Technical Memorandum*, 1995.



**Fig. 6.** 10 % outage capacity with long-term CSI at the transmitter as a function of the receive SNR<sub>r</sub>. Each user is equipped with four antennas and at the BS/AP there are eight antennas.

- [4] M. T. Ivrlac and J. A. Nossek, "Correlated fading in MIMO-systems - Blessing or Curse?," in *Proc. of the 39th Allerton Conference on Communication, Control, and Computing, Monticello, IL, USA*, October 2001.
- [5] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in Multiuser MIMO channels," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 461–471, February 2004.
- [6] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 2003.
- [7] L. U. Choi and R. D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, pp. 20–24, January 2004.
- [8] G. Del Galdo and M. Haardt, "Comparison of zero-forcing methods for downlink spatial multiplexing in realistic multi-user MIMO channels," in *Proc. IEEE Vehicular Technology Conference 2004-Spring, Milan, Italy*, May 2004.
- [9] M. Fuchs, G. Del Galdo, and M. Haardt, "A novel tree-based scheduling algorithm for the downlink of multi-user MIMO systems with ZF beamforming," in *Proc. IEEE Int. Conf. Acoust., Speech, and Signal Processing (ICASSP), Philadelphia, PA, USA*, March 2005.