Widely-Linear Distributed Beamforming for Weak-Sense Non-Circular Sources Based on Relay Power Minimization

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(invited paper)

***Abstract*—In this paper, we present a widely-linear (WL) distributed beamforming algorithm that takes full advantage of weak-sense second-order (SO) non-circular source signals. We consider a single-antenna source-destination pair in an ad-hoc relay network that suffers from strong interference. Assuming that perfect channel state information (CSI) is available at the receiver, we design our algorithm based on the total relay transmit power minimization subject to a signal-to-interference-plus-noise ratio (SINR) constraint after applying WL processing at the relays. We find key properties, analyze the computational complexity, and show by simulations that the proposed WL distributed beamforming algorithm provides a substantial relay power reduction compared to its linear counterpart for different degrees of non-circularity.**

***Index Terms*—Widely-linear processing, weak-sense non-circular sources, distributed beamforming, relay networks.**

I. Introduction

In distributed ad-hoc networks, the establishment of reliable transmissions between nodes has recently received considerable attention among researchers in the field of wireless communications. It is known that the performance of a wireless network, i.e., its coverage, capacity, energy-efficiency, and reliability can be dramatically improved by exploiting the concept of user cooperation diversity [1], [2]. In such cooperation scheme, passive relay nodes assist source-destination pairs in their communications by relaying signals through multiple independent paths in the network. Thus, they form a virtual array of transmit antennas to provide spatial diversity at the receiver without the need of multiple antennas at the users.

One of the most effective techniques to benefit from cooperative diversity capabilities is distributed relay beamforming [3], [4] based on the amplify-and-forward (AF) relaying protocol [2], which is often preferred due to its simplicity and low complexity. The authors in [3] and [4] consider a single source-destination pair and determine the beamforming weights based on the assumption that the channel state information (CSI) is available at the receiver. Thus, the weights have to be fed back to the relays. The studied criteria to determine the beamforming weights minimize the total relay power subject to a certain target signal-to-noise ratio (SNR) constraint at the receiver [4] or maximize the receiver SNR subject to certain power constraints [3], i.e., either individual relay power constraints or a total relay power constraint.

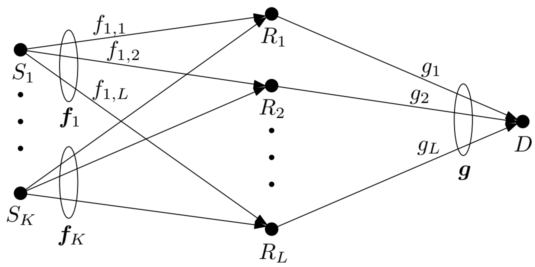

Fig. 1. A network of K sources, L relays, and one destination.

Recently, the concept of widely-linear (WL) processing originally applied to the conventional beamforming problem [5]-[8] has been extended to distributed beamforming [9] to take advantage of the strict second-order (SO) non-circularity of source signals [10]. Some examples of such signals include the BPSK, ASK, PAM, and the Offset-QPSK modulation schemes. It was shown that processing the strictly non-circular data and its complex conjugate version separately at the relays provides additional degrees of freedom that lead to significant performance improvements [9]. However, WL processing for distributed beamforming has so far not been exploited for the case of weak-sense SO non-circular signals. An example of such signals is the rectangular 8-QAM modulation scheme.

In this paper, we propose a WL distributed beamforming (WL-DB) algorithm that fully exploits the SO statistics of weak-sense non-circular source signals at the relays, where only the magnitudes of the non-circularity coefficients are required. We consider a single-antenna source-destination communication pair assisted by multiple relays using the AF protocol that is subject to interference and assume that perfect CSI is available at the receiver. We minimize the total relay power under a target signal-to-interference-plus-noise ratio (SINR) constraint. Moreover, the special cases of circular and strictly non-circular transmit signals are considered and the computational complexity is analyzed. Simulations results show that depending on the degree of non-circularity, the presented WL-DB algorithm provides a substantial reduction in the required total relay power and a lower bit error rate (BER) compared to its linear counterpart (L-DB) [4].

II. System Model and Widely-Linear Processing

We consider a distributed network of nodes with one source-destination pair, L relays and $K-1$ interfering sources, as illustrated in Fig. 1. All the nodes in the network are single-antenna units that operate in a common frequency band and work in half-duplex mode, i.e., they can either transmit or receive information. We assume that there is no direct link between the K sources and the destination such that the communication between them is assisted by all the relays. Furthermore, we have flat-fading channel realizations and the network is perfectly synchronized. Each transmission from the sources to the destination is implemented in a two-step procedure, e.g., in two consecutive time-slots. In the first slot, all the sources simultaneously broadcast their signals to the relays and in the second time-slot, the received signals at the relays are processed by a complex beamforming weight and retransmitted to the destination.

In the first stage of the transmission, the noise-corrupted source signals received at the relays are modeled as

$$\boldsymbol{x} = \boldsymbol{F}\boldsymbol{P}^{1/2}\boldsymbol{s} + \boldsymbol{\mu} \ \in \mathbb{C}^{L\times 1}, \tag{1}$$

where $\boldsymbol{F} = [\boldsymbol{f}_1, \ldots, \boldsymbol{f}_K] \in \mathbb{C}^{L\times K}$ is the channel matrix between the sources and the relays, and $\boldsymbol{f}_i = [f_{i,1}, \ldots, f_{i,L}]^T$, $i = 1, \ldots, K$, contains the channel coefficients from the i-th source to all the relays. The vector $\boldsymbol{s} = [s_1, \ldots, s_K]^T \in \mathbb{C}^{K\times 1}$ represents the uncorrelated signals from the K sources with $\mathbb{E}\{|s_i|^2\} = 1$, $\boldsymbol{P} \in \mathbb{R}^{K\times K}$ is the diagonal matrix with the source transmit powers P_i on its diagonal, and $\boldsymbol{\mu} \in \mathbb{C}^{L\times 1}$ is the zero-mean circularly symmetric complex Gaussian noise at the relays with variance σ_μ^2.

Next, we introduce the concept of WL processing for distributed relay beamforming to exploit the additional information contained in the pseudo-covariance matrix, when processing both the SO non-circular data and its complex conjugate version at the relays separately. Therefore, using (1), we define the augmented relay vector

$$\boldsymbol{x}_\mathrm{a} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}\boldsymbol{P}^{1/2}\boldsymbol{s} \\ \boldsymbol{F}^*\boldsymbol{P}^{1/2}\boldsymbol{s}^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\mu}^* \end{bmatrix} \in \mathbb{C}^{2L\times 1} \tag{2}$$

to be processed in the sequel. Due to the assumption of weak-sense SO non-circular source signals, a measure for the non-circularity degree of the i-th source is defined by the non-circularity coefficient [10]

$$\rho_i = \frac{\mathbb{E}\{s_i^2\}}{P_i} = |\rho_i| e^{\mathrm{j}\psi_i}, \quad 0 \leq |\rho_i| \leq 1, \tag{3}$$

where $|\rho_i| = 0$ and $|\rho_i| = 1$ represent a circular source and a strictly non-circular (rectilinear) source, respectively.

For $|\rho_i| \neq 0 \ \forall i$, it was shown in [5] and [6] that $\boldsymbol{s}^*$ is correlated with $\boldsymbol{s}$ and can be modeled as the orthogonal decomposition into a signal component and an interference component [6], given by

$$\boldsymbol{s}^* = \boldsymbol{K}^H\boldsymbol{s} + (\boldsymbol{I}_K - \boldsymbol{K}\boldsymbol{K}^H)^{1/2}\boldsymbol{s}', \tag{4}$$

where $\boldsymbol{K} = \mathrm{diag}\{\rho_i\}_{i=1}^K$ contains the non-circularity coefficients for the K signals on its diagonal and $\boldsymbol{s}'$ is orthonormal to $\boldsymbol{s}$, i.e., $\mathbb{E}\{\boldsymbol{s}'^H\boldsymbol{s}\} = 0$ and $\mathbb{E}\{\boldsymbol{s}'^H\boldsymbol{s}'\} = 1$. The special cases of strictly non-circular and circular sources for this model are analyzed in Section IV. Next, using (2) and (4), the augmented relay vector $\boldsymbol{x}_\mathrm{a}$ can be extended as

$$\boldsymbol{x}_\mathrm{a} = \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{P}^{1/2}\boldsymbol{s} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{F}^* \end{bmatrix} \boldsymbol{P}^{1/2}\boldsymbol{s}^* + \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\mu}^* \end{bmatrix} \tag{5}$$

$$= \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{F}^*\boldsymbol{K}^* \end{bmatrix} \boldsymbol{P}^{1/2}\boldsymbol{s} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{F}^* \end{bmatrix} \boldsymbol{P}^{1/2}\bar{\boldsymbol{s}} + \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\mu}^* \end{bmatrix}$$

$$= \boldsymbol{F}_\mathrm{a}\boldsymbol{P}^{1/2}\boldsymbol{s} + \bar{\boldsymbol{F}}_\mathrm{a}\boldsymbol{P}^{1/2}\bar{\boldsymbol{s}} + \boldsymbol{\mu}_\mathrm{a}, \tag{6}$$

where $\boldsymbol{F}_\mathrm{a} = [\boldsymbol{f}_{\mathrm{a}_1}, \ldots, \boldsymbol{f}_{\mathrm{a}_K}] \in \mathbb{C}^{2L\times K}$, $\bar{\boldsymbol{F}}_\mathrm{a} = [\bar{\boldsymbol{f}}_{\mathrm{a}_1}, \ldots, \bar{\boldsymbol{f}}_{\mathrm{a}_K}] \in \mathbb{C}^{2L\times K}$, and $\bar{\boldsymbol{s}} = (\boldsymbol{I}_K - \boldsymbol{K}\boldsymbol{K}^H)^{1/2}\boldsymbol{s}'$. The new dimensions of $\boldsymbol{x}_\mathrm{a}$ can be seen as a virtual doubling of the number of relays.

In the second stage of the transmission, the retransmitted augmented signal from the relays can be expressed as

$$\boldsymbol{r}_\mathrm{a} = \boldsymbol{W}_\mathrm{a}^H\boldsymbol{x}_\mathrm{a} \ \in \mathbb{C}^{2L\times 1}, \tag{7}$$

where $\boldsymbol{W}_\mathrm{a} = \mathrm{diag}\{\boldsymbol{w}_\mathrm{a}\}$ and $\boldsymbol{w}_\mathrm{a} \in \mathbb{C}^{2L\times 1}$ contains the $2L$ virtual beamforming weights to be designed. The physical relays transmit the widely-linear combination [9]

$$\boldsymbol{r} = \boldsymbol{W}_1^H\boldsymbol{x} + \boldsymbol{W}_2^H\boldsymbol{x}^* \ \in \mathbb{C}^{L\times 1}, \tag{8}$$

where $\boldsymbol{W}_l = \mathrm{diag}\{\boldsymbol{w}_l\}$, $l = 1, 2$, with $\boldsymbol{w}_l \in \mathbb{C}^{L\times 1}$ given by $\boldsymbol{w}_\mathrm{a} = [\boldsymbol{w}_1^T, \boldsymbol{w}_2^T]^T$. Next, we define the augmented channel vector $\boldsymbol{g}_\mathrm{a} = [\boldsymbol{g}^T, \boldsymbol{g}^T]^T \in \mathbb{C}^{2L\times 1}$, where $\boldsymbol{g} = [g_1, \ldots, g_L]^T$ is the channel vector between the L relays and the destination. Thus, $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ are not necessarily complex conjugates of each other due to the stacking in $\boldsymbol{g}_\mathrm{a}$ [9]. Combining (6) and (7), the received signal at the destination is obtained as

$$y = \underbrace{\sqrt{P_d}\, \boldsymbol{g}_\mathrm{a}^T\boldsymbol{W}_\mathrm{a}^H\boldsymbol{f}_{\mathrm{a}_d}s_d}_{\text{augm. desired signal}} + \underbrace{\boldsymbol{g}_\mathrm{a}^T\boldsymbol{W}_\mathrm{a}^H\sqrt{P_d}\bar{\boldsymbol{f}}_{\mathrm{a}_d}\bar{s}_d}_{\text{augm. interference}} \tag{9}$$

$$+ \underbrace{\boldsymbol{g}_\mathrm{a}^T\boldsymbol{W}_\mathrm{a}^H \sum_{k=1,k\neq d}^{K} \sqrt{P_k}(\boldsymbol{f}_{\mathrm{a}_k}s_k + \bar{\boldsymbol{f}}_{\mathrm{a}_k}\bar{s}_k)}_{\text{augm. interference}} + \underbrace{\boldsymbol{g}_\mathrm{a}^T\boldsymbol{W}_\mathrm{a}^H\boldsymbol{\mu}_\mathrm{a} + n}_{\text{augm. effective noise}},$$

where n is the zero-mean noise at the destination with variance σ_n^2 and the subscript d denotes the desired signal component.

III. Power Minimization

In this section, we design the $2L$ beamforming weights using (9) such that the total relay transmit power P_r in the WL case is minimized while maintaining the receiver SINR at least at the threshold γ. The presented development is inspired by the derivation in [4] that only uses linear processing. In contrast to [4], we incorporate WL processing and consider $K-1$ additional interferers in the network as shown in Fig. 1. The optimization problem is stated as

$$\begin{aligned} &\min_{\boldsymbol{w}_\mathrm{a}} \quad P_\mathrm{r} \\ &\text{subject to} \quad \mathrm{SINR}_\mathrm{WL} \geq \gamma. \end{aligned} \tag{10}$$

Here, $\mathrm{SINR}_\mathrm{WL}$ is the SINR in the WL case, which is defined by $\mathrm{SINR}_\mathrm{WL} = \frac{P_\mathrm{s}}{P_\mathrm{i}+P_\mathrm{n}}$, where P_s, P_i, and P_n represent the power of the desired signal, the interference power, and the noise power at the receiver, respectively. For the expressions of these powers, we have

$$\begin{aligned} P_{\mathrm{r}} &= \mathbb{E}\{\|\boldsymbol{r}_{\mathrm{a}}\|^2\} = \mathbb{E}\{\boldsymbol{x}_{\mathrm{a}}^H \boldsymbol{W}_{\mathrm{a}} \boldsymbol{W}_{\mathrm{a}}^H \boldsymbol{x}_{\mathrm{a}}\} \\ &= \mathrm{Tr}\{\boldsymbol{W}_{\mathrm{a}}^H \mathbb{E}\{\boldsymbol{x}_{\mathrm{a}} \boldsymbol{x}_{\mathrm{a}}^H\} \boldsymbol{W}_{\mathrm{a}}\} = \mathrm{Tr}\{\boldsymbol{W}_{\mathrm{a}}^H \boldsymbol{T}_{\mathrm{a}} \boldsymbol{W}_{\mathrm{a}}\} \\ &= \boldsymbol{w}_{\mathrm{a}}^H \boldsymbol{D}_{\mathrm{a}} \boldsymbol{w}_{\mathrm{a}}, \end{aligned} \tag{11}$$

where $\mathrm{Tr}\{\cdot\}$ is the trace operator and $\boldsymbol{D}_{\mathrm{a}} \in \mathbb{R}^{2L\times 2L}$ is a diagonal matrix with $\boldsymbol{D}_{\mathrm{a}} = \boldsymbol{T}_{\mathrm{a}} \odot \boldsymbol{I}_{2L}$, where $\odot$ is the Schur-Hadamard (element-wise) matrix product. The augmented covariance matrix $\boldsymbol{T}_{\mathrm{a}} \in \mathbb{C}^{2L\times 2L}$ can be expressed as

$$\begin{aligned} \boldsymbol{T}_{\mathrm{a}} &= \mathbb{E}\{\boldsymbol{x}_{\mathrm{a}} \boldsymbol{x}_{\mathrm{a}}^H\} \\ &= \boldsymbol{F}_{\mathrm{a}} \boldsymbol{P} \boldsymbol{F}_{\mathrm{a}}^H + \bar{\boldsymbol{F}}_{\mathrm{a}} (\boldsymbol{P}(\boldsymbol{I}_K - \boldsymbol{K}\boldsymbol{K}^H)) \bar{\boldsymbol{F}}_{\mathrm{a}}^H + \sigma_\mu^2 \boldsymbol{I}_{2L}. \end{aligned} \tag{12}$$

For the power of the desired signal component, we obtain

$$\begin{aligned} P_{\mathrm{s}} &= \mathbb{E}\{|\sqrt{P_d}\, \boldsymbol{g}_{\mathrm{a}}^T \boldsymbol{W}_{\mathrm{a}}^H \boldsymbol{f}_{\mathrm{a}_d} s_d|^2\} \\ &= P_d\, \boldsymbol{g}_{\mathrm{a}}^T \boldsymbol{W}_{\mathrm{a}}^H \boldsymbol{f}_{\mathrm{a}_d} \boldsymbol{f}_{\mathrm{a}_d}^H \boldsymbol{W}_{\mathrm{a}} \boldsymbol{g}_{\mathrm{a}}^* \\ &= P_d\, \boldsymbol{w}_{\mathrm{a}}^H \mathrm{diag}\{\boldsymbol{g}_{\mathrm{a}}\} \boldsymbol{f}_{\mathrm{a}_d} \boldsymbol{f}_{\mathrm{a}_d}^H \mathrm{diag}\{\boldsymbol{g}_{\mathrm{a}}^*\} \boldsymbol{w}_{\mathrm{a}} \\ &= \boldsymbol{w}_{\mathrm{a}}^H \boldsymbol{R}_{\mathrm{a}} \boldsymbol{w}_{\mathrm{a}}, \end{aligned} \tag{13}$$

where $\boldsymbol{R}_{\mathrm{a}} = P_d\, \boldsymbol{h}_{\mathrm{a}_d} \boldsymbol{h}_{\mathrm{a}_d}^H \in \mathbb{C}^{2L\times 2L}$ and $\boldsymbol{h}_{\mathrm{a}_d} = \boldsymbol{g}_{\mathrm{a}} \odot \boldsymbol{f}_{\mathrm{a}_d}$. The interference power can be written as

$$\begin{aligned} P_{\mathrm{i}} &= \mathbb{E}\Big\{\Big|\boldsymbol{g}_{\mathrm{a}}^T \boldsymbol{W}_{\mathrm{a}}^H \Big(\sqrt{P_d} \bar{\boldsymbol{f}}_{\mathrm{a}_d} \bar{s}_d \\ &\qquad + \sum_{k=1,k\neq d}^{K} \sqrt{P_k} (\boldsymbol{f}_{\mathrm{a}_k} s_k + \bar{\boldsymbol{f}}_{\mathrm{a}_k} \bar{s}_k)\Big|^2\Big)\Big\} \\ &= \boldsymbol{w}_{\mathrm{a}}^H \mathrm{diag}\{\boldsymbol{g}_{\mathrm{a}}\} \Big(P_d (1 - |\rho_d|^2) \bar{\boldsymbol{f}}_{\mathrm{a}_d} \bar{\boldsymbol{f}}_{\mathrm{a}_d}^H \\ &\quad + \sum_{k=1,k\neq d}^{K} P_k \big(\boldsymbol{f}_{\mathrm{a}_k} \boldsymbol{f}_{\mathrm{a}_k}^H + (1 - |\rho_k|^2) \bar{\boldsymbol{f}}_{\mathrm{a}_k} \bar{\boldsymbol{f}}_{\mathrm{a}_k}^H)\Big) \mathrm{diag}\{\boldsymbol{g}_{\mathrm{a}}^*\} \boldsymbol{w}_{\mathrm{a}} \\ &= \boldsymbol{w}_{\mathrm{a}}^H \boldsymbol{Q}_{\mathrm{i}_{\mathrm{a}}} \boldsymbol{w}_{\mathrm{a}}, \end{aligned} \tag{14}$$

where $\boldsymbol{Q}_{\mathrm{i}_{\mathrm{a}}} = P_d (1 - |\rho_d|^2) \bar{\boldsymbol{h}}_{\mathrm{a}_d} \bar{\boldsymbol{h}}_{\mathrm{a}_d}^H + \sum_{k=1,k\neq d}^{K} P_k\, (\boldsymbol{h}_{\mathrm{a}_k} \boldsymbol{h}_{\mathrm{a}_k}^H + (1 - |\rho_k|^2) \bar{\boldsymbol{h}}_{\mathrm{a}_k} \bar{\boldsymbol{h}}_{\mathrm{a}_k}^H) \in \mathbb{C}^{2L\times 2L}$ with $\bar{\boldsymbol{h}}_{\mathrm{a}_d} = \boldsymbol{g}_{\mathrm{a}} \odot \bar{\boldsymbol{f}}_{\mathrm{a}_d}$, $\boldsymbol{h}_{\mathrm{a}_k} = \boldsymbol{g}_{\mathrm{a}} \odot \boldsymbol{f}_{\mathrm{a}_k}$, and $\bar{\boldsymbol{h}}_{\mathrm{a}_k} = \boldsymbol{g}_{\mathrm{a}} \odot \bar{\boldsymbol{f}}_{\mathrm{a}_k}$. The expression for the noise power at the receiver can be written as

$$\begin{aligned} P_{\mathrm{n}} &= \mathbb{E}\{|\boldsymbol{g}_{\mathrm{a}}^T \boldsymbol{W}_{\mathrm{a}}^H \boldsymbol{\mu}_{\mathrm{a}} + n|^2\} \\ &= \sigma_\mu^2 \boldsymbol{g}_{\mathrm{a}}^T \boldsymbol{W}_{\mathrm{a}}^H \boldsymbol{W}_{\mathrm{a}} \boldsymbol{g}_{\mathrm{a}}^* + \sigma_n^2 \\ &= \sigma_\mu^2 \boldsymbol{w}_{\mathrm{a}}^H \mathrm{diag}\{\boldsymbol{g}_{\mathrm{a}}\} \mathrm{diag}\{\boldsymbol{g}_{\mathrm{a}}^*\} \boldsymbol{w}_{\mathrm{a}} + \sigma_n^2 \\ &= \boldsymbol{w}_{\mathrm{a}}^H \boldsymbol{Q}_{\mathrm{n}_{\mathrm{a}}} \boldsymbol{w}_{\mathrm{a}} + \sigma_n^2, \end{aligned} \tag{15}$$

where $\boldsymbol{Q}_{\mathrm{n}_{\mathrm{a}}} = \sigma_\mu^2 ((\boldsymbol{g}_{\mathrm{a}} \boldsymbol{g}_{\mathrm{a}}^H) \odot \boldsymbol{I}_{2L}) \in \mathbb{R}^{2L\times 2L}$. Whereas $\boldsymbol{D}_{\mathrm{a}}$ and $\boldsymbol{Q}_{\mathrm{n}_{\mathrm{a}}}$ are real-valued diagonal matrices, the augmented covariance matrices $\boldsymbol{R}_{\mathrm{a}}$ and $\boldsymbol{Q}_{\mathrm{i}_{\mathrm{a}}}$ contain the additional information in form of the pseudo-covariance matrices placed in their off-diagonal blocks as shown in [9]. This information will be exploited by the proposed WL-DB algorithm based on the relay power minimization criterion.

Finally, using (11)-(15) and following the lines in [4], the optimization problem (10) can be rewritten as

$$\begin{aligned} \min_{\boldsymbol{w}_{\mathrm{a}}} \quad & \boldsymbol{w}_{\mathrm{a}}^H \boldsymbol{D}_{\mathrm{a}} \boldsymbol{w}_{\mathrm{a}} \\ \text{subject to} \quad & \frac{\boldsymbol{w}_{\mathrm{a}}^H \boldsymbol{R}_{\mathrm{a}} \boldsymbol{w}_{\mathrm{a}}}{\boldsymbol{w}_{\mathrm{a}}^H (\boldsymbol{Q}_{\mathrm{i}_{\mathrm{a}}} + \boldsymbol{Q}_{\mathrm{n}_{\mathrm{a}}}) \boldsymbol{w}_{\mathrm{a}} + \sigma_n^2} \geq \gamma \end{aligned} \tag{16}$$

and simplified to

$$\begin{aligned} \min_{\tilde{\boldsymbol{w}}_{\mathrm{a}}} \quad & \|\tilde{\boldsymbol{w}}_{\mathrm{a}}\|^2 \\ \text{subject to} \quad & \tilde{\boldsymbol{w}}_{\mathrm{a}}^H \boldsymbol{D}_{\mathrm{a}}^{-1/2} (\boldsymbol{R}_{\mathrm{a}} - \gamma \boldsymbol{Q}_{\mathrm{a}}) \boldsymbol{D}_{\mathrm{a}}^{-1/2} \tilde{\boldsymbol{w}}_{\mathrm{a}} \geq \gamma \sigma_n^2, \end{aligned} \tag{17}$$

where $\boldsymbol{Q}_{\mathrm{a}} = \boldsymbol{Q}_{\mathrm{i}_{\mathrm{a}}} + \boldsymbol{Q}_{\mathrm{n}_{\mathrm{a}}}$ and we have substituted the optimization variable by $\tilde{\boldsymbol{w}}_{\mathrm{a}} = \boldsymbol{D}_{\mathrm{a}}^{1/2} \boldsymbol{w}_{\mathrm{a}}$. Note that the threshold γ has to be chosen such that $\boldsymbol{R}_{\mathrm{a}} - \gamma \boldsymbol{Q}_{\mathrm{a}}$ is not negative definite in order for the optimization problem in (17) to be feasible. Otherwise it becomes infeasible and cannot be solved.

To further simplify the optimization problem in (17), we find that at optimality the inequality constraint has to be satisfied with equality [4], which can easily be proven by contradiction. Therefore, we can apply the method of Lagrange multipliers, which yields the Lagrangian

$$\begin{aligned} L(\tilde{\boldsymbol{w}}_{\mathrm{a}}, \lambda) = \|\tilde{\boldsymbol{w}}_{\mathrm{a}}\|^2 - \lambda \big(\tilde{\boldsymbol{w}}_{\mathrm{a}}^H \boldsymbol{D}_{\mathrm{a}}^{-1/2} \big(\boldsymbol{R}_{\mathrm{a}} \\ - \gamma \boldsymbol{Q}_{\mathrm{a}}\big) \boldsymbol{D}_{\mathrm{a}}^{-1/2} \tilde{\boldsymbol{w}}_{\mathrm{a}} - \gamma \sigma_n^2\big). \end{aligned} \tag{18}$$

Taking the derivative with respect to $\tilde{\boldsymbol{w}}_{\mathrm{a}}^*$ and equating the Lagrange function to zero, we obtain

$$\boldsymbol{D}_{\mathrm{a}}^{-1/2} (\boldsymbol{R}_{\mathrm{a}} - \gamma \boldsymbol{Q}_{\mathrm{a}}) \boldsymbol{D}_{\mathrm{a}}^{-1/2} \tilde{\boldsymbol{w}}_{\mathrm{a}} = \frac{1}{\lambda} \tilde{\boldsymbol{w}}_{\mathrm{a}}. \tag{19}$$

It is evident from (19) after multiplying both sides with $\lambda \tilde{\boldsymbol{w}}_{\mathrm{a}}^H$ that minimizing $\|\tilde{\boldsymbol{w}}_{\mathrm{a}}\|^2$ corresponds to minimizing λ and hence, maximizing $1/\lambda$. Thus, the objective function in (17) is globally minimized when $\tilde{\boldsymbol{w}}_{\mathrm{a}}$ is chosen as

$$\tilde{\boldsymbol{w}}_{\mathrm{a}} = \alpha \bar{\boldsymbol{w}}_{\mathrm{a}} = \alpha \mathcal{P} \left\{ \boldsymbol{D}_{\mathrm{a}}^{-1/2} (\boldsymbol{R}_{\mathrm{a}} - \gamma \boldsymbol{Q}_{\mathrm{a}}) \boldsymbol{D}_{\mathrm{a}}^{-1/2} \right\}, \tag{20}$$

where $\mathcal{P}\{\cdot\}$ is the normalized principal eigenvector operator and $\alpha = (\gamma \sigma_n^2 / (\bar{\boldsymbol{w}}_{\mathrm{a}}^H \boldsymbol{D}_{\mathrm{a}}^{-1/2} (\boldsymbol{R}_{\mathrm{a}} - \gamma \boldsymbol{Q}_{\mathrm{a}}) \boldsymbol{D}_{\mathrm{a}}^{-1/2} \bar{\boldsymbol{w}}_{\mathrm{a}}))^{1/2}$ is the scaling to fulfill the SINR constraint in (17). Eventually, after resubstituting $\tilde{\boldsymbol{w}}_{\mathrm{a}}$ by $\boldsymbol{w}_{\mathrm{a}}$, we obtain the optimal WL beamforming vector as

$$\boldsymbol{w}_{\mathrm{a}} = \alpha \boldsymbol{D}_{\mathrm{a}}^{-1/2} \mathcal{P} \left\{ \boldsymbol{D}_{\mathrm{a}}^{-1/2} (\boldsymbol{R}_{\mathrm{a}} - \gamma \boldsymbol{Q}_{\mathrm{a}}) \boldsymbol{D}_{\mathrm{a}}^{-1/2} \right\}, \tag{21}$$

which is then processed according to (8).

IV. Key Properties and Analysis

In this section, we analyze the special cases of strictly SO non-circular sources and SO circular sources, study the impact of the non-circularity phases on the performance, and assess the computational complexity of the WL-DB algorithm.

1) In the case of strictly non-circular source signals considered in [9], $|\rho_i| = 1\ \forall i$ holds and the second term in (4) vanishes. Thus, the augmented relay vector in (6) reduces to $\boldsymbol{x}_{\mathrm{a}} = \boldsymbol{F}_{\mathrm{a}} \boldsymbol{P}^{1/2} \boldsymbol{s} + \boldsymbol{\mu}_{\mathrm{a}}$. In [9], the rectilinear symbol vector $\boldsymbol{s}$ is modeled as $\boldsymbol{s} = \boldsymbol{\Psi} \boldsymbol{s}_0$, where $\boldsymbol{s}_0 \in \mathbb{R}^{K\times 1}$ is a real-valued symbol vector and $\boldsymbol{\Psi} = \mathrm{diag}\{e^{\mathrm{j}\varphi_i}\}_{i=1}^K$ contains arbitrary complex phase shifts on its diagonal that are different for each signal. It is now straightforward to see that in this case $\boldsymbol{K} = \boldsymbol{\Psi}\boldsymbol{\Psi}$ and specifically $\psi_i = 2\varphi_i\ \forall i$. Furthermore, it is evident that the spectral norms of the covariance matrices and pseudo-covariance matrices contained in $\boldsymbol{R}_{\mathrm{a}}$ are equal. The same holds true for $\boldsymbol{Q}_{\mathrm{i}_{\mathrm{a}}}$, which consequently provides the maximum achievable gain for the WL-DB algorithm.

2) In the case of circular sources, $|\rho_i| = 0\ \forall i$ holds, such that $\boldsymbol{K} = \mathbf{0}$, and the model in (4) reduces to $\boldsymbol{s}^* = \boldsymbol{s}'$. Considering (6), this leads to the new definition $\boldsymbol{F}_{\rm a} = [\boldsymbol{F}^T, \mathbf{0}_{L\times K}^T]^T$, which causes the pseudo-covariance matrices contained in $\boldsymbol{R}_{\rm a}$ and $\boldsymbol{Q}_{\rm i_a}$ to be zero. Thus, the WL beamforming vector is given by $\boldsymbol{w}_{\rm a} = [\boldsymbol{w}_1^T, \mathbf{0}_{L\times 1}^T]^T$ and the achieved performance matches the one for the linear processing [4].

3) The non-circularity phases ψ_i have no effect on the output performance of the WL-DB algorithm as it only requires the knowledge of $|\rho_i|$ at the receiver. This property is also apparent from the fact that the flat-fading channel vectors in $\boldsymbol{F}$ are statistically independent from each other. Hence, the scaling of the columns of $\boldsymbol{F}^*$ by the phase components of $\boldsymbol{K}$ in $\boldsymbol{F}_{\rm a}$ merely corresponds to a new realization of the channel vectors without affecting their variances. Only the scaling by $|\rho_i| \neq 0$ translates into a performance change. This important property was not yet established in [9].

4) The computational complexity of the proposed WL-DB algorithm is dominated by the computation of the matrices in (16) and the eigendecomposition in (21). Therefore, the computational cost is $\mathcal{O}((2L)^3 + (2L)^2)$, where the factor 2 is due to the virtual doubling of the relays achieved by the WL processing. Compared to its linear counterpart, which is of cost $\mathcal{O}(L^3 + L^2)$, the WL-DB algorithm only requires a slight computational increase by a single factor. Note that in the special case of rectilinear sources, the cost further reduces as some terms in $\boldsymbol{Q}_{\rm i_a}$ vanish.

V. Simulation Results

In this section, we show simulation results that illustrate the performance of the proposed WL-DB algorithm based on the total relay power minimization criterion for different degrees of SO non-circularity of the sources. For comparison purposes, we use its linear counterpart (L-DB) [4] to demonstrate the performance gains achieved by WL processing at the relays. We have $L = 10$ relays and adopt Rayleigh flat-fading channels with unit-variance channel coefficients. We also assume that the variances of the relay and the destination noise are equal to each other and SNR = 10 dB. Furthermore, the desired user transmits with a power of 0 dBW and the signal-to-interference ratio (SIR) for the $K-1$ interferers is set to -20 dB to simulate the strong interference case. The source signals are described by their non-circularity coefficient, where the non-circularity phases are drawn from a uniform distribution in the interval $[0, 2\pi]$ since they have no impact on the performance as discussed in the previous section. All the curves are obtained by averaging over 1000 Monte Carlo trials.

In the first experiment, we evaluate the minimum required total relay transmit power as a function of the target SINR γ for various degrees of non-circularity, where we have $K = 3$ sources. The non-circularity coefficient is assumed to be the same for all sources and the curves are plotted for $|\rho_i|\ \forall i$ values of 0, 0.5, 0.8, and 1. Fig. 2 demonstrates the performance. It is evident that the curves of L-DB and WL-DB coincide for SO circular signals, where $|\rho_i| = 0\ \forall i$, and that the required total relay power decreases with increasing $|\rho_i|\ \forall i$. Eventually, the total relay power reduction is maximal for strict SO non-circularity when $|\rho_i| = 1\ \forall i$.

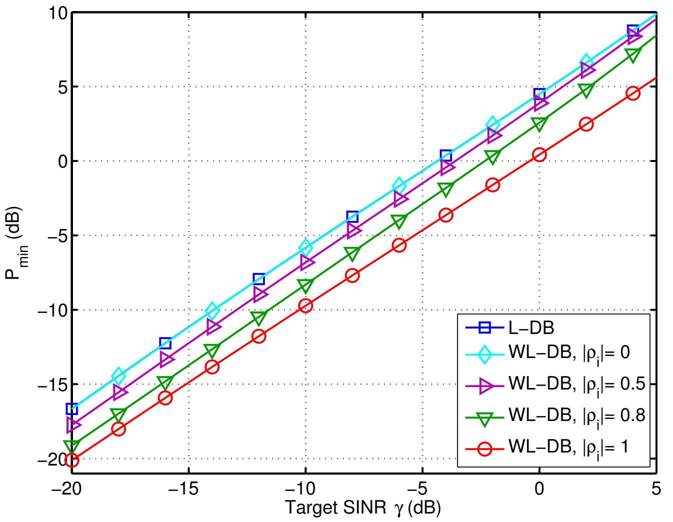

Fig. 2. Minimum total relay power versus SINR threshold γ for $K = 3$, $L = 10$, SNR = 10 dB, and SIR = -20 dB.

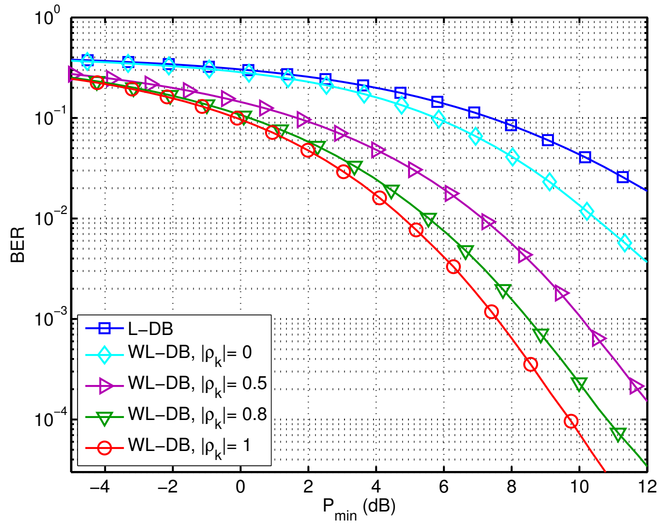

Fig. 3. BER versus minimum total relay power for $K = 3$, $L = 10$, SNR = 10 dB, and SIR = -20 dB.

In Fig. 3, we display the BER as a function of the minimum total relay power. The signal of the desired user is BPSK modulated, i.e., $|\rho_d| = 1$ and we vary the non-circularity coefficient of the interferers. The weak-sense non-circular signals are obtained by modifying a circularly symmetric complex Gaussian distribution for a given real-valued non-circularity coefficient. We keep the same parameter for K and the values of $|\rho_k|\ \forall k,\ k = 1, \dots, K,\ k \neq d$, from the previous plot. To obtain the BER curves, a symbol-by-symbol maximum likelihood (ML) decoder is used at the receiver. It can be seen that the BER decreases when $|\rho_k|\ \forall k$ grows. Again, the BER achieves its lowest curve when all the interfering sources are strictly SO non-circular, i.e., $|\rho_k| = 1\ \forall k$.

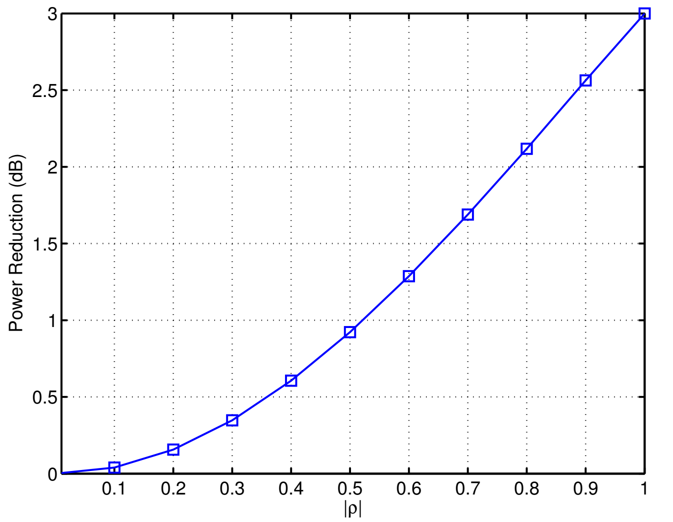

Fig. 4. Power reduction versus $|\rho|$ for $K = 1$, $L = 10$, SNR = 10 dB, SIR = -20 dB, and γ = -10 dB.

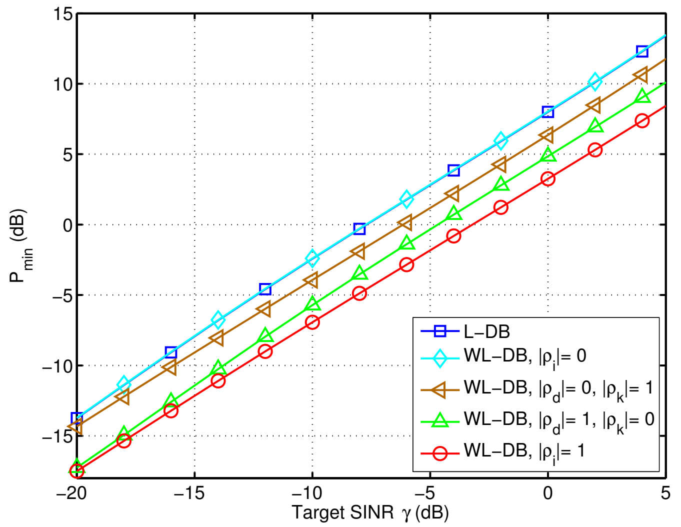

Fig. 5. Minimum total relay power versus SINR threshold γ for $K = 4$, $L = 10$, SNR = 10 dB, and SIR = -20 dB.

In the second experiment, we assess the maximum gain in terms of the relay power reduction as a function of $|\rho|$. The power reduction is computed as the difference of the minimum relay powers achieved by the WL-DB and the L-DB algorithms in dB. Here, we only analyze the non-interference case with $K = 1$. The target SINR is chosen to be $\gamma = -10$ dB. The curve depicted in Fig. 4 shows the increase of the gain when $|\rho|$ rises. In the non-interference scenario a maximum gain of 3 dB can be attained for strictly SO non-circular sources. This property was analytically proven in [9] for the SINR maximization criterion.

In the last experiment, we consider the minimum total relay power as a function of the target SINR γ but assume different non-circularity coefficients for the desired user and the $K-1$ interferers. We now assume $K = 4$ users in the network and all the other parameters are kept the same. Specifically, we analyze the extreme cases where we have a rectilinear desired user and SO circular interferers, and vice versa. We also include the case of only rectilinear and circular users for the comparison. Fig. 5 illustrates the behavior of the proposed WL-DB algorithm. It turns out that a strictly SO non-circular desired user provides the largest performance gain, whereas rectilinear interferers only contribute to a moderate performance increase.

VI. Conclusion

In this paper, we have presented a WL-DB algorithm that fully exploits the weak-sense SO non-circularity of sources in a network, which consists of a single-antenna source-destination pair, multiple relays and multiple interferers. The weight computation is based on the total relay transmit power minimization subject to a target SINR constraint after applying WL processing at the relays, assuming perfect CSI at the receiver. We have analyzed the special cases of strictly non-circular and circular sources, and simulation results demonstrate the significant performance gains in terms of the SINR and the BER, requiring only a slight increase in the computational complexity.

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