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Direction-of-Arrival Estimation for Coprime Arrays via Coarray Correlation Reconstruction: A One-Bit Perspective

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***Abstract*—In this paper, we consider the problem of under-determined direction-of-arrival (DOA) estimation using coprime arrays from a ont-bit perspective, where the coarray correlations of the quantized sparse measurements are explored for augmented covariance matrix reconstruction. To fully utilize the coarray signals calculated from the one-bit coprime array measurements for DOA estimation, a correlation reconstruction problem is formulated to obtain the quantized covariance matrix corresponding to a filled coarray containing the discontiguous one, where the one-bit quantization transforms the possibilities of correlations from an infinite to a finite number. The performance of the proposed method is validated from the aspects of degrees-of-freedom (DOFs), estimation accuracy, as well as the resolution performance. Simulation results demonstrate that the proposed method not only retains full achievable DOFs of the coprime array, but is also capable of presenting a better DOA estimation performance than the non-quantization approaches.**

***Keywords*— Coprime array, correlation reconstruction, DOA estimation, finite, one-bit quantization.**

I. Introduction

Direction-of-arrival (DOA) estimation using arrays plays a fundamental role in a variety of applications including radar, sonar and wireless communications [1]–[6]. Compared with the uniform linear array (ULA), sparse arrays have proven to achieve better angular resolution and more degrees-of-freedom (DOFs) given a fixed number of antennas, among which the coprime array is a typical one because of its special coprime structure [7]. The existing coprime array DOA estimation algorithms can be classified into three categories, namely, coarray-based approaches [8]–[11], physical array decomposition-based approaches [12]–[15], and the generalized array configuration design-based approaches [16]–[19]. However, they usually assume that there are ideal analog-to-digital converts (ADCs) available at the receiver.

More recently, one-bit quantization has found several applications in wireless communications, especially in massive MIMO systems equipped with a very large number of antennas [20]–[22]. By preserving the signs of real and imaginary parts of the complex-valued signals with low-resolution ADCs, both system cost and power consumption are greatly reduced. Compared with the abundant work on one-bit DOA estimation using ULAs [23]–[26], the potential of one-bit DOA estimation has also been explored in sparse array DOA estimation [27]. It has been revealed that for the DOA estimation, the sparse array is more robust to the one-bit quantization than the ULA. However, for the coprime array, the performance loss resulting from the discontiguous coarray structure has not been adequately addressed. On the other hand, a more comprehensive study on the coarray-based models with one-bit measurements that tailored for the coprime array is still a pending task.

In this paper, we propose a novel DOA estimation method using one-bit coprime array measurements. With one-bit quantization, there are four possibilities for the correlation of each snapshot. As such, the second-order coarray signals with infinite number of possibilities in the existing non-quantization approaches are transformed into a finite number of possibilities for coarray signal processing. While the non-linear transformation based on the arcsine law does not affect the low-rank property of the quantized covariance matrix, we can formulate a correlation reconstruction problem with minimized rank penalty to estimate the covariance matrix corresponding to the one-bit measurements of an augmented ULA, where all the elements in the discontiguous coarray are included. The retrieved covariance matrix maintains the full number of DOFs offered by the coprime array for DOA estimation. Numerical simulations are performed to evaluate and compare the DOA estimation performance, and the simulation results demonstrate the effectiveness of the proposed one-bit coprime array DOA estimation method.

II. Signal Model

The coprime array $\mathbb{S}$ is an interleaving of two sparse uniform linear subarrays $\mathbb{S}_{\mathbb{M}} = \{\tilde{m}Nd | 0 \leq \tilde{m} \leq M-1\}$ and $\mathbb{S}_{\mathbb{N}} = \{\tilde{n}Md | 0 \leq \tilde{n} \leq N-1\}$, i.e., $\mathbb{S} = \mathbb{S}_{\mathbb{M}} \cup \mathbb{S}_{\mathbb{N}}$, where M and N are a pair of coprime integers, and d equals a half-wavelength. While the first sensor of each subarray is positioned at the zeroth position as the reference, the coprime array totally contains $M+N-1$ sensors.

Supposing that the coprime array is illuminated by K far-field narrowband and uncorrelated sources from directions $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_K]^{\mathrm{T}}$, where $(\cdot)^{\mathrm{T}}$ denotes the transpose, the array received signal vector at the l-th snapshot can be modeled as

$$\boldsymbol{x}(l) = \sum_{k=1}^{K} \boldsymbol{a}_{\mathbb{S}}(\theta_k) s_k(l) + \boldsymbol{n}_{\mathbb{S}}(l), \tag{1}$$

where $\boldsymbol{a}_{\mathbb{S}}(\theta_k)$ and s_k denote the steering vector and the signal waveform of the k-th source, respectively, and $\boldsymbol{n}_{\mathbb{S}}(l) \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \boldsymbol{I})$ is the additive white Gaussian noise that is independent of the incident sources. Here, σ_n^2 is the noise power, and $\boldsymbol{I}$ is the identity matrix with appropriate dimension. Accordingly, the one-bit coprime array measurement vector can be expressed as

$$\boldsymbol{y}(l) = \mathcal{Q}(\boldsymbol{x}(l)) = \frac{1}{\sqrt{2}} \Big(\mathrm{sgn}\big[\Re(\boldsymbol{x}(l))\big] + j \cdot \mathrm{sgn}\big[\Im(\boldsymbol{x}(l))\big] \Big), \tag{2}$$

where the quantization operator $\mathcal{Q}(\cdot)$ preserves the sign of the real part $\Re(\cdot)$ and the imaginary part $\Im(\cdot)$ separately, and the sign function $\mathrm{sgn}[\cdot]$ returns 1 and -1 for the plus sign and the minus sign, respectively. Here, $1/\sqrt{2}$ is the normalization parameter, and $j = \sqrt{-1}$ is the imaginary part. As such, the array received signal $\boldsymbol{x}(l)$ with an infinite number of possible values is quantized into four possibilities for each component, i.e., $\boldsymbol{y}(l) \in \{(1+j, 1-j, -1+j, -1-j)/\sqrt{2}\}$.

The quantized covariance matrix of the one-bit measurements $\boldsymbol{y}(l)$ follows the arcsine law [28] as

$$\boldsymbol{Q}_{\mathbb{S}} = \mathrm{E}\left[\boldsymbol{y}(l)\boldsymbol{y}^{\mathrm{H}}(l)\right] = \frac{2}{\pi} \Bigg(\arcsin\Big(\boldsymbol{\Sigma}^{-\frac{1}{2}} \Re(\boldsymbol{R}_{\mathbb{S}}) \boldsymbol{\Sigma}^{-\frac{1}{2}}\Big) + j \cdot \arcsin\Big(\boldsymbol{\Sigma}^{-\frac{1}{2}} \Im(\boldsymbol{R}_{\mathbb{S}}) \boldsymbol{\Sigma}^{-\frac{1}{2}}\Big) \Bigg), \tag{3}$$

where $\boldsymbol{R}_{\mathbb{S}} = \mathrm{E}\left[\boldsymbol{x}(l)\boldsymbol{x}^{\mathrm{H}}(l)\right]$ is the covariance matrix of $\boldsymbol{x}(l)$, $\boldsymbol{\Sigma} = \mathrm{diag}(\boldsymbol{R}_{\mathbb{S}})$ is the diagonal matrix containing the diagonal entries of $\boldsymbol{R}_{\mathbb{S}}$. Here, $\mathrm{E}[\cdot]$ is the expectation operator, and $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose. In practice, $\boldsymbol{Q}_{\mathbb{S}}$ is usually estimated by the sample covariance matrix

$$\hat{\boldsymbol{Q}}_{\mathbb{S}} = \frac{1}{L} \sum_{l=1}^{L} \boldsymbol{y}(l)\boldsymbol{y}^{\mathrm{H}}(l), \tag{4}$$

where L denotes the number of one-bit snapshots.

III. The Proposed Method

The increased DOFs of coprime array DOA estimation can be realized by coarray signal processing. With the correlations vectorized from $\boldsymbol{R}_{\mathbb{S}}$, the second-order coarray signals corresponding to an augmented difference coarray

$$\mathbb{D} = \{v_\imath - v_\jmath | v_\imath, v_\jmath \in \mathbb{S}\} \tag{5}$$

can be obtained. In the one-bit case, the quantization from $\boldsymbol{R}_{\mathbb{S}}$ to $\boldsymbol{Q}_{\mathbb{S}}$ in (3) does not change the coarray structure $\mathbb{D}$, and the quantized coarray signals can be represented as

$$\boldsymbol{q}_{\mathbb{D}} = \mathrm{vec}(\boldsymbol{Q}_{\mathbb{S}}) = \frac{2}{\pi} \Bigg(\arcsin\Big((\boldsymbol{\Sigma}^{-\frac{1}{2}} \otimes \boldsymbol{\Sigma}^{-\frac{1}{2}}) \Re(\boldsymbol{r}_{\mathbb{D}}) \Big) + j \cdot \arcsin\Big((\boldsymbol{\Sigma}^{-\frac{1}{2}} \otimes \boldsymbol{\Sigma}^{-\frac{1}{2}}) \Im(\boldsymbol{r}_{\mathbb{D}}) \Big) \Bigg), \tag{6}$$

where $\mathrm{vec}(\cdot)$ denotes the vectorization operator, $\otimes$ is the Kronecker product, and

$$\boldsymbol{r}_{\mathbb{D}} = \sum_{k=1}^{K} \sigma_k^2 \boldsymbol{a}_{\mathbb{D}}(\theta_k) + \sigma_n^2 \mathrm{vec}(\boldsymbol{I}) \tag{7}$$

is the coarray signals without quantization. Here, $\boldsymbol{a}_{\mathbb{D}}(\theta_k) = \boldsymbol{a}_{\mathbb{S}}^*(\theta_k) \otimes \boldsymbol{a}_{\mathbb{S}}(\theta_k)$, σ_k^2 is the power of the k-th source, and $(\cdot)^*$ denotes the conjugate operator.

Unlike most existing DOA estimation methods which only extract the contiguous segment of the discontiguous coarray $\mathbb{D}$ [8], [27], [29], we incorporate the idea of interpolation to generate a filled virtual ULA

$$\mathbb{U} = \{ud \,|\, u = 0, 1, 2, \cdots, \max(\mathbb{D})/d\}, \tag{8}$$

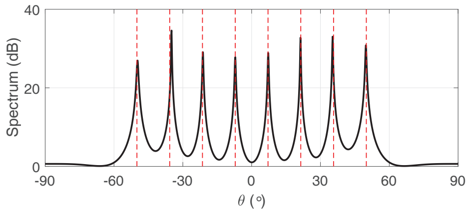
where all the non-negative elements in $\mathbb{D}$ are included. Due to the fact that the correlations corresponding to the negative part and the positive part of $\mathbb{D}$ are mutually conjugate, only the non-negative positions in $\mathbb{D}$ are included in $\mathbb{U}$. Accordingly, the quantized coarray signals $\hat{\boldsymbol{q}}_{\mathbb{U}} \in \mathbb{C}^{|\mathbb{U}|}$ corresponding to the discontiguous positions are set to zeros since there is no information available, whereas the remaining positions $ud \in \mathbb{D}$ keep the same lags as in $\hat{\boldsymbol{q}}_{\mathbb{D}} = \mathrm{vec}(\hat{\boldsymbol{Q}}_{\mathbb{S}})$. Here, $|\cdot|$ denotes the cardinality of a set. Considering that a certain lag in the sample covariance matrix $\hat{\boldsymbol{Q}}_{\mathbb{S}}$ usually does not have the same correlations, thus its corresponding coarray signal is an average over all these observed correlations. By incorporating the Hermitian Toeplitz structure, the quantized covariance matrix of $\mathbb{U}$ can be initialized as

$$\hat{\boldsymbol{Q}}_{\mathbb{U}} = \mathrm{Toep}\left(\hat{\boldsymbol{q}}_{\mathbb{U}}\right), \tag{9}$$

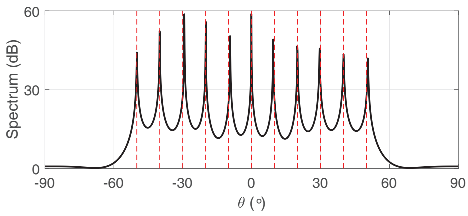
where $\mathrm{Toep}(\cdot)$ returns a Hermitian Toeplitz matrix with the contained vector as its first column. To perform DOA estimation with the coarray $\mathbb{U}$, it is necessary to recover the unknown correlations in $\hat{\boldsymbol{Q}}_{\mathbb{U}}$ corresponding to the discontiguous positions. Before proceeding, we have the following lemma for the following optimization problem design.

Lemma 1: The rank of the quantized covariance matrix $\boldsymbol{Q}_{\mathbb{S}}$ is the same as that of the non-quantized coprime array covariance matrix $\boldsymbol{R}_{\mathbb{S}}$. ■

Since the arcsine law-based transformation in (3) does not affect the maximum number of linearly independent columns of $\boldsymbol{R}_{\mathbb{S}}$, the quantized covariance matrix retains the matrix rank as the non-quantized one. Therefore, the low-rank property still holds for the quantized covariance matrix, and the unknown correlations in $\hat{\boldsymbol{Q}}_{\mathbb{U}}$ can be recovered from the following



(a)



(b)

Fig. 1. DOF performance of the proposed one-bit coprime array DOA estimation method. Vertical dashed lines represent true source directions. (a) $K = 8$; (b) $K = 11$.

correlation reconstruction problem

$$\begin{aligned} \boldsymbol{z}^{\star} = \arg \min_{\boldsymbol{z}\in\mathbb{C}^{|\mathbb{U}|}} & \|\mathcal{P}_{\Omega}(\mathrm{Toep}(\boldsymbol{z}))-\hat{\boldsymbol{Q}}_{\mathbb{U}}\|_F + \lambda \cdot \mathrm{rank}\left(\mathrm{Toep}(\boldsymbol{z})\right) \\ \text{subject to} \quad & \mathrm{Toep}(\boldsymbol{z}) \succeq \mathbf{0}, \end{aligned} \tag{10}$$

where $\mathcal{P}_{\Omega}(\cdot)$ is a projection operator onto Ω containing the indices of observed correlations, λ is the regularization parameter, $\|\cdot\|_F$ is the Frobenius norm, $\mathrm{rank}(\cdot)$ denotes the rank of a matrix, and the constraint $\mathrm{Toep}(\boldsymbol{z}) \succeq \mathbf{0}$ enforces a positive semi-definite matrix.

In contrast to the coarray interpolation-based algorithms without quantization [30]–[33], here the observed correlations in $\hat{\boldsymbol{Q}}_{\mathbb{U}}$ are calculated from one-bit measurements $\boldsymbol{y}(l)$ according to (4). Hence, the correlation of each individual snapshot only has four possibilities, i.e., 1, -1, j, and $-j$. In other words, the one-bit quantization transforms an *infinite* number of possibilities for the correlations in $\boldsymbol{r}_{\mathbb{D}}$ into a *finite* number of possibilities in $\boldsymbol{q}_{\mathbb{D}}$, effectively mitigating the sensitivity of the matrix approximation derivation in the correlation reconstruction procedure (10).

The non-convex optimization problem (10) can be efficiently solved by introducing a convex relaxation for the non-convex rank operator $\mathrm{rank}(\mathrm{Toep}(\boldsymbol{z}))$, e.g., the convex trace operator $\mathrm{Tr}(\mathrm{Toep}(\boldsymbol{z}))$. With the reconstructed covariance matrix $\mathrm{Toep}(\boldsymbol{z}^{\star})$, the normalized covariance matrix without quantization can be retrieved as

$$\hat{\boldsymbol{R}}_{\mathbb{U}} = \sin\left(\frac{\pi}{2}\Re\left(\mathrm{Toep}(\boldsymbol{z}^{\star})\right)\right) + \sin\left(\frac{\pi}{2}\Im\left(\mathrm{Toep}(\boldsymbol{z}^{\star})\right)\right), \tag{11}$$

where the inverse of the arcsine law is used. Then, using $\hat{\boldsymbol{R}}_{\mathbb{U}} \in \mathbb{C}^{|\mathbb{U}|\times|\mathbb{U}|}$ with off-the-shelf ULA-based DOA estimation methods enables the maximum achievable number of DOFs $|\mathbb{U}|$.

We specify the spectral MUSIC cost function $f(\theta)$ as an illustrative example, and the DOAs can be estimated by maximizing

$$f(\theta) = \frac{\|\boldsymbol{a}_{\mathbb{U}}(\theta)\|^2}{\boldsymbol{a}_{\mathbb{U}}^{\mathrm{H}}(\theta)\boldsymbol{E}_{\boldsymbol{n}}\boldsymbol{E}_{\boldsymbol{n}}^{\mathrm{H}}\boldsymbol{a}_{\mathbb{U}}(\theta)}, \tag{12}$$

where $\boldsymbol{a}_{\mathbb{U}}(\theta)$ is the steering vector of $\mathbb{U}$, $\boldsymbol{E}_{\boldsymbol{n}}$ is the noise eigenspace of $\hat{\boldsymbol{R}}_{\mathbb{U}}$, and $\|\cdot\|$ denotes the Euclidean norm.

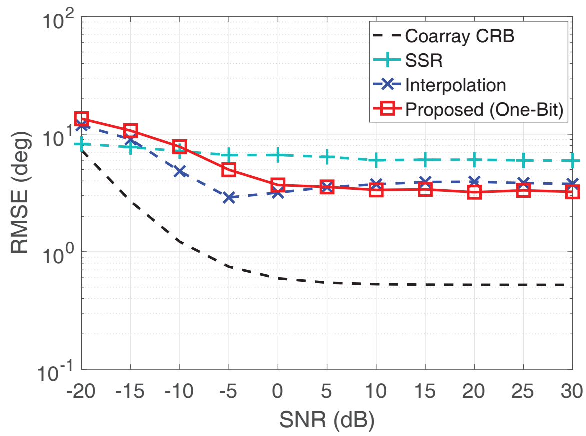


Fig. 2. RMSE performance versus SNR when $K > |\mathbb{S}|$.

IV. Simulation Results

In the simulations, $|\mathbb{S}| = 7$ sensors are utilized to form a coprime array with $M = 3$ and $N = 5$. K incident sources are assumed to be uniformly distributed in $[-50^\circ, 50^\circ]$, the number of snapshots is $L = 500$, and the regularization parameter is set to $\lambda = 0.25$. The MUSIC spectrum of the proposed method by operating $\hat{\boldsymbol{R}}_{\mathbb{U}}$ is shown in Fig. 1 to illustrate the DOFs performance, where the signal-to-noise ratio (SNR) is 0 dB. Clearly, the proposed method is capable of identifying all the sources in both $K = |\mathbb{S}|+1 = 8$ and $K = |\mathbb{D}| = 11$ scenarios, whereas the coarray-based methods only processing the contiguous segment of $\mathbb{D}$ cannot owing to the insufficient DOFs [8], [27], [29]. It demonstrates that the proposed correlation reconstruction-based optimization problem (10) makes full use of the DOFs offered by the discontiguous coarray $\mathbb{D}$.

We then compare the estimation accuracy of the proposed one-bit DOA estimation method with two non-quantization methods, namely, the sparse signal reconstruction (SSR) method [34] and the virtual array interpolation method [30]. All these methods fully utilize the discontiguous coarray. Here, the number of sources is $K = 10 > |\mathbb{S}|$, i.e., the underdetermined case. Both predefined grids for the SSR method and the MUSIC spectrum search are spaced in steps of 0.1°. Meanwhile, the Cramér-Rao Bound (CRB) for the coarray signal processing-based coprime array DOA estimation without quantization [30] is plotted to indicate the respective lower bound. For each scenario, $1,000$ Monte-Carlo trials are executed.

The root-mean-square error (RMSE) versus SNR is depicted in Fig. 2. It is clear that the performance of all the methods

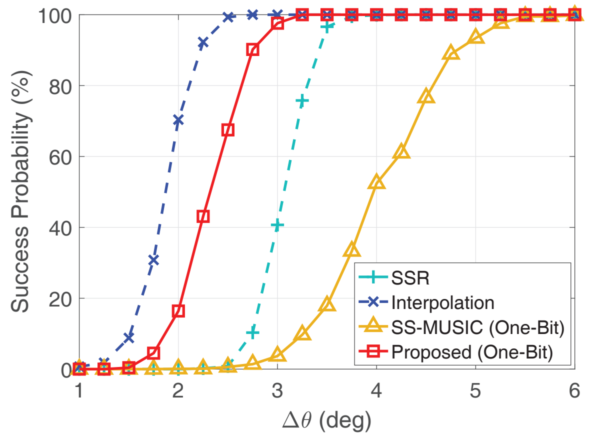


Fig. 3. Resolution performance comparison.

follows the same trend as the coarray CRB, which is a typical saturation behaviour [35] for the underdetermined case. The SSR method performs the worst in high SNR regions owing to the detrimental effect caused by the predefined grids and the spurious peaks. There is an interesting observation that the proposed one-bit DOA estimation method outperforms the virtual array interpolation method without quantization when the SNR is larger than 5 dB. It illustrates the effectiveness of the quantized correlation reconstruction with a finite number of possibilities in (10).

The resolution performance is compared in Fig. 3 with SNR = 0 dB, where the one-bit SS-MUSIC method [27] is also tested. Two closely-spaced sources are considered, where θ_1 is randomly chosen from $\mathcal{N}(0°, 1°)$ in each trial, and θ_2 = θ_1 + $\Delta\theta$ with $\Delta\theta$ representing the angular spacing. The success probability is defined as the percentage of successful trials among the whole Monte-Carlo trials, where the trial is regarded as successful if both estimated DOAs $\hat{\theta}_k$, $k = 1, 2$, satisfy $-\Delta\theta/2 < \hat{\theta}_k - \theta_k < \Delta\theta/2$.

It is shown in Fig. 3 that the resolution of the one-bit SS-MUSIC method is lower than the other tested methods, since only the quantized correlations corresponding to the contiguous segment of $\mathbb{D}$ is utilized, which has a smaller array aperture. The SSR method has a $0.75°$ resolution gap to the proposed method although the non-quantized signals are utilized in this scenario. Although the virtual array interpolation method outperforms the proposed one-bit method with $0.5°$, its ADCs present significant challenges with respect to system cost and power consumption. Therefore, the proposed one-bit DOA estimation method enjoys a better overall performance than the tested ones.

V. Conclusions

In this paper, we proposed a one-bit coprime array DOA estimation method, where the discontiguous coarray can be fully utilized to retrieve DOAs with low resolution ADCs. Based on the observed one-bit statistics with a finite number of possibilities, the correlations corresponding to the ULA with the same aperture as the discontiguous coarray are reconstructed through a convex optimization problem. Simulation results show that the proposed method outperforms that methods use non-quantized measurements in certain scenarios.

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