

Comparison of Zero-Forcing Methods for Downlink Spatial Multiplexing in realistic Multi-User MIMO Channels

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Abstract— The use of Space-Division Multiple Access (SDMA) in the downlink of a multi-user MIMO wireless communications system can offer a substantial gain in system throughput. In this contribution we compare the throughput achieved by employing recently developed algorithms for transmit beamforming in multi-user MIMO systems. The Block-Diagonalization algorithm is a generalization of the channel inversion scheme which suppresses completely all interference among the users although allowing interference among different data streams. The Successive Optimization algorithm, on the other hand, solves the power minimization problem one user at a time so that the n -th user will produce no interference with respect to the m -th user, where $1 \leq m < n$. This paper tests these algorithms on several realistic multi-user MIMO channels generated with the IlmProp, a flexible Geometry Based propagation model for wireless communications developed at Ilmenau University of Technology. This analysis reveals the potentials and limits of these algorithms. In order to overcome the drawbacks of these beamforming schemes, a subspace-based grouping algorithm is proposed. Its performance is evaluated in a realistic multi-user synthetic scenario, especially taking into account how the algorithms adapt to the different conditions of the channels as time progresses.

I. INTRODUCTION

The use of Space-Division Multiple Access (SDMA) in the downlink of a multi-user MIMO wireless communications system can offer a substantial gain in system throughput. In this paper we compare the throughput achieved by employing two recently developed algorithms for transmit beamforming in multi-user MIMO systems. These algorithms are briefly reviewed in section II and can be found in [1], [2]. Both algorithms aim to maximize the overall capacity taking into account the interference among users. In order to study the performance of these algorithms in correlated channels as time evolves, we construct several synthetic channels generated with the IlmProp, a flexible propagation modelling tool developed at Ilmenau University of Technology. The IlmProp is described in section III. Section IV shows simulation results of the two algorithms applied to realistic channels, elucidating their limits. In particular it reveals how sensible the algorithms are with respect to spatially correlated users. Finally, section V proposes a subspace-based algorithm to solve the grouping

issue, which successfully overcomes the drawbacks of the downlink beamforming algorithms.

II. THE BLOCK DIAGONALIZATION AND SUCCESSIVE OPTIMIZATION ALGORITHMS

In the downlink of a multi-user MIMO system the received vector \mathbf{x}_i at the i -th mobile at a certain instant in time can be calculated as

$$\mathbf{x}_i = \mathbf{H}_i \mathbf{M}_i \mathbf{d}_i + \mathbf{n}_i, \quad (1)$$

where \mathbf{H}_i is the flat-fading channel matrix for the i -th user, $\mathbf{M}_i \in \mathbb{C}^{M_T \times S_i}$ is its modulation matrix, which performs the beamforming, and \mathbf{d}_i is a data vector of arbitrary dimension S_i , which then represents the number of parallel data streams sent to the i -th user [3]. The noise vector \mathbf{n}_i contains i.i.d. complex Gaussian random numbers with variance σ_n^2 . The channel matrix \mathbf{H}_i has dimension $M_{R,i} \times M_T$, where M_T and $M_{R,i}$ are the antennas at the base station and at the i -th mobile, respectively.

The goal of the Block-Diagonalization (BD) and of the Successive Optimization (SO) algorithms is to find the modulation matrices for all users served by the base station.

The *Block-Diagonalization* is a generalization of the channel inversion which suppresses completely all interference among the users, although allowing interference among the different data streams directed to the same user. This means that potentially the BS station can send up to $M_{R,i}$ parallel data streams to the i -th user, which will use its $M_{R,i}$ antennas to separate them. On the other hand the i -th user will not receive any interference from the data streams sent to other mobiles. The diagonalization of the channel occurs then only in blocks, each block belonging to one user only. In case of $S_i \geq 2$ the remaining equalization steps are left to the i -th user.

The columns of the modulation matrix \mathbf{M}_i are the beamforming vectors towards the i -th user. The matrix $\tilde{\mathbf{H}}_i$, containing all channels but the i -th, is defined as

$$\tilde{\mathbf{H}}_i = [\mathbf{H}_1^T \quad \dots \quad \mathbf{H}_{i-1}^T \quad \mathbf{H}_{i+1}^T \quad \dots \quad \mathbf{H}_K^T]^T, \quad (2)$$

where K is the total number of users. In order to fulfill the zero-interference constraint the columns of \mathbf{M}_i must lie in

the null space of $\tilde{\mathbf{H}}_i$. The BD algorithm constructs the i -th modulation matrix taking the strongest eigenvectors from the i -th channel subspace projected into the null space of $\tilde{\mathbf{H}}_i$. The total power allocated for the i -th user is assigned to the different eigenbranches via the waterfilling strategy.

The *Successive Optimization* algorithm, on the other hand, follows a slightly different philosophy. The modulation matrix for the i -th user is chosen so that no interference is given to the user $1, \dots, i-1$, while optimizing the transmit power to compensate for the interference received from these very users.

The detailed description of these algorithms can be found in [1], [2].

The user rate for the i -th user can be calculated as

$$R_i = \log_2 \left| \mathbf{I} + \frac{P}{\sigma^2 M_T} \mathbf{H}_i \mathbf{M}_i (\mathbf{R}_{nn})^{-1} \mathbf{M}_i^H \mathbf{H}_i^H \right| \quad (3)$$

where H denotes the Hermitian transpose. The correlation matrix of the noise \mathbf{R}_{nn} is simply $\sigma_n^2 \mathbf{I}$ for the BD because the other users do not interfere. For the SO algorithm, however, the noise correlation matrix must consider the interference generated by the user $1, \dots, i-1$ and thus becomes

$$\mathbf{R}_{nn} = \sigma_n^2 \mathbf{I} + \sum_{j=1}^{i-1} \mathbf{M}_j^H \mathbf{H}_i^H \mathbf{H}_i \mathbf{M}_j. \quad (4)$$

The Signal to Noise Ratio is defined as $\text{SNR} = \frac{P}{\sigma^2}$ where P , the total power available at the base station, is set to M_T .

III. THE ILMPROP

The IImProp is a Geometry Based channel model developed at Ilmenau University of Technology. It is capable of simulating a MIMO multi-user multi-hop scenario. It includes a geometrical representation of the environment surrounding the experiment and a precise representation of the transmitting and receiving antennas, which can be set in an arbitrary array geometry.

Furthermore it includes a complete collection of tools aimed to analyze the synthetic channels in order to extract their most characterizing features, such as time variance and frequency selectiveness. More details can be found in [4].

Clusters of scatterers can be arbitrarily positioned in the 3D reconstruction of the environment. Each scatterer corresponds to a single ray and is characterized by a complex coefficient which, for simplicity, is independent of the Direction of Arrival and the Direction of Departure (DoA and DoD), but which may vary in time with an arbitrary law. The coefficient determines the phase shift and power attenuation introduced by the scatterer. The receiver thus receives a number of rays equal to the number of scatterers plus the Line Of Sight (LOS).

Its intrinsic geometrical approach facilitates the generation of a realistic channel with correlation in time, frequency and space. In other words, the statistics of the Angles of Arrival (AoA), which are dependent on the Angles of Departures (AoD), will change in time accordingly to the trajectory followed by the mobile.

The fact that more than one user is defined in the same space (common space) generates channels which will be more or less

correlated, depending on the relative position of the users.

IV. SIMULATION RESULTS

A. Uncorrelated channels

The first scenario that we illustrate is a weakly correlated channel, which represents a Non-Line of Sight (NLOS) link in a very rich scattering environment. Such a scenario is shown in Figure 1.

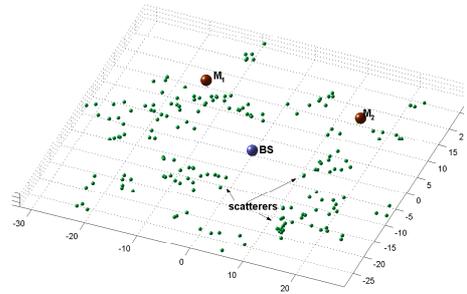


Fig. 1. A weakly correlated scenario generated with the IImProp. The LOS component is suppressed.

This model emulates the totally uncorrelated channel, often denoted in the literature as the \mathcal{H}_w channel [5]. This channel is usually taken as benchmark to test all sorts of beamforming algorithms, partly due to its simplicity. In fact, the \mathcal{H}_w channel, is a matrix whose elements are independent ZMCSCG (Zero Mean Circular Symmetric Complex Gaussians) random numbers with variance 1.

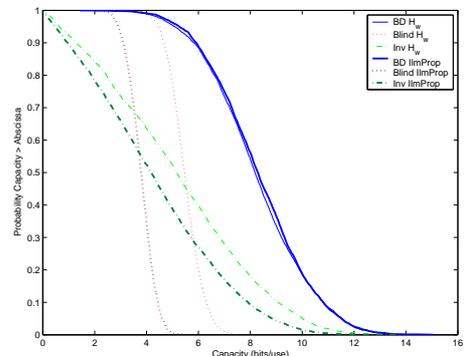


Fig. 2. Complementary Cumulative Distribution Functions of Sum for Gaussian channels for two mobiles with $M_R = 2$ antennas each at 10 dB SNR. The BS has $M_R = 4$ antennas.

In Figure 2 the BD algorithm is compared with the blind beamformer and the channel inversion solution on both the \mathcal{H}_w channel and the IImProp channel.

The latter is normalized in order to compare the capacity curves with the ones derived from the \mathcal{H}_w channel. The normalization is performed so that the total power received in the synthetic channel matches the one present in \mathcal{H}_w . If $\mathcal{H}_{IImProp}$ denotes the synthetic channel for all users and for all time snapshots T , of size $\sum_{i=1}^K M_{R,i} \times M_T \times T$, then the

whole channel matrix must be scaled so that

$$\sum_{\Theta} |\mathcal{H}_w|^2 = \sum_{\Theta} |\mathcal{H}_{IImProp}|^2 \quad (5)$$

where Θ spans the complete domain of all dimensions. Exploiting the statistical properties of the elements of \mathcal{H}_w , it is possible to express the sum in equation (5) as

$$E \left\{ \sum_{\Theta} |\mathcal{H}_w|^2 \right\} = \gamma, \quad (6)$$

where γ is the total number of samples present in \mathcal{H}_w , i.e., $\gamma = \sum_{i=1}^K M_{R,i} \cdot M_T \cdot T$.

For the blind beamforming the modulation matrix is chosen equal to a scaled identity matrix, $M_i = \eta \mathbf{I}$, so that the total power sent is P and with the assumption that only one user is served at a time.

For the channel inversion solution the modulation matrix M_S at each time snapshot is derived from the Moore-Penrose pseudo-inverse of H_S so that $M_S = H_S^\dagger$ where

$$\begin{aligned} M_S &= [M_1 \ M_2 \ \dots \ M_K] \text{ and} \\ H_S &= [H_1^T \ H_2^T \ \dots \ H_K^T]^T. \end{aligned}$$

Due to the intrinsic geometric nature of the model, the channel appears to be vaguely more correlated than \mathcal{H}_w , leading to worse performances for both the pseudo-inverse and the blind beamforming. On the other hand, the performance of the BD beamformer does not change. For this reason, the gain of the BD with respect to the blind beamforming is significantly larger in the weakly correlated scenario.

B. Correlated channels

Figure 3 shows a model which corresponds to a correlated channel. Two users M_1 and M_2 move in a space rich of scatterers in the proximity of a fixed base station (BS). Each mobile has $M_R = 2$ antennas while the base station has $M_T = 4$ antennas. All antenna arrays are Uniform Linear Arrays (ULA's). The trajectories of the two users are also visible. The first user moves on an elliptical trajectory and eventually crosses the path travelled by M_2 . At one instant in time the two users occupy the same space. This is of course physically not possible, but the model lets us experience what effects on the algorithms will have two totally correlated channels.

The geometrical representation intrinsically gives a realistic time variance to the channel letting us investigate the performance of the algorithms as a function of time.

The upper plot of Figure 4 shows the user rates, while the bottom one shows the Rician K -factors. The user rate R_i of the i -th user is calculated as in equation (3) for both algorithms. The reflection coefficients of the scatterers have been kept constant for the whole duration of the simulation. The trends for the Rician K -factors are thus a result of the changing path-losses deriving simply from the movement of the mobiles. The Block Diagonalization algorithm, as expected, allows better transfer rates to the mobile which has a better channel, in

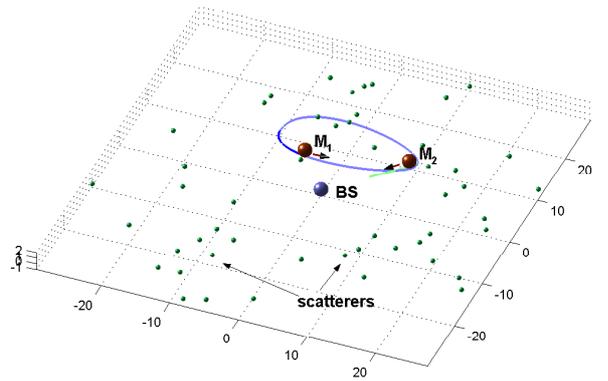


Fig. 3. A correlated scenario generated with the IImProp.

order to maximize the sum capacity of the system. Between 15 and 25 seconds we see a significant drop in the user rates. This is due to the fact that the two channels are perfectly correlated since the mobiles are at the same position. The BD algorithm cannot give all the power to one user because it cannot avoid interference with the other. Thus it chooses to give no power to all. The Successive Optimization algorithm performs better than the BD when calculated for the first user first. In this case the modulation matrix of the first user is calculated independently from the second user. For this reason the user rate does not drop to zero as for the BD. However the second modulation matrix is chosen not to interfere with the first user and thus achieves an extremely low rate. In fact the curves for the second user for both BD and SO overlap.

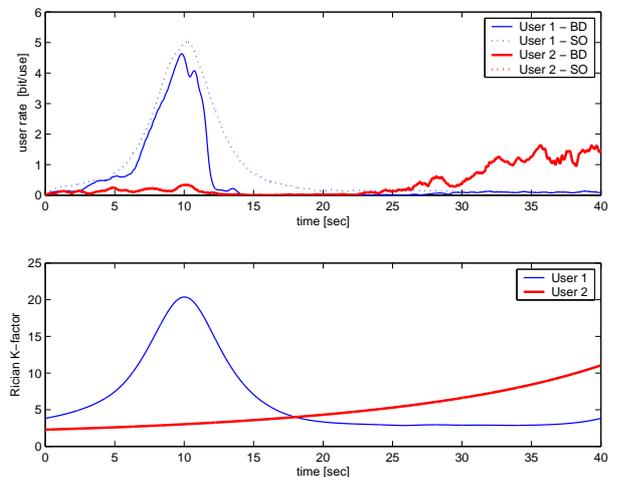


Fig. 4. User rates (top) and the Rician K -factor (below) plotted against time for the correlated scenario previously shown.

V. THE SUBSPACE-BASED GROUPING ALGORITHM

The BD and SO algorithms suffer from two major drawbacks: the dimensionality constraint and, as seen in section IV, the difficulty in handling spatially correlated users. The dimensionality constraint forces the base station to have

at least the same number of antennas as the sum of all antennas of the mobiles it intends to serve. Both problems can be efficiently solved by grouping the users in different sets, whereas each group will be treated in separate time slots. The resulting system will be a hybrid of TDMA and SDMA. This solution requires an efficient algorithm which determines how the groups are formed. One criterion would be to consider the Angles of Arrival (AoA) of the different users. Unfortunately this approach requires a greater effort since parameter estimation has to be performed. Furthermore only certain kinds of antenna arrays support such a processing, the beam patterns of the antennas must be known and a strong LOS must be present. Thus an antenna independent criterion is preferable.

The grouping algorithm that we hereby propose works in the subspaces and therefore does not require any information on the antennas whatsoever.

The channel matrix H_i describes the channel between the i -th user and the BS. For a given time snapshot, it is a matrix of dimensions $M_{R,i} \times M_T$. Let the columns of the matrix $\mathbf{V} \in \mathbb{C}^{M_T \times K}$ contain the strongest right singular vector of each users. Each singular vector represent is the best rank-1 approximation of the corresponding user's subspace. The more correlated the subspaces of users i and j are, the smaller will be the angle $\theta_{i,j}$ between their singular vectors.

The angle $\theta_{i,j}$ can be calculated as

$$\theta_{i,j} = \arccos(|\mathbf{V}(:,i)^H \cdot \mathbf{V}(:,j)|) = \arccos(|\gamma_{i,j}|), \quad (7)$$

where $\Gamma(:,i)$ is the i -th column of the matrix Γ . Note that the singular vectors have norm 1. Since the function $\arccos(\cdot)$ is monotonic in the range of interest, we take $\gamma_{i,j}$ as a measure of the spatial correlation between the users. If the i -th and j -th users are spatially uncorrelated, then $\gamma_{i,j} = 0$ whereas if they are totally correlated, i.e., they have the same subspace, then $\gamma_{i,j} = 1$. The absolute value of the Gramian of \mathbf{V} , namely $\Gamma = \|\mathbf{V}^H \cdot \mathbf{V}\|$ is a matrix whose upper and lower triangular part contains the indices $\gamma_{i,j}$.

In the case of two groups, the following steps define the subspace-based grouping algorithm:

- *Step 1*: the most correlated pair of users are identified, determining the maximum in the upper triangular section of Γ . The two users are put in separate groups.
- *Step 2*: the users already assigned to a group are not considered in following steps.
- *Step 3*: among the remaining users the most correlated pair is identified. The users are assigned to different groups so that their correlation with the other users in the groups is minimized.
- *Step 4*: iteration, back to *Step 2* until all users have been allocated.

In case of more than two groups, for instance 3, the algorithm can be extended as follows.

- *Step 1*: the strongest pair is identified and the users are put in different groups.
- *Step 2*: among all remaining users the one having highest

correlation with the first pair is determined and put in the remaining group.

- *Step 3*: the users already allocated are excluded from following steps.
- *Step 4*: among the remaining users the most correlated pair is identified. The users are assigned to different groups so that their correlation with the other users in the groups is minimized.
- *Step 5*. among all remaining users the one having highest correlation with the pair found in *Step 4* is determined and put in the remaining group.
- *Step 6*. iteration, back to *Step 3* until all users have been allocated.

The latter algorithm can be easily extended to cover more than 3 groups iterating *Step 2* and *5* as needed to cover all groups present.

Testing all possible combinations (in case of two groups) proved that our algorithm achieves the highest data rates possible.

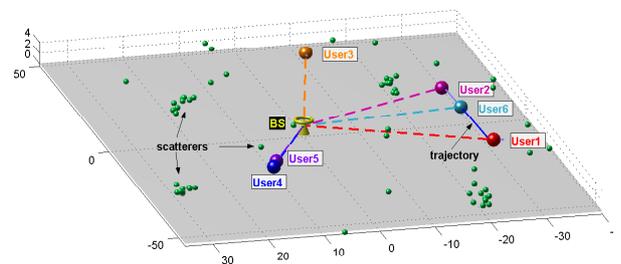


Fig. 5. A multi-user scenario generated with the *IlmProp*. The Line Of Sight (LOS) components are visible.

Figure 5 shows an exemplary scenario generated with the *IlmProp*. A Base Station has a UCA (Unifrom Circular Array) with 12 antennas. A total number of 6 users surround the BS, each one having 2 antennas, $\frac{\lambda}{2}$ spaced. The orientation of the mobiles' arrays is approximately orthogonal to their trajectory. All users but the sixth move only a few meters during the duration of the simulation. User 6, on the other hand, moves from a position very close to the first user to one which is close to the second. Potentially the BS could support all 6 users at the same time. However, at some time snapshots some mobiles are very close one another, thus being strongly spatially correlated. These users are separated with difficulty by the beamforming algorithms investigated here. This issue can be avoided by grouping the users in two or more groups, allowing to put strongly correlated users in separate groups.

The narrowband channels for all users seen in Figure 5 have been calculated with the *IlmProp* at a frequency of 2 GHz with an average SNR of 20 dB for a total time of 2.8 seconds and 400 time snapshots. Assuming perfect channel knowledge at the transmitter, at each time snapshot the grouping algorithms have been applied, and consequentially the BD algorithm on each group, dividing for simplicity the available power equally among the users of the same group. This simplification is

acceptable for this specific scenario since the mobiles have comparable path-losses, i.e., there is no near-far problem. The different groups are then multiplexed in time. The user rates are compared for different grouping strategies in Figure 6, which specifically shows the rates for 1, 2, 3 and 6 groups. Note that the curve for 6 groups corresponds to handling each user in a separate time slot. For this specific scenario, the

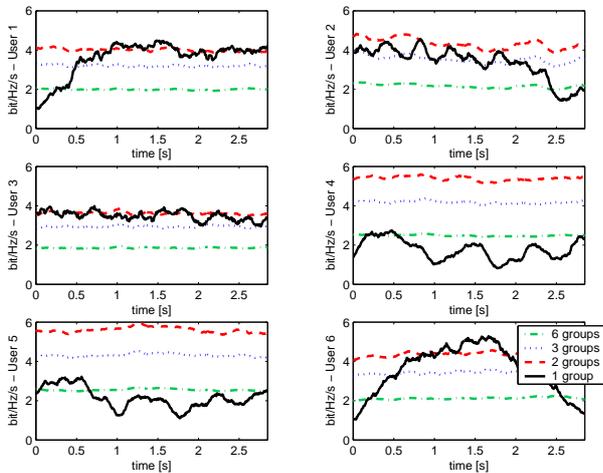


Fig. 6. User rates for the scenario seen in Figure 5 for different grouping strategies.

best grouping strategy is two groups of three users each. In fact, with this configuration the BD is capable of separating the users while assuring no interference among them. With 3 groups of two users each, the BD has an even easier job in separating the users. However the resulting user rates are lower due to the higher time multiplexing which decreases the length of each transmission time slot. When setting the spatial multiplexing to its maximum, i.e., with 1 group, in some cases it is impossible for the BD algorithm to assure high user rates. In fact, the sixth user is strongly correlated with the first at the beginning of the simulation, while it almost overlaps with the second in the final part. The BD algorithm inherently avoids interference among all users. Under this constraint it generates very inefficient modulation matrices for strongly correlated channels leading to low user rates, as already seen in section IV. User 4 and 5 are strongly spatially correlated for the whole duration of the simulations. For this reason they always achieve low user rates. With two groups it is possible to separate these unfavourable pairs putting the users in separate groups. The gain achieved from the exploitation of the spatial diversity is clearly visible comparing the rates of the users when treated separately (6 groups) and when grouped in 2 or 3 groups.

The grouping algorithm, when applied at each time snapshot, could provide a very unstable solution, i.e., users jump from one group to another very frequently. In a real system this is of course undesirable because it would generate a great deal of overhead traffic. A simple solution consists in calculating the users' subspaces on the long term correlation matrices. The long term spatial correlation matrix at the transmitter for

the i -th user at the n -th time snapshot is

$$\mathbf{R}_{T,i}^{(n)} = \rho \cdot \mathbf{R}_{T,i}^{(n-1)} + (1 - \rho) \cdot (\mathbf{H}_i^{(n)})^H \cdot \mathbf{H}_i^{(n)}, \quad (8)$$

where the matrix $\mathbf{H}_i^{(n)} \in \mathbb{C}^{M_{R,i} \times M_T}$ describes the channel towards the i -th user at the n -th time snapshot and ρ is a forgetting factor. From an EigenValue Decomposition (EVD) it is possible to extract the strongest eigenvector, i.e., the vector corresponding to the strongest eigenvalue. This vector represents the rank-1 approximation of the i -th user's subspace and can be fed to the grouping algorithm.

Setting a proper forgetting factor ρ , which depends on the time variance of the channels, it is possible to achieve the user rates seen in Figure 6 with extremely stable groups.

More sophisticated algorithms for the resource allocation problem in SDMA systems can be found in [6], [7].

VI. CONCLUSION

Two downlink beamformers, the Block Diagonalization and the Successive Optimization algorithms, have been tested on realistic channels generated with a Common Space Geometry Based channel model, the IlmProp. This flexible channel model allows to reveal the limits of the two algorithms, namely the dimensionality of the antenna arrays and the difficulty in handling strongly spatially correlated users. We propose a subspace-based grouping algorithm which allows to surpass both drawbacks. The grouping algorithm has been tested on a multi-user synthetic MIMO channel, especially taking in consideration how the scenario evolves in time. This permits us to study the algorithms as they adapt to the different channel conditions. Our study illuminates the advantages of these algorithms and provides solutions which allow to use these beamforming schemes in real applications.

ACKNOWLEDGMENT

The authors would like to thank Quentin Spencer for numerous fruitful discussions on the Block-Diagonalization and Successive Optimization algorithms.

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