ANALYSIS OF MIMO CHANNEL MEASUREMENTS

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Abstract: In this paper, we introduce a new method to estimate the Rice factor in MIMO (Multiple Input - Multiple Output) systems with a line of sight path (LOS) and omnidirectional scattering. This method is based on eigenvalue decompositions of the spatial correlation matrices at the transmitter and the receiver. To improve the performance, the correlation between subsequent temporal snapshots is used to get a more reliable estimate of the subspaces. The performance of the new technique is illustrated with MIMO channel sounder measurements obtained from a measurement campaign in Ilmenau.

Keywords: Ricean channel, subspace projection, MIMO measurements, Rice factor estimation

1. INTRODUCTION

The trend towards MIMO systems in mobile communications promises huge capacity gains for the users. In many publications the optimistic Rayleigh channel model is used for MIMO simulations. When looking at measurement data it can be observed that in most cases a Ricean channel model is better suited.

In this paper we analyze MIMO channel measurements obtained with the multidimensional MIMO RUSK channel sounder (Thomä *et al.*, 2001) in Ilmenau, Germany. In Section 2 we define a four-dimensional measurement channel transfer array in space, frequency, and time. Section 3 reviews the data model and basic properties of the Ricean K-factor. Section 4 describes how the subspaces are determined and tracked. This information is used in Section 5 to remove the LOS (line of sight) component from the measured channel transfer functions via the spatial subtraction. In Sections 6 and 7 the method is verified and tested with MIMO channel sounder measurements. The paper concludes with Section 8.

2. STRUCTURE OF THE CHANNEL

The multidimensional MIMO channel sounder uses multiple antennas at the transmitter as well as at the receiver (Thomä *et al.*, 2001). By measuring the channel transfer function between any pair of the antennas on both sides, a four-dimensional channel transfer array $\mathcal{H} \in \mathbb{C}^{M_\mathrm{R} \times M_\mathrm{T} \times W \times N}$ can be constructed as depicted in Fig. 1, where M_R denotes the number of receive antennas, M_T is the number of transmit antennas, W is the number of samples in the frequency domain, and N denotes the number of temporal snapshots. The

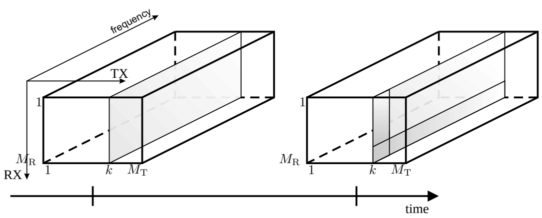

Fig. 1. Illustration of the channel array $\mathcal{H}$

high measurement repetition rate of the sounder hardware and its long-term recording capability enable the resolution of fast fading and the assessment of the long-term variations of the channel as well.

3. DATA MODEL AND BASIC PRINCIPLES

The Ricean K-factor is defined as the power of the line of sight (LOS) component divided by the total power of the scattered components.

$$K = \frac{P_{\text{LOS}}}{P_{\text{scatt}}} \tag{1}$$

Therefore it is convenient to model the channel as a sum of two different processes, one characterized by a strong LOS component and the other by diffuse scattering.

If we consider a single snapshot in time and flat-fading, the channel can be completely described by a single matrix $\boldsymbol{H}$ of size $M_{\text{R}} \times M_{\text{T}}$, where M_{R} and M_{T} represent the number of antennas. The two processes are additive and the final channel matrix can thus be expressed as a sum of two channel matrices, representing the LOS component and the scattering, respectively.

$$\boldsymbol{H} = \boldsymbol{H}_{\text{LOS}} + \boldsymbol{H}_{\text{scatt}} \tag{2}$$

The K-factor, being a ratio of two powers, suggests that the absolute power levels are not significant and we will thus take only the normalized channel matrix $\tilde{\boldsymbol{H}}$

$$\tilde{\boldsymbol{H}} = \frac{\boldsymbol{H}}{\sqrt{P_{\text{LOS}} + P_{\text{scatt}}}} \tag{3}$$

A similar normalization of the matrices $\boldsymbol{H}_{\text{LOS}}$ and $\boldsymbol{H}_{\text{scatt}}$ leads to

$$\tilde{\boldsymbol{H}} = \frac{\sqrt{P_{\text{LOS}}} \cdot \tilde{\boldsymbol{H}}_{\textbf{LOS}} + \sqrt{P_{\text{scatt}}} \cdot \tilde{\boldsymbol{H}}_{\textbf{scatt}}}{\sqrt{P_{\text{LOS}} + P_{\text{scatt}}}}, \tag{4}$$

where $\tilde{\boldsymbol{H}}_{\textbf{LOS}}$ and $\tilde{\boldsymbol{H}}_{\textbf{scatt}}$ are the normalized matrices representing a fully correlated channel and a fully uncorrelated channel, respectively. Equation (4) suggests that if we could remove from $\tilde{\boldsymbol{H}}$ the first term, the one corresponding to the LOS component, we would be able to extract the normalized scattering power $\tilde{P}_{\text{scatt}}$.

$$\tilde{P}_{\text{scatt}} = \frac{P_{\text{scatt}}}{P_{\text{LOS}} + P_{\text{scatt}}} \tag{5}$$

It is possible to rewrite equation (4) in terms of K as

$$\tilde{\boldsymbol{H}} = \sqrt{\frac{K}{1+K}} \cdot \tilde{\boldsymbol{H}}_{\textbf{LOS}} + \sqrt{\frac{1}{1+K}} \cdot \tilde{\boldsymbol{H}}_{\textbf{scatt}} \tag{6}$$

revealing that

$$\sqrt{\frac{1}{1+K}} = \frac{\sqrt{P_{\text{scatt}}}}{\sqrt{P_{\text{LOS}} + P_{\text{scatt}}}} = \sqrt{\tilde{P}_{\text{scatt}}} \tag{7}$$

equation (7) leads directly to the calculation of K as

$$K = \frac{1 - \tilde{P}_{\text{scatt}}}{\tilde{P}_{\text{scatt}}} \tag{8}$$

The basic idea of the proposed K-factor estimation scheme (presented in Sections 4 and 5) is to subtract from $\tilde{\boldsymbol{H}}$ the best estimate of $\tilde{P}_{\text{LOS}} \cdot \tilde{\boldsymbol{H}}_{\textbf{LOS}}$ in order to extract from the remaining matrix an estimate of $\tilde{P}_{\text{scatt}}$ which leads directly to the Ricean K-factor. This can be achieved via a subspace analysis of the available data in two separate steps.

First, the LOS subspace has to be identified and has to be tracked in time. This procedure determines good estimates of $\tilde{\boldsymbol{H}}_{\text{LOS}}$ for every time snapshot. The second step is the actual subtraction. The main problem is the determinination of $\tilde{P}_{\text{LOS}}$ in order to subtract all the power contained in the LOS component. This scaling problem can be solved by a subtraction of the subspaces via projection matrices. This spatial subtraction method (Del Galdo, 2002) is described in Section 5.

4. SUBSPACE IDENTIFICATION AND TRACKING

In this section we show how to estimate the power of the LOS path and the scattering components, respectively. There are several ways to estimate the LOS component. One way is to estimate the angle of arrival (AoA) and the angle of departure (AoD) via a parameter estimation technique such as MUSIC (Schmidt, 1981), ESPRIT (Roy *et al.*, 1986) or a multidimensional extension (Haardt and Nossek, 1998). However to perform this estimation, the array manifold is needed and in some cases special array geometries are required. A higher order statistics-based solution for the estimation of the K-factor is presented in (Abdi *et al.*, 2001)

In the approach presented in this paper, we identify the strongest path in the correlation matrices at the receiver and the transmitter side. To assess the structure of the channel, eigenvalue decompositions (EVD) of the correlation matrices can be computed. As the Ricean fading channel models a LOS component and some scattering components impinging from uniformly distributed angles, there will be one large eigenvalue representing the LOS component. The corresponding eigenvectors span a rank one subspace.

In MIMO measurements we can benefit from the temporal correlations between successive snapshots (Haardt *et al.*, 2001). In case of frequency selective channels, the channel transfer functions can be averaged over the frequency bins, where W is the number of samples in the frequency domain. By applying an exponential window, the effect of the noise is reduced and a more stable estimate of the subspaces is obtained. As the channel array $\mathcal{H}$ is four dimensional, the following Matlab notation is used: indexing with the colon operator $(:)$ returns all elements of the corresponding dimension. Furthermore, let any singleton dimension be squeezed out, forming a new array of a smaller dimensionality. Then the long-term spatial correlation matrices at the transmitter and the receiver can be estimated as

$$\begin{aligned}\boldsymbol{R}_{\mathrm{R}}(n) &= \rho\, \boldsymbol{R}_{\mathrm{R}}(n-1) \\ &\quad + \frac{1-\rho}{W} \sum_{w=1}^{W} \mathcal{H}(:,:,w,n) \cdot \mathcal{H}(:,:,w,n)^H \\ \boldsymbol{R}_{\mathrm{T}}(n) &= \rho\, \boldsymbol{R}_{\mathrm{T}}(n-1) \\ &\quad + \frac{1-\rho}{W} \sum_{w=1}^{W} \mathcal{H}(:,:,w,n)^H \cdot \mathcal{H}(:,:,w,n).\end{aligned}$$

The forgetting factor ρ effects the window length of the averaging process. Its choice depends on the time variance of the channel and the resolution of the measurements in the time domain. To successfully use the strongest eigenbeam as the weight vector pointing only at the LOS path, it is crucial to assume that the scatterers are uniformly distributed around the arrays. In fact, if the scatterers were distributed in clusters, the first eigenbeam would map also these directions yielding to an inaccurate estimate of the LOS power. In this case, multidimensional high resolution parameter estimation schemes, e.g., (Haardt and Nossek, 1998), should be used to extract the LOS component.

5. SUBTRACTION OF THE SUBSPACE

Once the rank one subspace that corresponds to the LOS component has been identified for every time snapshot, it is possible to apply the spatial subtraction method outlined in this section. The spatial subtraction scheme removes all the power that is present in the estimated one-dimensional subspace. If $\boldsymbol{R}_{\mathrm{R}}(n)$ and $\boldsymbol{R}_{\mathrm{T}}(n)$ have been estimated as the spatial correlation matrices for a given time snapshot n, the eigenvectors corresponding to their largest eigenvalues are denoted as $\boldsymbol{u}_{\mathrm{R}}(n)$ and $\boldsymbol{u}_{\mathrm{T}}(n)$, respectively.[1] The corresponding projection matrices are given by

$$\begin{aligned}\boldsymbol{P}_{\mathrm{left}} &= \boldsymbol{u}_{\mathrm{R}}(n) \cdot \boldsymbol{u}_{\mathrm{R}}^H(n) \\ \boldsymbol{P}_{\mathrm{right}} &= \boldsymbol{u}_{\mathrm{T}}(n) \cdot \boldsymbol{u}_{\mathrm{T}}^H(n)\end{aligned}$$

Note that every matrix depends on the current time index n. The index has been omitted for a more convenient notation. Now we can remove from $\tilde{\boldsymbol{H}}$ the part of it which lies in the space spanned by $\boldsymbol{P}_{\mathrm{left}}$ and $\boldsymbol{P}_{\mathrm{right}}$, i.e., the space which contains all the power attributed to the LOS component.

$$\boldsymbol{H}_{\mathrm{sub}} = \tilde{\boldsymbol{H}} - \boldsymbol{P}_{\mathrm{left}} \cdot \tilde{\boldsymbol{H}} \cdot \boldsymbol{P}_{\mathrm{right}}. \tag{9}$$

[1] Note that if multidimensional high resolution parameter estimation scheme is used (as indicated at the end of Section 4), the vectors $\boldsymbol{u}_{\mathrm{T}}(n)$ and $\boldsymbol{u}_{\mathrm{R}}(n)$ should be replaced by the array steering vectors corresponding to the AoD and the AoA of the LOS component, respectively.

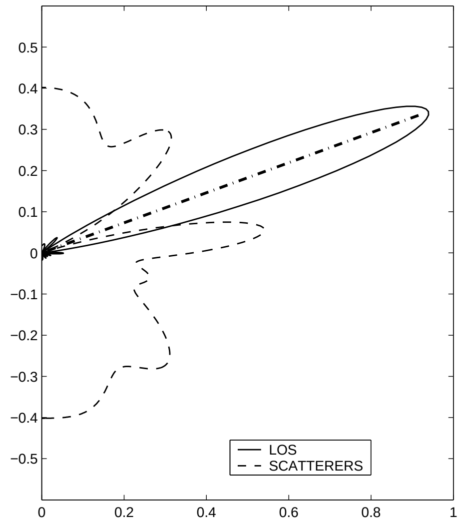

Fig. 2. Example of the spatial subtraction scheme. The dashed line shows the remaining normalized beampattern

The effect of this spatial subtraction procedure can be illustrated by analyzing the beampatterns. Fig. 2 shows a SIMO example. A strong line of sight component impinges on an eight-element uniform linear array (ULA) from an AoA of 20° (dash-dotted line). The one-dimensional subspace that is subtracted from the $\tilde{\boldsymbol{H}}$ matrix corresponds to the beampattern plotted with the solid line. With the spatial subtraction all the power collected by this beampattern will be subtracted. The power counted as power of the scatterers corresponds to the dashed beampattern. As Fig. 2 clearly shows, the power accredited to the LOS component is acquired mostly from the direction of the impinging wavefront while the scattering component is collected from all other directions. However, the estimate calculated in such a way is affected by a systematic bias. In fact, the solid beampattern will collect also the power of the scatterers within its main beam that will be added to the LOS component. In other words, the same amount of power will be subtracted from the scatterers and added to the power attributed to the LOS component leading to the following biased estimate

$$\hat{K} = \frac{P_{\mathrm{LOS}} + \alpha \cdot P_{\mathrm{scatt}}}{P_{\mathrm{scatt}} - \alpha \cdot P_{\mathrm{scatt}}}, \tag{10}$$

where $\alpha \cdot P_{\mathrm{scatt}}$ is the fraction of the power incorrectly attributed to the LOS component. With an increasing number of antenna elements, the LOS beam gets narrower such that the bias of the estimate gets smaller. Under the assumption that the scatterers are uniformly distributed it is possible to assume that α does not depend on K. The normalized power that remains after the subtraction is

$$\tilde{P}_{\mathrm{sub}} = \tilde{P}_{\mathrm{scatt}} \cdot (1-\alpha) = \frac{P_{\mathrm{scatt}}}{P_{\mathrm{scatt}} + P_{\mathrm{LOS}}} (1-\alpha)$$

and thus the correct estimate for the Ricean K-factor is equal to

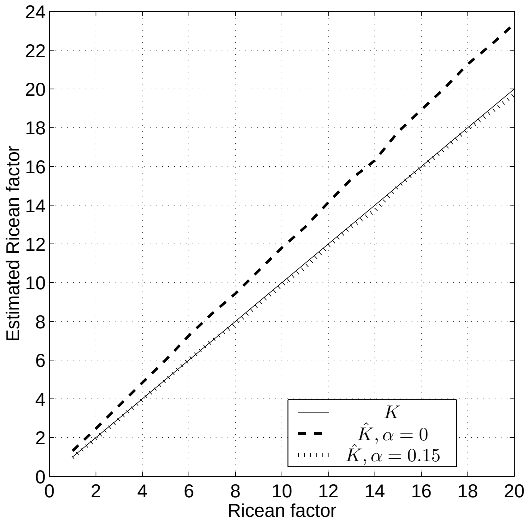

Fig. 3. Estimated Rice factors and the effect of α as a function of the K-factor used to generate the synthetic data, cf. Section 6

$$K = \frac{1 - \frac{\tilde{P}_{\text{sub}}}{1-\alpha}}{\frac{\tilde{P}_{\text{sub}}}{1-\alpha}}. \tag{11}$$

The factor α depends entirely on the beampattern and is independent of the actual powers involved, as seen in Fig. 3. A more detailed explanation of this figure is given in the next section. Under the assumption that the scatterers remain uniformly distributed around the array, that there is a LOS component, and that the array has a circular geometry, we can assume that any beampattern generated will have a main lobe with the same constant beamwidth. Such an assumption is realistic for UCAs (Uniform Circular Arrays) and CUBAs (Circular Uniform Beam Array). In other words, if the beamwidth of the main lobe remains constant for any angle, then α can be estimated and applied in any case. For other geometries, like ULAs (Uniform Linear Array) we observe that the beamwidth of the main lobe changes significantly with the AoA of the LOS path. Therefore, we have to estimate α for every eigenbeam obtained. If the array steering matrix is known, or in other words, if the antenna response is known for every angle θ, then it is possible to evaluate the fraction of power collected by the mainlobe of the array caused by the uniformly distributed scatterers around the antenna.

6. SYNTHETIC DATA ANALYSIS

To verify the accuracy of the estimation method it is appropriate to test the algorithm in a controlled scenario in which the Rice factor K is known. This can be easily be achieved by developing a synthetic channel model. The channel matrix $\tilde{\boldsymbol{H}}$ must match the initial assumptions. This means that it must be generated from two independent processes corresponding to a perfect LOS component and uniformly distributed scatterers around the receiver, respectively, i.e.,

$$\boldsymbol{H} = \boldsymbol{H}_{\text{LOS}} + \boldsymbol{H}_{\text{scatt}} \tag{12}$$

The matrix $\boldsymbol{H}_{\text{LOS}}$ can be built conveniently using a geometric model. Assuming θ_{R} and θ_{T} for the angle of arrival (AoA) and angle of departure (AoD), respectively, and ULAs (Uniform Linear Arrays) at both sides of the link (Rx and Tx) we can write the channel matrix as

$$\boldsymbol{H}_{\text{LOS}} = \boldsymbol{a}_{\text{R}} \cdot \sqrt{\boldsymbol{P}_{\text{LOS}}} \cdot \boldsymbol{a}_{\text{T}}^{H} \tag{13}$$

where $\boldsymbol{a}_{\text{R}}$ is the normalized array steering vector for the receiver

$$\boldsymbol{a}_{\text{R}} = \left[1 \; e^{j\mu} \; e^{j2\mu} \ldots \; e^{j(M_{\text{R}}-1)\mu}\right] \tag{14}$$

$$\mu = -j\frac{2\pi}{\lambda}\Delta \sin(\theta_{\text{R}}) \tag{15}$$

and $\boldsymbol{a}_{\text{T}}$ is constructed in the same way for the transmitter. The spacing between the antennas is Δ and the wavelength is equal to λ. To generate the matrix representing the scattering, a statistical model which better suits this scenario was used. In fact, the channel matrix for a uniform distribution of the scatterers corresponds to a full rank matrix in which the amplitude of every element follows the same Rayleigh distribution. To obtain such a distribution we ensure that every element has its real and imaginary parts generated from the same Gaussian random process. A proper normalization has to be applied in order to obtain $\tilde{\boldsymbol{H}}_{\textbf{scatt}}$. At this point the estimation algorithm proposed can be applied and the results can be seen in Fig. 3. The channel matrix was generated for different values of K and for 8 antennas both at the receiver and at the transmitter.

7. ESTIMATE WITH MIMO MEASUREMENTS

The data for the following simulation resulted from a measurement campaign acquired at Ilmenau University of Technology. The location was the courtyard of a big building as seen in Fig. 4.

Looking at Fig. 4 it can be seen that at the starting position the receiver is in a corner of the courtyard (Rx2). Therefore a lot of scattered power is present at the receiver, resulting in a low K-factor. Then the transmitter and the receiver are moved simultaneously along the two lines (Tx7 . . . Tx4 . . . Tx3 and Rx2 . . . Rx8 . . . Rx9). At the end of the paths, the receiver gets less reflected components such that the K-factor increases. Fig. 5 shows the estimated values of K as a function of the measurement time. The standard deviation is plotted as well. It is calculated across the different frequencies and, therefore, it should not be interpreted as an indication of the quality of the estimate.

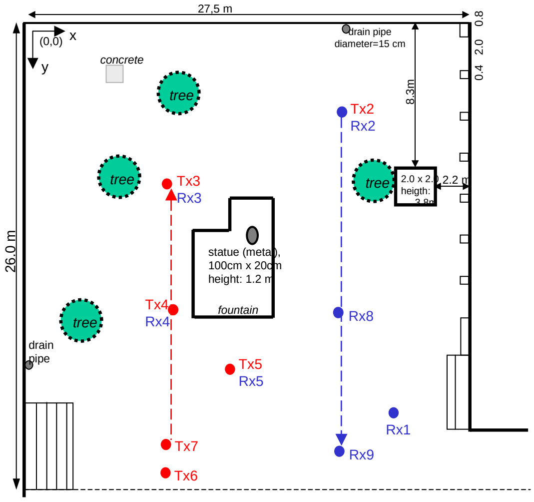

Fig. 4. Map of the measured scenario

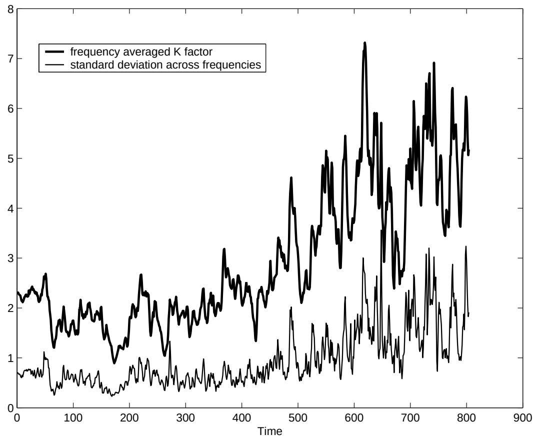

Fig. 5. The estimate for the K factor has been calculated for every frequency bin and time snapshot. The plot shows the mean and standard deviation across frequency.

8. CONCLUSIONS

The proposed algorithm yields to a good estimate of the Rice factor if the channel has a relatively strong LOS path compared to the scattered paths. If high resolution parameter estimation schemes are not used to determine the LOS component, the estimate of the K-factor still has a bias. This bias can be eliminated if the array responses are known or it might be neglected as the number of antennas increases, e.g., $M_{\mathrm{R/T}} \geq 16$.

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