

# A Tensor-Based Subspace Method for Blind Estimation of MIMO Channels

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**Abstract**—In this paper, we introduce a tensor-based subspace method for solving the blind channel estimation problem in a multiple-input multiple-output (MIMO) system. The current subspace methods of blind channel estimation require stacking the multidimensional measurement data into one highly structured vector and estimate the signal subspace via a singular value decomposition (SVD) of the correlation matrix of the measurement data. In contrast to this, we propose a 3-way measurement tensor to exploit the structure inherent in the measurement data and introduce a Higher-Order SVD (HOSVD) to obtain the signal subspace. This tensor-based subspace estimate is an improved estimate of the signal subspace, thereby leading to an improved estimate of the system channels. Numerical simulations demonstrate that the proposed method outperforms the current subspace based blind channel estimation methods in terms of the channel estimation accuracy. Furthermore, we show that the accuracy of the estimations is significantly improved by employing overlapping observed data windows at the receiver.

**Keywords**—Blind channel estimation, HOSVD, signal subspace, MIMO.

## I. INTRODUCTION

Tensor-based signal processing has become increasingly popular in many different areas of signal processing. This is because it offers several fundamental advantages compared to its matrix-based counterparts. For instance, since the structure of the data is preserved, structured denoising can be applied to obtain an improved tensor-based signal subspace estimate, thereby enhancing any subspace-based parameter estimation scheme [1]. As one of possible applications of tensors, blind and semi-blind channel estimations have attracted more attention recently. Several blind and semi-blind methods for channel estimation have been proposed by introducing tensor computations [2]–[4]. In [2], a tensor-based subspace method for blind estimation has been proposed for single-input multiple-output (SIMO) channels. The authors in [3], [4] utilize the parallel factor (PARAFAC) tensor decomposition for semi-blind channel estimation of orthogonal space-time block codes (OSTBC) based systems.

In this paper, we extend the existing matrix-based blind estimation techniques of MIMO channels in [5], [6] to the tensor case. In general, the matrix-based subspace method of

blind channel estimation stacks the multidimensional measurement data into one highly structured vector. Then, an SVD is performed on the correlation matrix of the measurement data to determine the signal and noise subspaces. With the obtained subspaces, the channel impulse responses are estimated as the solution of a quadratic form up to an invertible matrix for MIMO channels. However, the employed stacking operation does not consider the structure inherent in the measurement data for subspace estimation step. Therefore, we propose a tensor-based subspace method of blind channel estimation. The measurement data is modeled via a 3-way tensor. Consequently, a Higher-Order SVD (HOSVD) is utilized to obtain the signal subspace. This tensor-based subspace estimate achieves an improved estimate of the signal subspace which leads to a more accurate channel estimation.

This paper is organized as follows: The system description and notation are introduced in Section II. In Section III, the matrix-based subspace method for blind estimation of MIMO channels is shortly introduced. The proposed tensor-based subspace method follows in Section IV. In Section V and VI, simulation results and a conclusion are given, respectively.

## II. SYSTEM MODEL AND NOTATION

### A. System Model

The system is described as a MIMO system where the transmitter has  $M_T$  transmit antennas and receiver is equipped with  $M_R$  receive antennas. The channel between each transmit and receive antenna pair is modeled as an FIR filter with maximum of  $L + 1$  taps. Let  $\mathbf{s}(k) = [s_1(k), s_2(k), \dots, s_{M_T}(k)]^T$  denote the symbol vector emitted over  $M_T$  transmit antennas at time  $kT$ . Here  $T$  is the symbol duration. This discrete-time signal experiences an unknown communication channel which is assumed to be time-invariant during the observation interval. Then, the received signal at time  $kT$  is formulated as

$$\mathbf{y}(k) = \sum_{\ell=0}^L \mathbf{H}_\ell \mathbf{s}(k-\ell) + \mathbf{n}(k) \in \mathbb{C}^{M_R}, \quad (1)$$

where  $\mathbf{H}_\ell \in \mathbb{C}^{M_R \times M_T}$  contains the coefficients of the channel impulse responses corresponding to lag  $\ell$ . We assume that all subchannels have the same length  $L + 1$  for simplicity. The elements of the noise vector  $\mathbf{n}(k)$  are circularly symmetric complex Gaussian distributed with variance  $\sigma^2$  and assumed mutually uncorrelated in space and time.

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### B. Notation

To distinguish between scalars, vectors, matrices, and tensors, we use the following notation throughout the paper: Scalars are denoted as italic letters ( $a, \dots, A, \dots$ ), vectors are written as lower-case bold-faced letters ( $\mathbf{a}, \mathbf{b}, \dots$ ), matrices are indicated as upper-case bold-faced letters ( $\mathbf{A}, \mathbf{B}, \dots$ ), tensors are represented by bold-faced calligraphic letters ( $\mathcal{A}, \mathcal{B}, \dots$ ). We use  $\mathbf{A}(:, i)$  to denote the  $i$ th column of the matrix  $\mathbf{A}$ . The superscripts  $\text{T}$ ,  $\text{H}$ ,  $+$  indicate transposition, Hermitian transposition, and Moore-Penrose pseudoinverse, respectively.

An  $R$ -dimensional tensor  $\mathcal{A} \in \mathbb{C}^{M_1 \times M_2 \times \dots \times M_R}$  is an  $R$ -way array which has size  $M_r$  along mode  $r$ . The tensor operation we use are consistent with [7].

- **The  $r$ -mode vectors of  $\mathcal{A}$**  are obtained by varying the  $r$ -th index, while keeping all other indices fixed.
- **The  $r$ -mode unfolding of  $\mathcal{A}$**  is obtained by collecting all  $r$ -mode vectors into a matrix and represented by  $[\mathcal{A}]_{(r)} \in \mathbb{C}^{M_r \times M_{r+1} \times \dots \times M_R \cdot M_1 \times \dots \times M_{r-1}}$ . The ordering of the columns in  $[\mathcal{A}]_{(r)}$  is chosen in accordance with [7].
- **The  $r$ -rank of  $\mathcal{A}$**  is defined as the rank of  $[\mathcal{A}]_{(r)}$ . Note that in general, all the  $r$ -ranks of a tensor  $\mathcal{A}$  can be different.
- **The  $r$ -mode product of tensor  $\mathcal{A}$  and a matrix  $\mathbf{U}_r \in \mathbb{C}^{J_r \times M_r}$**  is denoted as  $\mathcal{B} = \mathcal{A} \times_r \mathbf{U}_r$ . It is visualized by multiplying all  $r$ -mode vectors of  $\mathcal{A}$  from left-hand side by the matrix  $\mathbf{U}_r$ , i.e.,  $[\mathcal{B}]_{(r)} = \mathbf{U}_r [\mathcal{A}]_{(r)}$ .
- **The HOSVD of  $\mathcal{A}$**  is given by

$$\mathcal{A} = \mathcal{S} \times_1 \mathbf{U}_1 \times_2 \mathbf{U}_2 \dots \times_R \mathbf{U}_R,$$

where  $\mathcal{S} \in \mathbb{C}^{M_1 \times M_2 \times \dots \times M_R}$  is the core tensor which satisfies the all-orthogonality conditions [7] and  $\mathbf{U}_r \in \mathbb{C}^{M_r \times M_r}$  are the unitary matrices of  $r$ -mode singular vectors for  $r = 1, \dots, R$ .

### III. MATRIX-BASED SUBSPACE METHOD FOR MIMO CHANNELS

Practically, the measurement data is observed by consecutive data windows over all receive antennas. Each window has the length  $W$ . The dimensions of  $M_R$  receive antennas and data window length are stacked into one highly structured vector by the matrix-based subspace method.

#### A. Matrix-Based Measurement Data Model

The measurement data  $\mathbf{y}_n \in \mathbb{C}^{M_R \cdot W \times 1}$  with respect to the  $n$ th observed data window is given by

$$\mathbf{y}_n = \mathbf{H}_T \mathbf{s}_n + \mathbf{n}. \quad (2)$$

The term  $\mathbf{s}_n = [\mathbf{s}_1^T, \mathbf{s}_2^T, \dots, \mathbf{s}_{M_T}^T]^T$  is the stacked  $M_T \cdot (W + L) \times 1$  vector of the input data sequences where  $\mathbf{s}_i = [s_i(nW), s_i(nW - 1), \dots, s_i(nW - W - L + 1)]^T$  is  $(W + L) \times 1$  dimensional input data sequence on  $i$ th transmit antenna for  $i = 1, \dots, M_T$ . The matrix  $\mathbf{H}_T \in \mathbb{C}^{M_R \cdot W \times M_T \cdot (W + L)}$  is the filtering matrix and structured as

$$\mathbf{H}_T = \begin{bmatrix} \mathbf{H}_T^{(1,1)} & \mathbf{H}_T^{(1,2)} & \dots & \mathbf{H}_T^{(1,M_T)} \\ \mathbf{H}_T^{(2,1)} & \mathbf{H}_T^{(2,2)} & \dots & \mathbf{H}_T^{(2,M_T)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_T^{(M_R,1)} & \mathbf{H}_T^{(M_R,2)} & \dots & \mathbf{H}_T^{(M_R,M_T)} \end{bmatrix}. \quad (3)$$

Here, the matrix  $\mathbf{H}_T^{(j,i)} \in \mathbb{C}^{W \times (W + L)}$  denotes a banded Toeplitz matrix associated to the channel impulse response  $\mathbf{h}^{(j,i)}$  between the  $j$ th receive antenna and  $i$ th transmit antenna. The vector  $\mathbf{h}^{(j,i)}$  is defined as

$$\mathbf{h}^{(j,i)} \stackrel{\text{def}}{=} [h_0^{(j,i)}, h_1^{(j,i)}, \dots, h_L^{(j,i)}]^T \\ \stackrel{\text{def}}{=} [h^{(j,i)}(t_0), h^{(j,i)}(t_0 + T), \dots, h^{(j,i)}(t_0 + LT)]^T. \quad (4)$$

Then, we have

$$\mathbf{H}_T^{(j,i)} = \begin{bmatrix} h_0^{(j,i)} & \dots & h_L^{(j,i)} & 0 & \dots & \dots & 0 \\ 0 & h_0^{(j,i)} & \dots & h_L^{(j,i)} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 & h_0^{(j,i)} & \dots & h_L^{(j,i)} \end{bmatrix}. \quad (5)$$

#### B. Estimate of the Signal Subspace

To observe  $N$  consecutive data windows at the receiver, the correlation matrix  $\mathbf{R}_{yy} \in \mathbb{C}^{M_R \cdot W \times M_R \cdot W}$  of the measurement data  $\mathbf{y}_n$  and its estimate  $\hat{\mathbf{R}}_{yy}$  are expressed as

$$\mathbf{R}_{yy} = \text{E} \left\{ \mathbf{y}_n \mathbf{y}_n^H \right\} = \mathbf{H}_T \mathbf{R}_{ss} \mathbf{H}_T^H + \sigma^2 \mathbf{I}_{M_R \cdot W} \\ \hat{\mathbf{R}}_{yy} \approx \frac{1}{N} \sum_{n=1}^N \mathbf{y}_n \mathbf{y}_n^H. \quad (6)$$

Here, the matrix  $\mathbf{R}_{ss} = \text{E} \left\{ \mathbf{s}_n \mathbf{s}_n^H \right\}$  indicates the correlation matrix of the input data with dimension  $M_T \cdot (W + L) \times M_T \cdot (W + L)$ . Applying SVD of the estimated correlation matrix  $\hat{\mathbf{R}}_{yy}$ , the signal subspace  $\hat{\mathbf{U}}_s \in \mathbb{C}^{M_R \cdot W \times M_T \cdot (W + L)}$  is estimated as the first  $M_T \cdot (W + L)$  dominant left singular vectors.

Due to the property that the column space of  $\hat{\mathbf{U}}_s$  is the linear space spanned by the columns of the filtering matrix  $\mathbf{H}_T$ , the unknown MIMO channel coefficients incorporated in the filtering matrix  $\mathbf{H}_T$  can be identified up to right multiplication to an invertible matrix  $\mathbf{A}$  by solving the maximization of the quadratic form  $q(\{\mathbf{H}_\ell\}_{\ell=0}^L) \stackrel{\text{def}}{=} \sum_{i=1}^{M_T \cdot (W + L)} \left\| \hat{\mathbf{U}}_s(:, i)^H \mathbf{H}_T \right\|_2^2$ . This signal subspace based parameter estimation and the further determination of the invertible matrix  $\mathbf{A}$  are not discussed in this paper. The details can be found in [5], [6].

The necessary conditions for the channel identifiability are listed as the following.

- 1) The correlation matrix  $\mathbf{R}_{ss}$  is full-rank but otherwise unknown, which requires  $N \geq M_T \cdot (W + L)$ .
- 2) The matrix  $\mathbf{H}_T$  has a full column rank.
- 3) The number of transmit antennas  $M_T$  is strictly less than the number of receive antennas  $M_R$ .
- 4) The observed data window length is greater than the channel order  $L$  (i.e.,  $W > L$ ).
- 5) The number of channel taps  $L + 1$  has been correctly estimated before.
- 6) The noise samples are uncorrelated with the input data.

#### IV. PROPOSED TENSOR-BASED SUBSPACE METHOD FOR MIMO CHANNELS

The dimension stacking operation employed in the definition of  $\mathbf{y}_n$  in equation (2) does not account for the structure inherent in the measurement data. Therefore, we introduce a 3-way tensor to model the measurement data. Instead of an SVD of the correlation matrix of the measurement data, we employ a HOSVD of the measurement tensor to obtain an enhanced estimate of the signal subspace.

##### A. Tensor-Based Measurement Data Model

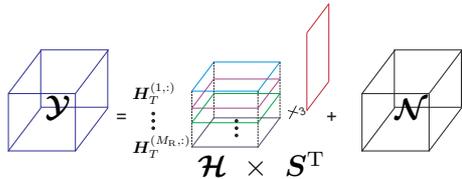


Fig. 1. Block diagram of the tensor based data model.

We model the observed receive signals as a 3-way tensor  $\mathbf{Y} \in \mathbb{C}^{M_R \times W \times N}$ . The three dimensions of the tensor  $\mathbf{Y}$  represent receive antennas, observed data window length, and the number of data windows, respectively. The corresponding input output data model can be expressed as

$$\mathbf{Y} = \mathcal{H} \times_3 \mathbf{S}^T + \mathcal{N}. \quad (7)$$

The matrix  $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_N] \in \mathbb{C}^{M_T \cdot (W+L) \times N}$  contains the input data sequences corresponding to the  $N$  sequentially observed data windows at the receiver. Each column of  $\mathbf{S}$  is  $\mathbf{s}_n = [\mathbf{s}_1^T, \mathbf{s}_2^T, \dots, \mathbf{s}_{M_T}^T]^T$  for  $n = 1, \dots, N$ . The filtering tensor  $\mathcal{H} \in \mathbb{C}^{M_R \times W \times M_T \cdot (W+L)}$  is constructed by aligning the slices of the block matrices defined in equation (3) along the first dimension as shown in Figure 1. We multiply all 3-mode vectors of  $\mathcal{H}$  from the left-hand side by the matrix  $\mathbf{S}^T$ . The tensor  $\mathcal{N}$  contains noise samples and has the same size as the tensor  $\mathbf{Y}$ . The channel identifiability conditions of the tensor-based subspace method are simply consistent with the necessary conditions for matrix-based subspace method.

##### B. Estimate of the Signal Subspace

In the tensor case, we directly compute the truncated HOSVD of the measurement tensor  $\mathbf{Y}$  as

$$\mathbf{Y} \approx \mathcal{S}^{[s]} \times_1 \mathbf{U}_1^{[s]} \times_2 \mathbf{U}_2^{[s]} \times_3 \mathbf{U}_3^{[s]}, \quad (8)$$

where  $\mathcal{S}^{[s]} \in \mathbb{C}^{r_1 \times r_2 \times r_3}$ ,  $\mathbf{U}_1^{[s]} \in \mathbb{C}^{M_R \times r_1}$ ,  $\mathbf{U}_2^{[s]} \in \mathbb{C}^{W \times r_2}$ , and  $\mathbf{U}_3^{[s]} \in \mathbb{C}^{N \times r_3}$ . Here,  $r_n$  ( $n = 1, 2, 3$ ) denotes the  $n$ -rank of the noiseless tensor  $\mathbf{Y}$  (i.e.,  $\tilde{\mathbf{Y}} = \mathcal{H} \times_3 \mathbf{S}^T$ ). In our application, we have  $r_1 = \min(M_R, M_T \cdot (L+1))$ ,  $r_2 = \min(W, N \cdot M_R)$ , and  $r_3 = \min(N, M_T \cdot (W+L))$ . According to the assumption of  $N \geq M_T \cdot (W+L)$ , the  $r_2$  and  $r_3$  can be simplified to  $r_2 = W$  and  $r_3 = M_T \cdot (W+L)$ , respectively.

From equation (8), the estimated signal subspace tensor  $\hat{\mathbf{U}}^{[s]} \in \mathbb{C}^{M_R \times W \times r_3}$  is defined as [1]

$$\hat{\mathbf{U}}^{[s]} = \mathcal{S}^{[s]} \times_1 \mathbf{U}_1^{[s]} \times_2 \mathbf{U}_2^{[s]}. \quad (9)$$

Then, the estimated signal subspace is spanned by the columns of  $\left[\hat{\mathbf{U}}^{[s]}\right]_{(3)}^T \in \mathbb{C}^{M_R \cdot W \times r_3}$ . By exploiting the inherent structure

TABLE I. COMPARISON OF THE REQUIRED NUMBER OF MULTIPLICATION FOR SIGNAL SUBSPACE ESTIMATION IN CASE OF THE MATRIX-BASED METHOD AND THE TENSOR-BASED METHOD,  $r_2 = W$  AND  $r_3 = M_T \cdot (W+L)$ .

Matrix-based method	
$r_1 < M_R$ :	$k_t \cdot M_R^2 \cdot W \cdot r_2 \cdot r_3$
$r_1 = M_R$ :	$k_t \cdot M_R \cdot W \cdot r_1 \cdot r_2 \cdot r_3$
Tensor-based method	
$r_1 < M_R$ :	$(M_R + W) \cdot r_1 \cdot r_2 \cdot r_3 + (k_t + 1) \cdot M_R \cdot W \cdot N \cdot (r_1 + r_2 + r_3)$
$r_1 = M_R$ :	$(M_R + W) \cdot r_1 \cdot r_2 \cdot r_3 + (k_t + 1) \cdot M_R \cdot W \cdot N \cdot (r_1 + r_2 + r_3)$

in subspace estimation step,  $\left[\hat{\mathbf{U}}^{[s]}\right]_{(3)}^T$  can provide a more accurate estimate than  $\hat{\mathbf{U}}_s$  from the matrix-based method under the conditions that the measurement tensor  $\mathbf{Y}$  is rank-deficient in the first or second mode (i.e.,  $M_R > r_1$  or  $W > r_2$ ) [1], [8]. Otherwise, both the tensor-based and matrix-based signal subspace estimation yield exactly the same estimate. Since  $r_2$  is always equal to  $W$  for our model, a benefit of the tensor-based signal subspace estimation is achieved under the condition  $M_R > r_1 = M_T \cdot (L+1)$ .

**Computational complexity:** We compare the computational complexity of the truncated SVD and the truncated HOSVD in terms of the number of required multiplications for the computation of the signal subspace. There is a large variety of methods to compute the SVD with different complexities. [9] shows an efficient solution employing the method of orthogonal iterations which has a complexity in terms of the required number of multiplications of  $k_t \cdot M \cdot N \cdot r$  for an  $M \times N$  matrix truncated to rank  $r$ , where  $k_t$  is a constant that depends on the design of the algorithm. In matrix case, a single SVD of the estimated correlation matrix  $\hat{\mathbf{R}}_{yy}$  truncated to rank  $M_T \cdot (W+L)$  is computed to obtain  $\hat{\mathbf{U}}_s$ . In tensor case, the truncated HOSVD of the measurement tensor  $\mathbf{Y}$  is computed to obtain the estimated signal subspace  $\left[\hat{\mathbf{U}}^{[s]}\right]_{(3)}^T$ , which is equivalent to truncated SVDs of all its unfolding. Moreover, additional multiplications are required to compute the core tensor  $\mathcal{S}^{[s]}$  and the signal subspace tensor  $\hat{\mathbf{U}}^{[s]}$ . The total number of required multiplications is compared in Table I. It indicates that the computational complexity of the tensor method is higher than the matrix method but of the same order. However, the performance improvement demonstrated in Section V justifies this increase of the computational complexity.

Since the column spaces of  $\left[\hat{\mathbf{U}}^{[s]}\right]_{(3)}^T$  and  $[\mathcal{H}]_{(3)}^T$  coincide, the unknown MIMO channel coefficients incorporated in the filtering tensor  $\mathcal{H}$  can be identified up to right multiplication to an invertible matrix  $\mathbf{A}$  by solving the maximization of the quadratic form  $q(\{\mathbf{H}_\ell\}_{\ell=0}^L) \stackrel{\text{def}}{=} \sum_{i=1}^{M_T \cdot (W+L)} \left\| \hat{\mathbf{U}}_{s_T}(:, i)^H [\mathcal{H}]_{(3)}^T \right\|_2^2$ . Here, we use  $\hat{\mathbf{U}}_{s_T}$  to indicate the estimated signal subspace of the tensor case for notational simplicity (i.e.,  $\hat{\mathbf{U}}_{s_T} = \left[\hat{\mathbf{U}}^{[s]}\right]_{(3)}^T$ ). The maximization problem follows the same signal subspace based

parameter estimation procedure as the current matrix-based method which is shown in [5], [6].

### C. Oversampled Antenna Array

As we mentioned above, the performance benefit of the tensor based subspace method is constrained by the condition  $M_R > M_T \cdot (L + 1)$ . In order to maintain the performance benefit for the case  $M_R \leq M_T \cdot (L + 1)$ , we introduce an oversampling of the receive signals with a factor  $P = T/\Delta$ . Then, a set of  $P$  sequences are constructed from the received signal of the  $j$ th receive antenna in the  $n$ th observed data window  $\mathbf{y}_n^{(j)}$  as

$$\mathbf{y}_n^{(j,m)} = \left[ y_n^{(j)}(k + \frac{m}{P}), y_n^{(j)}(k + 1 + \frac{m}{P}), \dots, y_n^{(j)}(k + W - 1 + \frac{m}{P}) \right]^T, \quad (10)$$

for  $m = 0, 1, \dots, P - 1$ . Each sequence  $\mathbf{y}_n^{(j,m)}$  depends on the discrete-time impulse responses  $\mathbf{h}^{(j,i,m)}$  for  $i = 1, \dots, M_T$ . We have

$$\begin{aligned} \mathbf{h}^{(j,i,m)} &\stackrel{\text{def}}{=} [h_0^{(j,i,m)}, h_1^{(j,i,m)}, \dots, h_L^{(j,i,m)}] \\ &\stackrel{\text{def}}{=} [h^{(j,i)}(t_0 + m\Delta), h^{(j,i)}(t_0 + m\Delta + T), \\ &\quad \dots, h^{(j,i)}(t_0 + m\Delta + LT)]. \end{aligned} \quad (11)$$

To this end, the filtering matrix associated to the  $j$ th receive antenna is defined as

$$\mathbf{H}_T^{(j,:)} = \begin{bmatrix} \mathbf{H}_T^{(j,1,0)} & \mathbf{H}_T^{(j,2,0)} & \dots & \mathbf{H}_T^{(j,M_T,0)} \\ \mathbf{H}_T^{(j,1,1)} & \mathbf{H}_T^{(j,2,1)} & \dots & \mathbf{H}_T^{(j,M_T,1)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_T^{(j,1,P-1)} & \mathbf{H}_T^{(j,2,P-1)} & \dots & \mathbf{H}_T^{(j,M_T,P-1)} \end{bmatrix}, \quad (12)$$

where each matrix  $\mathbf{H}_T^{(j,i,m)} \in \mathbb{C}^{W \times (W+L)}$  has a banded Toeplitz structure associated to the channel impulse response  $\mathbf{h}^{(j,i,m)}$  as equation (5). The filtering matrix associated to all receive antennas  $\mathbf{H}_{T_P} \in \mathbb{C}^{M_R \cdot P \cdot W \times M_T \cdot (W+L)}$  is a cumulation of  $\mathbf{H}_T^{(j,:)}$  for  $j = 1, \dots, M_R$  as  $\mathbf{H}_{T_P} = [\mathbf{H}_T^{(1,:)}]^T, \mathbf{H}_T^{(2,:)}]^T, \dots, \mathbf{H}_T^{(M_R,:)}]^T]^T$ .

We still can use a 3-way tensor  $\mathcal{Y}_P \in \mathbb{C}^{M_R \cdot P \times W \times N}$  to model the oversampled receive signals. Only the size of first dimension changes due to the oversampling compared to the previous tensor  $\mathcal{Y}$ . The corresponding input output data model is given by

$$\mathcal{Y}_P = \mathcal{H}_P \times_3 \mathbf{S}^T + \mathcal{N}_P. \quad (13)$$

As shown in Figure 2, the filtering tensor  $\mathcal{H}_P \in \mathbb{C}^{M_R \cdot P \times W \times M_T \cdot (W+L)}$  is organized by aligning the slices of the block matrices in equation (12) along the first dimension for  $j = 1, 2, \dots, M_R$ . The noise tensor  $\mathcal{N}_P$  has the same size as the tensor  $\mathcal{Y}_P$ . Notice that the noise samples are not necessarily temporally uncorrelated due to the oversampling.

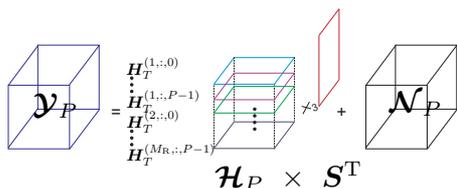


Fig. 2. Block diagram of the tensor based data model with oversampling.

By computing the truncated HOSVD of the measurement tensor  $\mathcal{Y}_P$ , the signal subspace tensor  $\hat{\mathcal{U}}_P^{[s]} \in \mathbb{C}^{M_R \cdot P \times W \times r_3}$  can be estimated as

$$\hat{\mathcal{U}}_P^{[s]} = \mathcal{S}^{[s]} \times_1 \mathbf{U}_{1_P}^{[s]} \times_2 \mathbf{U}_2^{[s]}. \quad (14)$$

Here, the ranks of the second and third modes (i.e.,  $r_2$  and  $r_3$ ) maintain as before. Only the rank of the first mode changes to  $r_{1_P} = \min(M_R \cdot P, M_T \cdot (L + 1))$ . Now, the rank-deficient condition for achieving the benefit of the tensor case is loosened to  $M_R \cdot P > r_{1_P} = M_T \cdot (L + 1)$  due to the oversampling.

### V. SIMULATION RESULTS

In this section, we demonstrate the performance improvement introduced by the tensor-based subspace method for blind estimation of MIMO channels. The evaluation is performed to show the root mean square error (RMSE) of the estimated normalized channels. This RMSE is defined as

$$\text{RMSE} = \frac{1}{P} \sqrt{\mathbb{E} \left\{ \sum_{\ell=0}^L \left\| \hat{\mathbf{H}}_{\ell} \mathbf{A}_{\ell} - \mathbf{H}_{\ell} \right\|_F^2 \right\}}, \quad (15)$$

where  $\mathbf{A}_{\ell}$  is the invertible matrix and is computed as  $\mathbf{A}_{\ell} = \hat{\mathbf{H}}_{\ell}^+ \mathbf{H}_{\ell}$ . The channel matrix  $\{\mathbf{H}_{\ell}\}_{\ell=0}^L$  is normalized to unit Frobenius norm and the RMSE is averaged over 500 channel realization. The emitted signal is in 4-QAM format. The signal to noise ratio (SNR) is defined as  $10 \log_{10} \frac{\mathbb{E}\{\|\mathbf{s}(k)\|^2\}}{\mathbb{E}\{\|\mathbf{n}(k)\|^2\}}$ . To simulate a multipath environment, we adopt a commonly used model [10] to construct an  $(L + 1)$ -ray multipath continuous-time channel  $h^{(j,i)}(t)$  between the  $j$ th receive antenna and  $i$ th transmit antenna from a raised cosine pulse shaping filter  $g_c(t - \gamma_{\ell}, \beta)$ . We have  $h^{(j,i)}(t) = \sum_{\ell=0}^L \alpha_{\ell} g_c(t - \gamma_{\ell}, \beta)$ , where the roll-off factor  $\beta$  is set to 0.5 for simulations and  $\alpha_{\ell}$  are zero mean, i.i.d., unit variance complex Gaussian random variables. The term  $\gamma_{\ell}$  indicates the delay of the  $\ell$ th path. The discrete-time channel is obtained by sampling  $h^{(j,i)}(t)$  at a rate of  $T/P$ . The length of observed data window is  $W = 10$ . We introduce a smoothing window to observe the measurement data with a smoothing parameter  $\eta$  as shown in Figure 3.

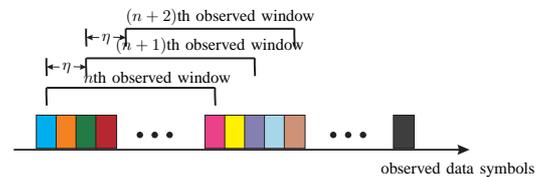


Fig. 3. The smoothing window with the smoothing parameter  $1 \leq \eta < W$ .

First, we evaluate the case that satisfies the condition  $M_R > r_1$  and  $\eta = W$ . We consider 2 taps MIMO channels and transmitter has 2 transmit antennas. The first mode rank of the measurement tensor is  $M_T \cdot (L + 1)$  (i.e.,  $r_1 = 4$ ). The number of receive antennas is greater than  $r_1$ . In this case, we do not employ oversampling at the receiver, the value  $P$  is set to 1. White Gaussian noise is added to the output. The performance improvement introduced by the tensor method is obviously shown from the Figure 4. It is noticed that the performance improvement increases with the larger difference between  $M_R$  and  $r_1$ .

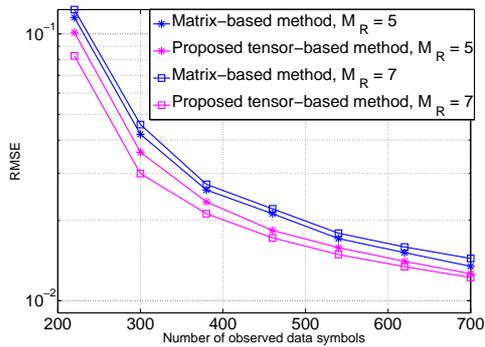


Fig. 4. RMSE for MIMO channels:  $M_T = 2$ ,  $L + 1 = 2$ ,  $r_1 = 4$ , SNR = 20 dB.

Then, we consider the case  $M_R \leq r_1$  with  $\eta = W$ . We assume 4 taps MIMO channels. The transmitter is equipped with 2 transmit antennas and the receiver has 5 receive antennas, which leads to  $M_R < M_T \cdot (L + 1)$ . Under this condition, both tensor-based subspace method and matrix-based subspace method achieve the same performance as shown in Figure 5. In order to maintain the benefit of the tensor method, we introduce oversampling at the receiver. For fair comparisons, the oversampling is utilized for both matrix-based and tensor-based methods. It is observed that performance improvement is achieved by the tensor-based method with the oversampling factor  $P > 1$ , since the benefit condition of the tensor method is loosened to  $M_R \cdot P > M_T \cdot (L + 1)$ . Larger  $P$  leads to more significant improvement.

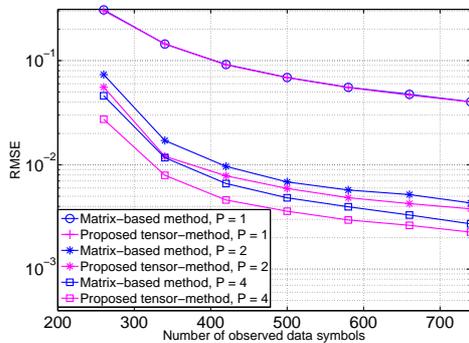


Fig. 5. RMSE for MIMO channels:  $M_R = 5$ ,  $M_T = 2$ ,  $L + 1 = 4$ , SNR = 20 dB.

Furthermore, we vary the smoothing parameter within  $1 \leq \eta < W$  for both matrix-based and tensor-based subspace methods. In Figure 6, it is shown that the accuracy of the estimate improves with the decrease of the parameter  $\eta$ . The proposed tensor-based method always outperforms the current matrix-based subspace method for different  $\eta$ . But notice that for the same number of observed data symbols, the smaller  $\eta$  results in an increased number of observed data windows.

## VI. CONCLUSIONS

In this paper, a tensor-based subspace method for blind estimation of MIMO channels is proposed. Compared to the matrix-based subspace method, the proposed method models the measurement data with a 3-way tensor which allows us to exploit the structure inherent in the measurement data.

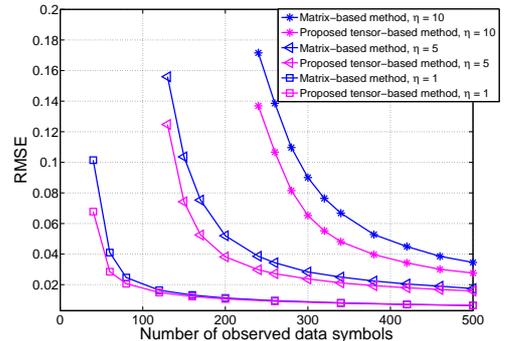


Fig. 6. RMSE for blind estimation of MIMO channels with varied smoothing parameter  $\eta$ .  $M_R = 10$ ,  $M_T = 2$ ,  $L + 1 = 3$ ,  $r_1 = M_T \cdot (L + 1) = 6$ , and SNR = 20dB.

The truncated HOSVD is employed of the tensor-structured measurement data to estimate the signal subspace. For the case that the measurement tensor has a low rank in the first mode (i.e.,  $M_R > M_T \cdot (L + 1)$ ), the tensor-based subspace method leads to a more accurate channel estimate than the matrix-based subspace method. Otherwise, the tensor-based subspace method achieves the same performance as the matrix-based counterpart. However, we can introduce oversampling to maintain the performance improvement of the tensor method for  $M_R \leq M_T \cdot (L + 1)$  case. Then, the condition for the improved estimate is loosened to  $M_R \cdot P > M_T \cdot (L + 1)$ , where  $P$  indicates the oversampling factor.

## REFERENCES

- [1] M. Haardt, F. Roemer, and G. Del Galdo, "Higher-order SVD based subspace estimation to improve the parameter estimation accuracy in multi-dimensional harmonic retrieval problems," *IEEE Trans. on Signal Processing*, vol. 56, pp. 3198–3213, 2008.
- [2] B. Song, F. Roemer, and M. Haardt, "Blind estimation of SIMO channels using a tensor-based subspace method," in *Proc. of the 44th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, 2010, pp. 8–12.
- [3] N. D. Sidiropoulos and R. S. Budampati, "Khatri-rao space-time codes," *IEEE Trans. on Signal Processing*, vol. 50, pp. 2396–2407, 2002.
- [4] F. Roemer, N. Sarmadi, B. Song, M. Haardt, M. Pesavento, and A. B. Gershman, "Tensor-based semi-blind channel estimation for mimo ostbc-coded systems," in *Proc. of the 45th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, 2011, pp. 449–453.
- [5] N. Nefedov, "On subspace channel estimation in multipath SIMO and MIMO channels," in *11th International Conference on Telecommunications (ICT)*, Fortaleza, Brazil, 2004, pp. 532–540.
- [6] A. Gorokhov and P. Loubaton, "Subspace-based techniques for blind separation of convolutive mixtures with temporally correlated sources," *IEEE Trans. on Circuits and Systems*, vol. 44(9), pp. 813–820, 1997.
- [7] L. de Lathauwer, B. de Moor, and J. Vanderwalle, "A multilinear singular value decomposition," *SIAM J. Matrix Anal. Appl.*, vol. 21, pp. 1253–1278, 2000.
- [8] F. Roemer, H. Becker, M. Haardt, and M. Weis, "Analytical performance evaluation for HOSVD-based parameter estimation schemes," in *IEEE Int. Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP 2009)*, Aruba, Dutch Antilles, 2009, pp. 77–80.
- [9] G. H. Golub and C. F. V. Loan, "Matrix computations," 3rd ed. Baltimore, MD: The Johns Hopkins Univ. Press, 1996.
- [10] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *25th Asilomar Conf.*, Pacific Grove, CA, 1991, pp. 856–860.