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On the introduction of an extended coupling matrix for a 2D bearing estimation with an experimental RF system ☆

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Abstract

Narrow-band DOA (direction of arrival) estimation methods need an accurate modeling of the array manifold (response of the array of antennas to one source in all directions). In radio frequency (RF) systems, electromagnetic perturbations arising from the neighborhood of the array will bring differences between the ideal and the true or measured response. If the model of the array response used in the algorithms does not take this modeling error into account, the performance of the bearing estimation methods may degrade dramatically. Usually, either a data collection of true steering vectors or a mutual-coupling model are used to perform DOA estimation in an experimental setup. The purpose of this paper is to propose an alternative to the mutual-coupling model by deriving a more accurate analytic expression of the true response. We present a model using a new extending coupling matrix, which includes the polarization and the scattering elements of the array in addition to mutual-coupling effects. An estimation of these extended and mutual coupling matrices is also originally proposed when measurements of the true steering vectors are available. The true steering vectors are measured in an experimental setup. Based on these new analytical expressions of the steering vectors of the array response, we extend the MUSIC DOA estimation algorithm to polarization diversity.

Keywords: DOA estimation; Calibration; Polarization; Coupling matrix; High-resolution methods; MUSIC

1. Introduction

Over the last three decades, a large number of high-resolution direction finding techniques have been developed in order to estimate the DOAs (directions of arrival) of sources impinging on an array of antennas [1–6]. These techniques can be used in a radio-communication context to estimate the DOAs of correlated or coherent sources such as the propagation paths of the different emitters. In such a context, accurate DOA estimation methods as high-resolution methods are required to provide efficient estimates.

These high-resolution direction finding algorithms need an accurate knowledge of the spatial array response (array manifold) of the sources. If this array manifold is not accurately known, the performance of the high-resolution methods will degrade dramatically [7]. In experiments, one of the

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Nomenclature

a	scalar
$\mathbf{a}$	column vector
a_i	ith component of the column vector $\mathbf{a}$
$(.)^{\mathrm{T}}$	transpose
$(.)^{\dagger}$	Hermitian transpose
$\det(\mathbf{A})$	determinant of matrix $\mathbf{A}$
$\mathbf{I}_N$	$N \times N$ identity matrix
θ	azimuth
Δ	elevation
$(.)^{(\mathrm{H}),(\mathrm{V})}$	scalar, vector or matrix associated to the horizontal or vertical polarization
$\mathbf{\Phi}$	polarization vector of horizontal and vertical components $\Phi^{(\mathrm{H})}$ and $\Phi^{(\mathrm{V})}$: $\mathbf{\Phi} = [\Phi^{(\mathrm{H})}\ \Phi^{(\mathrm{V})}]^{\mathrm{T}}$
$\mathbf{\Theta} = (\theta, \Delta)$	direction of arrival
$\mathbf{a}(\mathbf{\Theta})$	geometrical steering vector without diverse polarization
$\mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi})$	$= \mathbf{a}^{(\mathrm{H})}(\mathbf{\Theta})\Phi^{(\mathrm{H})} + \mathbf{a}^{(\mathrm{V})}(\mathbf{\Theta})\Phi^{(\mathrm{V})}$: geometrical steering vector with diverse polarization
$\mathbf{a}_e(\mathbf{\Theta}, \mathbf{\Phi})$	$= \mathbf{a}_e^{(\mathrm{H})}(\mathbf{\Theta})\Phi^{(\mathrm{H})} + \mathbf{a}_e^{(\mathrm{V})}(\mathbf{\Theta})\Phi^{(\mathrm{V})}$: exact steering vector
$\widetilde{\mathbf{a}}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta})$	steering vector model associated to the horizontal (H) or vertical (V) polarization component
$\widetilde{\mathbf{a}}(\mathbf{\Theta}, \mathbf{\Phi})$	$= \widetilde{\mathbf{a}}^{(\mathrm{H})}(\mathbf{\Theta})\Phi^{(\mathrm{H})} + \widetilde{\mathbf{a}}^{(\mathrm{V})}(\mathbf{\Theta})\Phi^{(\mathrm{V})}$: steering vector modeling
$\mathbf{Z}_0$	mutual coupling matrix
$\mathbf{Z}_b$	body coupling matrix
$\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$	extended coupling matrix associated to the horizontal (H) or vertical (V) polarization component

main reason of such model miss-match is due to the electromagnetic perturbations on the antennas of the array. A typical example of perturbations are the electromagnetic reflections between elements of the array (sensors and/or structures) that lead to a distortion of the nominal ideal expression of the array manifold.

A first alternative to overcome these perturbations is to use a data collection of exact steering vectors that is the numerical recording in an experimental setup of the array response for different directions covering the electromagnetic field of view. It is the calibration process [8]. The main drawbacks of such a technique lie firstly in the cost of this data collection procedure and secondly in the fact that it leads to a non-continuous knowledge of the array manifold. Also, the resolution is limited by the angular sampling of the electromagnetic field of view.

A second alternative which is the purpose of this paper is to elaborate a "compensated" expression of the steering vectors of the array manifold. A first approach taking into account mutual-coupling (inter-sensors) array perturbations has been proposed in [9–13]. In the papers [10–12], adaptations of the MUSIC algorithm and spatial-smoothing techniques using the mutual-coupling model (MCM) are proposed for a uniform linear array. These methods only take into account the mutual-coupling between sensors of the array and perform only a single dimensional (1D) DOA estimation. In [9] mutual coupling electronic measurements have been proposed. This method consists in the effective electronic measure of the transfer function between two elementary antennas of the array. However, this approach requires the capability of each antenna to receive or transmit independently from the other antennas. In practice, this is not always possible. Moreover, this method does not allow an estimation of the coupling coefficients between the elements of the structure (mast and arms) and the elementary sensors. As an alternative, the MoM (method of moments) might be used to derive a closed form expression of the mutual coupling matrix [14–16]. However, this last numerical approach requires the knowledge of the precise antennas shape.

In addition, a bearing estimation setup generally uses an array composed of the antennas and the metallic structures that behave as external scatters. Unfortunately, electronic measurements proposed in [9] and the MoM cannot estimate the coupling parameters between the antennas of the arrays and the external scattering elements.

The aim of this paper is to identify and propose a model for the additional perturbations brought by the structure of the array. The resorting coupling will be called "extended-coupling" throughout this paper. The introduction of such an extended-coupling matrix will provide a more accurate analytical model of the array manifold (or array response) leading to a more efficient use of bearing estimation methods.

The paper is organized as follows: in Section 2 the formulation of the problem is stated. In Section 3 we propose an extended model for the perturbations. This new model incorporates polarization and parametrization with coupling coefficients. Two methods are developed for the estimation of the extended coupling matrix: a structured one and an unstructured one. In Section 4, the improvement brought by the bearing estimation methods will be analyzed with exact steering vectors collected by an experimental setup and we conclude in Section 5.

2. Model and problem formulation

2.1. Signal model

Let us consider an array of N sensors and let $\mathbf{x}(t)$ be the $N \times 1$ vector of the complex envelopes of the received signals $x_n(t)$ $(1 \leqslant n \leqslant N)$ at the output of the antennas. Each antenna is assumed to receive a linear mixture of M sources $(1 \leqslant m \leqslant M)$ of directions $\mathbf{\Theta}_m = (\theta_m, \Delta_m)$. Under these assumptions, the observation vector $\mathbf{x}(t)$ can be written as

$$\mathbf{x}(t) = \sum_{m=1}^{M} \mathbf{a}_e(\mathbf{\Theta}_m, \mathbf{\Phi}_m) s_m(t) + \mathbf{n}(t)$$
$$= \mathbf{A}_e \mathbf{s}(t) + \mathbf{n}(t) \text{ with } \mathbf{s}(t) = \begin{bmatrix} s_1(t) \\ \vdots \\ s_M(t) \end{bmatrix}, \tag{1}$$

where $\mathbf{a}_e(\mathbf{\Theta}, \mathbf{\Phi})$ is the exact steering vector of a source from direction (azimuth θ, elevation Δ) and polarization vector $\mathbf{\Phi}$. The $N \times M$ matrix $\mathbf{A}_e$ is $\mathbf{A}_e = [\mathbf{a}_e(\mathbf{\Theta}_1, \mathbf{\Phi}_1) \ \ldots \ \mathbf{a}_e(\mathbf{\Theta}_M, \mathbf{\Phi}_M)]$ and the $N \times 1$ vector $\mathbf{n}(t)$ is the noise vector. $s_m(t)$ is the complex envelope of the signal of the mth emitter, $\mathbf{\Phi}_m$ is the polarization of the mth source. The polarization of the impinging sources is described by the horizontal (H) and vertical (V) components of the polarization vector $\mathbf{\Phi} = [\Phi^{(H)} \ \Phi^{(V)}]^{T}$. It is well known that the steering vector $\mathbf{a}_e(\mathbf{\Theta}, \mathbf{\Phi})$ satisfies

$$\mathbf{a}_e(\mathbf{\Theta}, \mathbf{\Phi}) = \mathbf{a}_e^{(H)}(\mathbf{\Theta})\Phi^{(H)} + \mathbf{a}_e^{(V)}(\mathbf{\Theta})\Phi^{(V)}, \tag{2}$$

where $\mathbf{a}_e^{(H)}(\mathbf{\Theta})$ and $\mathbf{a}_e^{(V)}(\mathbf{\Theta})$ are the steering vectors associated to the horizontal and vertical components, respectively. We assume that the noise vector $\mathbf{n}(t)$ is white, Gaussian, circular, spatially and temporally uncorrelated.

2.2. Problems formulation

The purpose of this paper is to provide an analytical expression of the exact steering vector $\mathbf{a}_e(\mathbf{\Theta}, \mathbf{\Phi})$ or more precisely an expression of the vectors $\mathbf{a}_e^{(H)}(\mathbf{\Theta})$ and $\mathbf{a}_e^{(V)}(\mathbf{\Theta})$ of Eq. (2). Using the narrow-band hypothesis [1] the array response $\mathbf{a}^{(H),(V)}(\mathbf{\Theta})$ ($\mathbf{a}^{(H),(V)}$ standing for $\mathbf{a}^{(H)}$ or $\mathbf{a}^{(V)}$) depends on the locations (x_n, y_n, z_n) of the antennas, their radiation patterns $G_n^{(H),(V)}(\mathbf{\Theta})$ and the wavelength λ:

$$\mathbf{a}^{(H),(V)}(\mathbf{\Theta}) = \begin{bmatrix} G_1^{(H),(V)}(\mathbf{\Theta}) a_1(\mathbf{\Theta}) \\ \vdots \\ G_N^{(H),(V)}(\mathbf{\Theta}) a_N(\mathbf{\Theta}) \end{bmatrix},$$
$$a_n(\mathbf{\Theta}) = e^{j(2\pi/\lambda)(x_n \cos\theta \cos\Delta + y_n \sin\theta \cos\Delta + z_n \sin\Delta)}. \tag{3}$$

In this paper, the nominal responses $\mathbf{a}^{(H),(V)}(\mathbf{\Theta})$ are called zero coupling steering vectors (related to the "zero coupling model" ZCM) since coupling effects are not taken into account in this model. According to expressions (2) and (3), the zero coupling array response $\mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi}) = \mathbf{a}^{(H)}(\mathbf{\Theta})\Phi^{(H)} + \mathbf{a}^{(V)}(\mathbf{\Theta})\Phi^{(V)}$ satisfies the following expression:

$$\mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi}) = \begin{bmatrix} G_1(\mathbf{\Theta}, \mathbf{\Phi}) a_1(\mathbf{\Theta}) \\ \vdots \\ G_N(\mathbf{\Theta}, \mathbf{\Phi}) a_N(\mathbf{\Theta}) \end{bmatrix},$$
$$G_n(\mathbf{\Theta}, \mathbf{\Phi}) = G_n^{(H)}(\mathbf{\Theta})\Phi^{(H)} + G_n^{(V)}(\mathbf{\Theta})\Phi^{(V)}. \tag{4}$$

If the antennas have the same radiation pattern $G(\mathbf{\Theta}, \mathbf{\Phi})$, $\mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi})$ can be factorized as

$$\mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi}) = G(\mathbf{\Theta}, \mathbf{\Phi})\mathbf{a}(\mathbf{\Theta}). \tag{5}$$

Fig. 1 shows the location of the antennas in the presence of a source with direction $\mathbf{\Theta}$ and polarization $\mathbf{\Phi}$. The wavefronts of the impinging sources are assumed to be planar and are characterized by the wave vector $k(\mathbf{\Theta})$.

To overcome the mismatch introduced by the steering vector $\mathbf{a}^{(H),(V)}(\mathbf{\Theta})$, we may use a recording procedure (calibration) of the exact steering vectors $\mathbf{a}_e^{(H)}(\mathbf{\Theta})$ and $\mathbf{a}_e^{(V)}(\mathbf{\Theta})$. Fig. 2 shows an example of a calibration process of an array of five vertical dipoles in vertical polarization with fixed emitter and rotating array.

The calibration depicted in Fig. 2 provides the recording of the vectors $\mathbf{a}_e^{(V)}(\mathbf{\Theta}_i)$ for K different calibration directions $\mathbf{\Theta}_i = (\theta_i, \Delta_i = 0)$ $(1 \leqslant i \leqslant K)$. The recording of $\mathbf{a}_e^{(V)}(\mathbf{\Theta}_i)$ requires a calibration with an emitter using vertical dipoles. However, the DOA estimation algorithms are limited by the angular steps of the calibration data collections.

An alternative is to build the MCM of the steering vector $\mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta})$ by introducing the vectors $\widetilde{\mathbf{a}}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}) = \mathbf{Z}_0\mathbf{a}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta})$ [9–12] where the square matrix $\mathbf{Z}_0$ is determined by electronic measurements [9] and the antennas of the array have the ability to transmit and receive signals. However, when exterior scattering elements (and especially the mast and arms of the structure) are close to the antennas their effects cannot be neglected in the model and the vector $\mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta})$ cannot be accurately modeled by $\mathbf{Z}_0\mathbf{a}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta})$. In order to combat these drawbacks, the problem is twofold:

(1) Building a linear "extended-coupling model" (ECM) of $\widetilde{\mathbf{a}}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}) = \mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}\mathbf{a}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta})$ with a non-square coupling matrix $\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$. This provides some more degrees of freedom to take into account the scattering elements in the propagation field.

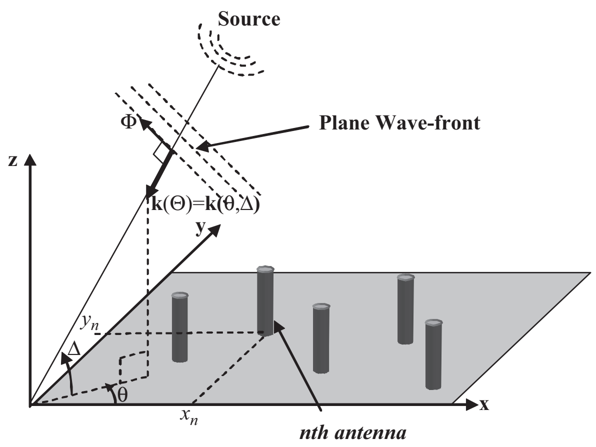

Fig. 1. Wavefront of an impinging source of direction.

(2) Estimating the coupling matrix $\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$ (or the mutual coupling matrix $\mathbf{Z}_0$) from the vectors $\mathbf{a}_e^{(\mathrm{H})}(\mathbf{\Theta}_i)$ and $\mathbf{a}_e^{(\mathrm{V})}(\mathbf{\Theta}_i)$ for $(1 \leqslant i \leqslant K)$ recorded after a calibration procedure. This approach avoids the delicate mutual coupling measurements [9].

3. Estimation of the coupling matrix

3.1. Coupling perturbations modelings for an experimental array

Usually papers dealing with coupling effects only take into account the mutual-coupling between the antennas of the array [9–13] and not the scattering due to the body structure (mast and arms). In order to overcome these drawbacks, this section provides an extended-coupling matrix valid for scattering elements behaving as antennas. To this end, two models will be investigated: an unstructured and a structured model.

3.1.1. Unstructured model

Let us consider an array of N antennas with L external scattering elements. Fig. 3 depicts the studied case for a circular array with $N = 5$ dipoles and $L = 6$ scattering elements. An image of the real antenna is given in Fig. 4. In this case, the scatterers are composed of the carrier mast and the five horizontal arms. This configuration induces a mutual-coupling between the N antennas of the array and a coupling between the N antennas and the L body structure elements. These L metallic elements behave like antennas with radiation patterns $G_n^b(\mathbf{\Theta}, \mathbf{\Phi})$.

Fig.5 illustrates the coupling effect either between two different antennas with respective radiation pattern $G_n(\mathbf{\Theta}, \mathbf{\Phi})$, $1 \leqslant n \leqslant N$ and $G_m(\mathbf{\Theta}, \mathbf{\Phi})$,

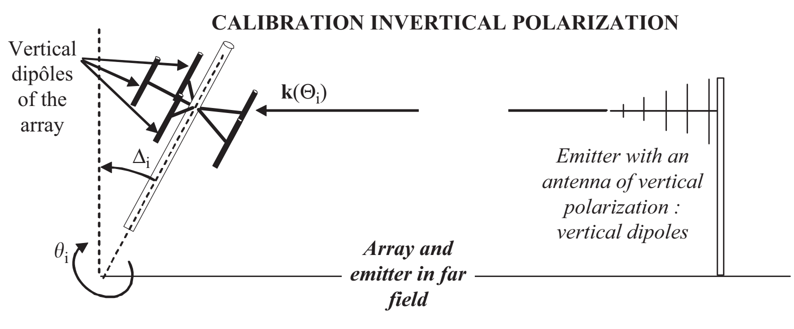

Fig. 2. Calibration procedure of an experimental array in vertical polarization.

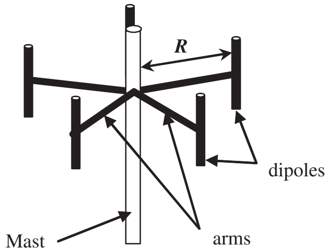

Fig. 3. Circular array of radius R.

Fig. 4. An example of circular array.

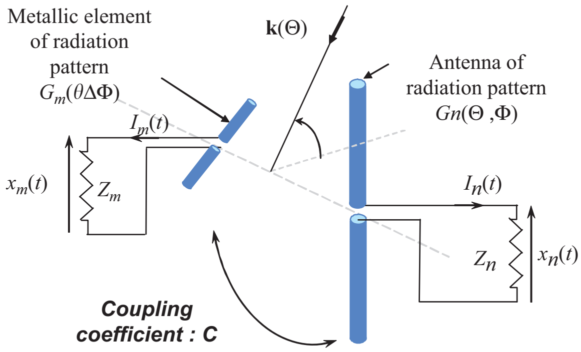

Fig. 5. Coupling effect between an antenna and a metallic element.

$1\leqslant m\leqslant N$ $(n\neq m)$ or between an antenna with radiation pattern $G_n(\mathbf{\Theta},\mathbf{\Phi})$ and a metallic element of the body with radiation pattern $G^b_m(\mathbf{\Theta},\mathbf{\Phi})$, $1\leqslant m\leqslant L$. The source's field that impinges on the first antenna with direction $\mathbf{\Theta}$ creates an electric current $I_m(t)$ in the load impedance Z_m. This current produces a new electromagnetic field that propagates and impinges on the antenna with an attenuation that depends on the distance between both elements.

This analysis provides an improved model for the signal $x_n(t)$ at the output of the nth antenna in the presence of a metallic element where the complex envelope of the arrival source is $s(t)$:

$$x_n(t) = Z_n a_n(\mathbf{\Theta},\mathbf{\Phi})s(t) + Ca_m(\mathbf{\Theta},\mathbf{\Phi})s(t) = \tilde{a}_n(\mathbf{\Theta},\mathbf{\Phi})s(t),$$

with

$$\tilde{a}_n(\mathbf{\Theta},\mathbf{\Phi}) = Z_n a_n(\mathbf{\Theta},\mathbf{\Phi}) + Ca_m(\mathbf{\Theta},\mathbf{\Phi}). \tag{6}$$

$a_j(\mathbf{\Theta},\mathbf{\Phi})$ is the zero coupling response of the jth metallic element that satisfies $a_j(\mathbf{\Theta},\mathbf{\Phi}) = G_j(\mathbf{\Theta},\mathbf{\Phi})a_j(\mathbf{\Theta})$, where $a_j(\mathbf{\Theta})$ and $G_j(\mathbf{\Theta},\mathbf{\Phi})$ are defined in (3) and (4), respectively. C is the coupling coefficient of the two elements and Z_n is the load impedance of the nth antenna.

Let us now sum for the nth antenna the coupling contribution of the $(N-1)$ others antennas and the L scattering elements. Using (6), the nth component $a_{e_n}(\mathbf{\Theta},\mathbf{\Phi})$ of the exact steering vector $\mathbf{a}_e$ can be approximated by

$$\tilde{a}_n(\mathbf{\Theta},\mathbf{\Phi}) = [Z_{n,1} \ \cdots \ Z_{n,N}]\mathbf{a}(\mathbf{\Theta},\mathbf{\Phi}) + [Z_{n,N+1} \ \cdots \ Z_{n,N+L}]\mathbf{a}_b(\mathbf{\Theta},\mathbf{\Phi}), \tag{7}$$

where $\mathbf{a}(\mathbf{\Theta},\mathbf{\Phi})$ is given by expression (4) and $\mathbf{a}_b(\mathbf{\Theta},\mathbf{\Phi})$ is associated to the array of body structure scattering elements:

$$\mathbf{a}_b(\mathbf{\Theta},\mathbf{\Phi}) = \begin{bmatrix} G^b_{N+1}(\mathbf{\Theta},\mathbf{\Phi})a_{N+1}(\mathbf{\Theta}) \\ \vdots \\ G^b_{N+L}(\mathbf{\Theta},\mathbf{\Phi})a_{N+L}(\mathbf{\Theta}) \end{bmatrix},$$

where $G^b_{N+i}(\mathbf{\Theta},\mathbf{\Phi})$, $1\leqslant i\leqslant L$ is defined similarly to Eq. (4) and takes into account the radiation pattern of body structure scattering elements. $a_{N+i}(\mathbf{\Theta})$ is defined similarly to Eq. (3), where $(x_{N+i}, y_{N+i}, z_{N+i})$ are the coordinates of the phase center of the ith body structure scattering elements. The parameters $Z_{n,i}$ $(i\leqslant N)$ and $Z_{n,N+i}$ $(1\leqslant i\leqslant L)$ are, respectively, the mutual-coupling coefficient between the nth and the ith antennas of the array and the body-coupling coefficient between the nth antenna and the ith external scattering element. We may thus deduce from (7) the following model $\tilde{\mathbf{a}}(\mathbf{\Theta},\mathbf{\Phi})$ of the steering vector:

$$\tilde{\mathbf{a}}(\mathbf{\Theta},\mathbf{\Phi}) = \mathbf{Z}_0\mathbf{a}(\mathbf{\Theta},\mathbf{\Phi}) + \underbrace{\begin{bmatrix} Z_{1,N+1} & \cdots & Z_{1,N+L} \\ \vdots & \ddots & \vdots \\ Z_{N,N+1} & \cdots & Z_{N,N+L} \end{bmatrix}}_{\mathbf{z}_b} \times\mathbf{a}_b(\mathbf{\Theta},\mathbf{\Phi}), \tag{8}$$

where $\mathbf{Z}_0$ and $\mathbf{Z}_b$ are defined as the mutual-coupling matrix and the body-coupling matrix, respectively. Let $\mathbf{Z} = [\mathbf{Z}_0\ \mathbf{Z}_b]$ be the extended coupling matrix. The expression (8) may be rewritten as

$$\tilde{\mathbf{a}}(\mathbf{\Theta}, \mathbf{\Phi}) = \mathbf{Z}\mathbf{b}(\mathbf{\Theta}, \mathbf{\Phi}) \quad \text{with } \mathbf{b}(\mathbf{\Theta}, \mathbf{\Phi}) = \begin{bmatrix} \mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi}) \\ \mathbf{a}_b(\mathbf{\Theta}, \mathbf{\Phi}) \end{bmatrix}. \tag{9}$$

Let us note that $\mathbf{Z}$ is a rectangular matrix. According to expressions (2)–(4) the steering vectors $\mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi})$ and $\mathbf{a}_b(\mathbf{\Theta}, \mathbf{\Phi})$ are given by

$$\begin{aligned} \mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi}) &= \Phi^{(\mathrm{H})}\mathbf{a}^{(\mathrm{H})}(\mathbf{\Theta}) + \Phi^{(\mathrm{V})}\mathbf{a}^{(\mathrm{V})}(\mathbf{\Theta}), \\ \mathbf{a}_b(\mathbf{\Theta}, \mathbf{\Phi}) &= \Phi^{(\mathrm{H})}\mathbf{a}_b^{(\mathrm{H})}(\mathbf{\Theta}) + \Phi^{(\mathrm{V})}\mathbf{a}_b^{(\mathrm{V})}(\mathbf{\Theta}), \end{aligned} \tag{10}$$

where

$$\mathbf{a}_b^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}) = \begin{bmatrix} G_{N+1}^{b(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}, \mathbf{\Phi}) a_{N+1}(\mathbf{\Theta}) \\ \vdots \\ G_{N+L}^{b(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}, \mathbf{\Phi}) a_{N+L}(\mathbf{\Theta}) \end{bmatrix},$$

and where $\mathbf{a}_i(\mathbf{\Theta})$ is defined in expression (3). According to the expressions (8)–(10) we define the unstructured extended coupling model (UECM) where the steering vector $\tilde{\mathbf{a}}(\mathbf{\Theta}, \mathbf{\Phi})$ can be rewritten as

$$\tilde{\mathbf{a}}(\mathbf{\Theta}, \mathbf{\Phi}) = \Phi^{(\mathrm{H})}\tilde{\mathbf{a}}^{(\mathrm{H})}(\mathbf{\Theta}) + \Phi^{(\mathrm{V})}\tilde{\mathbf{a}}^{(\mathrm{V})}(\mathbf{\Theta}) \quad \text{(UECM model)}, \tag{11}$$

with

$$\tilde{\mathbf{a}}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}) = \mathbf{Z}\mathbf{b}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}), \quad \mathbf{b}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}) = \begin{bmatrix} \mathbf{a}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}) \\ \mathbf{a}_b^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}) \end{bmatrix}.$$

In the UECM of expression (11), the $N \times (N + L)$ coupling matrix $\mathbf{Z}$ depends on $N \times (N + L)$ parameters.

3.1.2. Structured model with an array of identical antennas

Taking into account some prior knowledge on the experimental array, we can obtain a structured model and reduce the number of unknown parameters of the coupling matrix. The introduction of such parameters will also improve the estimation of the coupling matrix.

In most applications, the antennas of an array are identical. Under this assumption their radiation patterns are equal:

$$\begin{aligned} G^{(\mathrm{H})}(\mathbf{\Theta}) &= G_1^{(\mathrm{H})}(\mathbf{\Theta}) = \cdots = G_N^{(\mathrm{H})}(\mathbf{\Theta}), \\ G^{(\mathrm{V})}(\mathbf{\Theta}) &= G_1^{(\mathrm{V})}(\mathbf{\Theta}) = \cdots = G_N^{(\mathrm{V})}(\mathbf{\Theta}). \end{aligned} \tag{12}$$

From Fig. 2, by choosing the polarization subspaces adapted to the antennas of the array such that $G^{(\mathrm{H})}(\mathbf{\Theta}) = 0$ and according to (3), expression (10) becomes

$$\begin{aligned} \mathbf{a}(\mathbf{\Theta}, \mathbf{\Phi}) &= G^{(\mathrm{V})}(\mathbf{\Theta})\Phi^{(\mathrm{V})}\mathbf{a}(\mathbf{\Theta}), \\ \mathbf{a}_b(\mathbf{\Theta}, \mathbf{\Phi}) &= \Phi^{(\mathrm{H})}\mathbf{a}_b^{(\mathrm{H})}(\mathbf{\Theta}) + \Phi^{(\mathrm{V})}\mathbf{a}_b^{(\mathrm{V})}(\mathbf{\Theta}). \end{aligned} \tag{13}$$

Replacing expression (13) in Eq. (8),

$$\begin{aligned} \tilde{\mathbf{a}}^{(\mathrm{V})}(\mathbf{\Theta}) &= G^{(\mathrm{V})}(\mathbf{\Theta})\mathbf{Z}_0\mathbf{a}(\mathbf{\Theta}) + \mathbf{Z}_b\mathbf{a}_b^{(\mathrm{V})}(\mathbf{\Theta}), \\ \tilde{\mathbf{a}}^{(\mathrm{H})}(\mathbf{\Theta}) &= \mathbf{Z}_b\mathbf{a}_b^{(\mathrm{H})}(\mathbf{\Theta}). \end{aligned} \tag{14}$$

This new model will be called the structured extended coupling model denoted SECM subsequently. Let $\mathbf{Z}^{(\mathrm{H})}$ and $\mathbf{Z}^{(\mathrm{V})}$ be the extended coupling matrices associated to the horizontal and vertical polarization components. According to Eq. (11), we may deduce from expression (14) the expression of the vectors $\tilde{\mathbf{a}}^{(\mathrm{H})}(\mathbf{\Theta})$ and $\tilde{\mathbf{a}}^{(\mathrm{V})}(\mathbf{\Theta})$:

$$\tilde{\mathbf{a}}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}) = \mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}\mathbf{b}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta}),$$

with

$$\begin{aligned} \mathbf{b}^{(\mathrm{H})}(\mathbf{\Theta}) &= \mathbf{a}_b^{(\mathrm{H})}(\mathbf{\Theta}), \\ \mathbf{b}^{(\mathrm{V})}(\mathbf{\Theta}) &= \begin{bmatrix} G^{(\mathrm{V})}(\mathbf{\Theta})\mathbf{a}(\mathbf{\Theta}) \\ \mathbf{a}_b^{(\mathrm{V})}(\mathbf{\Theta}) \end{bmatrix}, \\ \mathbf{Z}^{(\mathrm{H})} &= \mathbf{Z}_b, \quad \mathbf{Z}^{(\mathrm{V})} = \mathbf{Z}. \end{aligned} \tag{15}$$

Notice that $\tilde{\mathbf{a}}^{(\mathrm{H}),(\mathrm{V})}(\mathbf{\Theta})$ are $N \times 1$ vectors, $\mathbf{b}^{(\mathrm{H})}(\mathbf{\Theta})$ is a $L \times 1$ vector and $\mathbf{b}^{(\mathrm{V})}(\mathbf{\Theta})$ is a $(N + L) \times 1$ vector. Finally with an ad hoc polarization base, the initial $N \times (N + L)$ matrix $\mathbf{Z}^{(\mathrm{H})}$ may be reduced into an $N \times L$ matrix.

Matrices $\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$ depend on $N \times L$ and $N \times (N + L)$ parameters, respectively. For particular array geometries and scattering elements we can reduce the number of parameters to be estimated. Let $\mathbf{c}^{(\mathrm{H}),(\mathrm{V})}$ be the $Q^{(\mathrm{H}),(\mathrm{V})} \times 1$ parameter vector of the coupling matrix $\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$. For both SECM and UECM the relation between $\mathbf{c}^{(\mathrm{H}),(\mathrm{V})}$ and $\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$ can be written as

$$\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})} = \begin{bmatrix} \mathbf{z}_1^{(\mathrm{H}),(\mathrm{V})} \\ \vdots \\ \mathbf{z}_N^{(\mathrm{H}),(\mathrm{V})} \end{bmatrix} \quad \text{and} \quad \mathbf{z}_n^{(\mathrm{H}),(\mathrm{V})^{\mathrm{T}}} = \mathbf{D}_n^{(\mathrm{H}),(\mathrm{V})}\mathbf{c}^{(\mathrm{H}),(\mathrm{V})}. \tag{16}$$

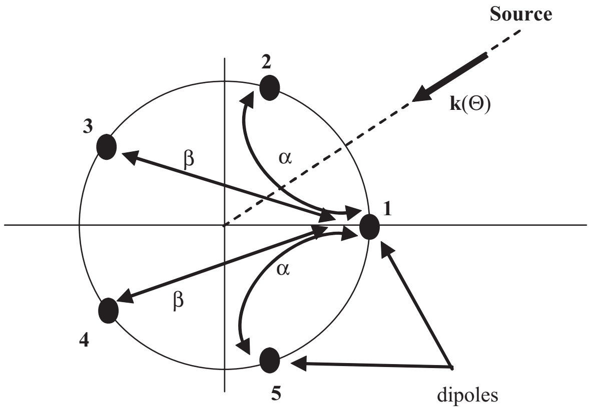

Fig. 6. Mutual-coupling in the case of a five elements circular array for the first antenna.

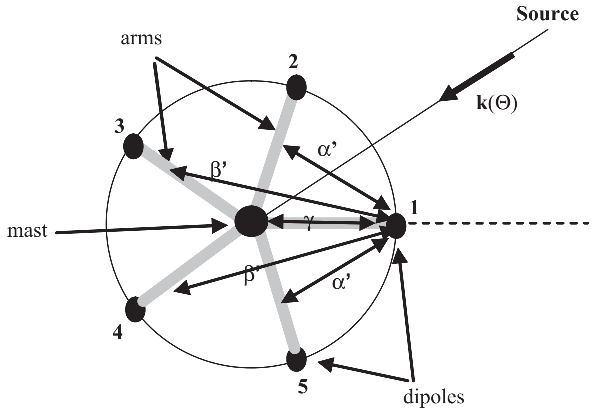

Fig. 7. Scatter-coupling in the case of a circular array for the 1st antenna.

In the case of the UECM, matrix $\mathbf{D}_n^{(\mathrm{H}),(\mathrm{V})}$ satisfies

$$\mathbf{D}_n^{(\mathrm{H}),(\mathrm{V})} = \left[\underset{(n-1)P^{(\mathrm{H}),(\mathrm{V})}}{\overset{\mathbf{0}}{\leftrightarrow}} \quad \mathbf{I}_{P^{(\mathrm{H}),(\mathrm{V})}} \quad \mathbf{0} \right], \tag{17}$$

where $Q^{(\mathrm{H}),(\mathrm{V})} = NP^{(\mathrm{H}),(\mathrm{V})}$ with $P^{(\mathrm{H})} = L$ and $P^{(\mathrm{V})} = N + L$.

In the case of the experimental circular array of Fig. 3, the mutual-coupling matrix $\mathbf{Z}_0$ depends only on two parameters α and β: Fig. 6 shows that α is the coupling coefficient between the nth and $(n+1)$th antennas and that β is the coupling between the nth and $(n+2)$th antennas.

From Fig. 5 we may deduce that the mutual coupling matrix $\mathbf{Z}_0$ is given by

$$\mathbf{Z}_0 = \begin{bmatrix} 1 & \alpha & \beta & \beta & \alpha \\ \alpha & 1 & \alpha & \beta & \beta \\ \beta & \alpha & 1 & \alpha & \beta \\ \beta & \beta & \alpha & 1 & \alpha \\ \alpha & \beta & \beta & \alpha & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{0_1} \\ \vdots \\ \mathbf{z}_{0_N} \end{bmatrix},$$

where

$$\mathbf{z}_{0n}^{\mathrm{T}} = \mathbf{D}_n \mathbf{c}, \quad \mathbf{c} = \begin{bmatrix} 1 \\ \alpha \\ \beta \end{bmatrix}.$$

For example,

$$\mathbf{D}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{z}_{0_1}^{\mathrm{T}} = \begin{bmatrix} 1 \\ \alpha \\ \beta \\ \beta \\ \alpha \end{bmatrix}. \tag{18}$$

Fig. 7 shows that the body-coupling matrix $\mathbf{Z}_b$ depends on four parameters α_0, α', β' and γ: α_0 is the coupling coefficient between an antenna and its arm, α' is the coupling coefficient between the nth antenna and the $(n+1)$th arm, β' is the coupling coefficient between the nth antenna and the $(n+2)$th arm and γ is the coupling coefficient between the mast and an antenna of the array.

If we consider that the first body-structure scattering elements are the arms and the last one is the mast, we deduce from Fig. 7 that the body coupling matrix $\mathbf{Z}_b$ equals

$$\mathbf{Z}_b = \begin{bmatrix} \alpha_0 & \alpha' & \beta' & \beta' & \alpha' & \gamma \\ \alpha' & \alpha_0 & \alpha & \beta & \beta' & \gamma \\ \beta' & \alpha' & \alpha_0 & \alpha' & \beta' & \gamma \\ \beta' & \beta & \alpha' & \alpha_0 & \alpha' & \gamma \\ \alpha' & \beta' & \beta' & \alpha' & \alpha_0 & \gamma \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{b_1} \\ \vdots \\ \mathbf{z}_{b_N} \end{bmatrix},$$

where

$$\mathbf{z}_{b_n}^{\mathrm{T}} = \mathbf{D}_{b_n} \mathbf{c}_b, \quad \mathbf{c}_b = \begin{bmatrix} \alpha_0 \\ \alpha' \\ \beta' \\ \gamma \end{bmatrix}.$$

For example,

$$\mathbf{D}_{b_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{z}_{b_1}^{\mathrm{T}} = \begin{bmatrix} \alpha_0 \\ \alpha' \\ \beta' \\ \beta' \\ \alpha' \\ \gamma \end{bmatrix}. \tag{19}$$

According to (15)–(19) we may deduce the coupling matrix $\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$ of the SECM:

$$\mathbf{Z}^{(\mathrm{H})} = \begin{bmatrix} \mathbf{z}_1^{(\mathrm{H})} \\ \vdots \\ \mathbf{z}_N^{(\mathrm{H})} \end{bmatrix} \quad \text{with } \mathbf{z}_n^{(\mathrm{H})^{\mathrm{T}}} = \mathbf{D}_n^{(\mathrm{H})}\mathbf{c}^{(\mathrm{H})},$$
$$\mathbf{c}^{(\mathrm{H})} = \mathbf{c}_b \quad \text{and} \quad \mathbf{D}_n^{(\mathrm{H})} = \mathbf{D}_{b_n}, \tag{20}$$

$$\mathbf{Z}^{(\mathrm{V})} = \begin{bmatrix} \mathbf{z}_1^{(\mathrm{V})} \\ \vdots \\ \mathbf{z}_N^{(\mathrm{V})} \end{bmatrix} \quad \text{with } \mathbf{z}_n^{(\mathrm{V})^{\mathrm{T}}} = \mathbf{D}_n^{(\mathrm{V})}\mathbf{c}^{(\mathrm{V})},$$
$$\mathbf{c}^{(\mathrm{V})} = \begin{bmatrix} \mathbf{c} \\ \mathbf{c}_b \end{bmatrix} \quad \text{and} \quad \mathbf{D}_n^{(\mathrm{V})} = \begin{bmatrix} \mathbf{D}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{b_n} \end{bmatrix}. \tag{21}$$

Finally, the coupling matrices $\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$ depend on a reduced number of parameters which are the components of the vectors $\mathbf{c}^{(\mathrm{H}),(\mathrm{V})}$ that depend on the reduced vectors $\mathbf{c}$ and $\mathbf{c}_b$.

According to (21) and (20), the coupling matrices of the array of Fig. 3 depend on $2+4=6$ parameters. Moreover, since arms and dipoles are orthogonal (as it can be noticed in Fig. 4), we can reduce the number of parameters in case of vertical polarization. Indeed, $\alpha_0 = \alpha' = \beta' = 0$.

3.2. Estimation of the coupling matrices

In this section we propose the use of a mean square error (MSE) method to estimate the coupling matrices from a data collection of exact steering vectors that consists of the vectors $\mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}_i)$, $1 \leqslant i \leqslant K$. Let us notice that the estimation of the coupling matrix $\mathbf{Z}^{(\mathrm{H}),(\mathrm{V})}$ requires only $Q^{(\mathrm{H}),(\mathrm{V})}$ vectors $\mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}_i)$ collected by an experimental setup, reducing the complexity of the initial calibration process.

The relation between the vector $\mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})$ of the calibration records and the vector $\widetilde{\mathbf{a}}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})$ of the model is given by $\mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}) \approx \rho e^{\mathrm{j}\varphi}\widetilde{\mathbf{a}}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})$ where ρ and φ are the amplitude and phase ambiguity, respectively. Using Eqs. (15) and (16), the expression of the nth component of the vector $\mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})$ is

$$a_{e_n}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}) \approx \rho e^{\mathrm{j}\varphi}\mathbf{z}_n^{(\mathrm{H}),(\mathrm{V})}\mathbf{b}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}). \tag{22}$$

We may deduce from (22) and (16), the following relation between the vectors $\mathbf{c}'^{(\mathrm{H}),(\mathrm{V})} = \rho e^{\mathrm{j}\varphi}\mathbf{c}^{(\mathrm{H}),(\mathrm{V})}$ of unknown parameters and $a_{e_n}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})$:

$$a_{e_n}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}) \approx \mathbf{t}_n^{(\mathrm{H}),(\mathrm{V})^{\mathrm{T}}}(\boldsymbol{\Theta})\mathbf{c}'^{(\mathrm{H}),(\mathrm{V})},$$
$$\mathbf{t}_n^{(\mathrm{H}),(\mathrm{V})^{\mathrm{T}}}(\boldsymbol{\Theta}) = \mathbf{b}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})^{\mathrm{T}}\mathbf{D}_n^{(\mathrm{H}),(\mathrm{V})}. \tag{23}$$

Notice that the expression of the vector $\mathbf{t}_n^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})$ is analytical since the matrix $\mathbf{D}_n^{(\mathrm{H}),(\mathrm{V})}$ is known and $\mathbf{b}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})$ has been defined before. A MSE technique can be used to estimate the vector $\mathbf{c}'^{(\mathrm{H}),(\mathrm{V})}$ using the vectors $\mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}_i)$ of the calibration records such that

$$\widehat{\mathbf{c}}'^{(\mathrm{H}),(\mathrm{V})} = \underset{\mathbf{c}^{(\mathrm{H}),(\mathrm{V})}}{\arg\min}\left\{\sum_{n=1}^{N}\sum_{i=1}^{K}|\mathbf{t}_n^{(\mathrm{H}),(\mathrm{V})^{\mathrm{T}}}(\boldsymbol{\Theta}_i)\mathbf{c}^{(\mathrm{H}),(\mathrm{V})} - a_{e_n}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}_i)|^2\right\}. \tag{24}$$

Remembering that the $a_{e_n}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta})$ are provided by the calibration process, the solution of Eq. (24) is given by

$$\widehat{\mathbf{c}}'^{(\mathrm{H}),(\mathrm{V})} = (\mathbf{Q}^{(\mathrm{H}),(\mathrm{V})^{\dagger}}\mathbf{Q}^{(\mathrm{H}),(\mathrm{V})})^{-1}\mathbf{Q}^{(\mathrm{H}),(\mathrm{V})^{\dagger}}\mathbf{q}^{(\mathrm{H}),(\mathrm{V})},$$
$$\mathbf{Q}^{(\mathrm{H}),(\mathrm{V})} = \begin{bmatrix} \mathbf{T}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}_1) \\ \vdots \\ \mathbf{T}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}_K) \end{bmatrix},$$
$$\mathbf{q}^{(\mathrm{H}),(\mathrm{V})} = \begin{bmatrix} \mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}_1) \\ \vdots \\ \mathbf{a}_e^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}_K) \end{bmatrix},$$
$$\mathbf{T}^{(\mathrm{H}),(\mathrm{V})}(\boldsymbol{\Theta}) = \begin{bmatrix} \mathbf{t}_1^{(\mathrm{H}),(\mathrm{V})^{\mathrm{T}}}(\boldsymbol{\Theta}) \\ \vdots \\ \mathbf{t}_N^{(\mathrm{H}),(\mathrm{V})^{\mathrm{T}}}(\boldsymbol{\Theta}) \end{bmatrix}. \tag{25}$$

Using the assumption that $c_1^{(\mathrm{V})}$ is equal to one, we deduce from (23) that $\widehat{c}_1'^{(\mathrm{V})} = \rho e^{\mathrm{j}\varphi}$ and that $\widehat{\mathbf{c}}^{(\mathrm{V})}$ satisfies the following expression:

$$\widehat{\mathbf{c}}^{(\mathrm{V})} = \frac{\widehat{\mathbf{c}}'^{(\mathrm{V})}}{\widehat{c}_1'^{(\mathrm{V})}}. \tag{26}$$

Factorizing α_0 in the expression of $\mathbf{c}^{(\mathrm{H})} = \mathbf{c}_b$, we deduce from (23) that $\widehat{c}_1'^{(\mathrm{H})} = \alpha_0\rho e^{\mathrm{j}\varphi}$ and that $\widehat{\mathbf{c}}^{(\mathrm{H})}$ satisfies the following expression:

$$\widehat{\mathbf{c}}^{(\mathrm{H})} = \frac{\widehat{\mathbf{c}}'^{(\mathrm{H})}}{\widehat{c}_1'^{(\mathrm{H})}}. \tag{27}$$

Using expressions (16) and (26), we can deduce the rows $\widehat{\mathbf{z}}_n^{(H),(V)}$ $(n = 1, \ldots, N)$ of the estimated coupling matrix $\widehat{\mathbf{Z}}^{(H),(V)}$:

$$\widehat{\mathbf{Z}}^{(H),(V)} = \begin{bmatrix} \widehat{\mathbf{z}}_1^{(H),(V)} \\ \vdots \\ \widehat{\mathbf{z}}_N^{(H),(V)} \end{bmatrix}$$

where $\widehat{\mathbf{z}}_n^{(H),(V)^{\mathrm{T}}} = \mathbf{D}_n^{(H),(V)} \widehat{\mathbf{c}}^{(H),(V)}$. (28)

3.3. Steering vectors model accuracy

In [7] the authors quantified the DOA estimation performances in presence of modeling errors on the exact steering vectors. They have shown the fast degradation of the bearing estimation performances with respect to the phase and amplitude errors of the exact steering vectors. It is therefore important to provide a very accurate modeling of the exact steering vectors. This section investigates the performances of the proposed modeling of steering vectors by using the exact steering vectors of the circular array of Fig. 3. These vectors have been collected outdoors with an experimental setup using the calibration procedure illustrated in Fig. 2, with $\Delta = 0$ $(\mathbf{\Theta}_i = (\theta_i, 0),\ 1 \leqslant i \leqslant K)$. In these conditions, these vectors $\mathbf{a}_e^{(H),(V)}(\mathbf{\Theta}_i)$ have been collected with an emitter in vertical polarization because the dipoles of the array are vertical and adapted to this polarization. Knowing that with $\Delta = 0$, the arms of the array have a null radiation pattern in vertical polarization and the mast behaves as a vertical dipole, we deduce from Figs. 5 and 6 that the mutual coupling coefficients depends on α and β and the scatter-coupling with the mast only depends on γ $(\alpha_0 = \alpha' = \beta' = 0)$. According to (18)–(21) the vector $\mathbf{c}^{(V)}$ satisfies the following expression:

$$\mathbf{c}^{(V)} = \begin{bmatrix} \mathbf{c} \\ \mathbf{c}_b \end{bmatrix} \quad \text{with } \mathbf{c} = \begin{bmatrix} 1 \\ \alpha \\ \beta \end{bmatrix} \text{ and } \mathbf{c}_b = \gamma, \tag{29}$$

where the mutual coupling matrix $\mathbf{Z}_0$ only depends on $\mathbf{c}$ and the extended coupling matrix $\mathbf{Z}^{(V)}$ depends on $\mathbf{c}$ and $\mathbf{c}_b$. In this example, the coupling matrices are estimated from $K = 90$ exact steering vectors provided by the calibration process (with $\mathbf{\Theta}_i = \theta_i = 360(i-1)/K,\ 1 \leqslant i \leqslant K$). Moreover, we also computed in Figs. 8–10 the case $K = 6$ in order to validate on actual data the good behavior of SECM in case of a reduced number of calibration measurements. The calibration method has been described in Fig. 2. The aim of this part is the comparison of the modeling of the steering vectors $\widetilde{\mathbf{a}}^{(V)}(\mathbf{\Theta})$ in the three following cases:

- Zero Coupling Model (ZCM): $\mathbf{a}(\mathbf{\Theta})$
- Mutual Coupling Model (MCM): $\mathbf{Z}_0\mathbf{a}(\mathbf{\Theta})$
- Structured Extended Coupling Model without diverse polarization (SECM-N): $\mathbf{Z}^{(V)}\mathbf{b}^{(V)}(\mathbf{\Theta})$ of the expression (15).

In order to evaluate the performances, we compare the steering vector provided by the model $\widetilde{\mathbf{a}}^{(V)}(\mathbf{\Theta})$ to the exact vector $\mathbf{a}_e^{(V)}(\mathbf{\Theta})$ provided by the file recorded in the calibration process.

In Figs. 8 and 9, $a_{e_1}^{(V)}(\mathbf{\Theta})$ is compared to $\widetilde{a}_1^{(V)}(\mathbf{\Theta})$ for the three models SECM, MCM and ZCM. The two cases $K = 90$ and 6 have been plotted for SECM and MCM models. Fig. 8 gives the ratio amplitude error of the two components, whereas Fig. 9 provides the phase error of the two quantities. We can notice the very good behavior of the SECM even in the case of $K = 6$ (amplitudes and phases of $a_{e_1}^{(V)}(\mathbf{\Theta})$ and $\widetilde{a}_1^{(V)}(\mathbf{\Theta})$ are quasi-equal). In Fig. 10, the comparison is conducted with the following distance function:

$$c(\mathbf{\Theta}) = \frac{|\mathbf{a}_e(\mathbf{\Theta})^{\dagger}\widetilde{\mathbf{a}}(\mathbf{\Theta})|^2}{(\mathbf{a}_e(\mathbf{\Theta})^{\dagger}\mathbf{a}_e(\mathbf{\Theta}))(\widetilde{\mathbf{a}}(\mathbf{\Theta})^{\dagger}\widetilde{\mathbf{a}}(\mathbf{\Theta}))}. \tag{30}$$

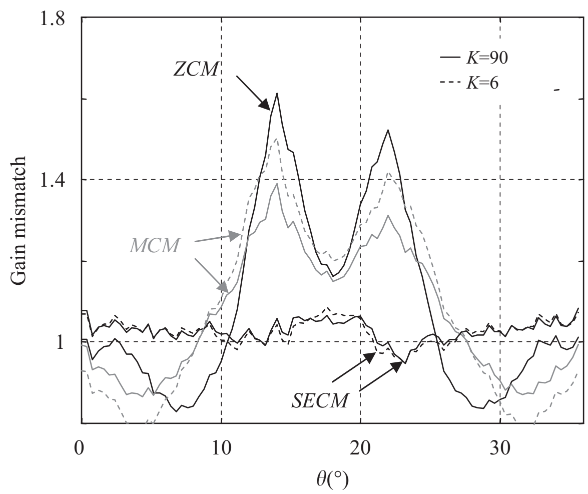

Fig. 8. Gain mismatch of $a_{e_1}^{(V)}(\mathbf{\Theta})/\widetilde{a}_1^{(V)}(\mathbf{\Theta})$.

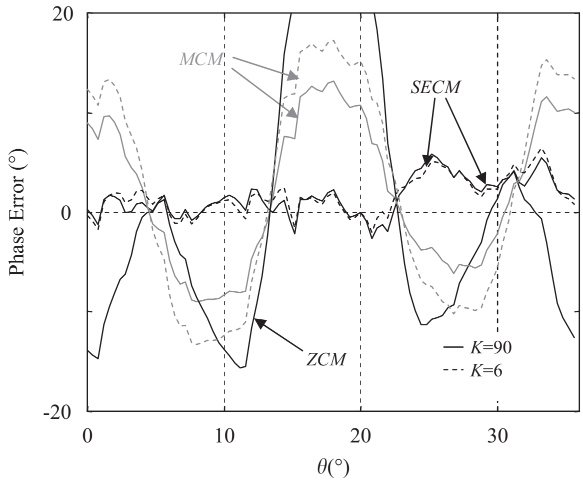

Fig. 9. Phase error of $a_{e_1}^{(V)}(\boldsymbol{\Theta})/\widetilde{a}_1^{(V)}(\boldsymbol{\Theta})$.

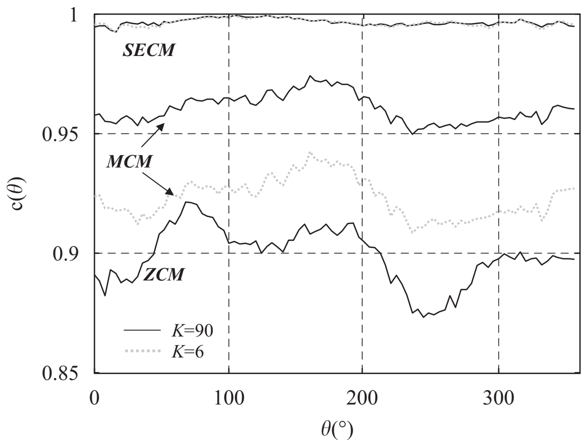

Fig. 10. Correlation function $c(\boldsymbol{\Theta})$.

We can notice that $c(\boldsymbol{\Theta})$ is a normalized function such that: $0 \leqslant c(\boldsymbol{\Theta}) \leqslant 1$. The coefficient $c(\boldsymbol{\Theta})$ must be near to one because $c(\boldsymbol{\Theta})$ satisfies $c(\boldsymbol{\Theta}) = 1$ when $a_{e_n}^{(V)}(\boldsymbol{\Theta}) = \widetilde{a}_n^{(V)}(\boldsymbol{\Theta})$.

Figs. 8, 9, and 10 emphasize the increment of performances taking coupling effects into account. The drastic diminution of the number of calibration measurements (only 6 measures instead of 90) significantly degrades the performances of the MCM, which has been implemented using the particular structure of $\mathbf{Z}_0$ in a similar way than for the SECM. On the contrary, the SECM is slightly affected by this diminution, showing the interest of the extended structured model. Let us define another distance between the models $\widetilde{\mathbf{a}}^{(H),(V)}(\boldsymbol{\Theta})$ and the exact steering vector $\mathbf{a}_e^{(H),(V)}(\boldsymbol{\Theta})$ in order to quantify the accuracy. Let $\Delta\varphi^{(H),(V)}$ and $\Delta\rho^{(H),(V)}$ be, respectively, the phase and amplitude error for the horizontal or vertical component of polarization given by

Table 1
Performances of the three models

	ZCM	MCM	SECM
$\Delta\varphi$ (deg)	17.44	8.85	3.41
$\Delta\rho$ (dB)	0.78	0.44	0.04

$$(\Delta\varphi^{(H),(V)})^2 = \frac{1}{KN}\sum_{n=1}^{N}\sum_{i=1}^{K}|\Delta\varphi_n^{(H),(V)}(\boldsymbol{\Theta}_i)|^2,$$

$$\Delta\varphi_n^{(H),(V)}(\boldsymbol{\Theta}_i) = \arg\left(\frac{a_{e_n}^{(H),(V)}(\boldsymbol{\Theta}_i)}{\widetilde{a}_n^{(H),(V)}(\boldsymbol{\Theta}_i)}\right), \tag{31}$$

$$\Delta\rho^{(H),(V)} = 10\log\left[\frac{1}{KN}\sum_{n=1}^{N}\sum_{i=1}^{K}|\Delta\rho_n^{(H),(V)}(\boldsymbol{\Theta}_i)|^2\right],$$

$$\Delta\rho_n^{(H),(V)}(\boldsymbol{\Theta}) = \left|\frac{a_{e_n}^{(H),(V)}(\boldsymbol{\Theta}_i)}{\widetilde{a}_n^{(H),(V)}(\boldsymbol{\Theta}_i)}\right|. \tag{32}$$

In Table 1, the errors of expressions (31) and (32) for the phase $\Delta\varphi$ and the amplitude $\Delta\rho$ are reported.

Table 1 shows that the best performance is obtained with the SECM model. Indeed, the estimated amplitude $|\gamma| = 0.19$ of the scattering coupling coefficient with the mast is in the same order than the mutual coupling parameters $|\alpha| = 0.21$ and $|\beta| = 0.1$: the influence of the mast cannot be neglected.

4. Application to bearing estimation

4.1. Adaptation of high-resolution method

High-resolution methods rely on the model of $\mathbf{x}(t)$ given by (1). In the general case, $\|\mathbf{a}_e(\boldsymbol{\Theta}, \boldsymbol{\Phi})\|$ depends on the directions $\boldsymbol{\Theta}$ and the polarization $\boldsymbol{\Phi}$. The covariance matrix of the noise $\mathbf{R}_{nn} = E[\mathbf{n}(t)\mathbf{n}(t)^\dagger]$ is equal to $\sigma^2\mathbf{I}_n$.

Using the steering vector $\widetilde{\mathbf{a}}(\boldsymbol{\Theta}, \boldsymbol{\Phi})$, the MUSIC algorithm [1] provides the M minima $(\boldsymbol{\Theta}_m, \boldsymbol{\Phi}_m)$ $(1 \leqslant m \leqslant M)$ of the following criterion:

$$J(\boldsymbol{\Theta}, \boldsymbol{\Phi}) = \frac{\widetilde{\mathbf{a}}^\dagger(\boldsymbol{\Theta}, \boldsymbol{\Phi})\boldsymbol{\Pi}_n\widetilde{\mathbf{a}}(\boldsymbol{\Theta}, \boldsymbol{\Phi})}{\widetilde{\mathbf{a}}^\dagger(\boldsymbol{\Theta}, \boldsymbol{\Phi})\widetilde{\mathbf{a}}(\boldsymbol{\Theta}, \boldsymbol{\Phi})}, \tag{33}$$

where the projector $\mathbf{\Pi}_n$ depends on the $(N-M)$ eigenvectors $\mathbf{e}_{M+i}$ $(1 \leqslant i \leqslant N-M)$ associated with the smallest eigenvalues of the covariance matrix $\mathbf{R}_{xx} = E[\mathbf{x}(t)\mathbf{x}(t)^{\dagger}]$: $\mathbf{\Pi}_n = \mathbf{E}_n\mathbf{E}_n^{\dagger}$ where $\mathbf{E}_n = [\mathbf{e}_{M+1} \cdots \mathbf{e}_N]$. Using the assumption that the polarization of all sources is $\mathbf{\Phi}_m = \mathbf{\Phi}$, we obtain the conventional MUSIC algorithm of [1]. In case of sources with different polarizations, vector $\widetilde{\mathbf{a}}(\theta, \Delta, \mathbf{\Phi})$ of Eq. (11) can be expressed as

$$\widetilde{\mathbf{a}}(\mathbf{\Theta}, \mathbf{\Phi}) = \mathbf{U}(\mathbf{\Theta})\mathbf{\Phi}, \tag{34}$$

where

$$\mathbf{U}(\mathbf{\Theta}) = [\widetilde{\mathbf{a}}^{(H)}(\mathbf{\Theta})\ \widetilde{\mathbf{a}}^{(V)}(\mathbf{\Theta})] \quad \text{and} \quad \mathbf{\Phi} = \begin{bmatrix} \Phi^{(H)} \\ \Phi^{(V)} \end{bmatrix}.$$

Inserting the expression (34) of $\widetilde{\mathbf{a}}(\mathbf{\Theta}, \mathbf{\Phi})$ in (33), we obtain the following ratio of quadratic forms:

$$J(\mathbf{\Theta}, \mathbf{\Phi}) = \frac{\mathbf{\Phi}^{\dagger}\mathbf{Q}_1(\mathbf{\Theta})\mathbf{\Phi}}{\mathbf{\Phi}^{\dagger}\mathbf{Q}_2(\mathbf{\Theta})\mathbf{\Phi}},$$

where

$$\mathbf{Q}_1(\mathbf{\Theta}) = \mathbf{U}(\mathbf{\Theta})^{\dagger}\mathbf{\Pi}_n\mathbf{U}(\mathbf{\Theta}) \quad \text{and}$$
$$\mathbf{Q}_2(\mathbf{\Theta}) = \mathbf{U}(\mathbf{\Theta})^{\dagger}\mathbf{U}(\mathbf{\Theta}). \tag{35}$$

The separate minimization of Eq. (35) with respect to the nuisance parameter $\mathbf{\Phi}$ leads to the following criterion as a function of (θ, Δ):

$$J_1(\mathbf{\Theta}) = \lambda_{\min}\{\mathbf{Q}_1(\mathbf{\Theta})\mathbf{Q}_2(\mathbf{\Theta})^{-1}\}, \tag{36}$$

where $\lambda_{\min}\{\mathbf{Q}\}$ is the smallest eigenvalue of the matrix $\mathbf{Q}$ and $J_1(\mathbf{\Theta}_m) = 0$ for $(1 \leqslant m \leqslant M)$.

4.2. Improvement analysis on the asymptotic DOA estimation

The MUSIC method is implemented with the three following steering vectors:

- ZCM: The steering vector of the model is the ideal vector $\mathbf{a}(\mathbf{\Theta})$ of expression (13) where the polarization $\mathbf{\Phi}$ and the matrix $\mathbf{U}(\mathbf{\Theta})$ of (34) satisfy:

  $$\mathbf{\Phi} = [1\ 0]^{\mathrm{T}}, \quad \mathbf{U}(\mathbf{\Theta}) = [\mathbf{a}(\mathbf{\Theta})\ \mathbf{0}].$$

- MCM: The steering vector of the model is the vector $\mathbf{Z}_0\mathbf{a}(\mathbf{\Theta})$ of expression (14) where the polarization $\mathbf{\Phi}$ and the matrix $\mathbf{U}(\mathbf{\Theta})$ satisfy

  $$\mathbf{\Phi} = [1\ 0]^{\mathrm{T}}, \quad \mathbf{U}(\mathbf{\Theta}) = [\mathbf{Z}_0\mathbf{a}(\mathbf{\Theta})\ \mathbf{0}].$$

- ECM: The steering vector of the model is the vector $\mathbf{\Phi}_1\mathbf{Z}^{(H)}\mathbf{b}^{(H)}(\mathbf{\Theta}) + \mathbf{\Phi}_2\mathbf{Z}^{(V)}\mathbf{b}^{(V)}(\mathbf{\Theta})$ of Eqs. (11)–(15) where the polarization $\mathbf{\Phi}_m$ and the matrix $\mathbf{U}(\mathbf{\Theta})$ satisfy

  $$\mathbf{U}(\mathbf{\Theta}) = [\mathbf{Z}^{(H)}\mathbf{b}^{(H)}(\mathbf{\Theta})\ \mathbf{Z}^{(V)}\mathbf{b}^{(V)}(\mathbf{\Theta})].$$

  We can distinguish two cases:
  - the nominal case (denoted SECM-N for the structured model) where

    $$\mathbf{\Phi}_1 = \cdots = \mathbf{\Phi}_M = \mathbf{\Phi} = [1\ 0]^{\mathrm{T}};$$

  - the diverse Polarization case where

    $$\mathbf{\Phi}_m = [\Phi^{(H)}\ \Phi^{(V)}]^{\mathrm{T}} \neq \mathbf{\Phi}.$$

The asymptotic performances of the DOA estimation methods are evaluated with the use of the steering vector $\widetilde{\mathbf{a}}^{(H),(V)}(\mathbf{\Theta})$ of the different models where $\mathbf{\Theta} = (\theta, \Delta = 0)$. The covariance matrix $\mathbf{R}_{xx}$ of the observation is built with the exact steering vectors of the calibration recording. Thus, in presence of M non-coherent sources, the asymptotic covariance matrix is according to (1) given by

$$\mathbf{R}_{xx} = \mathbf{A}_e\mathbf{R}_{ss}\mathbf{A}_e^{\dagger} + \sigma^2\mathbf{I}, \tag{37}$$

where the steering vectors of $\mathbf{A}_e = [\mathbf{a}_e(\mathbf{\Theta}_1, \mathbf{\Phi}_1) \cdots \mathbf{a}_e(\mathbf{\Theta}_M, \mathbf{\Phi}_M)]$ are provided by the exact steering vectors of the calibration recording, $\mathbf{R}_{ss}$ is the asymptotic covariance matrix of the sources vector $\mathbf{s}(t)$ and σ^2 is the noise power. Let us denote the estimated angles as $\widehat{\mathbf{\Theta}}_1 \ldots \widehat{\mathbf{\Theta}}_M$.

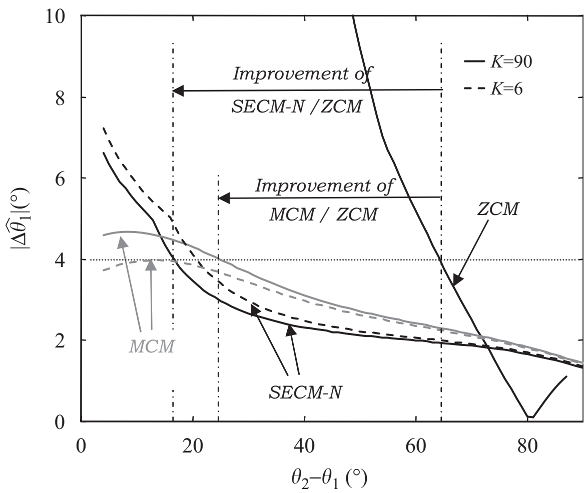

Fig. 11. MUSIC resolution performances with $\mathbf{\Theta}_1 = 19°$ and $\mathbf{\Phi}_1 = \mathbf{\Phi}_2 = \mathbf{\Phi} = [1\ 0]^{\mathrm{T}}$

In Fig. 11 the behavior of each model is compared in the case of equi-powered sources of azimuth $\theta_1 = 19^\circ$ and θ_2. The polarization of the sources are adapted to the dipole polarization and satisfy $\mathbf{\Phi}_1 = \mathbf{\Phi}_2 = \mathbf{\Phi} = [1\ 0]^{\mathrm{T}}$. The results of the nominal MUSIC algorithm with $\mathbf{\Phi} = [1\ 0]^{\mathrm{T}}$ are depicted in Fig. 11. The DOA θ_2 of the second source varied between θ_1 and 120°. For each value of $\theta_2 - \theta_1$ we plot the bias of $|\widehat{\theta}_1|$. We also measured the respective improvement of MCM and SECM-N relatively to ZCM in the case $K = 6$ (we arbitrarily chose an acceptable bias limit of 4°).

Fig. 11 shows that the DOA estimation methods have the best resolution when they use the SECM model that takes into account the coupling effect of the mast. Thus to provide accurate DOA estimates in the presence of scattering elements, the methods must use the steering vectors of the ECM model.

To complete this rough comparison, an investigation has now to be conducted for a finite number of snapshot in terms of bias and variance. But such an analysis is beyond the scope of this paper.

5. Conclusion

In this paper we have shown the benefits of introducing an extending coupling model (ECM) for the derivation of an analytical expression of the steering vectors. To build such an expression a set of calibration records is necessary. Using these data and the proposed parametric model, the ECM matrix is obtained via a MSE estimation. For this we have proposed an unstructured and a structured MSE estimation technique based on the prior information available on the RF system. In addition, the proposed method provides an alternative to [9–12] for the evaluation of the mutual-coupling coefficients.

Simulations confirm the improvement of ECM over MCM in terms of phase and amplitude errors. Indeed, the ECM model takes into account the coupling effect and the polarization properties of all the scattering elements in an experimental RF array.

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