ICASSP 2024 - 2024 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP) | 979-8-3503-4485-1/24/$31.00 ©2024 IEEE | DOI: 10.1109/ICASSP48485.2024.10447654

LEVERAGING TENSOR SUBSPACE PRIOR: ENHANCED SUM OF NUCLEAR NORM MINIMIZATION FOR TENSOR COMPLETION

Li Ge[⋆], *Xue Jiang*[⋆], *Lin Chen*[⋆], *Xingzhao Liu*[⋆], *and Martin Haardt*[†]

⋆ School of Electronic Information and Electrical Engineering,
Shanghai Jiao Tong University, Shanghai 200240, China
† Communications Research Laboratory, Ilmenau University of Technology, D-98684 Ilmenau, Germany

ABSTRACT

Tensor completion has attracted increasing attention in signal processing, computer vision, and biomedical engineering. By using nuclear norm minimization, a tensor completion problem can be converted into a convex program and enjoys properties gained from matrix completion. The low rank property has been widely used for tensor/matrix completion. However, the prior subspace information can also be utilized, which has been ignored and does not exhibit its full power in the existing formulation. In this paper, we propose a new framework leveraging tensor subspace prior for the sum of nuclear norm (SNN) minimization, which supports a range of tensor decompositions. By using the knowledge of the self-prior (SP)/nonself-prior (NSP) and further designing an efficient algorithm based on the Alternating Direction Method of Multipliers (ADMM), the performance of tensor completion can be enhanced. The superiority of the proposed method is verified by extensive numerical experiments.

Index Terms— Tensor completion, prior subspace information, tensor nuclear norm, SNN, ADMM

1. INTRODUCTION

Recovering data from partial observations has been studied for a long time. For one thing, a considerable progress has been made in matrix completion in the last decade. The organization of data by high dimensional matrices, i.e., tensors [1], provides more flexibility when dealing with color images/videos, hyperspectral data, communication signals, and EEG waveforms [2–4] and enjoys its analogy with matrices [5]. Thus, it is natural that tensor completion has received increasing attention in recent years.

Among the methods tackling tensor completion, nuclear norm minimization is an useful approach. Choosing a specific tensor decomposition, the tensor nuclear norm is defined as a convex relaxation of the tensor rank. A family among various tensor nuclear norms has the form as "sum of nuclear norms" (SNN) [6], which was first defined for the Tucker decomposition and was later considered for the low rank tensor completion (LRTC) in [7]. Later, nuclear norms defined by a tensor train (TT) decomposition [8] and a tensor ring (TR) decomposition [9] also resemble such a SNN formulation. In fact, TT and TR can be further generalized to the newly proposed fully-connected tensor network (FCTN) decomposition [10], which also has a SNN-like nuclear norm. It should also be noted that though theoretical guarantee of matrix nuclear norm minimization has been relatively well established [11, 12], the theories of tensor nuclear norm minimization still need further study.

In many applications, a portion of uncorrupted data is available and we are required to recover the others [13]. Fully exploiting such prior information of the column and row space can potentially enhance the recovery performance, which has already been verified in matrix recovery and matrix completion [14]. A multi-weight strategy is proposed in [15] to further enhance the flexibility of utilizing the subspace prior. In tensor completion, based on the t-product and the inherent low-tubal rank structure [16, 17], a priori information utilization specifically designed for three dimensional tensors is proposed in [18]. Inspired by these aforementioned works, a method leveraging tensor subspace prior for SNN minimization is proposed in this paper, which supports a range of tensor decomposition.

Our main contribution is two-fold. We propose a new framework that leverages tensor subspace prior for SNN minimization including SNN, TTNN, TRNN and FCTNNN, which yields sufficient flexibility. The proposed self-prior (SP) and nonself-prior (NSP) knowledge utilization promotes the recovery of the underlying low rank tensor structure. We have also designed an efficient algorithm by exploiting the Alternating Direction Method of Multipliers (ADMM) scheme to solve the modified framework of tensor completion incorporating subspace information.

The remainder of this paper is organized as follows. Section 2 introduces the notation and essential definitions. In Section 3, we introduce the new framework that leverages tensor subspace prior for SNN minimization. Section 4 presents an efficient algorithm based on the ADMM scheme to solve the new program. Section 5 provides numerical experiments and the conclusion is drawn in Section 6.

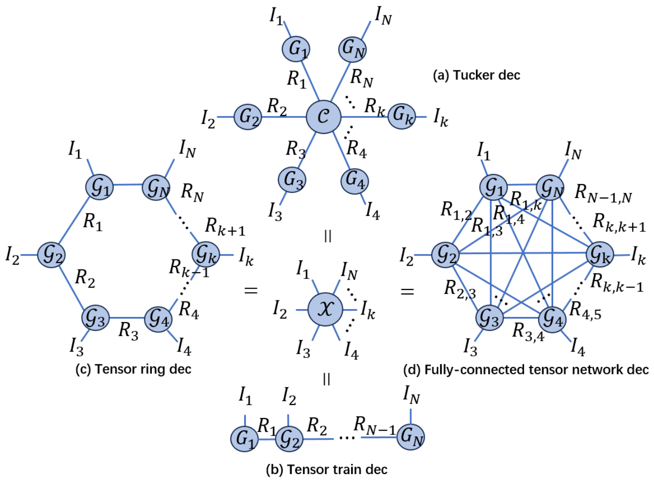

Fig. 1: tensor network representation [19] of Tucker, TT, TR and FCTN decomposition

2. PRELIMINARIES

The basic notation used in this paper is as follows: calligraphic, uppercase, and lowercase boldface letters denote tensors, matrices, and vectors, respectively; the Hermitian of the matrix $\mathbf{A}$ is written as $\mathbf{A}^{\mathrm{H}}$; $\|\cdot\|_*$ and $\|\cdot\|_{\mathrm{F}}$ denote the matrix nuclear norm and the Frobenius norm, respectively; $\Re\{\cdot\}$ and $\Im\{\cdot\}$ stand for taking the real and imaginary part of a complex number, vector or matrix, respectively.

Here we introduce some essential definitions to be used throughout the paper.

Definition 1 (Generalized Tensor Transposition) *Given a tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$ and a transposition $\mathbf{n}$ of $(1, 2, \cdots, N)$, the generalized tensor transposition of $\mathcal{X}$ can be attained as $\vec{\mathcal{X}}^{\mathbf{n}} = \textbf{permute}^1(\mathcal{X}, \mathbf{n}) \in \mathbb{C}^{I_{n_1} \times I_{n_2} \times \cdots \times I_{n_N}}$, the inverse operation is defined as $\mathcal{X} = \textbf{ipermute}\left(\vec{\mathcal{X}}^{\mathbf{n}}, \mathbf{n}\right)$*

Definition 2 (Generalized Tensor Unfolding) *Given a tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$, a transposition $\mathbf{n}$ of $(1, 2, \cdots, N)$, and a matricization step length d, the generalized tensor matricization / unfolding of $\mathcal{X}$ can be attained as $\mathbf{X}_{[\mathbf{n}_{1:d}, \mathbf{n}_{d+1:N}]} = \textbf{reshape}^1\left(\vec{\mathcal{X}}^{\mathbf{n}}, \prod_{i=1}^{d} I_{n_i}, \prod_{j=d+1}^{N} I_{n_j}\right) \in \mathbb{C}^{\prod_{i=1}^{d} I_{n_i} \times \prod_{j=d+1}^{N} I_{n_j}}$, the inverse operation is defined as $\mathcal{X} = \textbf{fold}\left(\mathbf{X}_{[\mathbf{n}_{1:d}, \mathbf{n}_{d+1:N}]}\right)$.*

Here we provide the FCTN unfoldings as an example, of which Tucker, TT, and TR unfoldings can be viewed as degraded cases. Details of these unfoldings can be found in [6, 8, 9] and the connection between these decompositions by tensor network graph is shown in Fig. 1.

Example 1 (Fully-connected Tensor Network Unfoldings) *In a fully-connected tensor network decomposition, the N-dimensional tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$ has $\binom{N}{\lfloor \frac{N}{2} \rfloor}$ unfoldings $\mathbf{X}_{[\mathbf{n}^k_{1:d_k}, \mathbf{n}^k_{d_k+1:N}]}$ by choosing any possible reordering vector $\mathbf{n}^k$ and perform unfolding as $\mathbf{X}_{[\mathbf{n}^k_{1:d_k}, \mathbf{n}^k_{d_k+1:N}]} = \textbf{reshape}\left(\vec{\mathcal{X}}^{\mathbf{n}^k}, \prod_{i=1}^{d_k} I_{n_i^k}, \prod_{j=d_k+1}^{N} I_{n_j^k}\right)$*

[1] Here the matlab command is used

Now we are ready to define the SNN-type tensor nuclear norm:

Definition 3 (SNN-type Tensor Nuclear Norm) *Given a tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times \cdots \times I_N}$, a set of specific matricizations $\{(\mathbf{n}^k, d_k)\}_{k=1}^{K}$, and the corresponding coefficients $\{\alpha_k\}_{k=1}^{K}$, the SNN-type of tensor nuclear norm can be defined as*

$$\sum_{k=1}^{K} \alpha_k \|\mathbf{X}_{[\mathbf{n}^k_{1:d_k}, \mathbf{n}^k_{d_k+1:N}]}\|_* \tag{1}$$

As can be observed from Def. 3, the tensor nuclear norms induced by the Tucker, TT, TR, and the recently proposed FCTN decompostion all inherit such a SNN-type of tensor nuclear norm.

3. EXPLOITING TENSOR SUBSPACE PRIOR FOR SNN MINIMIZATION

Consider that we have already chosen a specific set of unfoldings to tackle a tensor completion problem:

$$\begin{aligned} &\min\nolimits_{\mathcal{X}} \sum_{k=1}^{K} \alpha_k \|\mathbf{X}_{[k]}\|_* \\ &\text{s.t. } \mathcal{X}_\Omega = \mathcal{T}_\Omega \end{aligned} \tag{2}$$

where $\mathbf{X}_{[k]} \in \mathbb{C}^{p_k \times q_k}$ ($p_k = \prod_{i=1}^{d_k} I_{n_i^k}, q_k = \prod_{j=d_k+1}^{N} I_{n_j^k}$) is a simplified notation for $\mathbf{X}_{[\mathbf{n}^k_{1:d_k}, \mathbf{n}^k_{d_k+1:N}]}$, $\mathcal{T}$ stands for ground truth and Ω denotes the set of cell indices that can be observed.

Assume that we can access a tensor $\mathcal{X}_p$ of the same size as $\mathcal{X}$ (several strategies such as duplication and interpolation can be performed when their sizes do not match), which stands for the prior information at hand. Observing the matrix nuclear norm form contained in the SNN and inspired by the matrix subspace priori model [15], we can extract the subspace of $\mathcal{X}_p$ by applying singular value decompositions (SVDs) to its unfoldings $\mathbf{X}_{p[k]}, k = 1, 2, \cdots, K$.

Suppose that the SVD is given as $\mathbf{X}_{p[k]} = \mathbf{U}_k \mathbf{S}_k \mathbf{V}_k$, according to [15] we can use a linear combination of the subspace projectors upon $\mathbf{U}_k^{r_k}$/$\mathbf{V}_k^{r_k}$ [2] and upon $\mathbf{U}_k^{r_k\perp}$/$\mathbf{V}_k^{r_k\perp}$ [3] as

$$\begin{aligned} \mathcal{P}_{\mathbf{U}_k^{r_k}} &= \mathbf{U}_k^{r_k} \mathbf{\Lambda}_k \mathbf{U}_k^{r_k \mathrm{H}} + \mathbf{I}_k - \mathbf{U}_k^{r_k} \mathbf{U}_k^{r_k \mathrm{H}} \\ \mathcal{P}_{\mathbf{V}_k^{r_k}} &= \mathbf{V}_k^{r_k} \mathbf{\Gamma}_k \mathbf{V}_k^{r_k \mathrm{H}} + \mathbf{I}_k - \mathbf{V}_k^{r_k} \mathbf{V}_k^{r_k \mathrm{H}} \end{aligned} \tag{3}$$

where $\{\mathbf{\Lambda}_k\}_{k=1}^{K}, \{\mathbf{\Gamma}_k\}_{k=1}^{K}$ are diagonal weighting matrices. The values in the diagonal entries stand for the confidence of the corresponding subspace vectors and are constrained in [0, 1]. Intuitively, the more accurate the subspace estimate is, the closer the corresponding weight should be to 0.

[2] Superscript r stands for the first r column space of the matrix

[3] Superscript $\perp$ stands for the complement of the matrix column space

Utilizing prior information, problem (2) can be redefined as:

$$\begin{aligned}&\min_{\mathcal{X}} \sum_{k=1}^{K} \alpha_k \|\mathbf{L}_k \mathbf{X}_{[k]} \mathbf{R}_k\|_* \\ &\text{s.t. } \mathcal{X}_\Omega = \mathcal{T}_\Omega \\ &\mathbf{L}_k = \mathbf{U}_k^{r_k} \mathbf{\Lambda}_k \mathbf{U}_k^{r_k \mathrm{H}} + \mathbf{I}_k - \mathbf{U}_k^{r_k} \mathbf{U}_k^{r_k \mathrm{H}} \\ &\mathbf{R}_k = \mathbf{V}_k^{r_k} \mathbf{\Gamma}_k \mathbf{V}_k^{r_k \mathrm{H}} + \mathbf{I}_k - \mathbf{V}_k^{r_k} \mathbf{V}_k^{r_k \mathrm{H}}\end{aligned} \tag{4}$$

We have designed two methods that leverages the subspace prior information. When the prior information is available, we can directly construct the prior tensor $\mathcal{X}_p$ by aligning with the target tensor, which is referred as nonself-prior (abbr. NSP). Also note that the target tensor can be viewed as self-prior (SP), unsupervised subspace exploitation can be achieved by adding an small outer loop to problem (4). In each iteration $\mathcal{X}_p$ is set to $\mathcal{X}$ in the last iteration and $\mathcal{X}$ is obtained without prior information in the first iteration. The effectiveness of SP/NSP will be verified in the experiment part.

4. ALGORITHM FOR SOLVING TENSOR SUBSPACE PRIOR-AIDED SNN MINIMIZATION

Assuming that $\{\mathbf{\Lambda}_k\}_{k=1}^K$ and $\{\mathbf{\Gamma}_n\}_{k=1}^K$ are preset (the weights can also be changing during the iterations), we can use ADMM [20] to solve (4). First, we introduce auxiliary variables $\{\mathbf{M}_k\}_{k=1}^K$ and the corresponding Lagrangian multipliers $\{\mathbf{Y}_k\}_{k=1}^K$ to formulate the augmented Lagrangian function of (4)

$$\begin{aligned}&\mathcal{L}_{\mathcal{X}_\Omega = \mathcal{T}_\Omega}(\mathcal{X}, \{\mathbf{M}_k\}_{n=1}^K, \{\mathbf{Y}_k\}_{n=1}^K, \mu_1) = \sum_{n=1}^{N} \alpha_n \|\mathbf{L}_k \mathbf{M}_k \mathbf{R}_k\|_* \\ &+ \Re(\langle \mathbf{Y}_k, \mathbf{M}_k - \mathbf{X}_{[k]} \rangle) + \frac{\mu_1}{2} \|\mathbf{M}_k - \mathbf{X}_{[k]}\|_{\mathrm{F}}^2\end{aligned} \tag{5}$$

Next we solve each sub-problem.

- Computing $\{\mathbf{M}_k\}_{k=1}^K$

  Further introduce $\{\mathbf{A}_k\}_{k=1}^K$ and $\{\mathbf{Z}_k\}_{k=1}^K$,

$$\begin{aligned}\mathbf{M}_k = &\operatorname{argmin}_{\mathbf{M}_k, \mathbf{A}_k, \mathbf{Z}_k} \alpha_k \|\mathbf{A}_k\|_* \\ &+ \Re(\langle \mathbf{Y}_k, \mathbf{M}_k - \mathbf{X}_{[k]} \rangle) + \frac{\mu_1}{2} \|\mathbf{M}_k - \mathbf{X}_{[k]}\|_{\mathrm{F}}^2 \\ &+ \Re(\langle \mathbf{Z}_k, \mathbf{A}_k - \mathbf{L}_k \mathbf{M}_k \mathbf{R}_k \rangle) + \frac{\mu_2}{2} \|\mathbf{A}_k - \mathbf{L}_k \mathbf{M}_k \mathbf{R}_k\|_{\mathrm{F}}^2\end{aligned} \tag{6}$$

  - Computing $\mathbf{A}_k$

$$\begin{aligned}\mathbf{A}_k = &\operatorname{argmin}_{\mathbf{A}_k} \alpha_k \|\mathbf{A}_k\|_* \\ &+ \frac{\mu_2}{2} \|\mathbf{A}_k - \mathbf{L}_k \mathbf{M}_k \mathbf{R}_k + \frac{1}{\mu_2} \mathbf{Z}_k\|_{\mathrm{F}}^2\end{aligned} \tag{7}$$

    (7) has a closed form solution by singular value thresholding (SVT) [21]:

$$\mathbf{A}_k = \mathcal{D}_{\frac{\alpha_k}{\mu_2}}(\mathbf{L}_k \mathbf{M}_k \mathbf{R}_k + \frac{1}{\mu_2} \mathbf{Z}_k) \tag{8}$$

    where $\mathcal{D}_\tau(\cdot)$ is the SVT operator with threshold τ.

Algorithm 1: subspace prior-aided SNN minimization

```
Initialize X by linear interpolation;
Initialize {A_k}_{k=1}^K, {Y_k}_{k=1}^K, {Z_k}_{k=1}^K by all zeros;
while no convergence do
    update {α_k}_{k=1}^K by α_k = 1 / ||L_k X_[k] R_k||_*;
    for k = 1 to K do
        compute A_k by eq. (8);
        compute M_k by eq. (10) and eq. (11);
        compute Y_k by eq. (14);
        compute Z_k by eq. (12);
    end
    compute X by eq. (13);
end
```

  - Computing $\mathbf{M}_k$

$$\begin{aligned}\mathbf{M}_k = &\operatorname{argmin}_{\mathbf{M}_k} \frac{\mu_2}{2} \|\mathbf{A}_k - \mathbf{L}_k \mathbf{M}_k \mathbf{R}_k + \frac{1}{\mu_2} \mathbf{Z}_k\|_{\mathrm{F}}^2 \\ &+ \frac{\mu_1}{2} \|\mathbf{M}_k - \mathbf{X}_{[k]} + \frac{1}{\mu_1} \mathbf{Y}_k\|_{\mathrm{F}}^2\end{aligned} \tag{9}$$

    Note that $\mathbf{M}_k$ also has a closed-form solution, which satisfies the Sylvester equation after taking the derivatives of eq. (9):

$$\mathbf{A}_k^{syl} \mathbf{M}_k + \mathbf{M}_k \mathbf{B}_k^{syl} = \mathbf{C}_k^{syl} \tag{10}$$

    where

$$\begin{aligned}&\mathbf{A}_k^{syl} = \mu_1 (\mathbf{L}_k \mathbf{L}_k^{\mathrm{H}})^{-1}, \mathbf{B}_k^{syl} = \mu_2 \mathbf{R}_k \mathbf{R}_k^{\mathrm{H}}, \\ &\mathbf{C}_k^{syl} = (\mathbf{L}_k \mathbf{L}_k^{\mathrm{H}})^{-1} \cdot (\mu_1 (\mathbf{X}_{[k]} - \frac{1}{\mu_1} \mathbf{Y}_k) \\ &\quad + \mu_2 \mathbf{L}_k^{\mathrm{H}} (\mathbf{A}_k + \frac{1}{\mu_2} \mathbf{Z}_k) \mathbf{R}_k^{\mathrm{H}})\end{aligned} \tag{11}$$

    Obtaining a solution for (10) can be faster than solving a general Sylvester equation by noting that $\mathbf{A}_k^{syl}$ and $\mathbf{B}_k^{syl}$ remain fixed between iterations. Also the structure of $\mathbf{L}_k$ saves us from computing the inversions in $\mathbf{A}_k^{syl}, \mathbf{C}_k^{syl}$ while computing common matrix multiplications instead. Such observations can be beneficial to computational efficiency.

  - Computing $\mathbf{Z}_k$ – The Lagrangian $\mathbf{Z}_k$ can be computed as:

$$\mathbf{Z}_k = \mathbf{Z}_k + \mu_2 (\mathbf{A}_k - \mathbf{L}_k \mathbf{M}_k \mathbf{R}_k) \tag{12}$$

- Computing $\mathcal{X}$

$$\begin{aligned}\mathcal{X} = &\operatorname{argmin}_{\mathcal{X}_\Omega = \mathcal{T}_\Omega} \sum_{k=1}^{K} \|\mathbf{X}_{[k]} - \mathbf{M}_k - \frac{1}{\mu_1} \mathbf{Y}_k\|_{\mathrm{F}}^2 \\ = &\mathcal{T}_\Omega + \left(\frac{1}{K} \mathbf{fold}_k (\mathbf{M}_k + \frac{1}{\mu_1} \mathbf{Y}_k) \right)_{\Omega^c}\end{aligned} \tag{13}$$

- Computing $\mathbf{Y}_k$– The Lagrangian $\mathbf{Y}_k$ can be computed as:

$$\mathbf{Y}_k = \mathbf{Y}_k + \mu_1 (\mathbf{M}_k - \mathbf{X}_{[k]}) \tag{14}$$

The full procedures are summarized in Algorithm 1. Assuming T iterations in all, the time complexity of Algorithm 1 is $\mathcal{O}(\sum_{k=1}^K p_k q_k (p_k + q_k) + T \cdot (\sum_{k=1}^K p_k q_k (p_k + q_k)))$ and the first term denotes the preprocessing complexity of $\mathcal{X}_p$, compared to $\mathcal{O}(T \cdot (\sum_{k=1}^K p_k q_k \min(p_k, q_k)))$, which is the time complexity for SNN minimization without prior information.

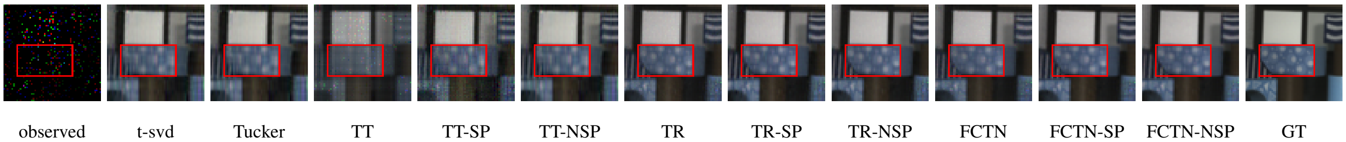

Fig. 2: Recovery results of the 1st frame of HSV with SR=0.05, band 8, 9, 10 are picked as R, G, B channels, respectively.

Table 1: mPSNR and mSSIM on CVs and HSV.

test video	SR	t-svd	Tucker	TT	TT-SP	TT-NSP	TR	TR-SP	TR-NSP	FCTN	FCTN-SP	FCTN-NSP
Shopping Mall	0.05	21.0562/0.5741	21.7537/0.6465	19.9115/0.5170	22.8134/0.5684	26.6930/0.7926	23.1046/0.6782	25.2294/0.7426	25.1273/0.7010	22.5960/0.6385	25.3865/0.7421	**27.9624/0.8460**
	0.1	25.7541/0.8077	24.1822/0.7446	23.0162/0.6491	27.0146/0.7877	28.0677/0.8223	26.2612/0.8099	28.0655/0.8518	27.3447/0.8372	25.7845/0.7793	28.5268/0.8573	**30.0200/0.8929**
Escalator	0.05	18.5506/0.7344	19.2578/0.7515	14.0346/0.3340	15.7847/0.4073	20.2290/0.7609	18.3747/0.6443	19.0279/0.6609	20.3923/0.7619	17.5408/0.5696	19.2438/0.6687	**21.7882/0.7991**
	0.1	21.8709/0.8515	20.2335/0.7937	16.6762/0.5011	18.1916/0.5855	21.6795/0.8030	21.3427/0.8017	22.4637/0.8100	22.7770/0.8614	20.5752/0.7544	22.5768/0.8284	**24.1028/0.8740**
HSV	0.05	29.499/0.9088	25.4651/0.8788	21.3141/0.7370	26.7266/0.8490	29.4439/0.9090	31.9739/0.9636	33.4652/0.9669	33.38/0.9718	31.6311/0.9578	34.7992/0.9771	**34.8775/0.9788**
	0.1	33.9263/0.9668	28.259/0.9238	27.0682/0.8914	29.5353/0.9031	32.0230/0.9474	37.2084/0.9865	38.8814/0.9897	37.7029/0.9875	37.6681/0.9873	**39.3964/0.9902**	39.2227/0.9901

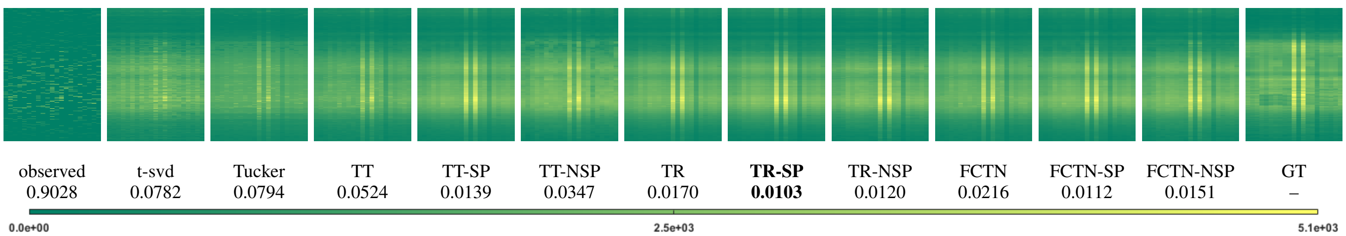

Fig. 3: Recovery results of traffic flow dataset on the 3rd day with SR=0.1. The subcaption of each image lists NMSE value.

5. EXPERIMENTS

To verify the proposed SP and NSP knowledge utilization, experiments have been conducted on two video datasets and a traffic flow dataset. Duplications are performed to attain prior tensor $\mathcal{X}_p$ in NSP when sizes mismatch. For parameter setting, α_k is adaptively set to $\frac{1}{\|\mathbf{L}_k \mathbf{X}_{[k]} \mathbf{R}_k\|_*}$ as suggested in [22]; singular vectors containing 85% energy are preserved for $\mathbf{U}_k^{r_k}$/$\mathbf{V}_k^{r_k}$; the diagonal entries in $\mathbf{\Lambda}_k$/$\mathbf{\Gamma}_k$ are set to $\{0.05, 0.03, 0.02\}$ sequentially in SP and are set to 0.1 in NSP; $\mu_1 = \mu_2 = 10^{-2}$ and the program is deemed converged when the normalized energy change of adjacent $\mathcal{X}$ is less than 10^{-7}.

color and hyperspectral video impainting We have conducted experiments to complete two color videos (CVs) *Shopping Mall* and *Escalator*[4] of size $128 \times 160 \times 3 \times 30$ (height × width × channel × frame) from the I2R dataset [23], and a hyperspectral video[5] (HSV) of size $60 \times 60 \times 20 \times 10$ (height × width × band × frame) under sampling rates (SR) of 0.05 and 0.1. The sampling scheme is i.i.d. Bernoulli sampling and the metrics are mean peak signal to noise ratio (mPSNR) and mean structural index similarity (mSSIM) of all test frames.

We have included t-svd [16], Tucker [6], TT [8], TR [9], FCTN [10] for comparison and applied the subspace technique onto TT, TR and FCTN as Tucker unfoldings are usually highly unbalanced. NSP takes 10 frames beforehand as prior in each video. Table. 1 lists the indicators and visual results of HSV recovery are given in Fig. 2. It can be observed from Table. 1 and Fig. 2 that both NSP and SP utilization can effectively enhance the performances.

traffic data completion The test data from the GTL traffic flow dataset[6] has been collected by sensors from 21 road segments for 10 days with intervals of 5 minutes thus can be organized as a tensor 12 x 25 x 10 x 21(minute × hour × day × segment). Let us consider a practical scenario: the sensors are deactivated most of the time and are only turned on during a small amount of time in order to save energy, which forms the need of tensor completion. Also note that these 21 segments are from one main road and the traffic flow contains regularities in the time domain, which indicates a low rank tensor completion problem. We set the duty ratio as 0.1 and assume clean data for 5 days before the test data can be obtained as supervised subspace information. We adopt the normalized mean squared error (NMSE) as the performance criterion and the result is listed in Fig. 3. As can be observed, the usage of subspace prior information greatly improves the completion accuracy.

6. CONCLUSION

In this paper, a method that leverages a tensor subspace prior for SNN minimization is proposed, which yields sufficient flexibility. We have also designed an efficient algorithm based on ADMM to solve the modified framework of tensor completion that incorporates subspace information. Extensive numerical experiments have been conducted to validate the superiority of the proposed method.

[4] *Escalator* is augmented from a greyscale video.

[5] The data is available at http://openremotesensing.net/kb/data/.

[6] The data is available at http://gtl.inrialpes.fr/.

7. REFERENCES

[1] T. G. Kolda and B. W. Bader, "Tensor decompositions and applications," *SIAM Rev.*, vol. 51, no. 3, pp. 455–500, 2009.

[2] R. Dian, S. Li, and L. Fang, "Learning a low tensor-train rank representation for hyperspectral image super-resolution," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 9, pp. 2672–2683, 2019.

[3] F. Roemer and M. Haardt, "Tensor-based channel estimation and iterative refinements for two-way relaying with multiple antennas and spatial reuse," *IEEE Trans. Signal Process.*, vol. 58, no. 11, pp. 5720–5735, 2010.

[4] H. Becker, L. Albera, P. Comon, M. Haardt, G. Birot, F. Wendling, M. Gavaret, C. G. Bénar et al., "EEG extended source localization: tensor-based vs. conventional methods," *NeuroImage*, vol. 96, pp. 143-157, Aug. 2014.

[5] L. Chen, X. Jiang, X. Liu, and Z. Zhou, "Logarithmic norm regularized low-rank factorization for matrix and tensor completion," *IEEE Trans. Image Process.*, vol. 30, pp. 3434–3449, 2021.

[6] J. Liu, P. Musialski, P. Wonka, and J. Ye, "Tensor completion for estimating missing values in visual data," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 35, no. 1, pp. 208–220, 2013.

[7] C. Mu, B. Huang, J. Wright, and D. Goldfarb, "Square deal: Lower bounds and improved relaxations for tensor recovery," in *Proc. Int. Conf. Mach. Learn.*, 2014, pp. 73–81.

[8] I. V. Oseledets, "Tensor-train decomposition," *SIAM J. Sci. Comput.*, vol. 33, no. 5, pp. 2295–2317, 2011.

[9] Q. Zhao, G. Zhou, S. Xie, L. Zhang, and A. Cichocki, "Tensor ring decomposition," *arXiv preprint arXiv:1606.05535*, 2016.

[10] Y.-B. Zheng, T.-Z. Huang, X.-L. Zhao, Q. Zhao, and T.-X. Jiang, "Fully-connected tensor network decomposition and its application to higher-order tensor completion," in *Proc. AAAI Conf. Artif. Intell.*, 2021, pp. 11071-11078.

[11] E. J. Candès and Y. Plan, "Matrix Completion With Noise," in *Proc. IEEE*, vol. 98, no. 6, pp. 925-936, 2010.

[12] Y. Chen, "Incoherence-Optimal Matrix Completion," *IEEE. Trans. Inf. Theory*, vol. 61, no. 5, pp. 2909-2923, 2015.

[13] N. Vaswani and W. Lu, "Modified-cs: Modifying compressive sensing for problems with partially known support," *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4595–4607, 2010.

[14] A. Eftekhari, D. Yang, and M. B. Wakin, "Weighted matrix completion and recovery with prior subspace information," *IEEE Trans. Inf. Theory*, vol. 64, no. 6, pp. 4044–4071, 2018.

[15] H. S. F. Ardakani, S. Daei, and F. Haddadi, "Multi-weight nuclear norm minimization for low-rank matrix recovery in presence of subspace prior information," *IEEE Trans. Signal Process.*, vol. 70, pp. 3000–3010, 2022.

[16] C. Lu, J. Feng, Y. Chen, W. Liu, Z. Lin, and S. Yan, "Tensor robust principal component analysis with a new tensor nuclear norm," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 42, no. 4, pp. 925–938, 2020.

[17] L. Chen, X. Jiang, X. Liu, and Z. Zhou, "Robust low-rank tensor recovery via nonconvex singular value minimization," *IEEE Trans. Image Process.*, vol. 29, pp. 9044–9059, 2020.

[18] F. Zhang, J. Wang, W. Wang, and C. Xu, "Low-tubal-rank plus sparse tensor recovery with prior subspace information," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 43, no. 10, pp. 3492–3507, 2021.

[19] A. Cichocki, "Tensor networks for big data analytics and large-scale optimization problems," *arXiv preprint arXiv:1407.3124*, 2014.

[20] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends in Mach. Learn.*, vol. 3, no. 1, pp. 1–122, 2011.

[21] J.-F. Cai, E. J. Candès, and Z. Shen, "A singular value thresholding algorithm for matrix completion," *SIAM J. Optimization*, vol. 20, no. 4, pp. 1956–1982, 2010.

[22] J. Xue, Y. Zhao, W. Liao, J. C.-W. Chan, and S. G. Kong, "Enhanced sparsity prior model for low-rank tensor completion," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 11, pp. 4567–4581, 2020.

[23] L. Li, W. Huang, I. Y.-H. Gu, and Q. Tian, "Statistical modeling of complex backgrounds for foreground object detection," *IEEE Trans. Image Process.*, vol. 13, no. 11, pp. 1459–1472, 2004.