

# Blind Reduced-rank Receiver with Column Adaptation for DS-UWB Systems Based on Joint Iterative Optimization and CCM Criterion

Sheng Li and Rodrigo C. de Lamare

Communications Research Group, Department of Electronics, University of York, UK, YO10 5DD

Email: {sl546,rcdl500}@ohm.york.ac.uk

**Abstract**—A novel linear blind reduced-rank receiver based on the Joint Iterative Optimization (JIO) and the constrained constant modulus (CCM) design criterion is proposed for interference suppression in direct-sequence ultra-wideband (DS-UWB) systems. The proposed receiver consists of a projection matrix that performs dimensionality reduction and a reduced-rank filter that produces the output. The columns of the projection matrix and the reduced-rank filter are updated jointly at each time instant or iteration to minimize the CM cost function subject to a constraint. Recursive least-squares (RLS) algorithms are developed for the adaptive implementation. Simulation results show that the proposed scheme has excellent performance in suppressing the inter-symbol interference (ISI) and multiple access interference (MAI) with a low complexity.

## I. INTRODUCTION

UWB technology [1],[2] is a short-range wireless communication technique that can achieve very high data rates. For DS-UWB systems, a high degree of diversity is available at the receiver because of the large number of resolvable multipath components (MPCs) [3]. In multiuser scenarios, receivers are required to efficiently suppress the ISI and the MAI.

Blind adaptive linear receivers [4]-[6] are promising techniques for interference suppression as they offer higher spectrum efficiency than the adaptive schemes that require training sequence. Low-complexity blind receiver designs can be obtained by solving constrained optimization problems based on the CCM or constrained minimum variance (CMV) criterion [6],[7]. The blind receiver designs based on the CCM criterion have shown better performance and increased robustness against signature mismatch over the CMV approaches [6]. Recently, blind full-rank adaptive filters based on constrained optimization have been proposed for multiuser detection in DS-UWB communications [7],[8]. For DS-UWB systems in which the received signal length is large due to the long channel delay spread, the interference sensitive full-rank adaptive schemes experience slow convergence rate. In the large filter scenarios, the reduced-rank algorithms can be adopted to accelerate the convergence and provide an increased robustness against interference and noise [9]-[14].

The well-known reduced-rank techniques include the eigen-decomposition methods such as the principal components (PC) [9] and the cross-spectral metric (CSM) [10], the Krylov subspace methods such as the powers of R (POR), the multistage Wiener filter (MSWF) [11] and the auxiliary vector filtering (AVF) [12]. Eigen-decomposition methods are based on the eigen-decomposition of the estimated covariance matrix of the received signal. These methods have very high computational

complexity and the performance is often poor in heavily loaded communication systems [11]. Compared with the full-rank linear filtering techniques, the MSWF and AVF methods have faster convergence speed with a small filter size.

In this work, we propose a novel CCM based blind reduced-rank scheme, named JIO with column adaptation (JIO-CA), in which a projection matrix is updated column by column with a reduced-rank filter to minimize the CM cost function subject to a constraint. The RLS adaptive algorithms are developed for the joint and iterative adaptation and an approximation is devised to achieve complexity reduction. We remark that the proposed reduced-rank scheme allows information exchange between the projection matrix and the reduced-rank filter. This feature leads to a more efficient adaptive implementation than the existing reduced-rank schemes.

The rest of this paper is structured as follows. Section II presents the DS-UWB system model. The design of the JIO CCM blind receiver is detailed in Section III. The proposed blind JIO-CA scheme with the RLS designs are described in Section IV. Simulation results are shown in Section V and conclusions are drawn in Section VI.

## II. DS-UWB SYSTEM MODEL

In this work, we consider the uplink of a synchronous binary phase-shift keying (BPSK) DS-UWB system with  $K$  users. For the  $k$ -th user, a random spreading code  $\mathbf{s}_k$  is assigned. The spreading gain is  $N_c = T_s/T_c$ , where  $T_s$  and  $T_c$  denote the symbol duration and chip duration, respectively. The transmit signal of the  $k$ -th user (where  $k = 1, 2, \dots, K$ ) is given by

$$x^{(k)}(t) = \sqrt{E_k} \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_c-1} p_t(t - iT_s - jT_c) s_k(j) b_k(i), \quad (1)$$

where  $b_k(i) \in \{\pm 1\}$  denotes the BPSK symbol for the  $k$ -th user at the  $i$ -th time instant,  $s_k(j)$  denotes the  $j$ -th chip of the spreading code  $\mathbf{s}_k$ .  $E_k$  denotes the transmission energy.  $p_t(t)$  is the pulse waveform of width  $T_c$ . Throughout this paper, the pulse waveform  $p_t(t)$  is modeled as the root-raised cosine (RRC) pulse with a roll-off factor of 0.5 [15]. The channel model considered is the IEEE 802.15.4a standard channel model for the indoor residential non-line of sight (NLOS) environment [16]. It should be noted that this standard channel model is valid for both low-data-rate and high-data-rate UWB systems [16]. For the  $k$ -th users, the channel impulse response

(CIR) of the standard channel model can be expressed as

$$h_k(t) = \sum_{u=0}^{L_c-1} \sum_{v=0}^{L_r-1} \alpha_{u,v} e^{j\phi_{u,v}} \delta(t - T_u - T_{u,v}), \quad (2)$$

where  $L_c$  denotes the number of clusters,  $L_r$  is the number of multipath components (MPCs) in one cluster.  $\alpha_{u,v}$  is the fading gain of the  $v$ -th MPC in the  $u$ -th cluster,  $\phi_{u,v}$  is uniformly distributed in  $[0, 2\pi)$ .  $T_u$  is the arrival time of the  $u$ -th cluster and  $T_{u,v}$  denotes the arrival time of the  $v$ -th MPC in the  $u$ -th cluster. For the sake of simplicity, we using the expression

$$h_k(t) = \sum_{l=0}^{L-1} h_{k,l} \delta(t - lT_\tau), \quad (3)$$

where  $h_{k,l}$  and  $lT_\tau$  present the complex-valued fading factor and the arrival time of the  $l$ -th MPC ( $l = uL_c + v$ ), respectively.  $L = T_{DS}/T_\tau$  denotes the total number of MPCs where  $T_{DS}$  is the channel delay spread. Assuming that the timing is acquired, the received signal can be expressed as

$$z(t) = \sum_{k=1}^K \sum_{l=0}^{L-1} h_{k,l} x^{(k)}(t - lT_\tau) + n(t),$$

where  $n(t)$  is the additive white Gaussian noise (AWGN) with zero mean and a variance of  $\sigma_n^2$ . At the receiver, this signal is first passed through a chip-matched filter and then sampled at the chip rate. A total number of  $M = (T_s + T_{DS})/T_c$  observation samples is selected for the detection of each data bit, where  $T_{DS}$  is the channel delay spread. Assuming the sampling starts at the zero-th time instant, then the  $m$ -th sample can be expressed as  $r_m = \int_{mT_c}^{(m+1)T_c} z(t) p_r(t) dt$ ,  $m = 1, 2, \dots, M$  where  $p_r(t) = p_r^*(-t)$  denotes the chip-matched filter and  $(\cdot)^*$  denotes the complex conjugation. After the chip-rate sampling, the discrete-time received signal for the  $i$ -th data bit can be expressed as  $\mathbf{r}(i) = [r_1(i), r_2(i), \dots, r_M(i)]^T$ , where  $(\cdot)^T$  is the transposition and we can further express it in a matrix form

$$\begin{aligned} \mathbf{r}(i) &= \sum_{k=1}^K \mathbf{P}_r \mathbf{H}_k \mathbf{P}_t \mathbf{s}_k b_k(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i), \\ &= \sum_{k=1}^K \mathbf{P}_r \mathbf{S}_{e,k} \mathbf{h}_k b_k(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i), \end{aligned} \quad (4)$$

where  $\mathbf{H}_k$  is the Toeplitz channel matrix for the  $k$ -th user with the first column being the CIR  $\mathbf{h}_k = [h_k(0), h_k(1), \dots, h_k(L-1)]^T$  zero-padded to length  $M_H = (T_s/T_\tau) + L - 1$ . The  $M$ -by- $M_H$  matrix  $\mathbf{P}_r$  represents the MF and chip-rate sampling.  $\mathbf{P}_t$  denotes the  $(T_s/T_\tau)$ -by- $N_c$  pulse shaping matrix.  $\mathbf{S}_{e,k}$  is the Toeplitz matrix with the first column being the vector  $\mathbf{s}_{e,k} = \mathbf{P}_t \mathbf{s}_k$  zero-padded to length  $M_H$ . The vector  $\boldsymbol{\eta}(i)$  denotes the ISI from  $2G$  adjacent symbols, where  $G$  denotes the minimum integer that is larger than or equal to the scalar term  $T_{DS}/T_s$ .

In order to estimate the transmitted signal from the noisy received signal, a blind receiver which only requires the spreading code of the desired user can be implemented in a

synchronized system. In what follows, we describe the design of the blind JIO reduced-rank receiver.

### III. PROPOSED BLIND JIO REDUCED-RANK RECEIVER DESIGN

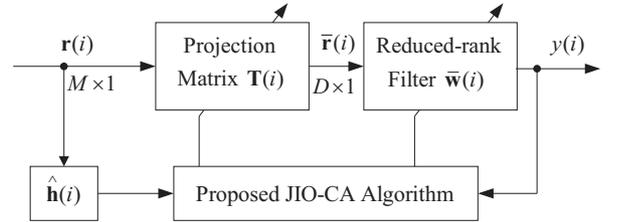


Fig. 1. Block diagram of the proposed blind reduced-rank receiver.

The block diagram of the proposed JIO reduced-rank receiver is shown in Fig.1. In the proposed receiver, the reduced-rank received signal is given by

$$\bar{\mathbf{r}}(i) = \mathbf{T}^H(i) \mathbf{r}(i), \quad (5)$$

where  $\mathbf{T}(i)$  is the  $M$ -by- $D$  (where  $D \ll M$ ) projection matrix. After the projection,  $\bar{\mathbf{r}}(i)$  is fed into the reduced-rank filter  $\bar{\mathbf{w}}(i)$  and the output signal is given by  $y(i) = \bar{\mathbf{w}}^H(i) \bar{\mathbf{r}}(i)$ . The decision of the desired data symbol is defined as  $\hat{b}(i) = \text{sign}(\Re[y(i)])$ , where  $\text{sign}(\cdot)$  is the algebraic sign function and  $\Re(\cdot)$  represents the real part of a complex number.

The optimization problem to be solved can be expressed as

$$[\bar{\mathbf{w}}(i), \mathbf{T}(i)] = \arg \min_{\bar{\mathbf{w}}(i), \mathbf{T}(i)} \frac{1}{2} E [ (|y(i)|^2 - 1)^2 ], \quad (6)$$

subject to the constraint  $\bar{\mathbf{w}}^H(i) \mathbf{T}^H(i) \mathbf{p} = v$ , where  $\mathbf{p} = \mathbf{P}_r \mathbf{S}_{e,k} \mathbf{h}_k$  is defined as the effective signature vector for the desired user and  $v$  is a real-valued constant to ensure the convexity of the cost function.

Let us now consider the problem through the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{JIO}}(\bar{\mathbf{w}}(i), \mathbf{T}(i)) &= \frac{1}{2} E [ (|y(i)|^2 - 1)^2 ] + \Re[\lambda(i) (\bar{\mathbf{w}}^H(i) \mathbf{T}^H(i) \mathbf{p} - v)], \end{aligned} \quad (7)$$

where  $\lambda(i)$  is a complex-valued Lagrange multiplier. In order to obtain the adaptation equation of  $\mathbf{T}(i)$ , we firstly assume that  $\bar{\mathbf{w}}(i)$  is fixed and the gradient of the Lagrangian with respect to  $\mathbf{T}(i)$  is given by

$$\nabla_{\mathbf{T}} \mathcal{L}_{\text{JIO}} = E [ e(i) y^*(i) \mathbf{r}(i) \bar{\mathbf{w}}^H(i) ] + \frac{\lambda_T(i)}{2} \mathbf{p} \bar{\mathbf{w}}^H(i), \quad (8)$$

where  $\lambda_T(i)$  is the complex-valued Lagrange multiplier for updating the projection matrix and  $e(i) = |y(i)|^2 - 1$  is defined as a real-valued error signal. Recalling the relationship  $y^*(i) = \mathbf{r}^H(i) \mathbf{T}(i) \bar{\mathbf{w}}(i)$  and setting (8) to a zero matrix, we obtain

$$\mathbf{T}_{\text{opt}} = \mathbf{R}_Y^{-1} \left( \mathbf{D}_T - \frac{\lambda_T(i)}{2} \mathbf{p} \bar{\mathbf{w}}^H(i) \right) \mathbf{R}_w^{-1}, \quad (9)$$

where  $\mathbf{R}_Y = E[|y(i)|^2 \mathbf{r}(i) \mathbf{r}^H(i)]$ ,  $\mathbf{D}_T = E[y^*(i) \mathbf{r}(i) \bar{\mathbf{w}}^H(i)]$  and  $\mathbf{R}_w = E[\bar{\mathbf{w}}(i) \bar{\mathbf{w}}^H(i)]$ . Using the constraint

$\bar{\mathbf{w}}^H(i)\mathbf{T}_{\text{opt}}^H\mathbf{p} = v$ , we obtain the Lagrange multiplier

$$\lambda_T(i) = 2 \left( \frac{\bar{\mathbf{w}}^H(i)\mathbf{R}_w^{-1}\mathbf{D}_T\mathbf{R}_Y^{-1}\mathbf{p} - v}{\bar{\mathbf{w}}^H(i)\mathbf{R}_w^{-1}\bar{\mathbf{w}}(i)\mathbf{p}^H\mathbf{R}_Y^{-1}\mathbf{p}} \right)^*. \quad (10)$$

Assuming that  $\mathbf{T}(i)$  is fixed, the gradient of the Lagrangian with respect to  $\bar{\mathbf{w}}(i)$  is obtained as

$$\nabla_w \mathcal{L}_{\text{JIO}} = E[e(i)\mathbf{T}^H(i)\mathbf{r}(i)y^*(i)] + \frac{\lambda_w(i)}{2}\mathbf{T}^H(i)\mathbf{p}, \quad (11)$$

where  $\lambda_w(i)$  is the complex-valued Lagrange multiplier for updating the reduced-rank filter. Rearranging the terms, we obtain

$$\bar{\mathbf{w}}_{\text{opt}} = \mathbf{R}_{\bar{y}}^{-1} \left( \mathbf{d}_{\bar{r}} - \frac{\lambda_w(i)}{2}\mathbf{T}^H(i)\mathbf{p} \right), \quad (12)$$

where  $\mathbf{R}_{\bar{y}} = E[|y(i)|^2\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)]$  and  $\mathbf{d}_{\bar{r}} = E[y^*(i)\bar{\mathbf{r}}(i)]$ . Using the constraint  $\bar{\mathbf{w}}_{\text{opt}}^H\mathbf{T}^H(i)\mathbf{p} = v$ , we obtain the Lagrange multiplier

$$\lambda_w(i) = 2 \left( \frac{\mathbf{d}_{\bar{r}}^H\mathbf{R}_{\bar{y}}^{-1}\mathbf{T}^H(i)\mathbf{p} - v}{\mathbf{p}^H\mathbf{T}(i)\mathbf{R}_{\bar{y}}^{-1}\mathbf{T}^H(i)\mathbf{p}} \right)^*. \quad (13)$$

It should be noted that the blind JIO reduced-rank receiver design requires the knowledge of the effective signature vector of the desired user. Here, we employ the variant of the power method introduced in [17] to estimate the channel as

$$\hat{\mathbf{h}}(i) = \left( \mathbf{I} - \hat{\mathbf{V}}(i)/\text{tr}[\hat{\mathbf{V}}(i)] \right) \hat{\mathbf{h}}(i-1), \quad (14)$$

where  $\hat{\mathbf{V}}(i) = \mathbf{S}_e^H\mathbf{P}_r^H\mathbf{R}^{-m}(i)\mathbf{P}_r\mathbf{S}_e$ ,  $\mathbf{R}(i)$  is the correlation matrix of the received signal and  $m$  is a finite power.  $\mathbf{I}$  is the identity matrix,  $\text{tr}[\cdot]$  stands for trace and we make  $\hat{\mathbf{h}}(i) \leftarrow \hat{\mathbf{h}}(i)/\|\hat{\mathbf{h}}(i)\|$  to normalize the channel. Hence, the estimate of the signature vector can be obtained as  $\hat{\mathbf{p}}(i) = \mathbf{P}_r\mathbf{S}_e\hat{\mathbf{h}}(i)$ , where  $\hat{\mathbf{h}}(i)$  is given in (14). In terms of computational complexity, the blind channel estimate in (14) requires  $O(L^2)$  operations.

#### IV. PROPOSED BLIND JIO-CA LINEAR REDUCED-RANK RECEIVER DESIGN AND RLS ALGORITHMS

The  $M$ -by- $D$  projection matrix can be expressed as  $\mathbf{T}(i) = [\mathbf{t}_1(i), \mathbf{t}_2(i), \dots, \mathbf{t}_D(i)]$  and the reduced-rank received signal is given by  $\bar{\mathbf{r}}(i) = \mathbf{T}^H(i)\mathbf{r}(i)$ , whose  $d$ -th element is  $\bar{r}_d(i) = \mathbf{t}_d^H(i)\mathbf{r}(i)$ . Since the projection matrix projects the received signal onto a small-dimensional subspace, these vectors  $\mathbf{t}_d(i)$  can be considered as the direction vectors on each dimension of the subspace. For each time instant, we compute these  $M$ -dimensional vectors  $\mathbf{t}_d(i)$  (where  $d = 1, 2, \dots, D$ ) one by one. After the projection,  $\bar{\mathbf{r}}(i)$  is fed into the reduced-rank filter  $\bar{\mathbf{w}}(i)$  and the output signal can be expressed as

$$y(i) = \bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\mathbf{r}(i) = \bar{\mathbf{w}}^H(i) \sum_{d=1}^D \mathbf{t}_d^H(i)\mathbf{r}(i)\mathbf{q}_d,$$

where  $\mathbf{q}_d$  (where  $d = 1, 2, \dots, D$ ) are the vectors whose  $d$ -th elements are ones, while all the other elements are zeros. In this section, RLS algorithms are developed to optimize  $\mathbf{t}_d(i)$  jointly and iteratively with  $\bar{\mathbf{w}}(i)$ .

#### A. RLS Algorithms

In the JIO-CA scheme, we need to solve the optimization problem

$$\begin{aligned} & [\bar{\mathbf{w}}(i), \mathbf{t}_1(i), \dots, \mathbf{t}_D(i)] \\ & = \arg \min_{\bar{\mathbf{w}}(i), \mathbf{t}_1(i), \dots, \mathbf{t}_D(i)} \frac{1}{2} \sum_{j=1}^i \alpha^{i-j} (|y(j)|^2 - 1)^2, \end{aligned} \quad (15)$$

subject to the constraint  $\bar{\mathbf{w}}^H(i) \sum_{d=1}^D \mathbf{t}_d^H(i)\hat{\mathbf{p}}(i)\mathbf{q}_d = v$ , where  $0 < \alpha \leq 1$  is the forgetting factor and  $y(i)$  is the output signal at the  $i$ -th time instant.  $\hat{\mathbf{p}}(i)$  is the estimated signature vector and  $v$  is a real-valued constant to ensure the convexity of the cost function. Let us now consider the problem through the Lagrangian

$$\begin{aligned} \mathcal{L}_{CA}(\bar{\mathbf{w}}(i), \mathbf{t}_1(i), \dots, \mathbf{t}_D(i)) &= \frac{1}{2} \sum_{j=1}^i \alpha^{i-j} (|y(j)|^2 - 1)^2 \\ &+ \Re \left[ \lambda(i) \left( \bar{\mathbf{w}}^H(i) \sum_{d=1}^D \mathbf{t}_d^H(i)\hat{\mathbf{p}}(i)\mathbf{q}_d - v \right) \right], \end{aligned} \quad (16)$$

where  $\lambda(i)$  is a complex-valued Lagrange multiplier. In the proposed CA scheme, for each time instant, we firstly update the vectors  $\mathbf{t}_d(i)$  (where  $d = 1, 2, \dots, D$ ) while assuming that  $\bar{\mathbf{w}}(i)$  and other column vectors are fixed. Then we adapt the reduced-rank filter with the updated projection matrix.

For the update of the column vectors of the projection matrix, we can divide the output signal as follows

$$y(i) = \bar{\mathbf{w}}^H(i) \sum_{d=1}^D \mathbf{t}_d^H(i)\mathbf{r}(i)\mathbf{q}_d = \bar{w}_d^*(i)\bar{r}_d(i) + \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}_e(i),$$

where the  $D$ -dimensional vector  $\bar{\mathbf{r}}_e(i)$  can be obtained by calculating the reduced-rank received signal  $\bar{\mathbf{r}}(i)$  and setting its  $d$ -th element to zero. By computing the gradient term of (16) with respect to  $\mathbf{t}_d(i)$  and setting it to a null vector, we have  $\nabla_{\mathbf{t}_d^*} \mathcal{L}_{CA} = \sum_{j=1}^i \alpha^{i-j} e(j)\mathbf{r}(j) (|\bar{w}_d(j)|^2 \mathbf{r}^H(j)\mathbf{t}_d(i) + \bar{w}_d^*(j)\bar{\mathbf{r}}_e^H(j)\bar{\mathbf{w}}(j)) + \lambda_{t,d}(i)\bar{w}_d^*(i)\hat{\mathbf{p}}(i) = 0$ , where  $e(i) = |y(i)|^2 - 1$  and  $\lambda_{t,d}(i)$  is the complex-valued Lagrange multiplier for updating the  $d$ -th column vector in the projection matrix. Rearranging the terms we obtain

$$\mathbf{t}_d(i) = -\mathbf{R}_d^{-1}(i) (\lambda_{t,d}(i)\bar{w}_d^*(i)\hat{\mathbf{p}}(i) + \mathbf{v}_r(i)), \quad (17)$$

where we define the  $M$ -dimensional vector  $\mathbf{v}_r(i) = \sum_{j=1}^i \alpha^{i-j} \bar{w}_d^*(j)\mathbf{r}(j) (e(j)\mathbf{r}_e^H(j)\bar{\mathbf{w}}(j) - \bar{w}_d(j)\bar{r}_d^*(j))$  and the  $M$ -by- $M$  matrix  $\mathbf{R}_d(i) = \sum_{j=1}^i \alpha^{i-j} |\bar{w}_d(j)|^2 |y(j)|^2 \mathbf{r}(j)\mathbf{r}^H(j)$ . Note that,  $\mathbf{R}_d(i)$  is dependent on  $\bar{w}_d(i)$ , which is the  $d$ -th element of the reduced-rank filter. Hence, for updating each  $\mathbf{t}_d(i)$ , we need to calculate the corresponding  $\mathbf{R}_d^{-1}(i)$  and that leads to high computational complexity. In our work, we devise an approximation  $\mathbf{R}_d(i) \approx |\bar{w}_d(i)|^2 \sum_{j=1}^i \alpha^{i-j} |y(j)|^2 \mathbf{r}(j)\mathbf{r}^H(j) = |\bar{w}_d(i)|^2 \mathbf{R}_y(i)$ . Then

we adopt the matrix inversion lemma [18] to recursively estimate  $\mathbf{R}_y^{-1}(i)$  as follows

$$\hat{\mathbf{R}}_y^{-1}(i) = \alpha^{-1} \left( \hat{\mathbf{R}}_y^{-1}(i-1) - (\phi(i)\boldsymbol{\kappa}(i))\boldsymbol{\kappa}^H(i) \right) \quad (18)$$

where  $\boldsymbol{\kappa}(i) = \hat{\mathbf{R}}_y^{-1}(i-1)\mathbf{r}(i)y(i)$  and  $\phi(i) = (\alpha + y^*(i)\mathbf{r}^H(i)\boldsymbol{\kappa}(i))^{-1}$ . We use  $\hat{\mathbf{R}}_y^{-1}(i)$  for all the adaptations of  $\mathbf{t}_d(i)$  to avoid the estimation of the  $\mathbf{R}_d^{-1}(i)$  (where  $d = 1, 2, \dots, D$ ) and the new update equation is given by

$$\mathbf{t}_d(i) = -\frac{\hat{\mathbf{R}}_y^{-1}(i)}{|\bar{w}_d(i)|^2} (\lambda_{t,d}(i)\bar{w}_d^*(i)\hat{\mathbf{p}}(i) + \mathbf{v}_r(i)). \quad (19)$$

Using the constraint  $\bar{\mathbf{w}}^H(i) \sum_{d=1}^D \mathbf{t}_d^H(i)\hat{\mathbf{p}}(i)\mathbf{q}_d = v$ , we obtain the expression of the Lagrange multiplier as

$$\lambda_{t,d}(i) = \left[ \frac{\bar{w}_d^*(i)\mathbf{v}_r^H(i)\hat{\mathbf{R}}_y^{-1}(i)\hat{\mathbf{p}}(i) + (v - \bar{\mathbf{w}}^H(i)\hat{\mathbf{p}}(i))|\bar{w}_d(i)|^2}{-|\bar{w}_d(i)|^2\hat{\mathbf{p}}^H(i)\hat{\mathbf{R}}^{-1}(i)\hat{\mathbf{p}}(i)} \right]^*, \quad (20)$$

where  $\hat{\mathbf{p}}_d(i)$  can be obtained by calculating the vector  $\mathbf{T}^H(i)\hat{\mathbf{p}}(i)$  and setting its  $d$ -th element to zero. Note that in the update equation (19), small values of  $|\bar{w}_d(i)|^2$  may cause numerical problems for the later calculation. This issue can be addressed by normalizing the column vector after each adaptation, which is given by  $\mathbf{t}_d(i) \leftarrow \mathbf{t}_d(i)/\|\mathbf{t}_d(i)\|$ .

After updating the projection matrix column by column, now we are going to adapt the reduced-rank filter  $\bar{\mathbf{w}}(i)$ . By assuming that the projection matrix is fixed, we can express the output signal in a simpler way as

$$y(i) = \bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\mathbf{r}(i), \quad (21)$$

where  $\mathbf{T}(i) = [\mathbf{t}_1(i), \dots, \mathbf{t}_D(i)]$  and the constraint can be expressed as  $\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) = v$ . Hence, the Lagrangian becomes

$$\mathcal{L}_{CA}(\bar{\mathbf{w}}(i), \mathbf{T}(i)) = \frac{1}{2} \sum_{j=1}^i \alpha^{i-j} \left( |y(j)|^2 - 1 \right)^2 + \Re[\lambda(i)(\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) - v)]. \quad (22)$$

By taking the gradient term of (22) with respect to  $\bar{\mathbf{w}}(i)$  and setting it to a null vector, we have  $\nabla_{\bar{\mathbf{w}}^*} \mathcal{L}_{CA} = \sum_{j=1}^i \alpha^{i-j} e(j)\mathbf{T}^H(j)\mathbf{r}(j)\mathbf{r}^H(j)\mathbf{T}(j)\bar{\mathbf{w}}(i) + \lambda_w(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) = 0$ , where  $e(i) = (|y(i)|^2 - 1)$  and  $\lambda_w(i)$  is the complex-valued Lagrange multiplier for updating the reduced-rank filter, rearranging the terms we obtain

$$\bar{\mathbf{w}}(i) = \mathbf{R}_T^{-1}(i) (-\lambda_w(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) + \bar{\mathbf{d}}(i)), \quad (23)$$

where  $\mathbf{R}_T(i) = \sum_{j=1}^i \alpha^{i-j} |y(j)|^2 \bar{\mathbf{r}}(j)\bar{\mathbf{r}}^H(j)$  and  $\bar{\mathbf{d}}(i) = \sum_{j=1}^i \alpha^{i-j} \bar{\mathbf{r}}(j)y^*(j) = \bar{\mathbf{d}}(i-1) + \alpha\bar{\mathbf{r}}(i)y^*(i)$ . The matrix inversion lemma [18] is used again to recursively estimate the inversion matrix  $\mathbf{R}_T^{-1}(i)$  as follows

$$\hat{\mathbf{R}}_T^{-1}(i) = \alpha^{-1} \left( \hat{\mathbf{R}}_T^{-1}(i-1) - (\phi_T(i)\boldsymbol{\kappa}_T(i))\boldsymbol{\kappa}_T^H(i) \right), \quad (24)$$

where  $\boldsymbol{\kappa}_T(i) = \hat{\mathbf{R}}_T^{-1}(i-1)\bar{\mathbf{r}}(i)y(i)$  and  $\phi_T(i) = (\alpha + y^*(i)\bar{\mathbf{r}}^H(i)\boldsymbol{\kappa}_T(i))^{-1}$ . For calculating the Lagrange multiplier, we use the constraint  $\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) = v$  and obtain

$$\lambda_w(i) = \left[ \frac{\bar{\mathbf{d}}^H(i)\mathbf{R}_T^{-1}(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) - v}{\hat{\mathbf{p}}^H(i)\mathbf{T}(i)\mathbf{R}_T^{-1}(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i)} \right]^*. \quad (25)$$

Note that the inversion of the matrix  $\mathbf{R}_y(i) = \sum_{j=1}^i \alpha^{i-j} |y(j)|^2 \mathbf{r}(j)\mathbf{r}^H(j)$  is obtained in the stage of adapting the projection matrix and the quantity  $|y(j)|^2$  tends to 1 as the number of received signal increase. Hence, we use (14) to estimate the channel and replace  $\mathbf{R}^{-m}$  with  $\hat{\mathbf{R}}_y^{-m}(i)$  to reduce its complexity.

### B. Complexity Analysis

As shown in Table. I, the complexity of the analyzed blind CCM full-rank stochastic gradient (SG) and RLS, MSWF-RLS [6] and the proposed JIO-CA-RLS scheme is compared with respect to the number of complex additions and complex multiplications for each adaptation. The quantity  $M$  is the length of the full-rank filter,  $D$  is the dimension of reduced-rank filter ( $D \ll M$ ). In the scenario of  $M = 59$ , the proposed JIO-CA-RLS algorithm with  $D = 3$  has lower complexity than the MSWF algorithms and the full-rank RLS.

## V. SIMULATIONS

In this section, we apply the proposed blind JIO-CA-RLS adaptive receiver to the uplink of a multiuser BPSK DS-UWB system and evaluate its uncoded bit-error rate (BER) performance against blind full-rank schemes and the blind MSWF-RLS. The performance of the RAKE receiver with the maximal-ratio combining (MRC) is also included for comparison. In all simulations, all the users are assumed to be transmitting continuously at the same power as the desired user. The pulse-width is 0.375ns and the spreading codes are generated randomly with a spreading gain of 32. The data rate of the communication is approximately 83Mbps. We assume that the channel is constant during the whole transmission and the delay spread is  $T_{DS} = 10$ ns. The ISI from 2 neighbor symbols are taken into account for the simulations. The sampling rate at the receiver is assumed to be 2.67GHz and the length of the discrete time received signal is  $M = 59$ . For all the experiments, all the adaptive filters are initialized as vectors with all the elements equal to 1. This allows a fair comparison between the analyzed techniques for their convergence performance. The phase ambiguity derived from blind channel estimation is eliminated in our simulations by using the phase  $\mathbf{h}(0)$  as a reference to remove the ambiguity. All the curves shown in this section are obtained by averaging 200 independent runs.

The first experiment we perform is to compare the performance of the algorithms in a 7-user system with a signal-to-noise ratio (SNR) of 20dB. Fig.2 shows the BER performance of different algorithms as a function of symbols transmitted. The proposed JIO-CA-RLS algorithm shows the fastest convergence and a noticeable improvement on the BER performance is obtained.

TABLE I  
COMPLEXITY ANALYSIS

Algorithm	Complex Additions	Complex Multiplications
Full-Rank SG	$3M + 2$	$4M + 3$
Full-Rank RLS	$5M^2 + 2M + 1$	$5M^2 + 3M + 1$
MSWF-RLS	$DM^2 + (2D + 2)M + 2D^2 - D$	$(D + 1)M^2 + (4D + 2)M + 2D^2 + 3D + 1$
JIO-CA-RLS	$DM^2 + 3DM + 4D^2 - 4D$	$DM^2 + 6DM + 4D^2 + 15D + 1$

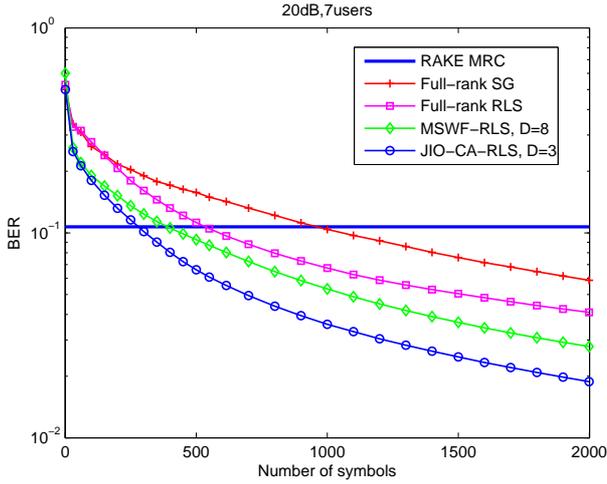


Fig. 2. BER performance of different algorithms. For full-rank SG:  $\mu = 0.025$ , full-rank RLS:  $\delta = 10$ ,  $\lambda = 0.9998$ . For MSWF-RLS:  $D = 8$ ,  $\lambda = 0.998$ . For JIO-CA-RLS:  $D = 4$ ,  $\lambda = 0.9998$ ,  $\delta = 10$ ,  $v = 0.5$ .

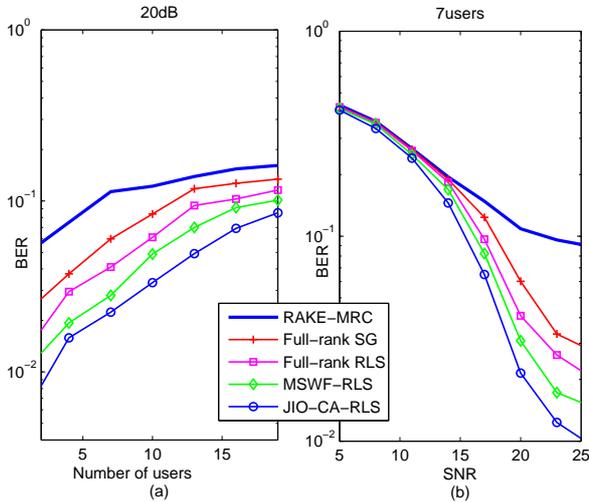


Fig. 3. BER performance of the proposed scheme in different scenarios.

Fig.3 (a) and (b) show the uncoded BER performances of algorithms with different numbers of users in a 20dB scenario and with different SNRs in a 7-user system, respectively. The setting of parameters for all adaptive algorithms are the same as in the first experiment. The proposed JIO-CA-RLS algorithm shows a better MAI and ISI canceling capability in all the simulated scenarios.

## VI. CONCLUSION

A novel blind adaptive reduced-rank receiver is presented based on the JIO and CCM criterion. RLS adaptive algorithms are developed for updating its parameters. In DS-UWB systems, the proposed receiver outperforms the analyzed CCM based full-rank receivers and MSWF with a low complexity.

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