prop Fachgebiet Prozessoptimierung

TECHNISCHE UNIVERSITÄT ILMENAU

Fakultät für Informatik und Automatisierung
Institut für Automatisierungs- und Systemtechnik

Systems Optimization

Winter Semester 2019

Exercise I

Prof. Dr.-Ing. habil. Pu Li
M.Sc. Xujiang Huang
Technische Universität Ilmenau
Department of Process Optimization (Prop)

1. (Circular Tank Design) A circular tank, closed at both ends, is to be designed with a volume of $200m^3$. The material cost is proportional to the surface area of the design, which is equal to 400 $\$/m^2$. The height of the tank is limited to $h \leq 12 - \frac{d}{2}$, where d is the diameter of the tank.

Question: Formulate an optimization problem for a cost optimal design.

2. (Product-Mix-Problem) A television manufacturer produces two types of TV sets A and C in two separate production lines. The daily production capacity for the A line is 60 units per day and for the C line 50 units per day. The A device requires 1, the C device 2 person working hours. Each working day, 120 hours of work are available, which can be used for both devices. Each A-device contributes 20$, each C-device 30$ to profit.

Question: What should the daily production look like from an entrepreneur's perspective? Formulate the problem?

3. (Adapted from S.S. Rao: Engineering Optimization (4th ed.)) A manufacturer of a particular product produces x_1 units in the first week and x_2 units in the second week. The number of units produced in the first and second weeks must be at least 200 and 400, respectively, to be able to supply the regular customers. The initial inventory is zero and the manufacturer ceases to produce the product at the end of the second week. The production cost, in dollars, is given by $4x_i^2$, where x_i is the number of units produced in week $i (i = 1, 2)$. In addition to the production costs, there is an inventory cost of 10$ per unit for each unit produced in the first week that is not sold by the end of the first week. If the selling price of the product in the first week is equal to 10$ per unit and that of the second week is 5$ per unit.
Question: How should the manufacturer produce to maximize his profit?

4. To supply water to a small town, the following simplified water supply system is available available, see Fig. 1. The aim is to achieve the most favorable pumping of water through

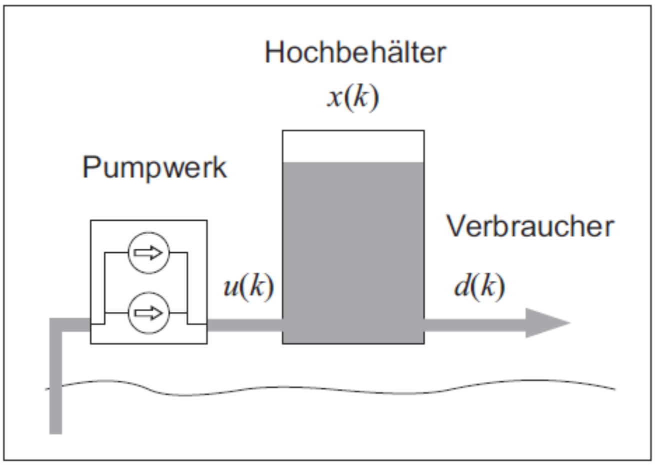

Figure 1: Simplified water supply system

the pumping station, in compliance with all boundary conditions (provision of a reserve amount of water in the tank of at least (case a) 700, case b) 500) m^3, prevention of overflow, complete demand coverage, maximum capacity of the high tank: $1000m^3$).

The dynamic behavior should be described in a discrete-time manner. The water tank is 95% filled at the beginning and the end of the daily cycle. Its level has no effect on the flow through the pumping station. In the pumping station, two parallel pumps of the same capacity are installed, which can work both individually (each $80m^3/h$) and together ($150m^3/h$). For simplicity, only three time intervals of 8 hours are considered. In the first two time intervals the electric energy high tariff (HT) applies, in the last one the low tariff (LT). The cost coefficients for the operation costs (variable costs) for one or two pumps in the high or low tariff are determined:

Number of the pump	cost coefficient of HT in $\$/h$	cost coefficient of LT in $\$/h$
1	9	6
2	18.75	12.5

The average of the water consumption in the three intervals are $105.6875m^3/h$, $102.1250m^3/h$ and $29.6875m^3/h$, respectively.
Question: Formulate the optimizatioin problem. Determine the water volume and the pump running times in the three time intervals!