Fachgebiet Prozessoptimierung

TECHNISCHE UNIVERSITÄT ILMENAU

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Systems Optimization

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Exercise VI

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1. **Energy optimal building design** (adapted from [1])
An architect considers to design a building with a cost optimal heating and cooling. The design specifications are:

   - the building should be rectangular and be partially buried under the ground
   - the total floor space needed is at least $20,000\ m^2$
   - the floor dimensions should not be longer than $50\ m$
   - the ratio of length to the width of the floor should be 1.618
   - each story must be $3.5\ m$ high
   - cost of heating and cooling the exposed areas of the building is $100\ \$/m^2$
   - the annual heating and cooling energy costs of the exposed areas of the building should not exceed $\$225,000$

   Objective: to determine the dimension of the building with minimum excavation costs.
   (Excavation costs are assumed to be proportional to the volume of the building below the ground.)

Table 1: Design parameters

n	Number of stories
d	Depth of building below ground
h	Hight of building above ground
ℓ	Length of building in plan
w	Width of building in plan

Mathematical Model:

$$\min_{(n,d,h,\ell,w)} \{f(n,d,h,\ell,w) = d\ell w\}$$

subject to:

$$\frac{d+h}{n} = 3.5$$
$$\ell = 1.618w$$
$$100\,(2h\ell + 2hw + \ell w) \leq 225,000$$
$$n\ell w \geq 20000$$
$$\ell \leq 50$$
$$w \leq 50$$
$$n \geq 1, d \geq 0, h \geq 0, \ell \geq 0, w \geq 0.$$

1.1 Re-write the above optimization problem as an optimization problem involving only the d, h, w as decision variables.

1.2 Solve the resulting optimization problem using Matlab's `fmincon.m`, `GAMS` and compare your results, and determine all design parameters.

2. Optimal Design of a pressure vessel. (Use Matlab and GAMS)

A pressure chamber is a closed vessel for transportation and storage of gases and liquids. The pressure in a such chamber can alter during operation is usually much greater than the ambient pressure. Areas of application are of pressure chambers

- in steam boilers,
- as hot water tankers,
- in distillation columns and for the storage of chemicals in chemical plants,
- in nuclear power plants,
- for the transport and storage of liquefied gases and petroleum products,
- as an engine cylinder, etc.,

The design, manufacture and use of pressure vessels requires high precision and precautions to avoid potential future risks, and for the reduction of manufacturing costs. A not precise designs of pressure vessels can lead to life-threatening and harmful explosions. Therefore, the engineer must make the best design decision. For example, the thickness of the vessel wall and the material costs are considered as decision variables.

Problem Formulation (see e.g. [3])

Following the pressure vessel which is capped at both ends by semi-spherical heads.

Figure 1: Some applications of pressure vessels (Image source: [7])

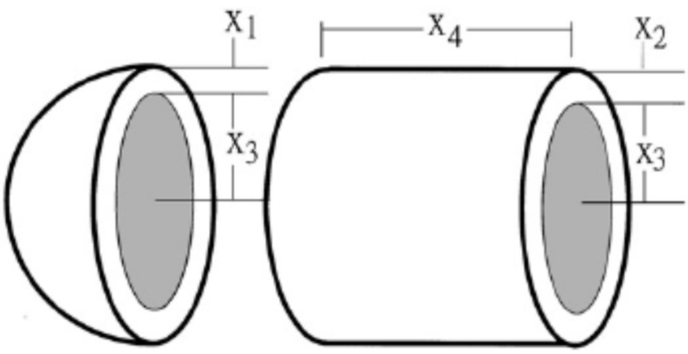

Figure 2: Design of a pressure vessel (figure taken from [3])

Objective :
To minimize the entire material and production costs.

<u>**Variablen**</u>:
- x_1 - Outer radius of the hemispherical shells
- x_2 - Outer radius the cylindrical shell
- x_3 - Interior radius of the hemispherical and cylindrical shells
- x_4 - length of the cylinder

Objective function:

$$f(x) = 0.6224x_1x_2x_3 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$$

Constraints:

$$\begin{aligned} g_1(x) &= -x_1 + 0.0193x_3 \leq 0 \\ g_2(x) &= -x_2 + 0.00954x_3 \leq 0 \\ g_3(x) &= -\pi x_3^2 x_4^2 - \frac{4}{3}\pi x_3^2 + 1,296,000 \leq 0 \\ g_4(x) &= x_4 - 240 \leq 0. \end{aligned}$$

Bounds on the design variables:

$$
\begin{aligned}
0.0625 \leq x_1 \leq 6.1875 \\
0.0625 \leq x_2 \leq 6.1875 \\
x_3 \geq 10.0 \\
x_4 \leq 200.0
\end{aligned}
$$

2.1. Explain the physical meaning of the constraints

2.2. Write the corresponding Lagrange function and state the necessary and sufficient conditions

2.3. Calculate the gradiant vector $\nabla f(x)$ and the Jacobian matrix $\nabla g(x)$

2.4. Solve the optimization problem using the function `fmincon.m` from Matlab's Optimization-Toolbox (see, e.g. [4]).

2.5. Solve the same optimization problem using GAMS (see e.g. [5]) and compare your result with one obtained from Matlab.

3. (Hanging chain).
A chain is suspended from to two hooks at (x_0, y_0) and (x_f, y_f). The chain consists of a set of links of stiff steel. The total length of the chain is given by L.

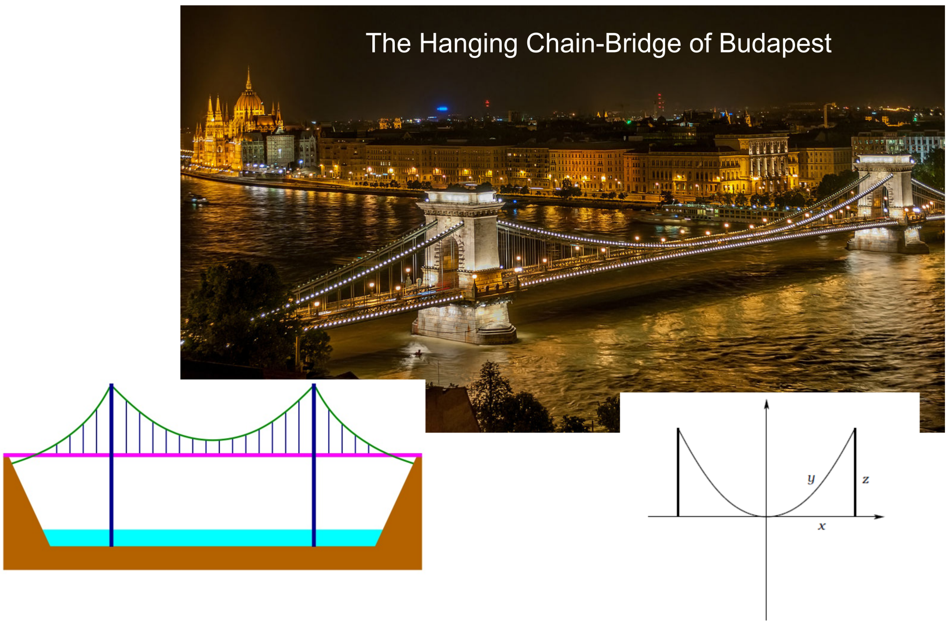

Figure 3: The hanging chain (image sources [8], [9])

<u>**Objective:**</u> To find the shape of the chain that guarantees the stable equilibrium of the chain despite external forces and the chain's own weight.

Assuming that the chain is represented by a curve, suppose the shape of the chain is given by a function $y(x)$. Hence, we need to find the function $y(x)$ that guarantees the minimum potential energy.

Optimization problem:

$$\min_{y}\left\{J[y]=\int_{x_0}^{x_f} y\sqrt{1+[y'(x)]^2}\,dx\right\} \qquad \text{:Potential energy}$$

subject to:

$$\int_{x_0}^{x_f}\sqrt{1+[y'(x)]^2}dx=L; \qquad \text{:total length of hanging chain}$$

$$y(x_0)=y_0,\quad y(x_f)=y_f. \qquad \text{:boundary conditions}$$

3.1. Use a finite-difference discretization to transform the problem into a constrained nonlinear optimization problem.

3.2. Use $L = 60\ m$, $(x_0, y_0) = (0, 20)$, and $(x_f, y_f) = (40, 20)$ to and solve the discretize problem using `fmincon.m`.

4. (Minimization of the material cost).
There are many industrial manufacturing processes involving the cutting out of certain geometrical shapes from a given raw material. However, after the cutting process is completed, the rest of the material is usually thrown away as waste. This causes a waste of material as well as the loss of invested money in the purchase of raw materials.

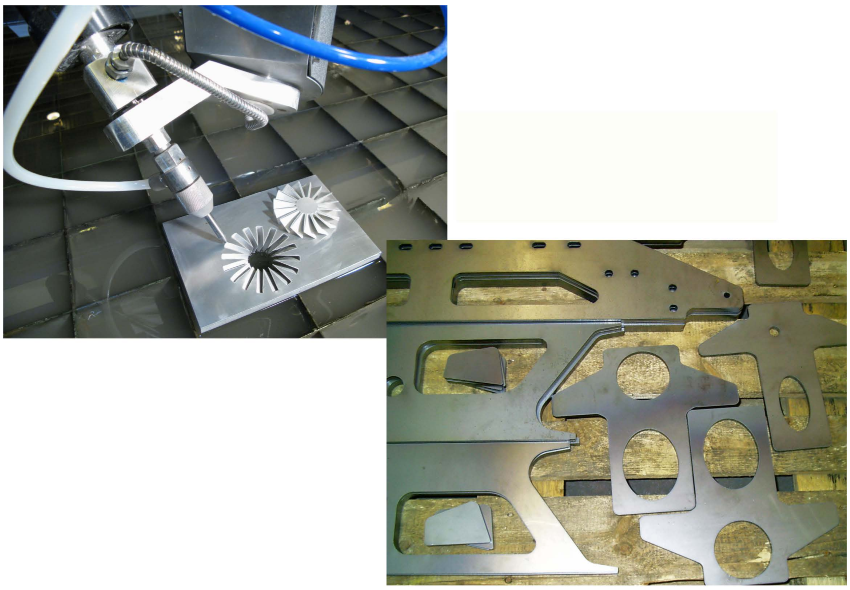

Figure 4: optimal cutting

Problem formulation (see [6])
It is necessary to cut out four circular disks with the variable radian R_1, R_2, R_3 and R_4 from a rectangular steel plate (see Figure 4).

Objective of the optimization task:
The goal is to minimize wastage of steel (i.e., cut out circular disks as large as possible so that the remaining steel is as small as possible). This requires the formulation and solution of an optimization problem.

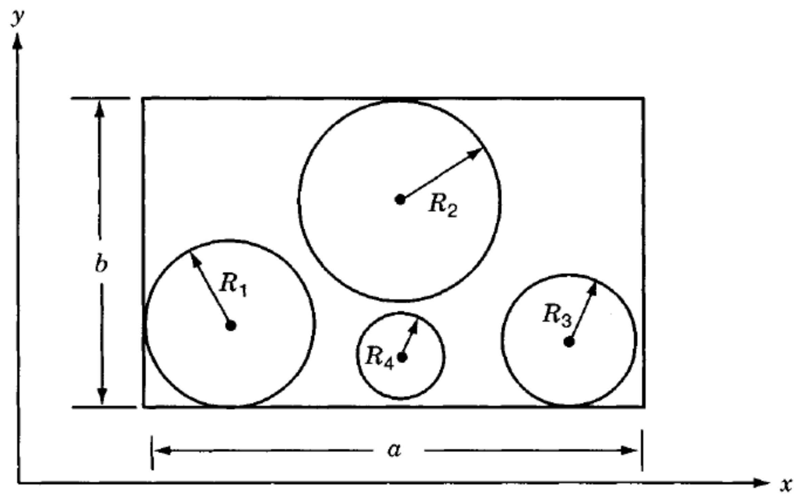

Figure 5:

4.1. Identify the design variable, objective function and constraints.

4.2. Formulate the problem as nonlinear optimization proble with constraints.

4.3. State the Lagrange function

4.4. Write the upper and lower bound of the constraints

4.5. Solve the optimization problem for $a = 0.3m$ and $b = 0.2m$ using MATLAB optimization toolbox

4.6. Analyze the numerical solution (the number of iteration steps, computation time)

4.7. Try to solve the problem with different initial values

4.8. How much is the material loss after the optimal cutting

4.9. Give suggestions on saving the loss of the material usage

References

[1] M. A. Bhatti: Practical optimization methods. Springer-Verlag, New York, 2000.

[2] A. D. Belegundu and T. R. Chandrupatla: Optimization concepts and applciations in engineering (2nd Ed.). Cambridge University Press, 2011

[3] L. C. Cagnina, S. C. Exquivle. Solving eignineering optimization problems with simple constrained particle swarm optimizer. *Informatica*, V. 32, 319 – 326, 2008.

[4] Optimization Toolbox^{TM} User's Guide, R2014a. The MathWorks, 2014.

[5] A. Geletu. GAMS - Modeling and Solving Optimization Problems. TU-Ilmenau, Faculty of Mathematics and Natural Sciences, Department of Operation Research & Stochastrics, 2008.

[6] A. Geletu. Solving Optimization Problems using the Matlab Optimization Toolbox - a Tutorial. TU-Ilmenau, Faculty of Mathematics and Natural Sciences, Department of Operation Research & Stochastrics, 2007.

[7] Indiamart, Oriental Engineering Co.
URL: www.indiamart.com/oriental-engineering-co/industrial-tank-vessels.html

[8] http://mathforum.org/mathimages/index.php/Image:Deckbridge.jpg

[9] http://de.wikipedia.org/wiki/Kettenbr