Network Security

Chapter 6
Random Number Generation

Tasks of Key Management (1)

- *Generation:*
  - It is crucial to security, that keys are generated with a truly random or at least a pseudo-random generation process (see below)
  - Otherwise, an attacker might reproduce the key generation process and easily find the key used to secure a specific communication
- *Distribution:*
  - Distribution of some initial keys usually has to be performed manually / out of band
  - Session key distribution is generally performed during an authentication exchange
  - Examples: Diffie-Hellman, Otway-Rees, Kerberos, X.509
- *Storage:*
  - Keys, especially authentication keys, should be securely stored:
    - either encrypted with a hard-to-guess pass-phrase, or better
    - in a secure device like a smart-card

Tasks of Key Management (2)

- *Revocation:*
  - If a key has been compromised, it should be possible to revoke that key, so that it can no longer be misused (cf. X.509)
- *Destruction:*
  - Keys that are no longer used (e.g. old session keys) should be safely destroyed (cf. media security in lecture 1)
- *Recovery:*
  - If a key has been lost (e.g. defect smart-card, floppy, accidentally erased) it should be possible to recover it, in order to to avoid loss of data
  - Key recovery is not to be mixed up with key escrow (see below):
- *Escrow:*
  - Mechanisms and architectures that shall allow government agencies (and only them) to obtain session keys in order to be able to eavesdrop on communications / to read stored data for law enforcement purposes
    - *"If I can get my key back it's key recovery, if you can get my key back it's key escrow..." :o)*

Random and Pseudo-Random Number Generation (1)

- Definition:

  A *random bit generator* is a device or algorithm, which outputs a sequence of statistically independent and unbiased binary digits.
- Remark:
  - A random bit generator can be used to generate uniformly distributed random numbers, e.g. a random integer in the interval [0, n] can be obtained by generating a random bit sequence of length $\lfloor \lg n \rfloor + 1$ and converting it into a number. If the resulting integer exceeds *n* it can be discarded and the process is repeated until an integer in the desired range has been generated.
- Definition:

  A *pseudo-random bit generator (PRBG)* is a deterministic algorithm which, given a truly random binary sequence of length *k*, outputs a binary sequence of length $m >> k$ which "appears" to be random.

  The input to the PRBG is called the *seed* and the output is called a *pseudo-random bit sequence*.

Random and Pseudo-Random Number Generation (2)

- Remarks:
  - The output of a PRBG is not random, in fact the number of possible output sequences of length *m* is at most all small fraction $2^k / 2^m$, as the PRBG produces always the same output sequence for one (fixed) seed
  - The motivation for using a PRBG is that it might be too expensive to produce true random numbers of length *m*, e.g. by coin flipping, so just a smaller amount of random bits is produced and then a pseudo-random bit sequence is produced out of the *k* truly random bits
  - In order to gain confidence in the "randomness" of a pseudo-random sequence, statistical tests are conducted on the produced sequences
- Example:
  - A linear congruential generator produces a pseudo-random sequence of numbers $y_1, y_2, \ldots$ According to the linear recurrence

    $$y_i = a \times y_{i-1} + b \bmod q$$

    with *a, b, q* being parameters characterizing the PRBG
  - Unfortunately, this generator is predictable even when *a, b* and *q* are unknown, and should, therefore, not be used for cryptographic purposes

Random and Pseudo-Random Number Generation (3)

- Security requirements of PRBGs for use in cryptography:
  - As a minimum security requirement the length *k* of the seed to a PRBG should be large enough to make brute-force search over all seeds infeasible for an attacker
  - The output of a PRBG should be statistically indistinguishable from truly random sequences
  - The output bits should be unpredictable for an attacker with limited resources, if he does not know the seed
- Definition:

  A PRBG is said *to pass all polynomial-time statistical tests*, if no deterministic polynomial-time algorithm can distinguish between an output sequence of the generator and a truly random sequence of the same length with probability significantly greater than 0.5
  - *Polynomial-time algorithm* means, that the running time of the algorithm is bound by a polynomial in the length *m* of the sequence

Random and Pseudo-Random Number Generation (4)

- Definition:

  A PRBG is said *to pass the next-bit test*, if there is no deterministic polynomial-time algorithm which, on input of the first *m* bits of an output sequence *s*, can predict the $(m + 1)^{st}$ bit s_{m+1} of the output sequence with probability significantly greater than 0.5
- Theorem (universality of the next-bit test):

  A PRBG passes the next-bit test

  $\Leftrightarrow$

  it passes all polynomial-time statistical tests
  - For the proof, please see section 12.2 in [Sti95a]
- Definition:

  A PRBG that passes the next-bit test – possibly under some plausible but unproved mathematical assumption such as the intractability of the factoring problem for large integers – is called a *cryptographically secure pseudo-random bit generator (CSPRBG)*

Random Number Generation (1)

- Hardware-based random bit generators are based on physical phenomena, as:
  - elapsed time between emission of particles during radioactive decay,
  - thermal noise from a semiconductor diode or resistor,
  - frequency instability of a free running oscillator,
  - the amount a metal insulator semiconductor capacitor is charged during a fixed period of time,
  - air turbulence within a sealed disk drive which causes random fluctuations in disk drive sector read latencies, and
  - sound from a microphone or video input from a camera
  - the state of an odd number of circular connected NOT gates
- A hardware-based random bit generator should ideally be enclosed in some tamper-resistant device and thus shielded from possible attackers

Random Number Generation (2)

- Software-based random bit generators, may be based upon processes as:
  - the system clock,
  - elapsed time between keystrokes or mouse movement,
  - content of input- / output buffers
  - user input, and
  - operating system values such as system load and network statistics
- Ideally, multiple sources of randomness should be "mixed", e.g. by concatenating their values and computing a cryptographic hash value for the combined value, in order to avoid that an attacker might guess the random value
  - If, for example, only the system clock is used as a random source, than an attacker might guess random-numbers obtained from that source of randomness if he knows about when they were generated

Random Number Generation (3)

- *De-skewing:*
  - Consider a random generator that produces biased but uncorrelated bits, e.g. it produces 1's with probability $p \neq 0.5$ and 0's with probability $1 - p$, where p is unknown but fixed
  - The following technique can be used to obtain a random sequence that is uncorrelated and unbiased:
    - The output sequence of the generator is grouped into pairs of bits
    - All pairs 00 and 11 are discarded
    - For each pair 10 the unbiased generator produces a 1 and for each pair 01 it produces a 0
  - Another practical (although not provable) de-skewing technique is to pass sequences whose bits are correlated or biased through a cryptographic hash function such as MD5 or SHA-1

Statistical Tests for Random Numbers

- The following tests allow to check, if a generated random or pseudo-random sequence inhibits certain statistical properties:
  - *Monobit Test:* Are there equally many 1's like 0's?
  - *Serial Test (Two-Bit Test):* Are there equally many 00-, 01-, 10-, 11-pairs?
  - *Poker Test:* Are there equally many sequences n_i of length q having the same value with q such that $\lfloor m / q \rfloor \geq 5 \times (2^q)$
  - *Runs Test:* Are the numbers of *runs* (sequences containing only either 0's or 1's) of various lengths as expected for random numbers?
  - *Autocorrelation Test:* Are there correlations between the sequence and (non-cyclic) shifted versions of it?
  - *Maurer's Universal Test:* Can the sequence be compressed?
  - *NIST SP 800-22:* Standardized test suite, includes above & more advanced tests
- The above descriptions just give the basic ideas of the tests. For a more detailed and mathematical treatment, please refer to sections 5.4.4 and 5.4.5 in [Men97a]

Secure Pseudo-Random Number Generation (1)

- There are a number of algorithms, that use cryptographic hash functions or encryption algorithms for generation of cryptographically secure pseudo random numbers
  - Although these schemes can not be proven to be secure, they seem sufficient for most practical situations
- One such approach is the ANSI X9.17 generator:
  - Input: a random and secret 64-bit seed s, integer m, and 3-DES key K
  - Output: m pseudo-random 64-bit strings $y_1, y_2, \ldots Y_m$

    1.) $q = E(K, Date\_Time)$

    2.) For i from 1 to m do

    2.1) $x_i = E(K, (q \oplus s))$

    2.2) $s = E(K, (x_i \oplus q))$

    3.) Return($x_1, x_2, \ldots x_m$)
  - This method is a U.S. Federal Information Processing Standard (FIPS) approved method for pseudo-randomly generating keys and initialization vectors for use with DES

Secure Pseudo-Random Number Generation (2)

- The RSA-PRBG is a CSPRBG under the assumption that the RSA problem is intractable:
  - Output: a pseudo-random bit sequence $z_1, z_2, ..., z_k$ of length k

    1.) Setup procedure:

    Generate two secret primes p, q suitable for use with RSA
    Compute $n = p \times q$ and $\Phi = (p - 1) \times (q - 1)$
    Select a random integer e such that $1 < e < \Phi$ and $\gcd(e, \Phi) = 1$

    2.) Select a random integer y_0 (the seed) such that $y_0 \in [1, n]$

    3.) For i from 1 to k do

    3.1) $y_i = (y_{i-1})^e \bmod n$

    3.2) z_i = the least significant bit of y_i
  - The efficiency of the generator can be slightly improved by taking the last j bits of every y_i, with $j = c \times \lg(\lg(n))$ and c is a constant
  - However, for a given bit-length m of n, a range of values for the constant c such that the algorithm still yields a CSPRBG has not yet been determined

Secure Pseudo-Random Number Generation (3)

- The Blum-Blum-Shub-PRBG is a CSPRBG under the assumption that the integer factorization problem is intractable:
  - Output: a pseudo-random bit sequence $z_1, z_2, ..., z_k$ of length k

    1.) Setup procedure:

    Generate two large secret and distinct primes p, q
    such that p, q are each congruent 3 modulo 4 and let $n = p \times q$

    2.) Select a random integer s (the seed) such that $s \in [1, n - 1]$
    such that $\gcd(s, n) = 1$ and let $y_0 = s^2 \bmod n$

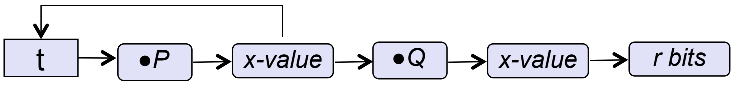
    3.) For i from 1 to k do

    3.1) $y_i = (y_{i-1})^2 \bmod n$

    3.2) z_i = the least significant bit of y_i
  - The efficiency of the generator can be improved using the same method as for the RSA generator with similar constraints on the constant c

Secure Pseudo-Random Number Generation (4)

- Dual Elliptic Curve Deterministic Random Bit Generator:
  - Based on the intractability of the elliptic curve discrete logarithm problem
  - Simplified version:



  - State t is multiplied with a generator P, the x-value of the new point becomes t'
  - Multiplied with a different point Q *r* bits of output can be generated, number of bits depend on curve (ranging between 240 and 504 bits)
  - Part of NIST 800-90A standard
  - Security:
    - It has been shown that if P is chosen to be eQ for a constant e then attackers can derive the state t
    - We do not know how the predefined points P and Q in NIST 800-90A are derived, so be careful ☺

CSPRNG security is a big thing!

- In September 2006 Debian was accidentally modified that only the process ID was used to feed the OpenSSL CSPRNG
  - Only 32,768 possible values!
  - Was not discovered until May 2008
- A scan of about 23 million TLS and SSH hosts showed that
  - At least 0.34% of the hosts shared keys because of faulty RNGs
  - 0.50% of the scanned TLS could be compromised because of low randomness
  - and 1.06% of the SSH hosts…
- Supervise your CSPRNG!
  - Do not generate random numbers right after booting your system
  - Use blocking RNGs, i.e. those that do not continue until having enough entropy