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Modeling Maximum Flow and Widest Path in Hybrid Multi-Channel Wireless Mesh Networks

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***Abstract*—High interest in Wireless Mesh Networks (WMNs) led to design problems for metrics to improve network capacity. They presume global coordination of transmissions in a slotted time model, a questionable presumption for IEEE 802.11. We propose a model for maximum flow and widest path in hybrid multi-channel WMNs based on shared airtime. It represents data rates and link correlations exactly and it supports wired connections. Experiments show increases in data rate when comparing maximum flow against widest paths with this model. We examine the widest path problem's complexity and a heuristic.**

***Index Terms*—Wireless Mesh Networks, Network Design Problem, Maximum Flow, Widest Path**

I. INTRODUCTION

Wireless Mesh Networks (WMNs) have been an active field of research for decades now, because they depict a valuable tool to provide wireless access for clients in a cost-efficient manner, if IEEE 802.11 components are employed for backbone communication. When considering multi-hop, multi-channel WMNs, network design techniques become especially important to provide high data rates, because locally available resources, i.e., multiple wireless interfaces tuned to different channels, need to be carefully instrumented to achieve high network capacity [1] and to provide maximum performance.

Concerning network performance, the *maximum flow* and *widest path* (a path yielding maximum data rate) between two nodes are suitable metrics (e.g. [2]) and many design problems have been proposed to calculate them for WMNs by means of linear programming. Most WMN design problems try to construct global link schedules based on a slotted time model in their pursuit to maximize network flow. However, a global schedule requires network-wide coordination of transmissions and time synchronization with relatively tight offsets. The necessity of global coordination on this level severely contradicts the distributed nature of WMNs, especially when they are based on IEEE 802.11 protocols and components. Another shortcoming of common models for WMN design problems in this field is their restriction to pure wireless communication. It is often worthwhile including other means of communication, e.g., some nodes could be interconnected using wired connections. In some use cases, a small number of wired connections could potentially be established due to geographic proximity of some nodes, or if two or more nodes (deliberately placed in very close vicinity) share their resources using cables – forming one virtual node with a large number of wireless interfaces. We refer to WMNs which include wired connections as *hybrid* WMNs.

Especially with WMN based on IEEE 802.11 principles, network design problems need to realistically model specifics of IEEE 802.11, i.e., data rates and correlation of links. These specifics are highly relevant for a model's validity. In particular, the medium access behavior [3] plays an important role when modeling network flow, because it already dictates which edges correlate with each other, i.e., simultaneous transmissions on correlating edges are impossible due to the medium access protocol. That means, many cases which state-of-the-art approaches would consider to be interference, are actively taken care of by means of medium access control mechanisms in IEEE 802.11.

Given the outlined deficits of WMN design problems, this paper proposes a model for maximum flow and widest path based on *shared airtime* specifically tailored for multi-channel IEEE 802.11 WMNs, which may include wired connections as well. Airtime is (without doubt) the most valuable resource when it comes to wireless communications; consequently, it should be modeled first in a network design problem. After airtime shares are sorted out, data rates can be obtained – which requires careful thought to produce accurate results for the maximum flow or widest path later on. Each node's individual share of airtime is distributed on wireless edges to other nodes and translates into an actual data rate based on Received Signal Strength Indicator (RSSI) and Signal-to-Noise Ratio (SNR), enabling the model to realistically represent network capacity. In particular, we provide the following contributions:

1) A novel formulation for the maximum flow as Linear Program (LP) and a formulation for widest path as Integer Linear Program (ILP) in hybrid multi-channel WMNs.
2) An evaluation study based on numerical experiments comparing achievable maximum flow, the capacity of an optimal widest path and the performance of heuristic path selection strategies on artificially generated hybrid multi-channel topologies resembling real-world WMNs.

The remainder of this paper is organized as follows: First, we motivate requirements for modeling maximum flow and widest path in WMNs and provide an overview of state-of-the-art network design problems and approaches (Sec. II). We present our model and (I)LP formulations in Sec. III and explain all relevant details. The evaluation in Sec. IV discusses our approach qualitatively and studies results using numerical experiments. To sum up, Sec. V presents some concluding thoughts and directions for future research.

II. Requirements and Related Work

Based on several shortcomings of state-of-the-art approaches for network design problems in WMNs, we identify the following crucial requirements:

- Realistic modeling of rates and correlating links based on IEEE 802.11 specifics
- Realistic channel access model (no slotted time model, as it would imply a need for network-wide synchronization)
- Support for multiple channels
- Support for wired parts of the topology (hybrid WMNs)
- Representation of maximum flow and widest path to obtain relevant network performance metrics

Concerning state-of-the-art approaches, some network design problems are part of a joint approach to channel selection and routing. Presuming their relevance, they are included in this study of existing approaches.

One approach [4] models a maximum concurrent flow problem, but does not consider correlation of wireless edges. Link capacities are generously approximated using one fixed rate multiplied by the number of available wireless interfaces.

With the additional constraint of computational efficiency, [5] derives a heuristic for the maximum concurrent flow (omitting link correlations). They aim at finding interference-free schedules to make up for their previous shortcuts.

A sophisticated interference model is used in [6], also including the use of multiple channels. However, data rates are the same (fixed) for all edges and their model presumes slotted time.

Authors of [7] optimize flows in a multi-channel setting. Link correlations are realized with a loose bound derived from the maximum number of interfaces based on fixed ranges. Transmission rates depend on distance between nodes, which is superior when compared to other approaches, but still insufficient in regard to IEEE 802.11. They try to obtain an interference-free schedule in a slotted time model.

In [2], a rather complex model for maximum flow and widest path is presented. It distinguishes active and inactive links (used for forwarding or not), resulting in binary variables in their constraints, which are furthermore used in products later on. Therefore, both their formulation of widest path and maximum flow are computationally complex and they are only able to provide upper bounds and heuristics. The model covers transmission ranges and RTS/CTS, but does not include multiple channels and does not incorporate wired edges. Furthermore, the rate model is quite simple, since an edge has a fixed data rate only presuming the incident nodes to be within communication range.

Summing up on the most notable state-of-the-art approaches, none of them could meet all of our stated requirements. Most important shortcomings are the use of an unrealistic slotted time model, insufficient representation of rates on wireless links and lack of wired or multi-channel communication support. To the best of our knowledge, this article is the first one to present a comprehensive model as a basis for network design problems for maximum flow and widest path tailored specifically for IEEE 802.11 in a hybrid multi-channel WMN context.

III. Modeling Hybrid Multi-Channel WMNs Based on IEEE 802.11

This section explains our model for hybrid multi-channel WMN in detail. It will not be dependent on slotted time, as this would imply globally and precisely synchronized transmission schedules, which contradicts the distributed character of WMNs. Instead, it will model the channel access of each node individually in a sense of airtime used for transmissions. Our explanations further include: Mapping of airtime to actual data rates, mesh peering, link correlations (that dictate how airtime needs to be shared) and wired links. The resulting model is a foundation to develop formulations for maximum flow and widest path as LP/ILP. Furthermore, a heuristic for widest path computation is presented as well.

A. Preliminaries

As linear programming does not allow exact representation of an actual protocol, i.e., transmission of individual (control) packets, we need to assume that overhead induced by medium access control mechanisms is negligible – and only constant. For the very same reason, we do not consider packet level traffic or bit error rates in this paper, but all aforementioned aspects could be reflected, e.g., when modeling data rates. Presumably, the network to be examined already possesses a static channel assignment following suitable optimality constraints. We include advice on how this can be achieved prior to presenting results in this paper. Medium access control mechanisms (clear channel assessment, RTS/CTS) work as expected and will therefore be used to determine link correlations.

B. Fundamentals of the Model

General Notation: A *bar* above an identifier (e.g. $\bar{G}$) is used for wired components, whereas a *tilde* (e.g. $\tilde{G}$) indicates wireless components. An instance of a hybrid multi-channel WMN can be formalized as a tuple $(\bar{G} = (\bar{V}, \bar{E}), \tilde{G} = (\tilde{V}, \tilde{E}, \mathcal{C}), \mathcal{A}, \mathcal{R})$. $\bar{G}$ is the wired and $\tilde{G}$ the wireless graph. $\mathcal{A} : \tilde{V} \times \tilde{V} \mapsto \mathbb{R}$ assigns an attenuation value in dB to each pair of wireless nodes. This function could be obtained using empirical measurements or (as in this paper) a path loss model based on Euclidean coordinates. The function $\mathcal{C}$ assigns each wireless node a set of channels to which its wireless interfaces are tuned to. Finally, $\mathcal{R}$ takes care of mapping edges to data rates. Please note that $\tilde{E}$ and $\mathcal{R}$ are highly dependent on $\mathcal{C}$ and $\mathcal{A}$ – as we will explain in detail.

Airtime Model: As mentioned before, the model in this paper does not use slotted time or shared data rates among wireless edges. Instead, we introduce variables $\Lambda_v^c \in [0.0, 1.0] \subseteq \mathbb{R}$ for the airtime a node v is able to transmit on channel c. This can be seen as a percentage from 0 % to 100 %. It is realistic to assume that nodes share airtime, as this is the actual resource all nodes compete for in wireless communication on a common channel.

A node may have several interfaces on different channels, hence, multiple Λ_v^c for each node are possible. Each nodes' airtime share is later used to obtain an achievable data rate.

Data Rates on Wireless Edges: In wireless communication, data rates depend on two important factors when assuming a clear channel: RSSI and SNR. The RSSI is usually dependent on transmission power, antenna gain, path loss, transceiver loss (a so called noise figure) and connector losses. Path loss is the only component dependent on which particular pair of nodes is examined, therefore, requiring pairwise attenuations. In our model, $\mathcal{A}$ represents attenuation induced by path loss and allows to derive an RSSI for a given node pair, as all other components can be easily parameterized. IEEE 802.11 standard [3] provides tables with minimum required RSSIs for certain coding schemes and coding rates, which then result in an achievable data rate. However, one particular coding scheme/rate combination requires a minimum SNR. The standard does not mention SNR levels, however, a theoretical SNR bound to transmit a given data rate can be obtained by employing Shannon's theorem [8] when assuming a certain level of (thermal) noise N_0 (dependent on the bandwidth of the wireless channel [9], but it can conservatively set to account for other losses). As Shannon's bound is not realistic for practical settings, we introduce a parameterizable safety margin (fade margin). This margin could be set high to exhibit stricter requirements (if needed). A given data rate can only be achieved, if both RSSI and SNR meet the minimum required value. The resulting rate can then be linked to a share of airtime.

Mesh Peering: Whether or not two Mesh Points (MPs) establish a connection depends on their channel selection (they both need at least one wireless interface on the same channel) and a preconfigured minimum RSSI value, the *mesh peering threshold*. This value could be quite conservative to avoid link flapping. If the RSSI between two nodes is too low, i.e., their distance is too large, there will be no mesh connection. Otherwise, they are said to be within *communication range*.

Correlating Links Share Airtime: Even though a mesh connection cannot be established between two nodes, their transmissions may still influence each other. In IEEE 802.11, all transmitted frames begin with a preamble [3] serving the purpose of announcing the length of the frame (among others) so that other stations are aware of the channel occupation. The preamble itself is transmitted using the slowest coding scheme and rate to provide backward compatibility to older standards. Therefore, it can be decoded with a much lower SNR than the actual transmission and consequently, inhibiting concurrent channel access attempts in a much wider range than the transmission range, the so called *correlation range*. Another mechanism contributing to the correlation range is the RTS/CTS dialog. Every station overhearing either a RTS or CTS frame will be aware of the channel occupation. The standard dictates RTS and CTS frames to be sent at the highest available *basic rate*, which determines the set of nodes in correlation range of an RTS/CTS dialog. The standard does not specify availability of basic rates in the case of mesh connections, but it is safe to assume that the RTS/CTS rate should not be greater than the lowest rate to any active mesh peer. Please note that concepts similar to correlation in the context of this paper are often referred to as binary interference models in the state of the art – a somewhat misleading nomenclature, because it does not refer to actual interference. It rather covers channel access mechanisms, only.

Interference: In the context of this paper, interference refers to simultaneous transmissions that could not be resolved using protocol mechanisms, i.e., from nodes out of each others communication and correlation range. For brevity, we do not cover interference here. However, exactly modeling interference from nodes beyond communication and correlation range will be subject to future work.

Incorporating Wired Links: Wired connections will naturally provide much higher rates than wireless ones do and they allow for full duplex communication. We assume a fixed wired rate for this model. Wired edges should not only be preferred because of higher rates, but they do not contribute to airtime consumption, therefore, they do not cause correlations whatsoever.

C. Maximum Flow in Hybrid Multi-Channel WMNs

We now develop an LP formulation for maximum flow in hybrid multi-channel WMNs based on the proposed model. Without loss of generality, flow from a source s to a target node t shall be maximized. The LP formulation is as follows:

$$\max \sum_{(s,x)\in\bar{E}} \bar{\varrho}_{s,x} + \sum_{(s,x)\in\tilde{E}} \tilde{\varrho}_{s,x} \tag{1}$$

$$\text{s.t.} \sum_{(v,y)\in\bar{E}} \bar{\varrho}_{v,y} + \sum_{(v,y)\in\tilde{E}} \tilde{\varrho}_{v,y}$$

$$= \sum_{(x,v)\in\bar{E}} \bar{\varrho}_{x,v} + \sum_{(x,v)\in\tilde{E}} \tilde{\varrho}_{x,v} \quad, \forall v \in (\bar{V}\cup\tilde{V})\setminus\{s,t\} \tag{2}$$

$$\sum_{(x,s)\in\bar{E}} \bar{\varrho}_{x,s} + \sum_{(x,s)\in\tilde{E}} \tilde{\varrho}_{x,s} = 0 \tag{3}$$

$$\sum_{(t,x)\in\bar{E}} \bar{\varrho}_{t,x} + \sum_{(t,x)\in\tilde{E}} \tilde{\varrho}_{t,x} = 0 \tag{4}$$

$$\bar{\varrho}_{x,y} \le \bar{r}_{x,y} \quad, \forall (x,y) \in \bar{E} \tag{5}$$

$$\tilde{\varrho}_{x,y} \le \sum_{c\in\mathcal{C}(x)\cap\mathcal{C}(y)} \tilde{r}_{x,y}\Lambda_{x,y}^c \quad, \forall (x,y) \in \tilde{E} \tag{6}$$

$$\sum_{(v,x)\in\tilde{E}} \Lambda_{v,x}^c = \Lambda_v^c \le 1.0 \quad, \forall v \in \tilde{V}, \forall c \in \mathcal{C}(v) \tag{7}$$

$$\Lambda_v^c + \sum_{(x,v)\in\tilde{E}\,:\,c\in\mathcal{C}(x)} \Lambda_{x,v}^c \le 1.0 \quad, \forall v \in \tilde{V}, \forall c \in \mathcal{C}(v) \tag{8}$$

$$\Lambda_v^c + \sum_{x\in\mathrm{PreambleRange}(c,v)} \Lambda_x^c \le 1.0 \quad, \forall v \in \tilde{V}, \forall c \in \mathcal{C}(v) \tag{9}$$

$$\Lambda_{x,y}^c + \sum_{z\in\mathrm{RTSCTSRange}(c,x,y)} \Lambda_z^c \le 1.0 \quad, \forall (x,y) \in \tilde{E}, \forall c \in \mathcal{C}(x)\cap\mathcal{C}(y) \tag{10}$$

$$\bar{r}_{x,y} \text{ constant}, \bar{\varrho}_{x,y} \in \mathbb{R}^+ \quad, \forall (x,y) \in \bar{E} \tag{11}$$

$$\tilde{r}_{x,y} \text{ constant}, \tilde{\varrho}_{x,y} \in \mathbb{R}^+ \quad, \forall (x,y) \in \tilde{E} \tag{12}$$

$$\Lambda_v^c \in \mathbb{R}^+ \quad, \forall v \in \tilde{V}, \forall c \in \mathcal{C}(v) \tag{13}$$

$$\Lambda^c_{x,y} \in \mathbb{R}^+ \qquad , \forall (x,y) \in \tilde{E}, \; \forall c \in \mathcal{C}(x) \cap \mathcal{C}(y) \qquad (14)$$

This formulation introduces non-negative variables for the airtime a node occupies (13), airtime on wireless edges (14), as well as rate on wired (11) and wireless edges (12) – the latter two include constant maximum rates derived as explained before. Naturally, we maximize the rate of the sum from all outgoing wired and wireless edges from s in the objective function (1). Constraint (2) depicts the flow conservation rule. Furthermore, flow into s and out of t is set to 0 in constraints (3) and (4). To bound rate variables, constraint (5) forces a maximum wired rate and in (6), airtime on each shared channel between two nodes is multiplied with the respective achievable wireless rate. The airtime for each node is bounded by 100 % or simply 1.0 in (7), as well as split into airtime for individual edges. In constraint (8), the sum of a node's airtime and airtime on incoming edges is bounded by 100 % to represent inevitable resource consumption implied by wireless reception. Finally, constraints (9) and (10) realize correlation induced by medium access mechanisms, i.e., preambles and RTS/CTS.

D. Widest Path for Hybrid Multi-Channel WMNs

Modeling the widest path problem adds significant complexity when compared to maximum flow, as one path is an unsplittable unit – contrary to a flow. This complexity increase may seem unintuitive, but is justified since we can show the problem's NP-completeness [10]. The severity of NP-completeness in realistic settings is subject to the quantitative evaluation in this paper. However, NP-completeness explains the use of binary variables in the following ILP formulation for the general case.

The ILP formulation for the widest path in hybrid multi-channel WMNs is an extension to the LP formulation for maximum flow, supplementing constraints to ensure the choice of one unsplittable path. Without loss of generality, we want to find the widest path between a source node s and a target node t. With M being a large constant, the ILP is as follows:

$$\max \sum_{(s,x) \in \bar{E}} \bar{\varrho}_{s,x} + \sum_{(s,x) \in \tilde{E}} \tilde{\varrho}_{s,x} \qquad \text{Same as (1)} \qquad (15)$$

$$\text{s.t. Constraints (2) - (14)}$$

$$\bar{\varrho}_{x,y} \leq M \cdot I_{x,y} \qquad , \forall (x,y) \in \bar{E} \qquad (16)$$

$$\Lambda^c_{x,y} \leq I^c_{x,y} \qquad , \forall (x,y) \in \tilde{E}, \; \forall c \in \mathcal{C}(x) \cap \mathcal{C}(y) \qquad (17)$$

$$\sum_{(v,x) \in \bar{E}} I_{v,x} + \sum_{(x,v) \in \tilde{E} \,:\, c \in \mathcal{C}(x)} I^c_{x,v} \leq 1, \; \forall v \in \bar{V} \cup \tilde{V} \qquad (18)$$

$$I_{x,y} \in \{0,1\} \qquad , \forall (x,y) \in \bar{E} \qquad (19)$$

$$I^c_{x,y} \in \{0,1\} \qquad , \forall (x,y) \in \tilde{E}, \; \forall c \in \mathcal{C}(x) \cap \mathcal{C}(y) \qquad (20)$$

In ensuring one unsplittable path to be result, binary indicator variables (19) and (20) are required that indicate whether or not a wired or wireless edge on a certain channel is chosen to be part of the resulting widest path. Each node is only allowed to have at most one active outgoing edge due to constraint (18). Finally, rate variables need to be properly bounded depending indicator variable values. If a variable $I_{x,y} = 1$, i.e., the wired edge is chosen, constraint (16) allows the edge to carry flow, presuming M (constant) is sufficiently large. Conversely, if $I_{x,y} = 0$ holds, no flow is allowed. Constraint (17) achieves the same for wireless connections, but restricting airtime for certain edges and channels (prior to rate deduction).

E. Widest Path Heuristic for Reference

One possible heuristic relies on Dijkstra's algorithm adapted to work with data rates, rather than usual minimum cost metrics. Beginning with a starting node s, initial rates to all other nodes are 0, whereas the rate from s to itself is ∞. Suppose a rate from s to an arbitrary node x during the algorithm is $r(x)$ and an edge $e = (x,y)$ with rate $r(e)$ is examined, then: $r(y) = \min\{r(x), r(e)\}$. To find the widest path, the neighbor providing the maximum rate needs to be chosen. These modifications to Dijkstra's algorithm have already been used in our previous work [11] on wired overlay networks.

To incorporate repeated use of the same channel in multi-hop wireless communication, we introduce a channel history of fixed length, maintained using the FIFO principle. Let e' be the currently inspected edge in a step of Dijkstra's algorithm. Naturally, wired edges are not effected, but if the channel of a wireless edge e', i.e., $c(e')$, is not in the history, it is simply added to it – without changes to metric computation of the data rate. If $c(e')$ is already $k > 0$ times in the history, i.e., edges $e_0, \ldots, e_{k-1}$ use the same channel, the data rate metric changes. The metric value of e' is modified to be $\min\{{}^{r(e_i)}/_{(k+1)} \mid 0 \leq i < k\}$, reflecting multiple use of the same channel. Taking the minimum of all affected edges is required, as any of these could be a bottleneck. If the channel history exceeds its maximum length, the oldest entries will be dropped. Please note that this is only a heuristic, because we implicitly change rates of already processed edges, which is (strictly speaking) a violation of Dijkstra's principles. Furthermore, as this heuristic works with a channel history of fixed length, the result will be affected easily if it is too long or too short. This issue will be subject to the quantitative evaluation in this paper.

IV. EVALUATION

We evaluated our models for maximum flow and widest path using numerical experiments. This section presents employed hybrid topologies, experiments and performance metrics of our evaluation in detail. Furthermore, we investigate consequences of the problem's NP-completeness in real-world-like scenarios. We discuss our model qualitatively first, before giving quantitative results from numerical experiments.

A. Qualitative Discussion

Our model fulfills all initially stated requirements: It does not rely on slotted time, as it rather uses airtime shares, which later translate into actual data rates. Correlation according to IEEE 802.11 is considered. It also includes multi-channel and hybrid topology support. We furthermore developed suitable representations for two important network performance metrics (maximum flow and widest path).

B. Topologies

This subsection describes all required steps to obtain suitable topologies for numerical experiments in this paper. We incorporate the NPART topology generator [12] in our evaluation to obtain network topologies that resemble real-world WMNs. However, these only depict a starting point, as they presume a different mesh peering distance and they exhibit poor connectivity. Furthermore, they depict single-channel WMNs, only. Please note that our modifications merely resemble optimizations a network planner would project to enhance connectivity and performance – making them more realistic.

Connectivity Augmentation: In pursuing improved network connectivity (which potentially enhances network performance), we place additional wireless nodes in an NPART topology in the following fashion: 1) Find articulation nodes (whose removal would lead to a disconnected graph). 2) Find all paths traversing one of the articulation nodes, sort them ascending by the distance between starting node (a) and end node (b) of the path. 3) Try to connect a and b by placing a chain of wireless nodes between them, if no other nodes are within communication range (except for a and b themselves). 4) Iterate until either a maximum of 10 nodes were placed or the set of articulation nodes is empty. Although this being a very pragmatic approach, it delivered good results with respect to network connectivity. Any leftover nodes were placed randomly in the topology, only assuring no influence to recent connectivity improvements.

Adaptation of Mesh Peering Distance: NPART presumes a fixed mesh peering distance of 90 meters. Therefore, we scaled NPART's coordinates to suit our mesh peering criteria of an RSSI of -75 dBm, which translates into 213.1 meters conforming to our path loss model (see Tab. I).

Channel Selection: The main idea of our channel optimization scheme is to reduce the number of cases in which the same channel on incident edges is used, as usual for state-of-the-art approaches (e.g. [13]). However, in the context of this paper, we developed an ILP formulation with the following (additional) constraints: 1) Minimize the same channel on incident edges. 2) Try to use different channels on all shortest three-hop paths. 3) Do the same for shortest paths with four hops. 4) Minimize the maximum number of times a channel is used in the network. These constraints are incorporated into the objective function of the resulting ILP formulation with descending importance. Constraints derived from paths are advantageous, because the correlation range includes more than just incident edges. Restriction to paths of length four makes sense since effects of correlation appear (topologically) localized. The last constraint tries to balance the load between all possible channels. Please note that the resulting formulation is omitted for brevity. Furthermore, since optimizing channel selection leads to quite complex formulations and impractical runtimes, the optimization process is canceled after 60 minutes and the intermediate result will be the basis for our topologies.

TABLE I
SETTINGS AND EXPERIMENT FACTORS (F)

Setting/Factor	**Value(s)**
Experiment Runs	32 per factor combination
Standard	IEEE 802.11a (for comparison purposes)
Node TX Power [dBm]	18
Background Noise [dBm]	-90 (also includes other constant losses)
Fade Margin [dB]	15
Mesh Peering Thres. [dBm]	-75
Individual Rates [Mbps]	$6, 9, 12, 18, 24, 36, 48, 54$
- Required SNRs [dB]	$8.6, 10.6, 12.1, 14.4, 16.1, 18.9, 21.3, 22.4$
Basic Rates [Mbps]	$6, 12, 24$
Preamble Rate [Mbps]	1
- Required SNR [dB]	5
Path Loss Model	Two Ray Ground Reflection
Antenna Elevation [m]	1.25
Topologies	Enhanced NPART
Topology Sizes (F)	$50, 60, \ldots, 100$
No. of Wireless Interfaces	4
No. of Available Channels	8
Metrics (F)	Maximum Flow, Widest Path
Maximum Flow Approach	LP
Widest Path Approach (F)	ILP, Heuristic
- Heuristic Setting (F)	Channel History Size: $0, 1, \ldots, 11$

Introduction of Wired Edges: Nodes with a maximum distance of 100 m are connected using Ethernet with a data rate of 1 Gbps [14]. At most 25 % of all connections are wired, i.e., we start to connect pairs of nodes with ascending distance to each other. Finally, we do not attach cables to nodes which were added during connectivity augmentation.

C. Numerical Results

We solved (I)LP's using Gurobi [15] to obtain results for maximum flow and widest path. All metrics were computed for 128 randomly sampled pairs of nodes (to avoid computation for all pairs) only making sure that nodes of one pair do not reside in the same connected component of the wired network – avoiding the trivial solution of a simple wired path. We averaged values from 128 pairs to obtain one value for each run. The results were then averaged over 32 runs including 99 % confidence intervals. All relevant settings for our experiments are displayed in Tab. I. In general, our model can be used with any particular IEEE 802.11 standard. The specific standard solely influences achievable data rates on wireless links and the set of available wireless channels. We build our model around IEEE 802.11a for two reasons: 1) Compatibility to prior works as well as tools and 2) Use of the 5 GHz band (in IEEE 802.11a) allows for a more diverse channel selection.

To obtain a more realistic data rate model, we choose fade margin of 15 dB to be on par with best-practice values from empirical results (e.g. from [16]). This margin may seem high, but is chosen conservatively because backhaul communication usually exhibits stricter requirements (for reasons of robustness). The mesh peering threshold is set to -75 dBm (as in [17]).

In our first experiment, we compare maximum flow and widest path rates for different topology sizes. Fig. 1 displays rates for both cases. Large topologies lead to a slight decrease for both maximum flow and widest path rates. However, the rate of flows is at least 132 % higher than rates from widest

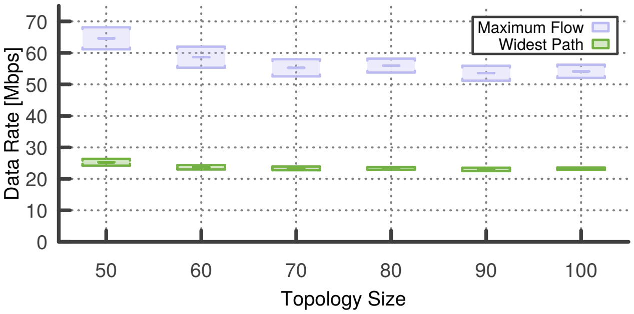

Fig. 1. Maximum flow vs. widest path data rates.

paths – more than double the rate. Whether or not particular rates can be achieved, i.e., the model yields accurate results, needs to be verified by means of packet-level simulation, which is out of scope for this paper, but subject to future work.

The second experiment covers implications of the widest path problem's NP-completeness in real-world-like settings. Therefore, we examine the performance of our Dijkstra-based heuristic using different history lengths and compare them to optimal results obtained from our ILP formulation for widest path. Fig. 2 presents heuristic and optimal results. Note that optimal results are not dependent on the history size and only displayed for each value of the X-axis for reference. Furthermore, the plot does not display data rates that the heuristic guesses. It rather shows the achievable data rate of the path obtained by the heuristic when considering correlation constraints. Only in case of a history size ≤ 5, the heuristic performs significantly worse than optimal paths obtained from the ILP. Confidence intervals overlap for history sizes ≥ 6, leading to the conclusion that the heuristic yields only insignificantly less data rate than optimal paths. However, a history size that is too large presumably causes issues, as it does not reflect the influence of correlations correctly – which occur localized in a wireless topology. We do not see this for large histories in Fig. 2 because even though the heuristic would underestimate a path's achievable rate, it can carry higher data rates when checked against realistic correlation constraints. The important takeaway is that even though the widest path problem is NP-complete, heuristic algorithms are completely sufficient in practical settings.

V. CONCLUSION AND FUTURE WORK

In this paper, we presented a novel model for network design problems targeting two important performance metrics: maximum flow and widest path. The model is tailored to incorporate IEEE 802.11 mechanisms in a realistic fashion and its main component (airtime sharing) reflects the resource wireless nodes compete for and allows accurate derivation of data rates later on. Evaluation results show a significant increase of maximum flow rates when comparing them to widest paths. Furthermore, we developed and examined heuristic solutions to the widest path problem in hybrid multi-channel WMNs.

In future work, we will incorporate multiple commodities in our formulation, as well as interference caused by nodes beyond communication and correlation range. Furthermore, we will check the model's results against packet-level simulation and develop a distributed protocol to establish maximum flows in hybrid multi-channel WMNs.

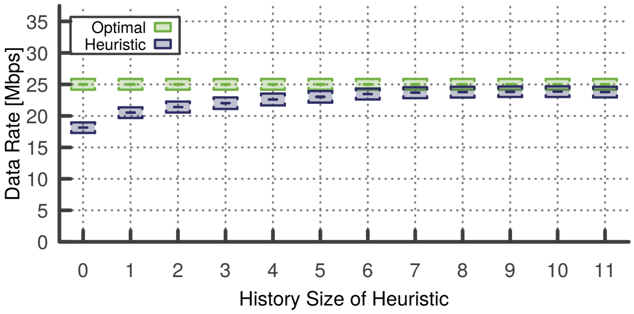

Fig. 2. Widest path heuristic and optimal results. Enhanced NPART. 50 nodes.

REFERENCES

[1] D. Benyamina, A. Hafid, and M. Gendreau, "Wireless Mesh Networks Design – A Survey," *IEEE Communications Surveys & Tutorials*, vol. 14, no. 2, pp. 299–310, 2012.

[2] C.-C. Hu, Y.-L. Kuo, C.-Y. Chiu, and Y.-M. Huang, "Maximum Bandwidth Routing and Maximum Flow Routing in Wireless Mesh Networks," *Telecommunication Systems*, vol. 44, pp. 125–134, 2010.

[3] *Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specification*, IEEE Std. 802.11, 2016.

[4] L. Zhang and X. H. Wang, "A Maximum Concurrent Network Flow Based Channel Assignment Algorithm for Wireless Mesh Networks," in *Mechanical Engineering, Industrial Electronics and Information Technology Applications in Industry*, ser. Applied Mechanics and Materials, vol. 427. Trans Tech Publications Ltd., 2013, pp. 2212–2216.

[5] G. Kuperman, N. M. Jones, J. Sun, and A. Narula-Tam, "A Fast and Scalable Algorithm for Calculating the Achievable Capacity of a Wireless Mesh Network," in *Intl. Symp. on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks*, 2016.

[6] S. Huang and R. Dutta, "Design of Wireless Mesh Networks Under the Additive Interference Model," in *Intl. Conference on Computer Communications and Networks*, 2006.

[7] M. Alicherry, R. Bhatia, and L. E. Li, "Joint Channel Assignment and Routing for Throughput Optimization in Multiradio Wireless Mesh Networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 11, pp. 1960–1971, 2006.

[8] C. E. Shannon, "Communication in the Presence of Noise," *Proceedings of the IRE*, vol. 37, no. 1, pp. 10–21, 1949.

[9] A. Molisch, *Wireless Communications*. Wiley-IEEE Press, 2005.

[10] M. Backhaus and G. Schaefer, "Towards the Complexity of the Widest Path Problem in Hybrid Multi-Channel WMNs," in *Intl. Conf. on Wireless and Mob. Computing, Netw. and Comm.*, 2020, currently under review. [Online]. Available: tu-ilmenau.de/telematik/hwmcwpnpc

[11] M. Backhaus and G. Schaefer, "Backup Paths for Multiple Demands in Overlay Networks," in *Global Information Infrastructure and Networking Symposium*, 2016.

[12] B. Milic and M. Malek, "NPART - Node Placement Algorithm for Realistic Topologies in Wireless Multihop Network Simulation," in *Simutools*, 2009.

[13] M. K. Marina, S. R. Das, and A. P. Subramanian, "A topology control approach for utilizing multiple channels in multi-radio wireless mesh networks," *Computer Networks*, vol. 54, no. 2, pp. 241 – 256, 2010.

[14] *IEEE Standard for Ethernet*, IEEE Std. 802.3, 2018.

[15] Gurobi Optimization, Inc., "Gurobi Optimizer Reference Manual," 2020. [Online]. Available: gurobi.com

[16] Cisco Systems, Inc. Cisco Wireless Mesh Access Points, Design and Deployment Guide. Accessed: 03-July-2020. [Online]. Available: cisco.com/c/en/us/td/docs/wireless/technology/mesh/7-3/design/guide/Mesh/Mesh_chapter_011.html

[17] Huawei Technologies Co., Ltd. Fat AP and Cloud AP V200R008C10 Command Reference. Accessed: 03-July-2020. [Online]. Available: support.huawei.com/enterprise/en/doc/EDOC1100008296/b0ed4d4a/link-rssi-threshold