For what it's worth:

An analyst's hunt for asymptotic heat transport in Rayleigh-Bénard convection

Charlie

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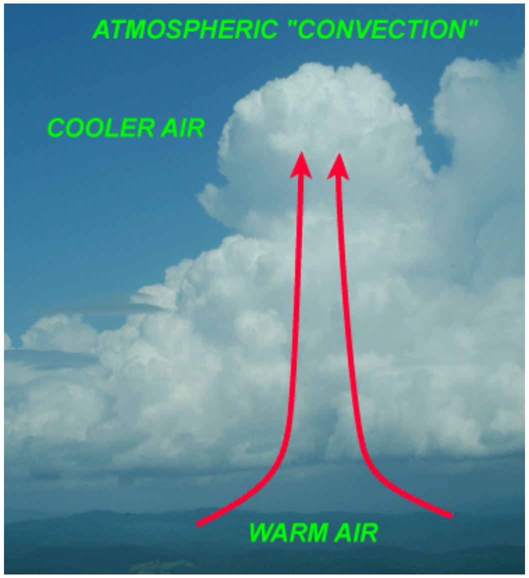
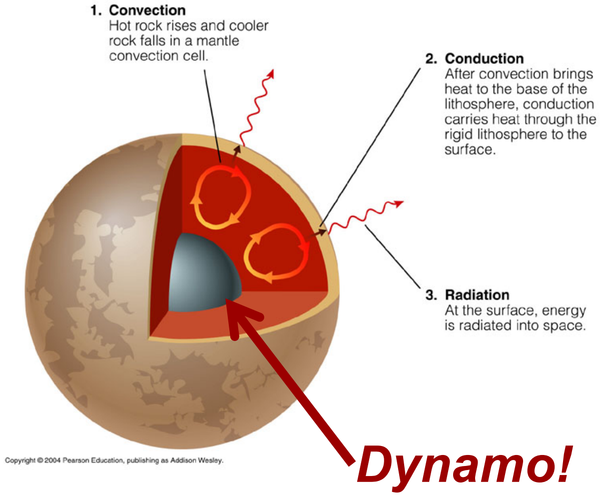
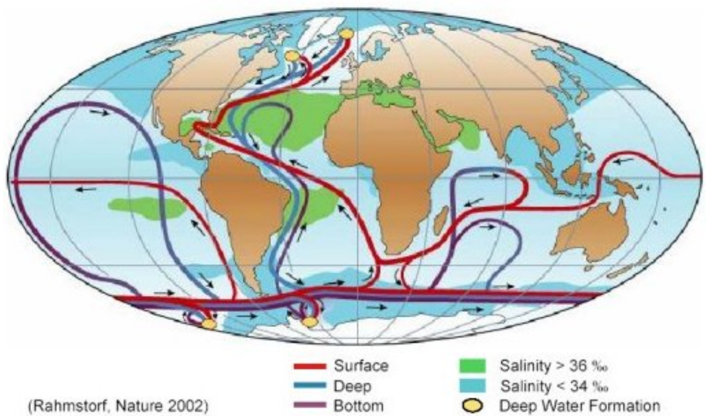
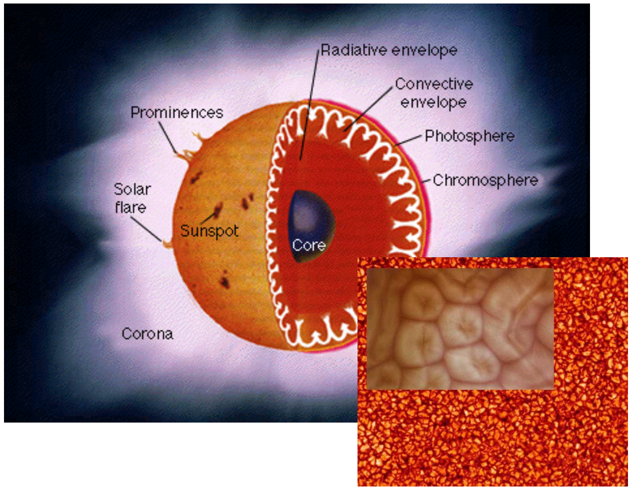
Awards DMS-1515161 and DMS-1813003


ATMOSPHERIC "CONVECTION"
COOLER AIR
WARM AIR


1. Convection
Hot rock rises and cooler rock falls in a mantle convection cell.
2. Conduction
After convection brings heat to the base of the lithosphere, conduction carries heat through the rigid lithosphere to the surface.
3. Radiation
At the surface, energy is radiated into space.
Copyright © 2004 Pearson Education, publishing as Addison Wesley.
Dynamo!


Surface
Deep
Bottom
Salinity > 36 ‰
Salinity < 34 ‰
Deep Water Formation
(Rahmstorf, Nature 2002)


Radiative envelope
Convective envelope
Prominences
Photosphere
Chromosphere
Solar flare
Sunspot
Core
Corona

THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

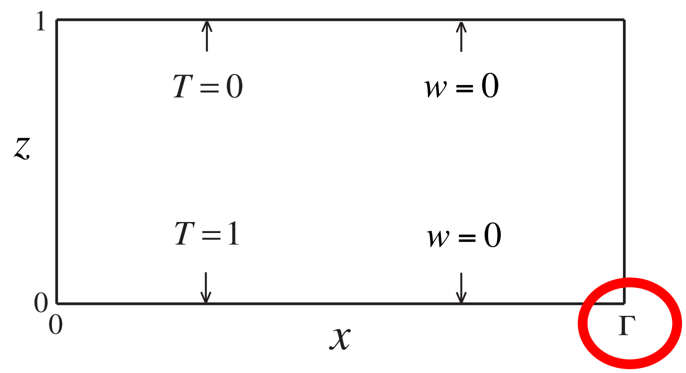
[SIXTH SERIES.]

DECEMBER 1916.

LIX. *On Convection Currents in a Horizontal Layer of Fluid, when the Higher Temperature is on the Under Side.* *By* Lord RAYLEIGH, *O.M., F.R.S.**

Dimensionless variables:

Rayleigh number: $\mathrm{Ra} = \dfrac{g\alpha(T_{hot} - T_{cold})h^3}{\nu\kappa}$ *Prandtl number*: $\mathrm{Pr} = \dfrac{\nu}{\kappa}$



$$\dot{T} + \vec{u} \cdot \vec{\nabla} T = \Delta T$$

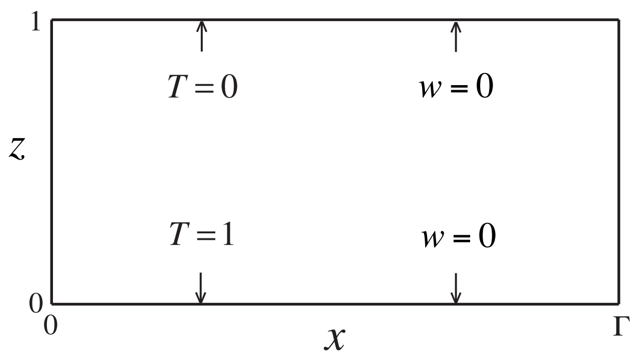
$$\frac{1}{\mathrm{Pr}}\left(\dot{\vec{u}} + \vec{u} \cdot \vec{\nabla}\vec{u}\right) + \vec{\nabla} p = \Delta \vec{u} + \mathrm{Ra}\hat{k}T$$

$$0 = \overrightarrow{\nabla} \cdot \vec{u}$$

Nusselt number: $\mathrm{Nu} \equiv \dfrac{J_z}{J_{conduction}} = 1 + \langle wT \rangle$

Dimensionless variables:

Rayleigh number: $\mathrm{Ra} = \dfrac{g\alpha(T_{hot} - T_{cold})h^3}{\nu\kappa}$ *Prandtl number*: $\mathrm{Pr} = \dfrac{\nu}{\kappa}$



Challenge:

find $\mathrm{Nu}(\mathrm{Ra},\mathrm{Pr},\Gamma)$

Nusselt number: $\mathrm{Nu} \equiv \dfrac{J_z}{J_{conduction}} = 1 + \langle wT \rangle$

Forward 85 years from Lord Rayleigh:

AUGUST 2001 PHYSICS TODAY

Leo P. Kadanoff

TURBULENT HEAT FLOW: STRUCTURES AND SCALING

For the Rayleigh–Nusselt relation, this guess is reflected in a power law of the form

$$Nu = A\, Ra^{\beta} + \ldots$$

where the ellipses represent corrections to the simple power law.

Other fits are possible. Joseph Niemela of the University of Oregon, and collaborators[10] note an excellent fit,

$$Nu = A\, [Ra^{3/2}\, (\ln Ra)]^{1/5} + \ldots$$

Siegfried Grossmann and Detlef Lohse[1]:

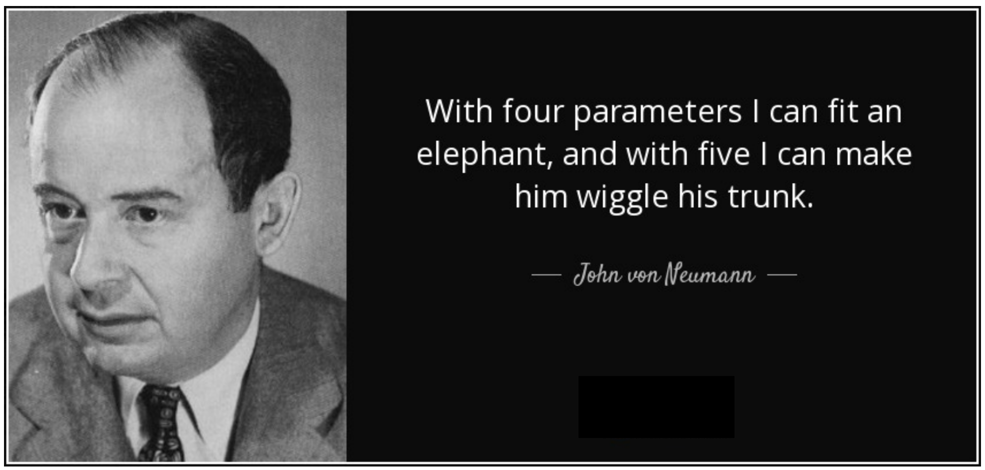
$$Nu = A\, Ra^{1/3} + B\, Ra^{1/4} + \ldots .$$

August 2001 Physics Today

Leo P. Kadanoff

Turbulent Heat Flow: Structures and Scaling

Before choosing it is reasonable to ask, “What are we trying to do?” If our goal is to get a decent representation of the facts observed in a very wide range of turbulence experiments, all three formulas are acceptably good.



Leo P. Kadanoff

Turbulent Heat Flow: Structures and Scaling

Before choosing it is reasonable to ask, “What are we trying to do?” If our goal is to get a decent representation of the facts observed in a very wide range of turbulence experiments, all three formulas are acceptably good.

A different perspective might be in order. None of these equations means anything in themselves. To give them meaning, you have to define the terms “+ . . .” appearing in all three equations. In my view, the right thing is to demand that the fit become *asymptotically accurate*. I mean that there should be some limiting process in which the proposed theory would be exactly true.[11] The limiting process would most likely involve having the Rayleigh number go to infinity, with maybe also having the Prandtl number going to some extreme value.

Today

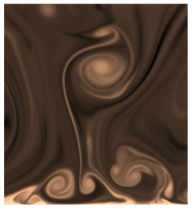
Confront the confounding question of asymptotically high Rayleigh number heat transport in Rayleigh-Bénard convection modeled by the Boussinesq approximation to the Navier-Stokes equations from some viewpoints of

- theory (models of the model),
- computation (simulations),
- experiment (laboratory tests),
- analysis (theorems).

Also remark on use of the word of ***ultimate*** vs. ***asymptotic*** …

Asymptotic theories

- **Malkus, Priestley, Howard:**

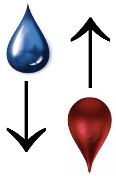


Maximal heat transport … finite nonzero $h \longrightarrow \infty$ limit …

Marginally stable boundary layer $\Rightarrow$ $Nu = \frac{1}{2}\left(\frac{Ra}{2Ra_c}\right)^{1/3}$

$$Ra_c \approx 500 \;\Rightarrow\; Nu \approx .05\, Ra^{1/3}$$

- **Spiegel v1.0:**



Finite nonzero $\kappa, \nu \rightarrow 0$ limit …

Fluid full of free falling blobs $\Rightarrow$ $Nu \sim (PrRa)^{1/2}$

… prefactor?

Asymptotic theories

- **Spiegel v2.0:**

Blobs lose heat at rate $\sim \frac{\boldsymbol{\kappa}}{\boldsymbol{\delta^2}} \Rightarrow Nu \sim (PrRa)^{1/3}$

Low Pr …

- **Spiegel v2.1:**

Blob momentum damps at rate $\sim \frac{\boldsymbol{\nu}}{\boldsymbol{\delta^2}} \Rightarrow Nu \sim Ra^{1/3}$

High Pr …

Asymptotic theories

- **Mean power balance:**

$$\frac{h^4}{\nu\kappa^2}\epsilon = \langle|\nabla\mathbf{u}|^2\rangle = Ra(Nu-1) \qquad \leftarrow exact$$

- **Introduce Reynolds number**

$$Re = \frac{Uh}{\nu}$$

- **and dissipation coefficient**

$$\epsilon = C\frac{U^3}{h}$$

Asymptotic theories

- **Mean power balance:**

$$C Pr^2 Re^3 = Ra(Nu - 1) \qquad \leftarrow exact$$

- **What *is* the Reynolds number?**

$$Re = Re(Ra, Pr)$$

- **and dissipation coefficient?**

$$C = C(Ra, Pr)$$

Asymptotic theories

- **Mean power balance:**

$$CPr^2Re^3 = Ra(Nu-1) \qquad \leftarrow exact$$

- ***Suppose*** **free-fall velocity scale**

$$assume \rightarrow \qquad Re = c\left(\frac{Ra}{Pr}\right)^{1/2}$$

- **and constant dissipation coefficient** **Spiegel v1.1*!***

$$assume \rightarrow \qquad C = \text{constant} \qquad \Rightarrow \qquad Nu - 1 = c^3C\times(PrRa)^{1/2}$$

Convection in a can:

Thanks to Susanne Horn and Olga Shishkina

http://www.lfpn.ds.mpg.de/RBC2015/

Asymptotic theories

- **Mean power balance:**

$$C Pr^2 Re^3 = Ra(Nu - 1) \qquad \leftarrow exact$$

- ***Suppose* free-fall velocity scale**

$$assume \rightarrow \qquad Re = c \left(\frac{Ra}{Pr}\right)^{1/2}$$

- **and *shear* flow dissipation coefficient**

$$assume \rightarrow \qquad C \equiv \frac{Ra(Nu - 1)}{Pr^2 Re^3} = C(Re)$$

PHYSICS OF FLUIDS VOLUME 13, NUMBER 5 MAY 2001

Turbulent Rayleigh–Bénard convection in gaseous and liquid He

X. Chavanne,[a)] F. Chillà,[b)] B. Chabaud,[c)] B. Castaing, and B. Hébral
Centre de Recherches sur les Très Basses Températures, associé àl'Université Joseph Fourier, CNRS, BP 166, 38042 Grenoble cedex 9, France

(Received 7 April 2000; accepted 23 January 2001)

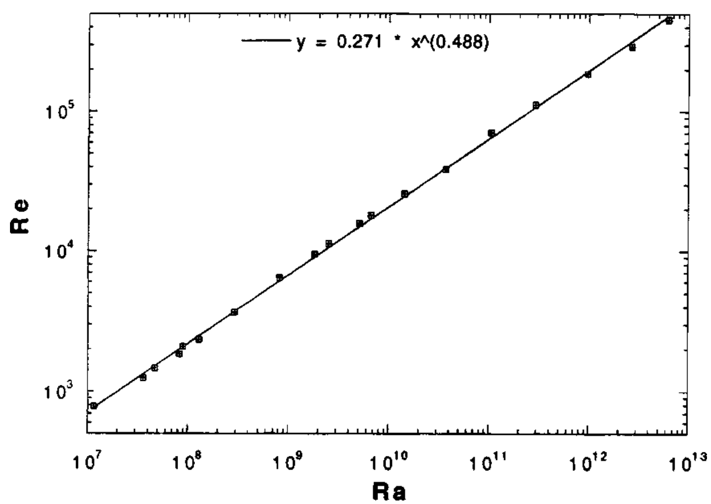


FIG. 11. The Re vs Ra for points with Pr fixed at 0.7. A power law fits the data with a 0.49 exponent.

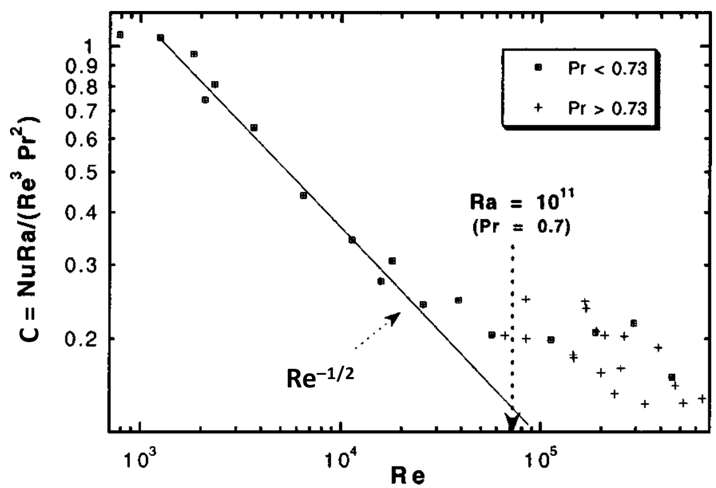


FIG. 16. Friction coefficient (normalized dissipation) vs Re for points of Table III under Boussinesq conditions and with $\Delta T \geqslant 40$ mK. Points with different Pr are distinguished.

PHYSICAL REVIEW E VOLUME 59, NUMBER 5 MAY 1999

Velocity structure functions, scaling, and transitions in high-Reynolds-number Couette-Taylor flow

Gregory S. Lewis* and Harry L. Swinney†
Center for Nonlinear Dynamics and Department of Physics, The University of Texas at Austin, Austin, Texas 78712

(Received 29 June 1998)

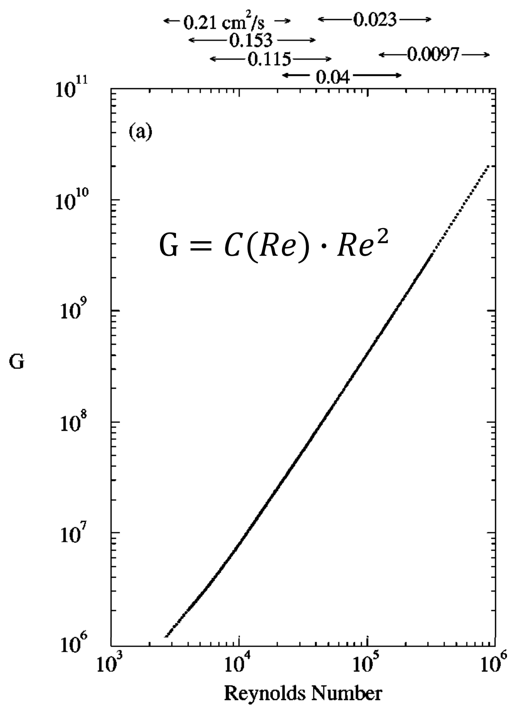
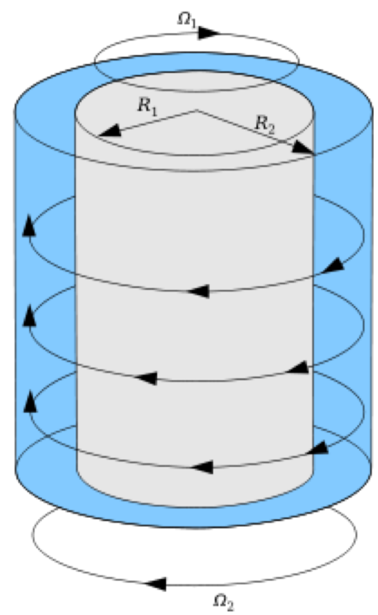


FIG. 1. (a) Experimental values of nondimensional torque in the eight-vortex state for $\Gamma = 11.4$; the average axial wavelength is 2.86 $\times$ the gap width. The viscosities of the fluids used in different runs are indicated at the top.



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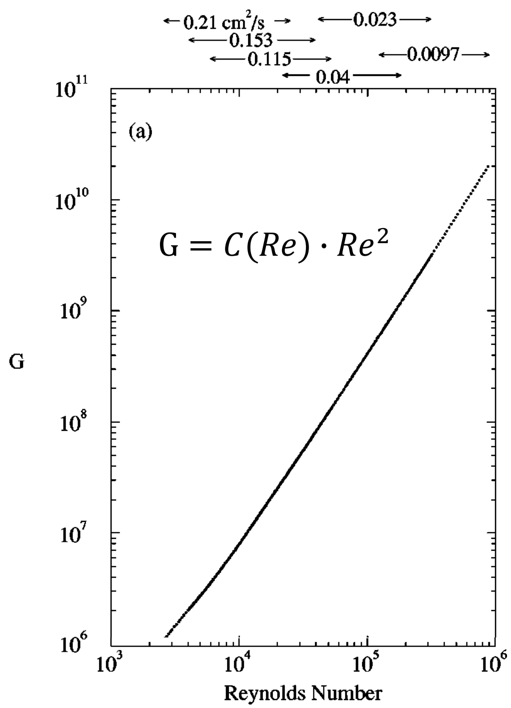

FIG. 1. (a) Experimental values of nondimensional torque in the eight-vortex state for $\Gamma = 11.4$; the average axial wavelength is 2.86 $\times$ the gap width. The viscosities of the fluids used in different runs are indicated at the top.

Prandtl-von Kármán (a ***one-parameter*** closure approximation):

$$\frac{1}{\sqrt{C(Re)}} = a \ln\left(\sqrt{C(Re)}\, Re\right) + b + \cdots$$

$$C(Re) \sim \frac{1}{[a \ln Re + \cdots]^2}$$ ← asymptotic prediction as $Re \to \infty$

$$\frac{d \log C(Re)}{d \log Re} \sim -\frac{2}{\ln Re}$$ ← ***parameter—free asymptotic prediction***

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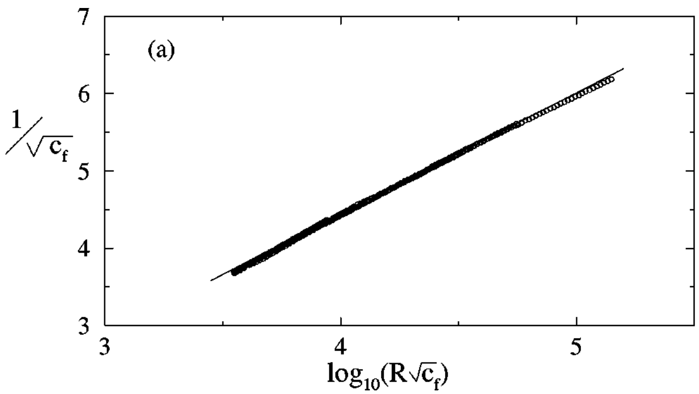
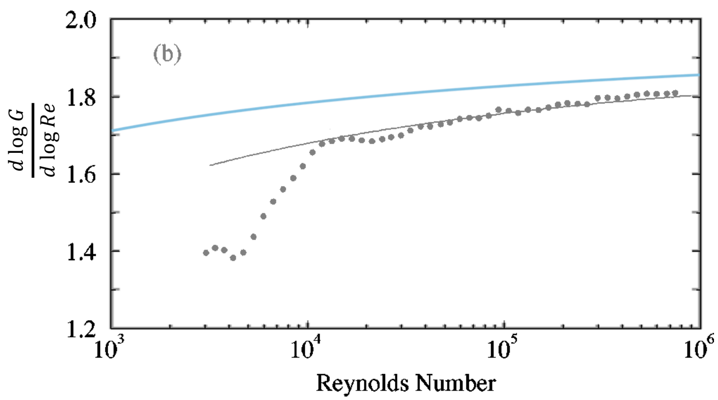


Prandtl-von Kármán (a ***one-parameter*** closure approximation):

$$\frac{1}{\sqrt{C(Re)}} = a \ln\left(\sqrt{C(Re)}\, Re\right) + b$$



← ***parameter—free asymptotic prediction***
← "**ultimate**" behavior … w/parameter fit

$$\frac{d \log G}{d \log Re} \sim 2 - \frac{2}{\ln Re}$$

$$\frac{d \log G}{d \log Re} = 2 + \frac{d \log C}{d \log Re}$$

Asymptotic theories

- **Chavanne *et al.* 2001:**

$$C \cdot Pr^2\, Re^3 = Ra(Nu - 1) \qquad \leftarrow \textit{exact}$$

$$\textit{assume} \rightarrow \qquad Re = c\left(\frac{Ra}{Pr}\right)^{1/2}$$

$$\textit{assume} \rightarrow \qquad C(Ra, Pr) = C(Re) \text{ from Prandtl–von Kármán}$$

- **Parameter free asymptotic prediction:**

$$\frac{d \log Nu}{d \log Ra} \sim \frac{1}{2} - \frac{2}{\ln Ra}$$

Asymptotic theories

- **Grossmann & Lohse 2011:**

$$C \cdot Pr^2\, Re^3 = Ra(Nu-1) \qquad \leftarrow \textit{exact}$$

assume → $Re \sim$ another $c{\times}Ra^{0.5}$

assume → $C(Ra,Pr) = C(Re)$ from Prandtl−von Kármán

- **Parameter free asymptotic prediction:** (***same*** as Chavanne *et al.*)

$$\frac{d\log Nu}{d\log Ra} \sim \frac{1}{2} - \frac{2}{\ln Ra}$$

Asymptotic theories

- **Kraichnan 1962:**

$$C \cdot Pr^2\, Re^3 = Ra(Nu - 1) \qquad \leftarrow exact$$

$$assume \rightarrow \qquad Re(\ln Re)^{1/2} = c\left(\frac{Ra}{Pr}\right)^{1/2}$$

$$(essentially)\ \ assume \rightarrow \qquad C(Ra, Pr) = constant$$

- **Parameter free asymptotic prediction:**

$$\frac{d \log Nu}{d \log Ra} \sim \frac{1}{2} - \frac{3}{2}\frac{1}{\ln Ra}$$

Experiments

Search for the "Ultimate State" in Turbulent Rayleigh-Bénard Convection

Denis Funfschilling,[1] Eberhard Bodenschatz,[2] and Guenter Ahlers[3]

[1]*LSGC CNRS - GROUPE ENSIC, BP 451, 54001 Nancy Cedex, France*

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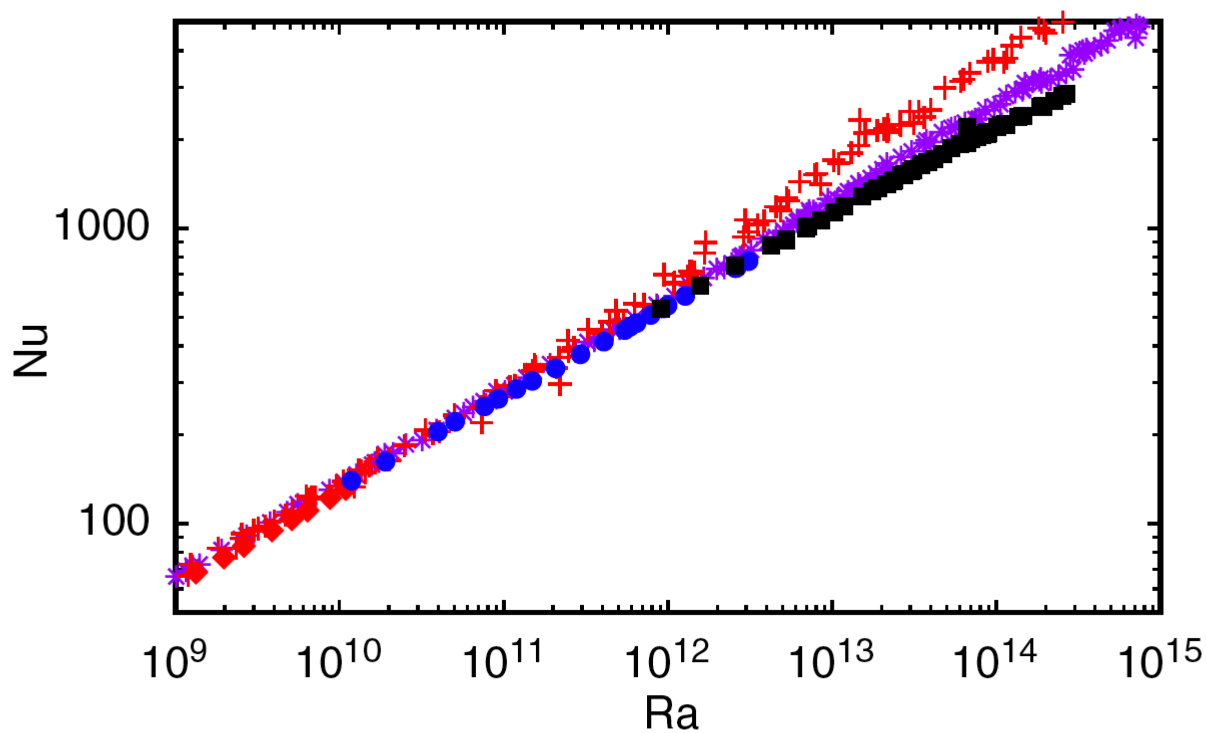
(Received 16 April 2009; published 1 July 2009)



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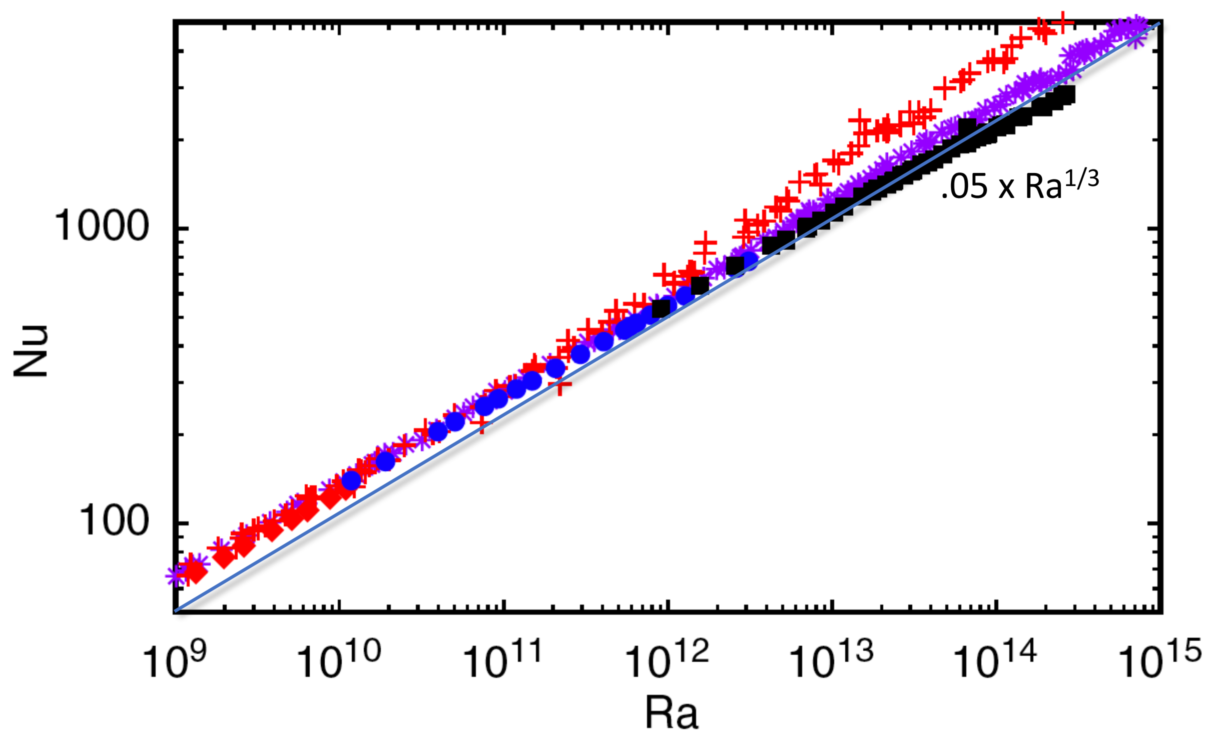

Malkus' marginally stable boundary layer theory $Nu \approx .05 \times Ra^{1/3}$ overlaid for reference

Search for the "Ultimate State" in Turbulent Rayleigh-Bénard Convection

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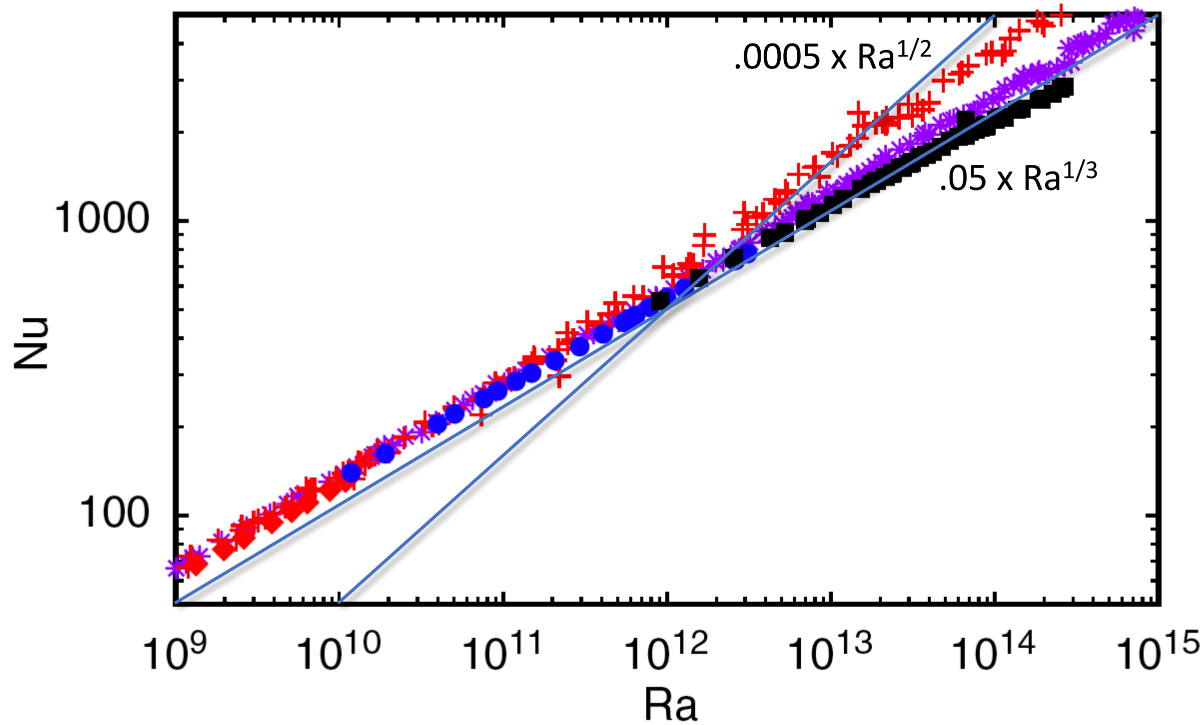

Malkus'
Nu ≈ .05 x Ra$^{1/3}$
plus a Nu ~ Ra$^{1/2}$
scaling guide for
reference

PRL **103,** 014503 (2009) PHYSICAL REVIEW LETTERS week ending 3 JULY 2009

Search for the "Ultimate State" in Turbulent Rayleigh-Bénard Convection

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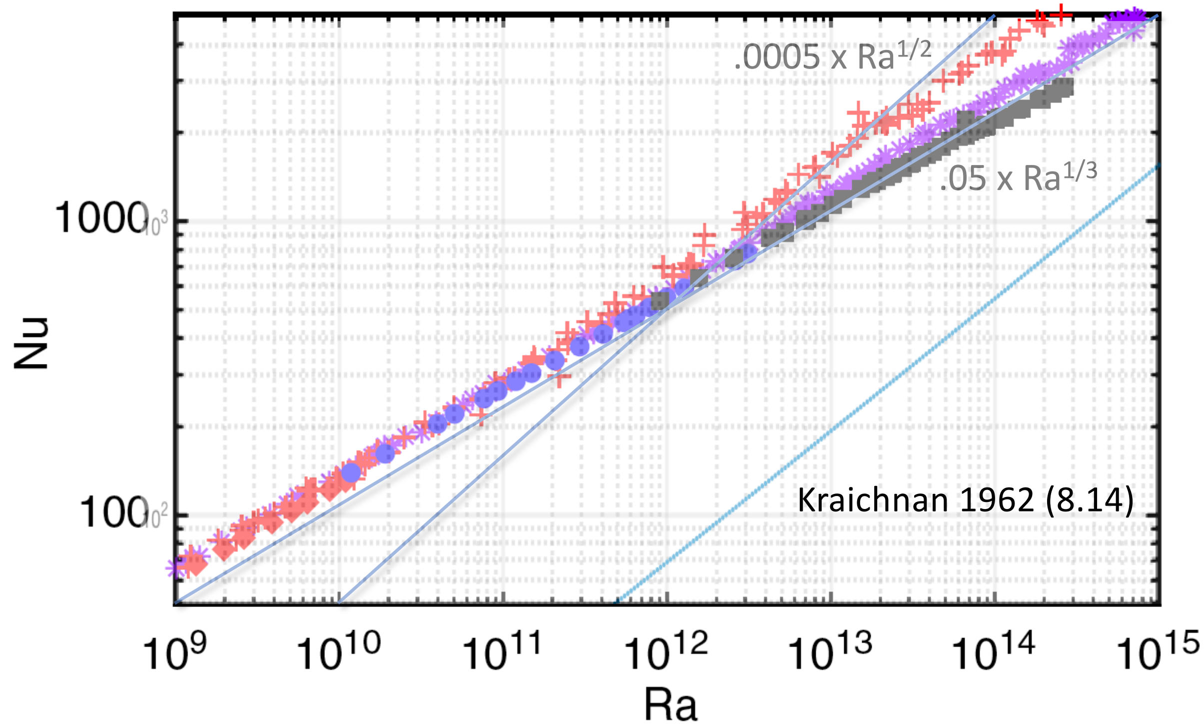

Malkus'
$Nu \approx .05 \times Ra^{1/3}$
plus a $Nu \sim Ra^{1/2}$
scaling guide for
reference
plus Kraichnan
theory for
$Pr = 0.7$

Transition to the Ultimate State of Turbulent Rayleigh-Bénard Convection

Xiaozhou He,[1] Denis Funfschilling,[2] Holger Nobach,[1] Eberhard Bodenschatz,[1,3,4] and Guenter Ahlers[5]

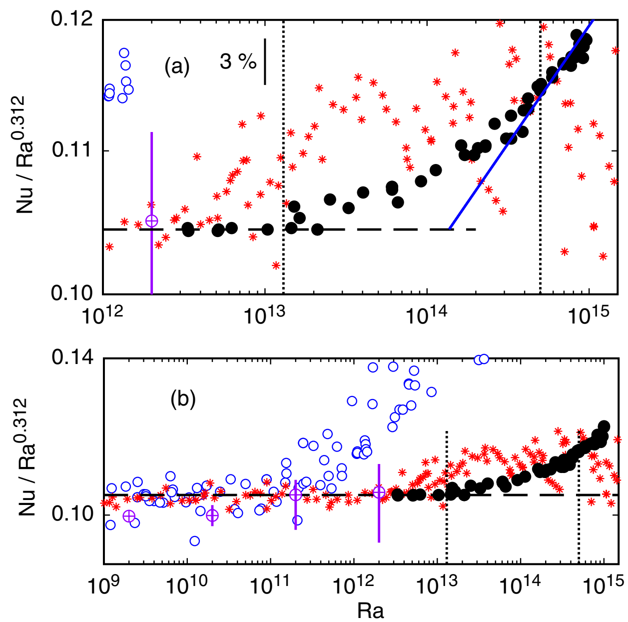

FIG. 1 (color online). $\mathrm{Nu}_{\mathrm{red}} \equiv \mathrm{Nu}/\mathrm{Ra}^{0.312}$ as a function of Ra for the "closed" sample. Black solid circles: $T_m - T_U \lesssim -3$ K. Solid line (blue) through the data at the largest Ra corresponds to $\gamma_{\mathrm{eff}} = 0.38$. Vertical dotted lines: $\mathrm{Ra}_1^* = 1.3 \times 10^{13}$ and $\mathrm{Ra}_2^* = 5 \times 10^{14}$. Small stars (red): Ref. [16]. Small open circles (blue): Ref. [9]. Circles with pluses and error bars (purple): DNS [14].

PRL **103,** 014503 (2009) PHYSICAL REVIEW LETTERS week ending 3 JULY 2009

Search for the "Ultimate State" in Turbulent Rayleigh-Bénard Convection

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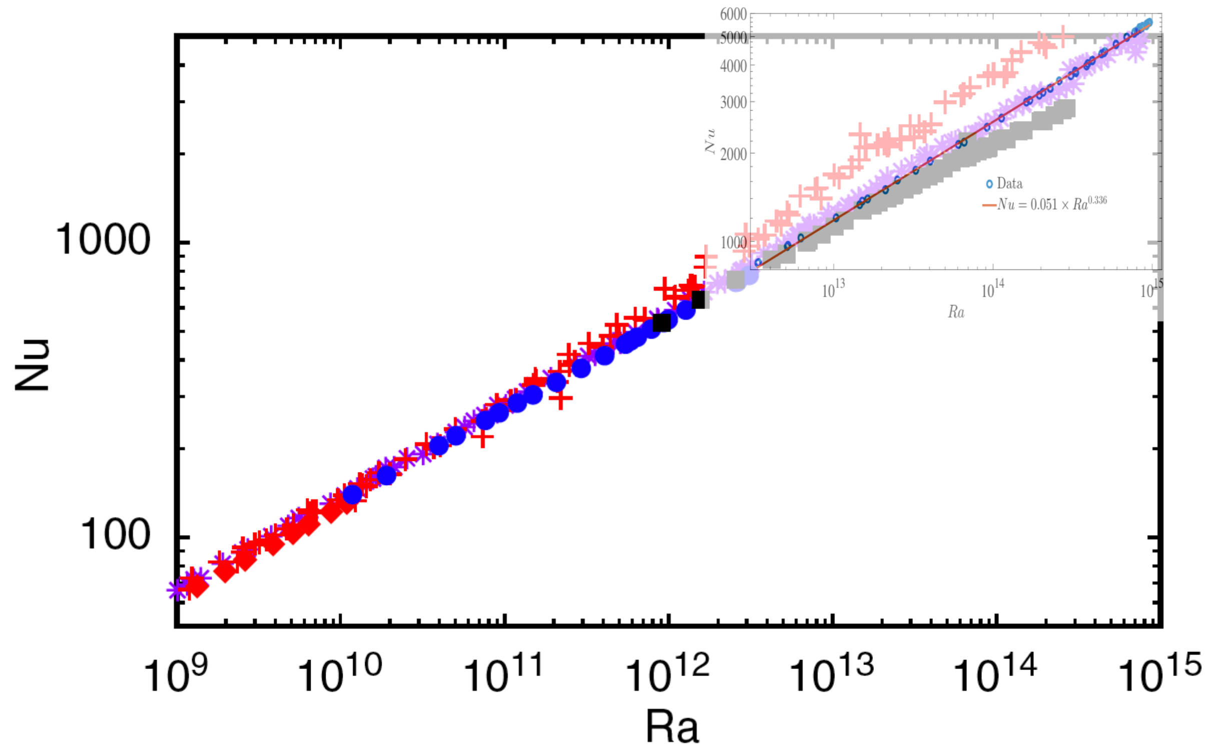
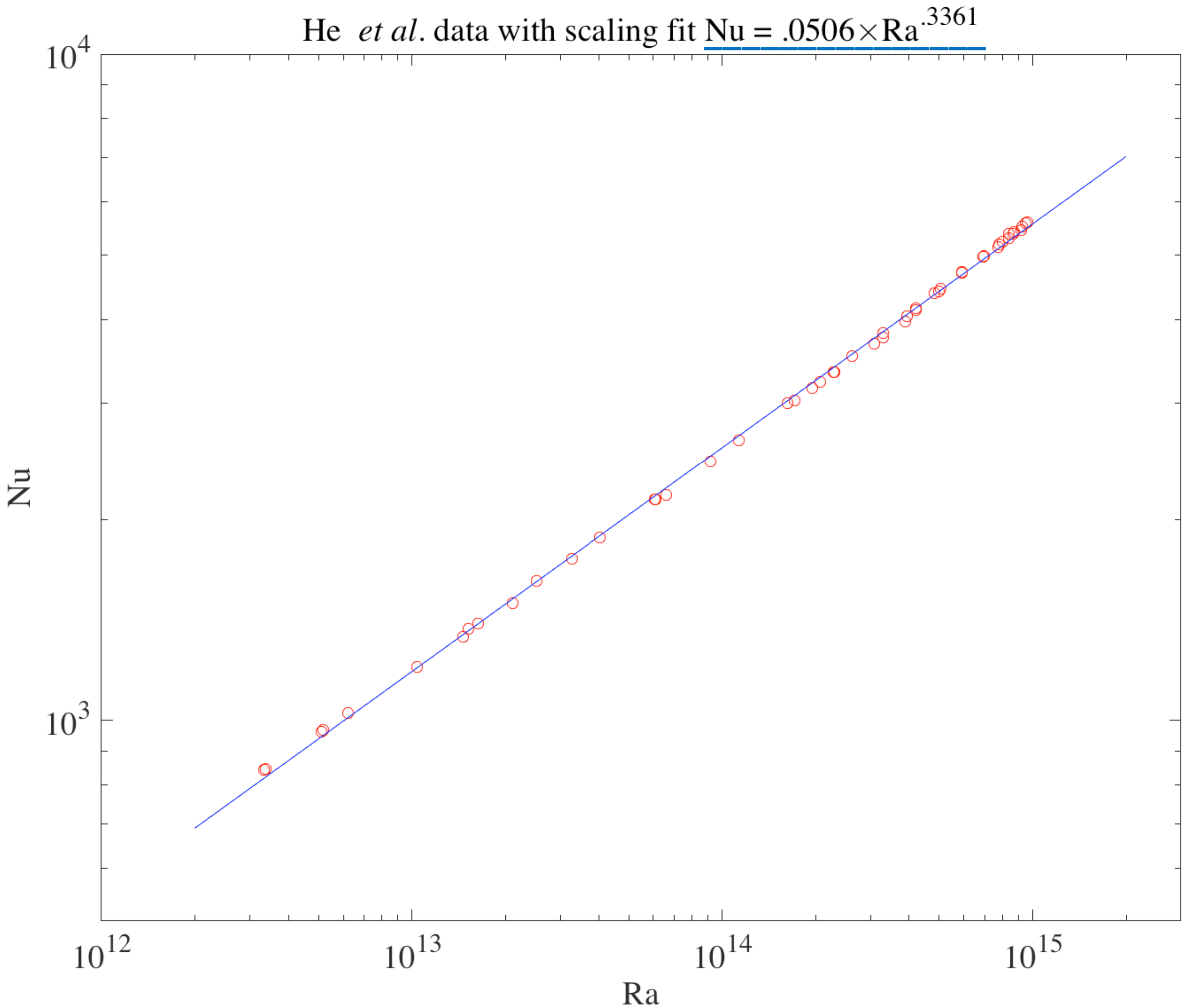
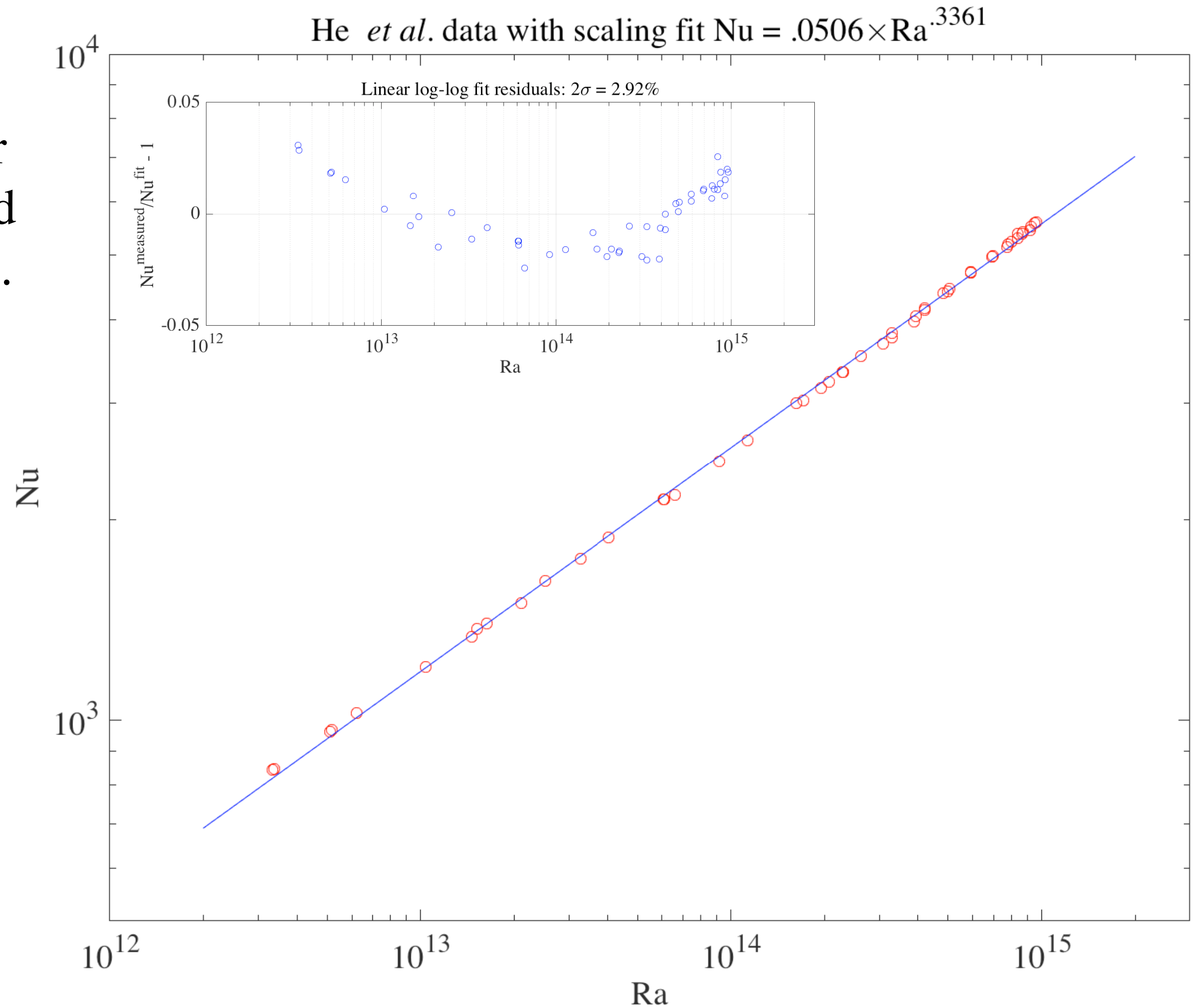


He *et al*. data overlaid …

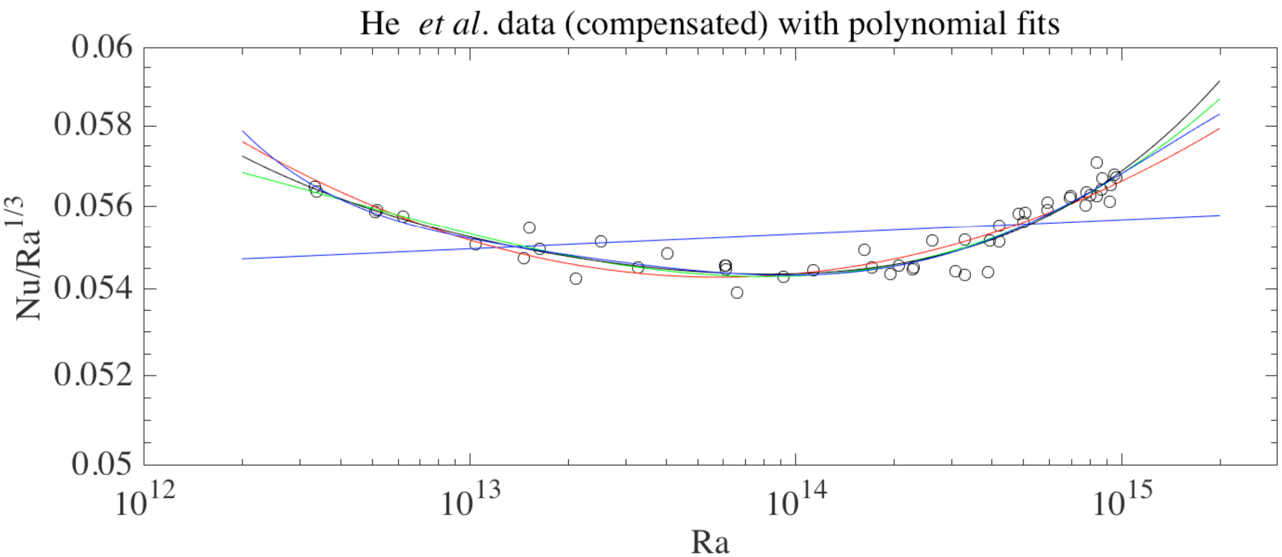
Log-log plot and scaling fit of full data set:



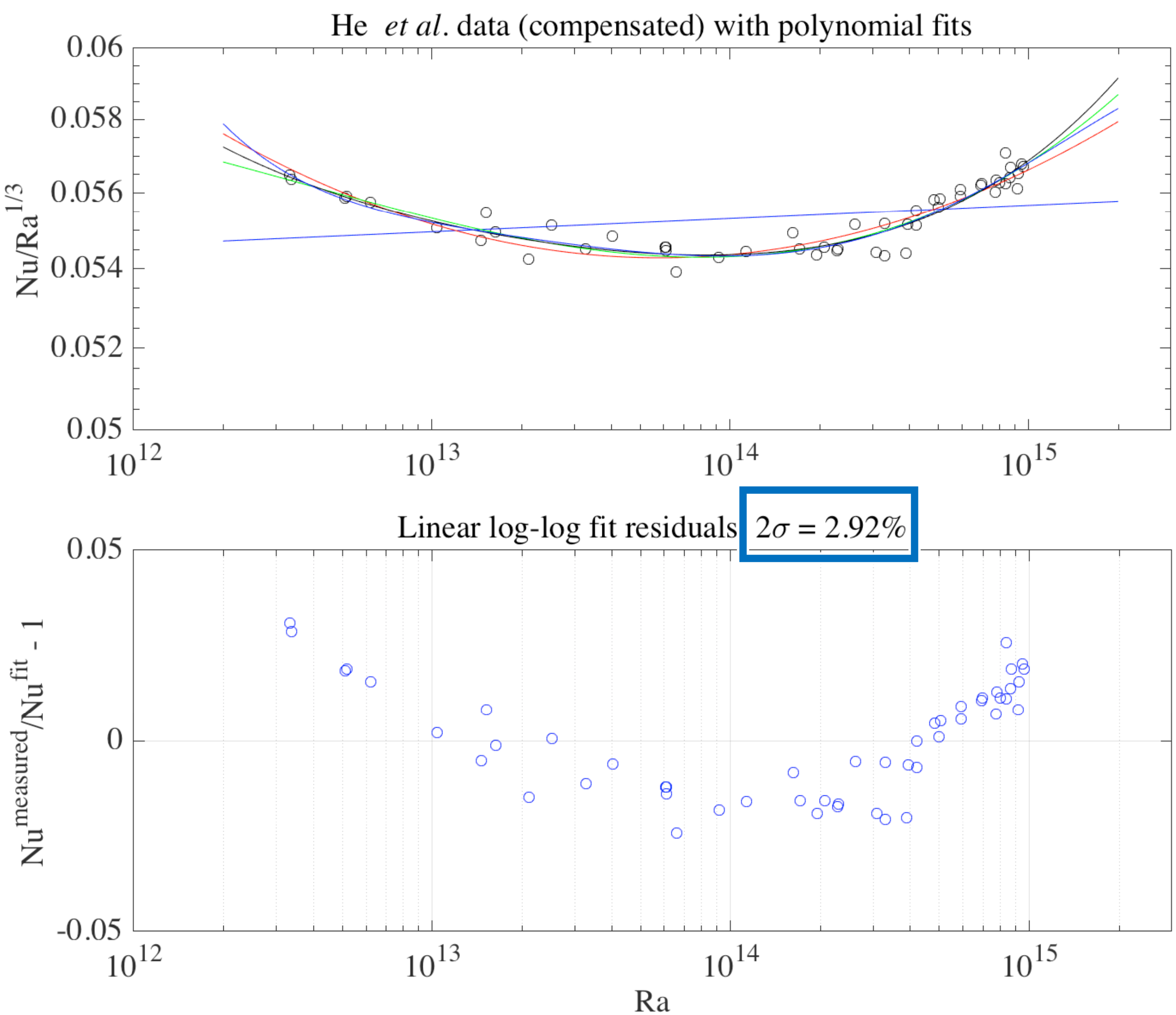
There is a clear systematic trend to the residuals.

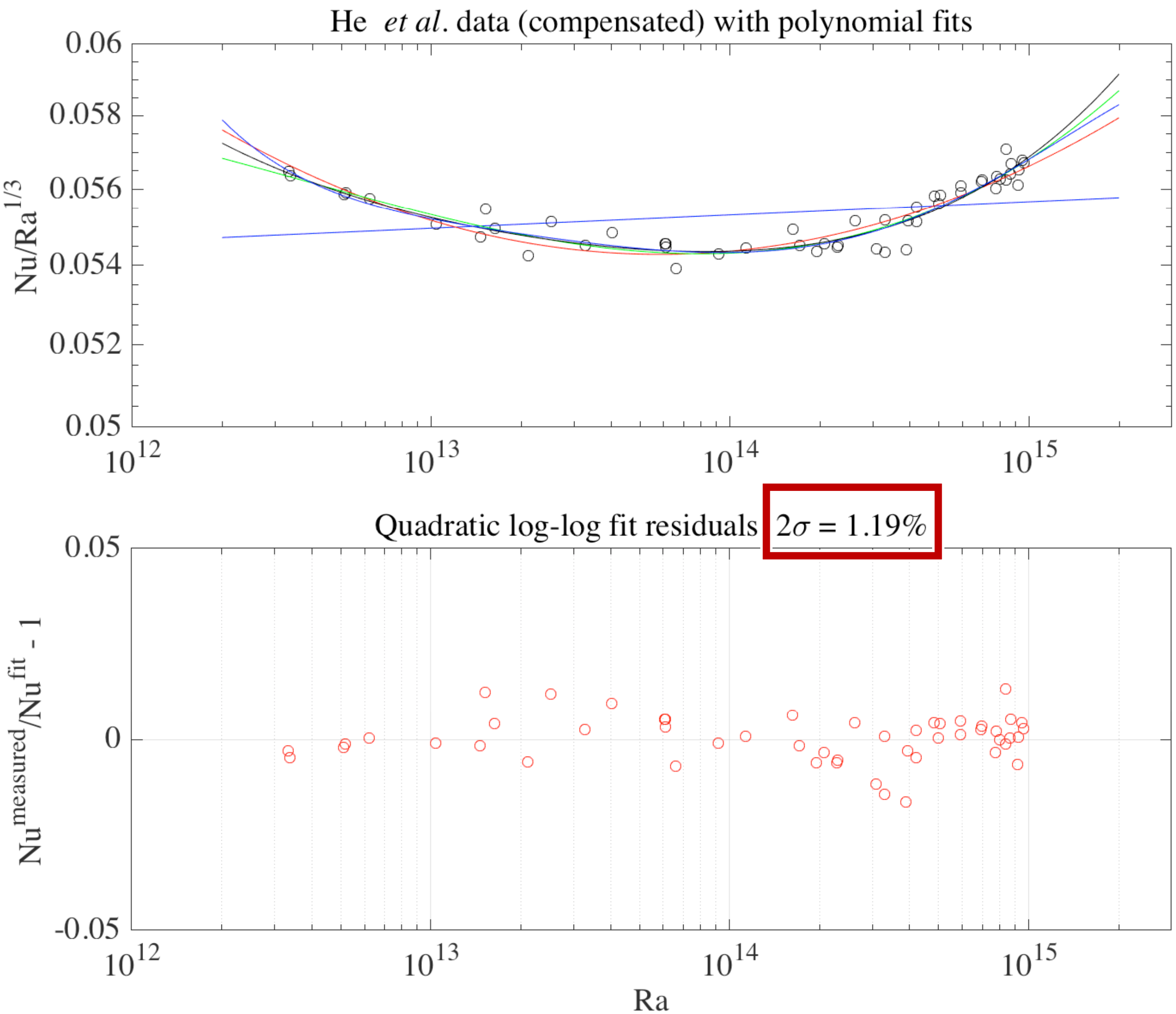


Fit data to higher order curves
(via least squares of logs)
and examine residuals.

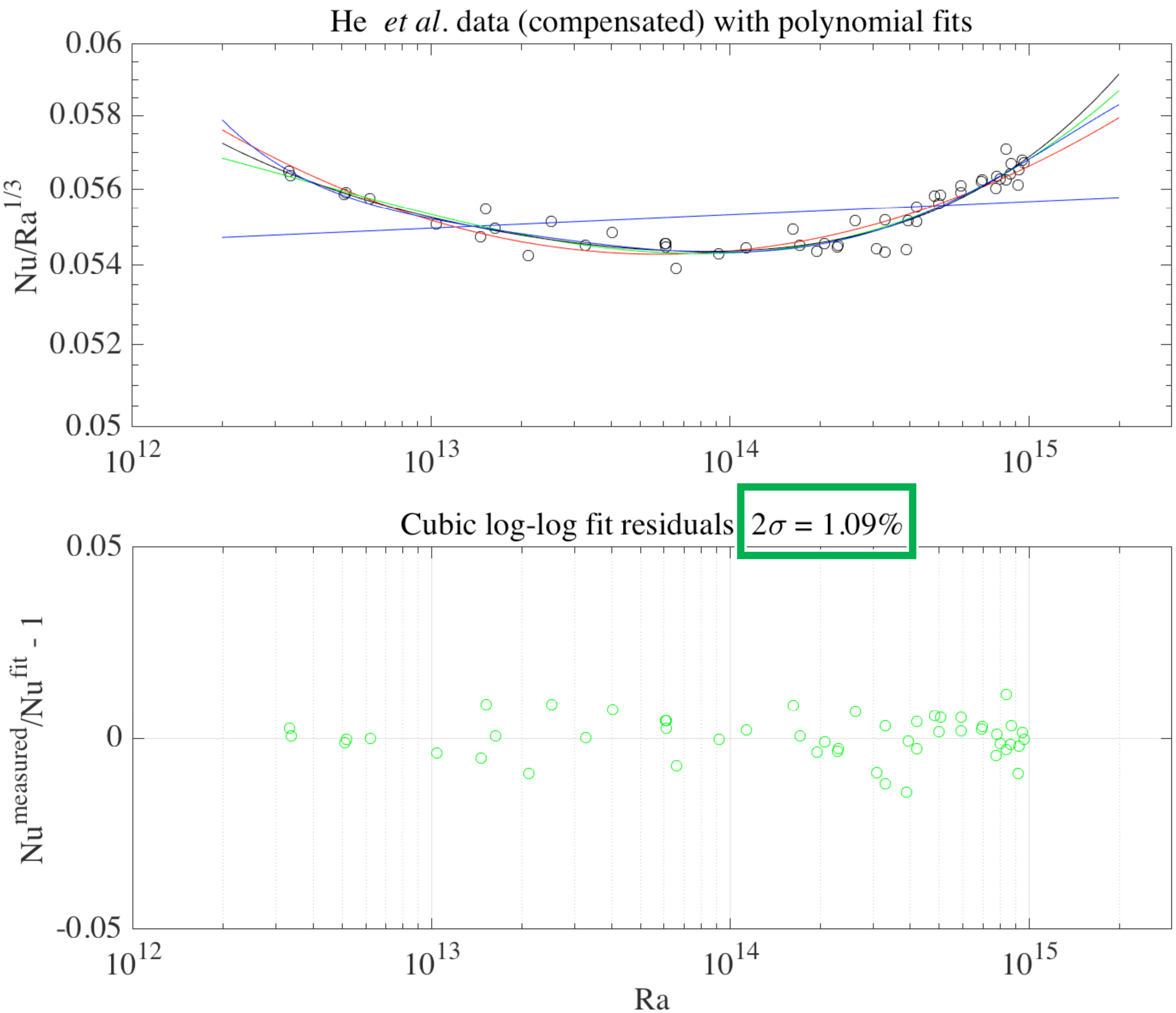


linear
quadratic
cubic
quartic
quintic


He et al. data (compensated) with polynomial fits
Nu/Ra^1/3
0.06
0.058
0.056
0.054
0.052
0.05
10^12
10^13
10^14
10^15
Linear log-log fit residuals 2σ = 2.92%
Nu^measured/Nu^fit - 1
0.05
0
-0.05
Ra

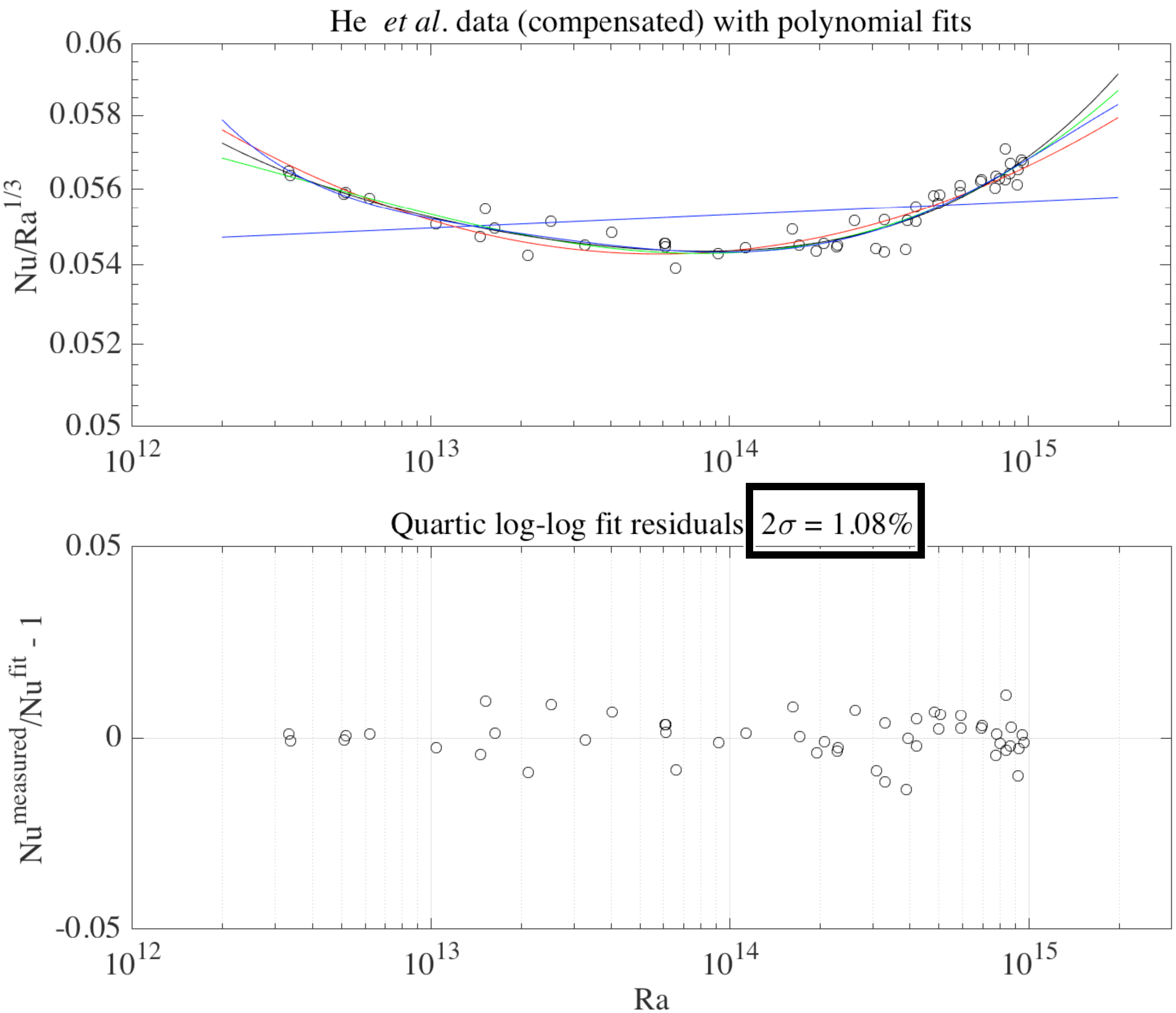


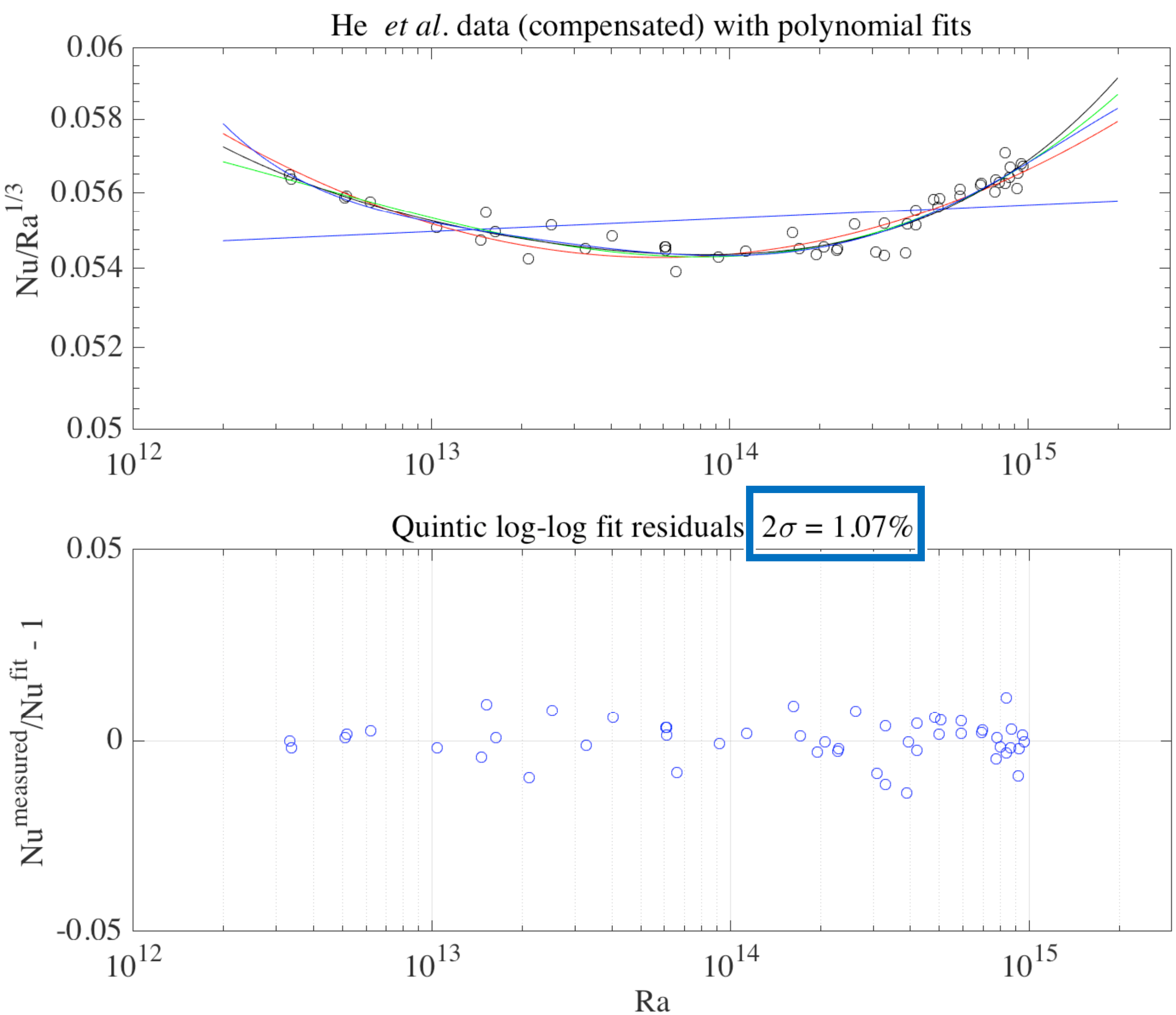
Much better fit allowing curvature.



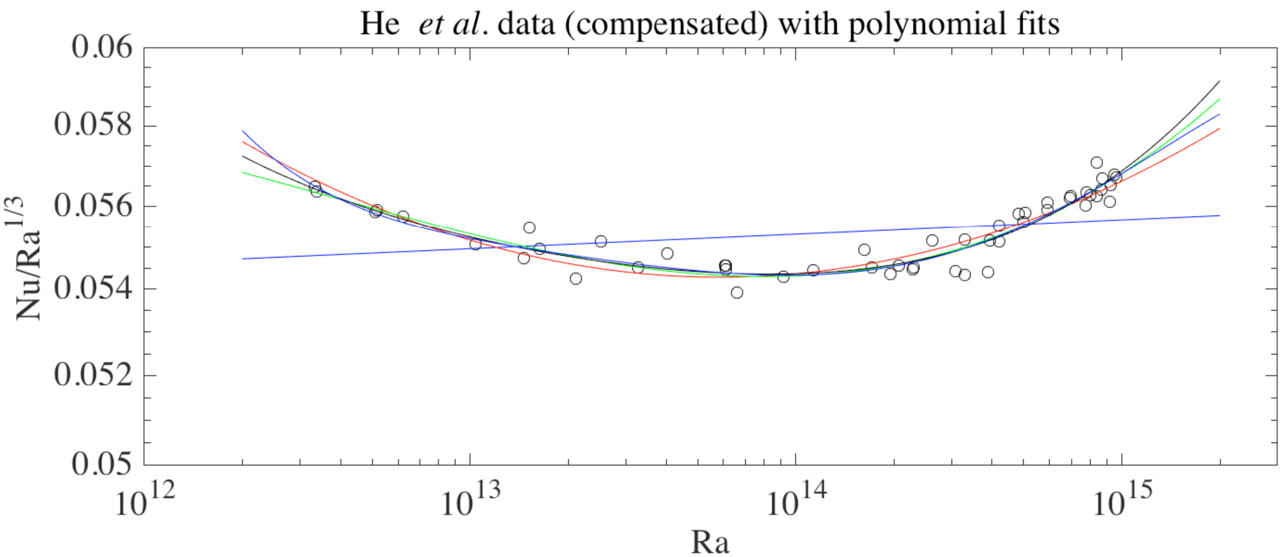
A little better but not so much.

Just a wee bit better …





And a wee bit more …



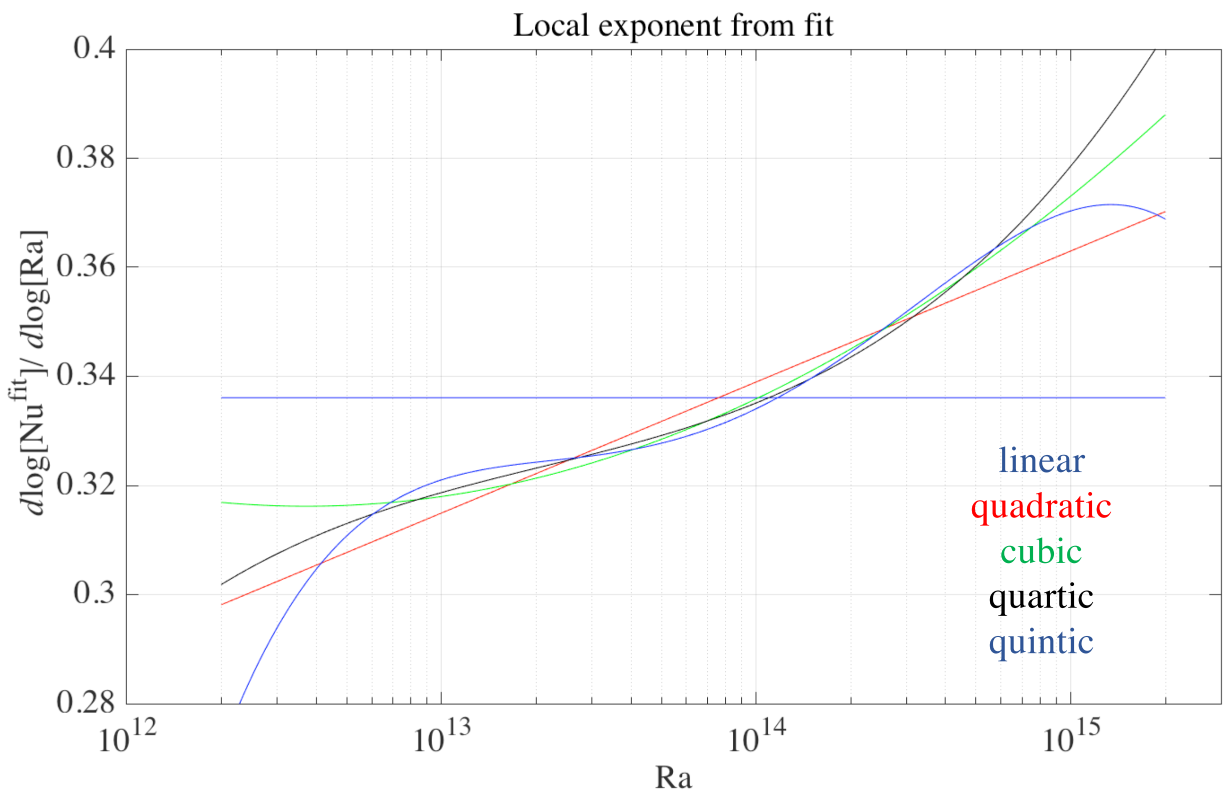
Point: the data cannot discriminate among quadratic, cubic, quartic and quantic fits …

… so the range of equivalent fits quantifies uncertainty in local exponent estimates.

Compute local exponents from the fits:

(Differentiate the polynomials.)

Compute local exponents from the fits:



Compare with a functional fit:

S. Grossmann and D. Lohse Phys. Fluids **23**, 045108 (2011)

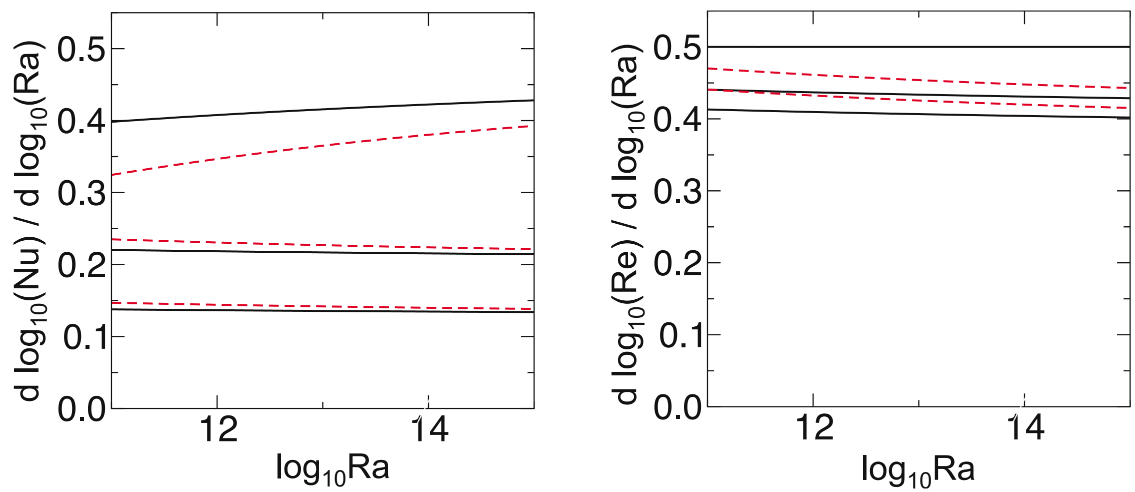
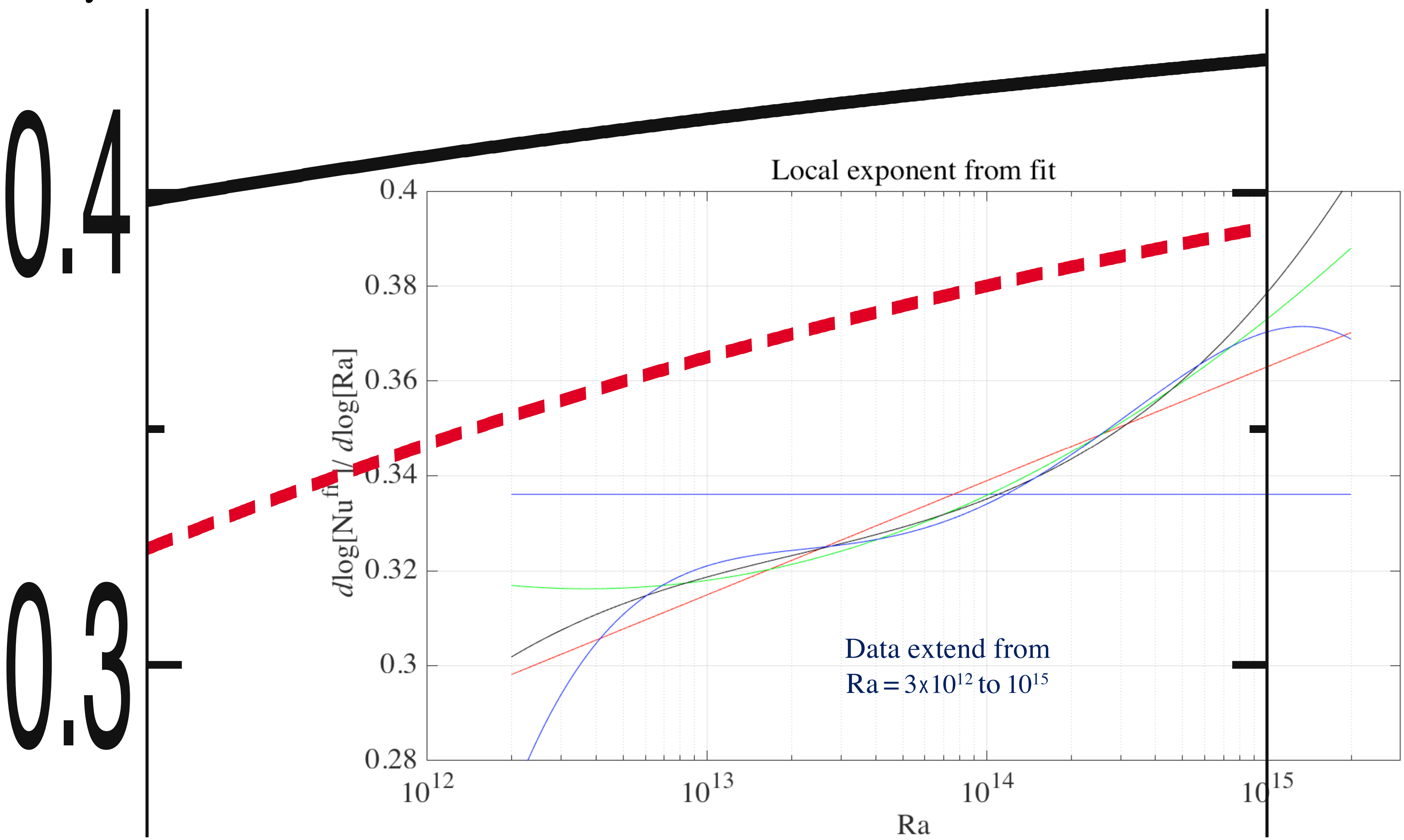


FIG. 2. (Color online) Solid black lines: Local scaling exponents $d \log \mathrm{Nu}/d \log \mathrm{Ra}$ (a) and $d \log \mathrm{Re}/d \log \mathrm{Ra}$ (b) for the three regimes of Sec. III A (bottom lines: plume dominated thermal transport in laminar thermal BL); Sec. III B (middle lines: background dominated thermal transport in laminar thermal BL); and Sec. III C (top lines: turbulent thermal BL) for Ra in the regime $10^{11}-10^{15}$. The dashed (red online) lines correspond to the case for which we have replaced $\mathrm{Re}_1(\mathrm{Ra})$ by $\mathrm{Re}_2(\mathrm{Ra})=\mathrm{Re}_1(\mathrm{Ra})/100$ in the calculation of the logarithmic correction factor $\mathcal{L}(\mathrm{Re})$, reflecting the weaker mean flow beyond the breakdown of the laminar kinetic boundary-layer. For all cases we have assumed $\mathrm{Pr}=0.7$, $b=1$, and $\bar{\kappa}=\bar{\kappa}_\theta=0.4$. lg means $\log_{10}$.

Overlay and zoom in:



For what it’s worth ...

There's something happening here
What it is ain't exactly clear

— *S. Stills* [1]

[1] <https://www.youtube.com/watch?reload=9&v=gp5JCrSXkJY>

New Journal of Physics
The open-access journal for physics

New Journal of Physics **14** (2012) 063030

Heat transport by turbulent Rayleigh–Bénard convection for $Pr \simeq 0.8$ and $4 \times 10^{11} \lesssim Ra \lesssim 2 \times 10^{14}$: ultimate-state transition for aspect ratio $\Gamma = 1.00$

Xiaozhou He[1,6], Denis Funfschilling[2,6], Eberhard Bodenschatz[1,3,4,6] and Guenter Ahlers[1,5,6,7]

Fit to the data with
$0.85 \le \mathrm{Pr} \le 0.87$
& $\mathrm{Ra} \ge 2 \times 10^{13}$

$$\mathrm{Nu} = .0444 \times \mathrm{Ra}^{0.3393}$$

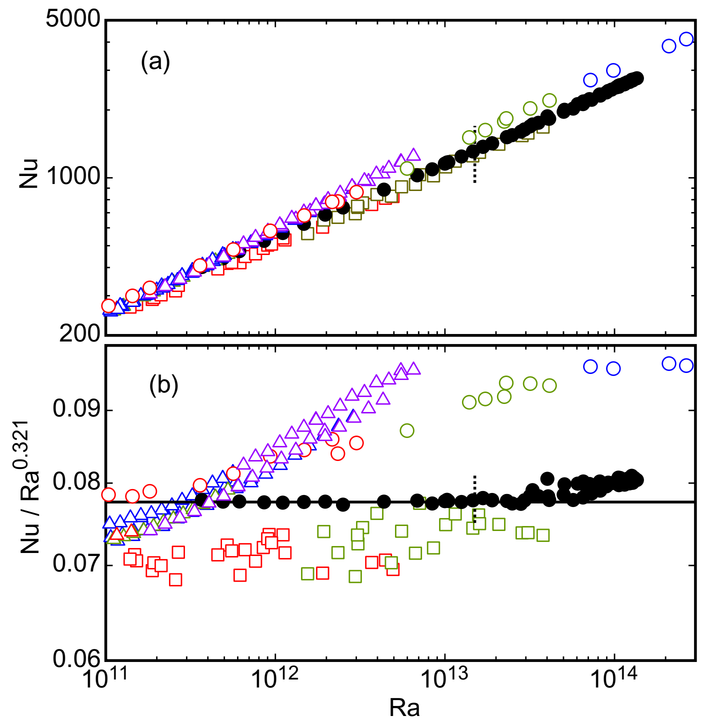


Figure 4. Comparison of our results (black solid circles, $0.79 \leqslant Pr \leqslant 0.86$) with the data of previous investigations. For the previous work we show data with $Pr < 1$ in red, data with $1 < Pr < 2$ in green, data with $2 < Pr < 4$ in blue, and data with $Pr > 4$ in purple. Open circles: Niemela and Sreenivasan [38]. Open up-pointing triangles: Roche *et al* [35]. Open squares: Urban *et al* [40]. The short vertical dotted line represents $Ra_1^* = 1.5 \times 10^{13}$ as determined for $\Gamma = 0.50$ [22, 36].

New Journal of Physics
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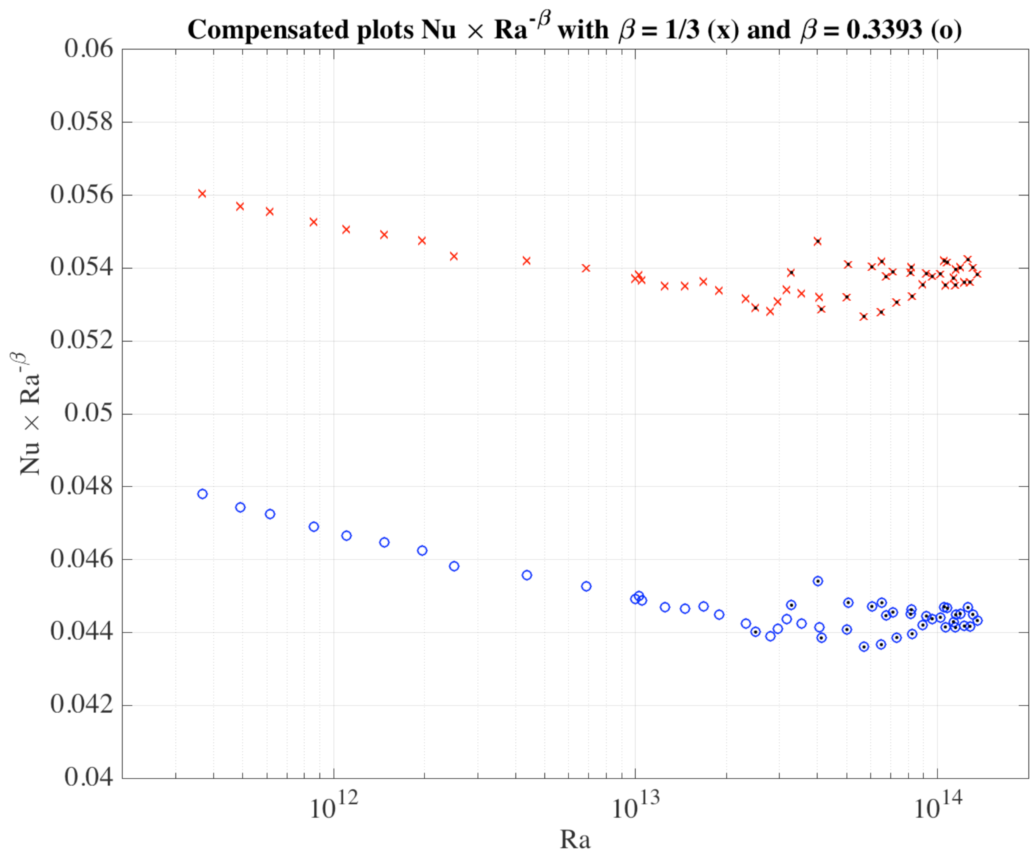


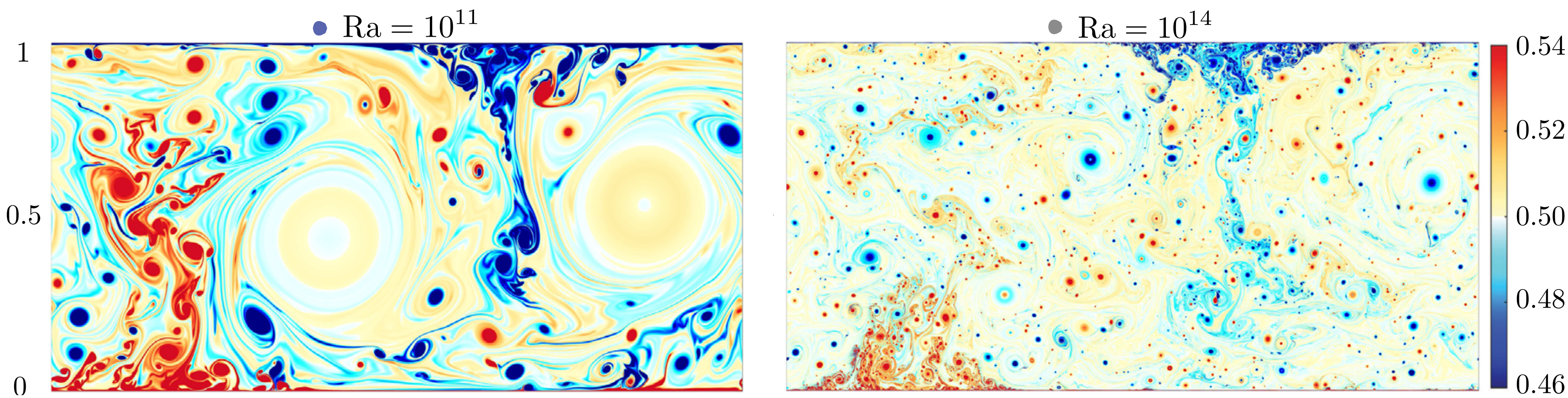
In view of the above it is our view that the ultimate-state transition has not yet been seen in any of the published data for $\Gamma \simeq 1$.

Simulations

PHYSICAL REVIEW LETTERS **120,** 144502 (2018)

Transition to the Ultimate Regime in Two-Dimensional Rayleigh-Bénard Convection

Xiaojue Zhu,[1,*] Varghese Mathai,[1] Richard J. A. M. Stevens,[1] Roberto Verzicco,[2,1] and Detlef Lohse[1,3,†]
[1]*Physics of Fluids Group and Max Planck Center Twente for Complex Fluid Dynamics, MESA+Institute and J. M. Burgers Centre for Fluid Dynamics, University of Twente, P.O. Box 217, 7500AE Enschede, The Netherlands*
[2]*Dipartimento di Ingegneria Industriale, University of Rome "Tor Vergata", Via del Politecnico 1, Roma 00133, Italy*
[3]*Max Planck Institute for Dynamics and Self-Organization, 37077 Göttingen, Germany*



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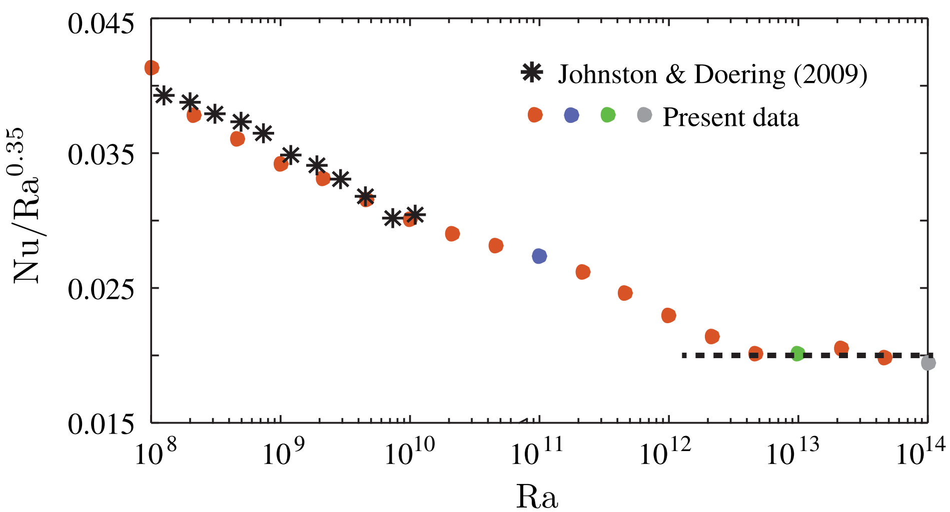

FIG. 1. Nu(Ra) plot compensated by $Ra^{0.35}$. The data agree well with the previous results in the low-Ra regime [39]. The flow structures of the three colored data points (blue for $Ra = 10^{11}$, green for $Ra = 10^{13}$, and gray for $Ra = 10^{14}$) are displayed in Fig. 2.

PHYSICAL REVIEW LETTERS **120,** 144502 (2018)

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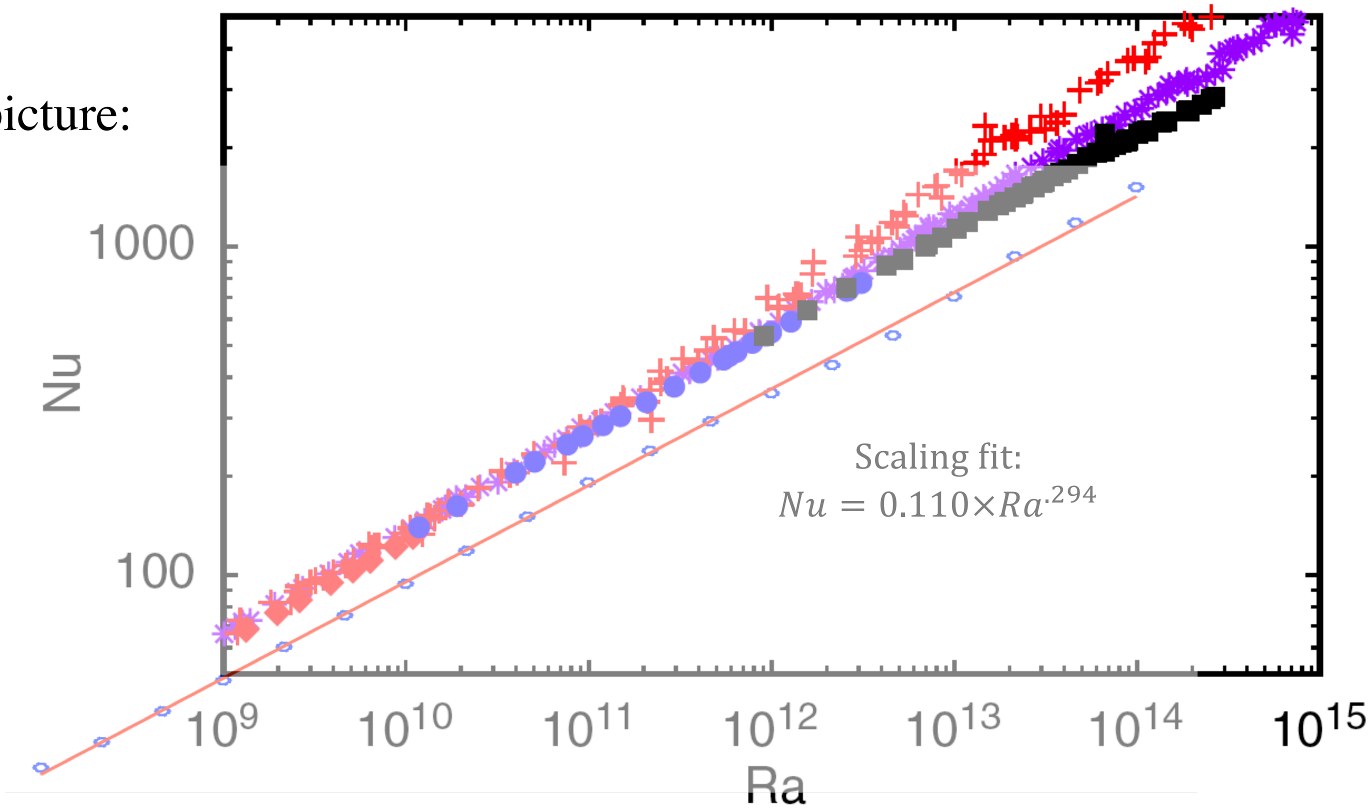
The ***big*** picture:



There are clear trends to the residuals.

PHYSICAL REVIEW LETTERS **120,** 144502 (2018)

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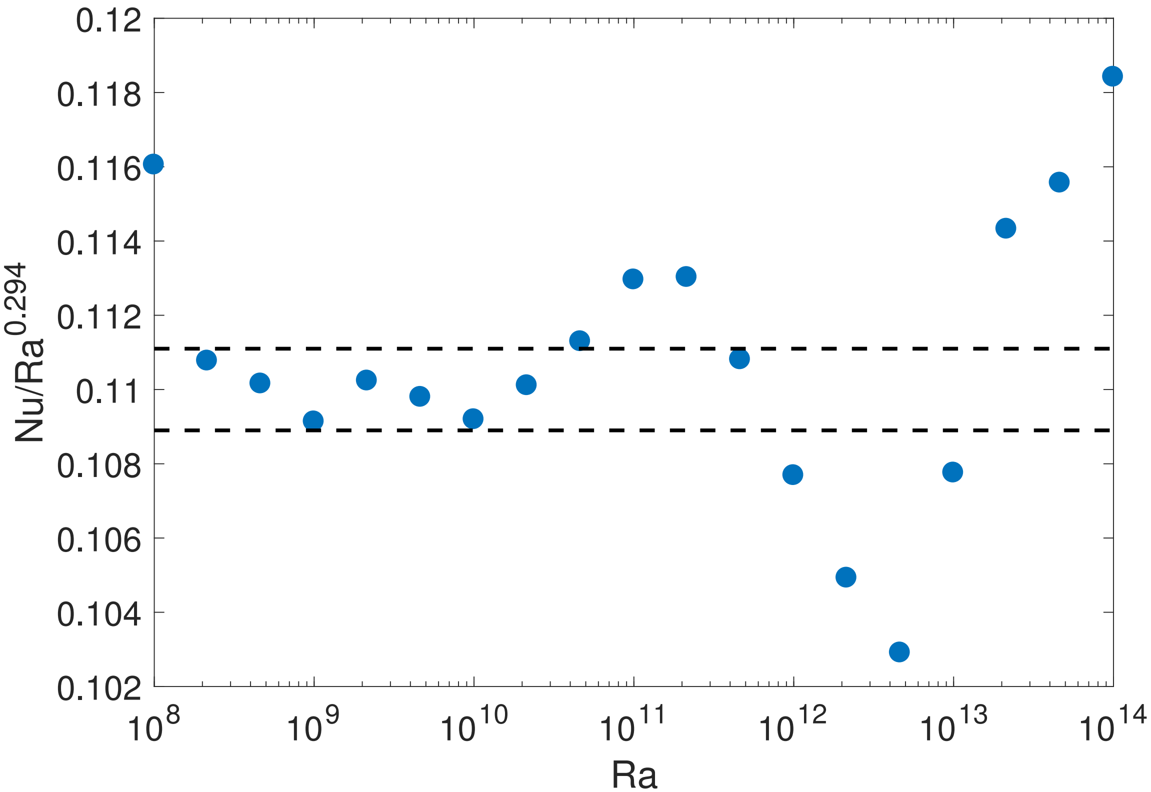
Xiaojue Zhu,[1,*] Varghese Mathai,[1] Richard J. A. M. Stevens,[1] Roberto Verzicco,[2,1] and Detlef Lohse[1,3,†]

Zhu *et al*.
(unpublished)



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PHYSICAL REVIEW LETTERS **120,** 144502 (2018)

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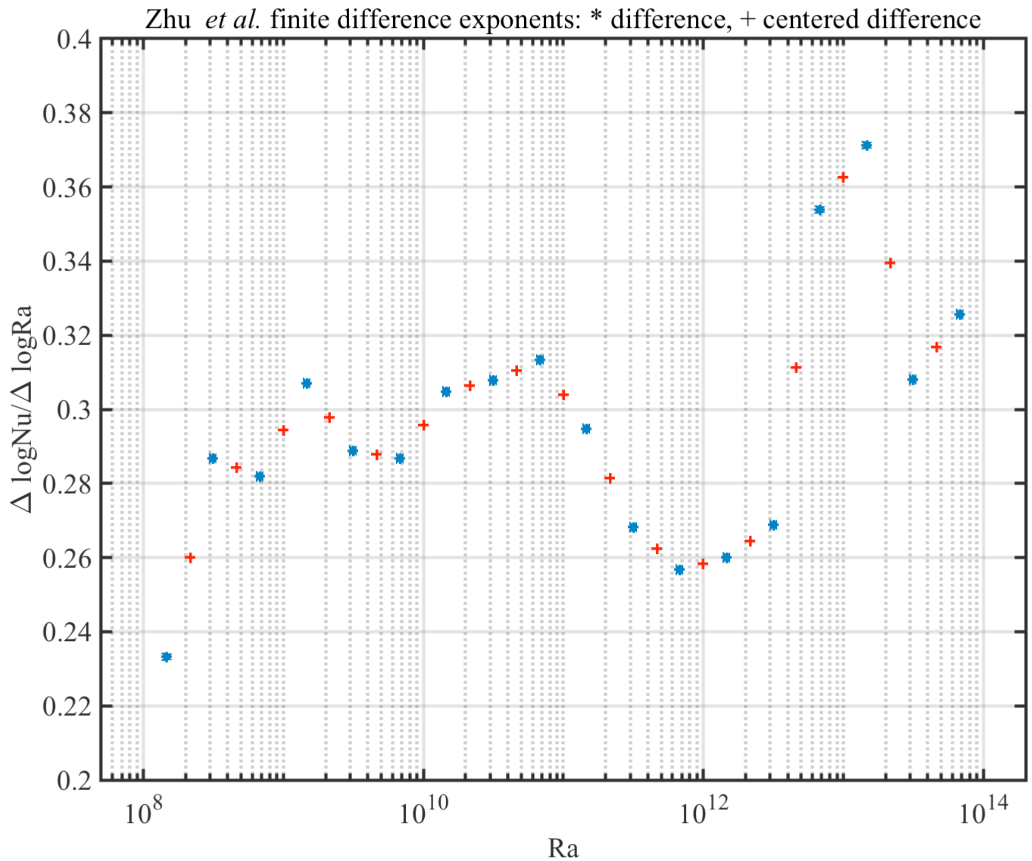
Local slopes differentiating the *Nu*-*Ra* data:



PHYSICAL REVIEW LETTERS **120,** 144502 (2018)

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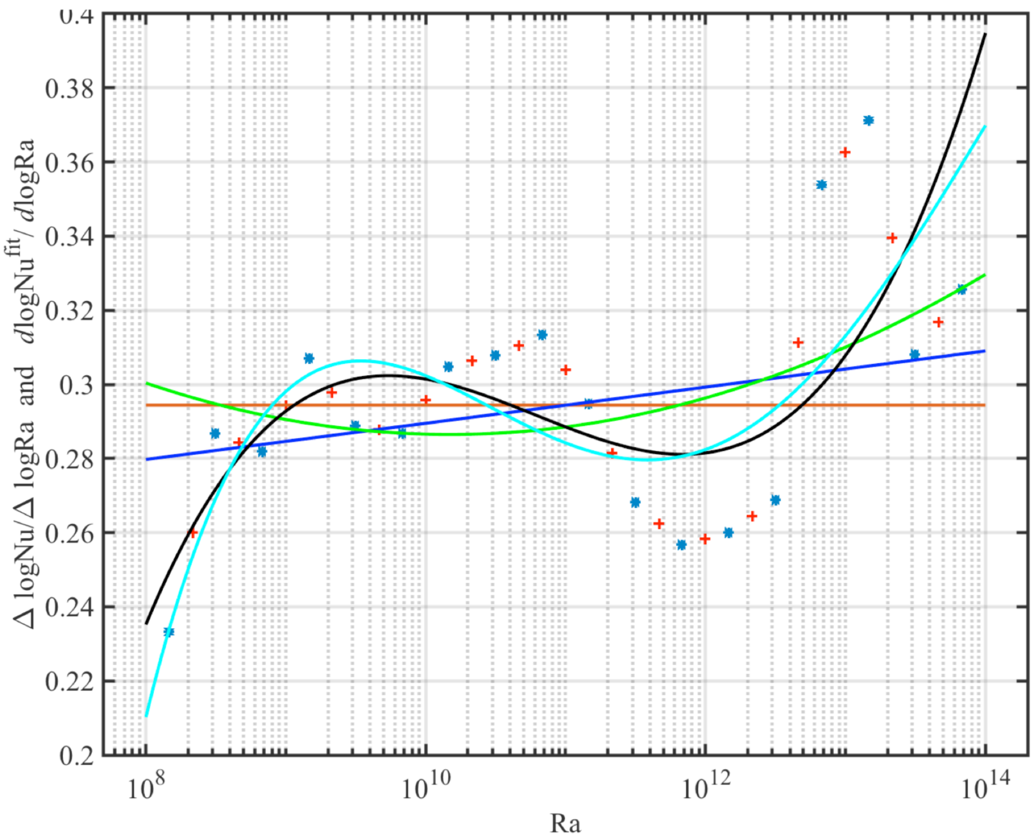
Xiaojue Zhu,[1,*] Varghese Mathai,[1] Richard J. A. M. Stevens,[1] Roberto Verzicco,[2,1] and Detlef Lohse[1,3,†]

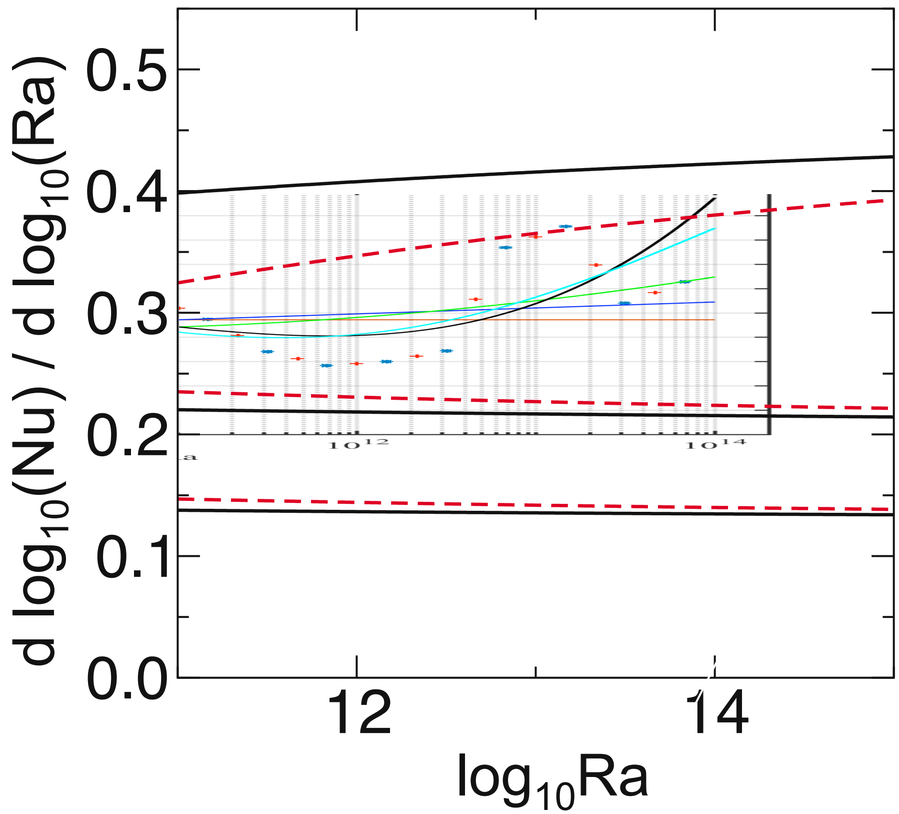
Local slopes differentiating polynomial fits to the *Nu-Ra* data:



Transition to the Ultimate Regime in Two-Dimensional Rayleigh-Bénard Convection

Xiaojue Zhu,[1,*] Varghese Mathai,[1] Richard J. A. M. Stevens,[1] Roberto Verzicco,[2,1] and Detlef Lohse[1,3,†]

Overlay …

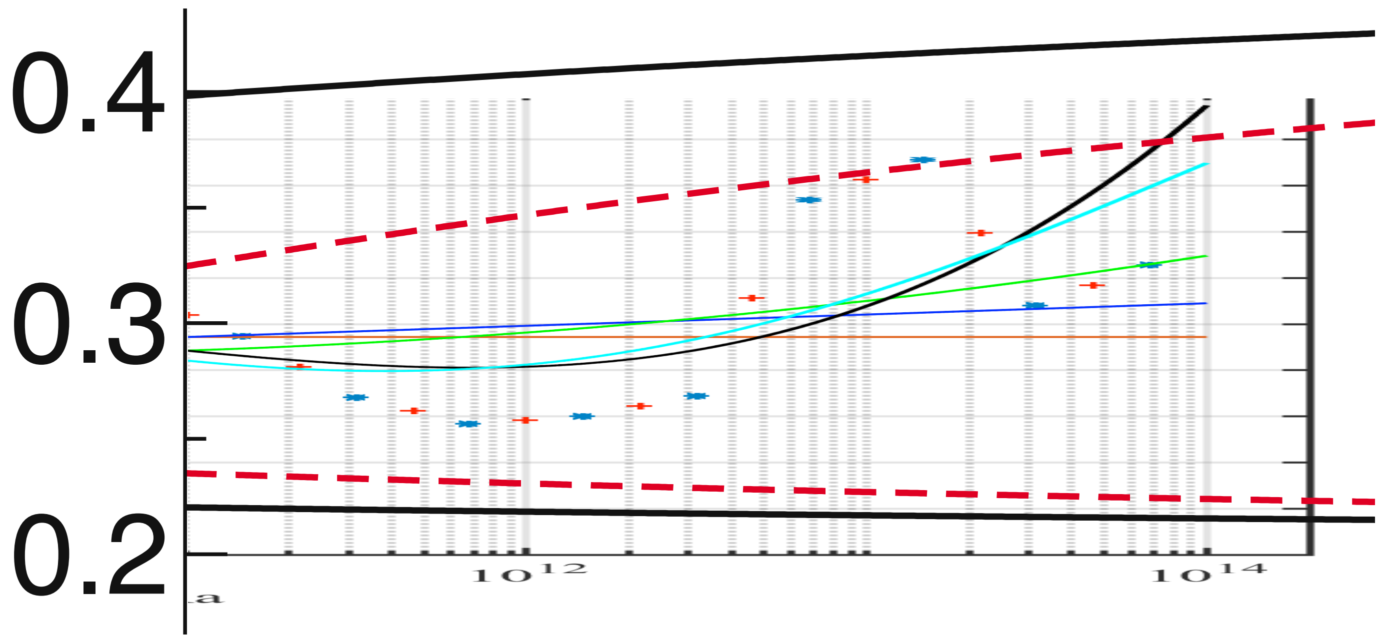


PHYSICAL REVIEW LETTERS **120,** 144502 (2018)

Transition to the Ultimate Regime in Two-Dimensional Rayleigh-Bénard Convection

Xiaojue Zhu,[1,*] Varghese Mathai,[1] Richard J. A. M. Stevens,[1] Roberto Verzicco,[2,1] and Detlef Lohse[1,3,†]

Overlay …
… and zoom in



PHYSICAL REVIEW LETTERS **120,** 144502 (2018)

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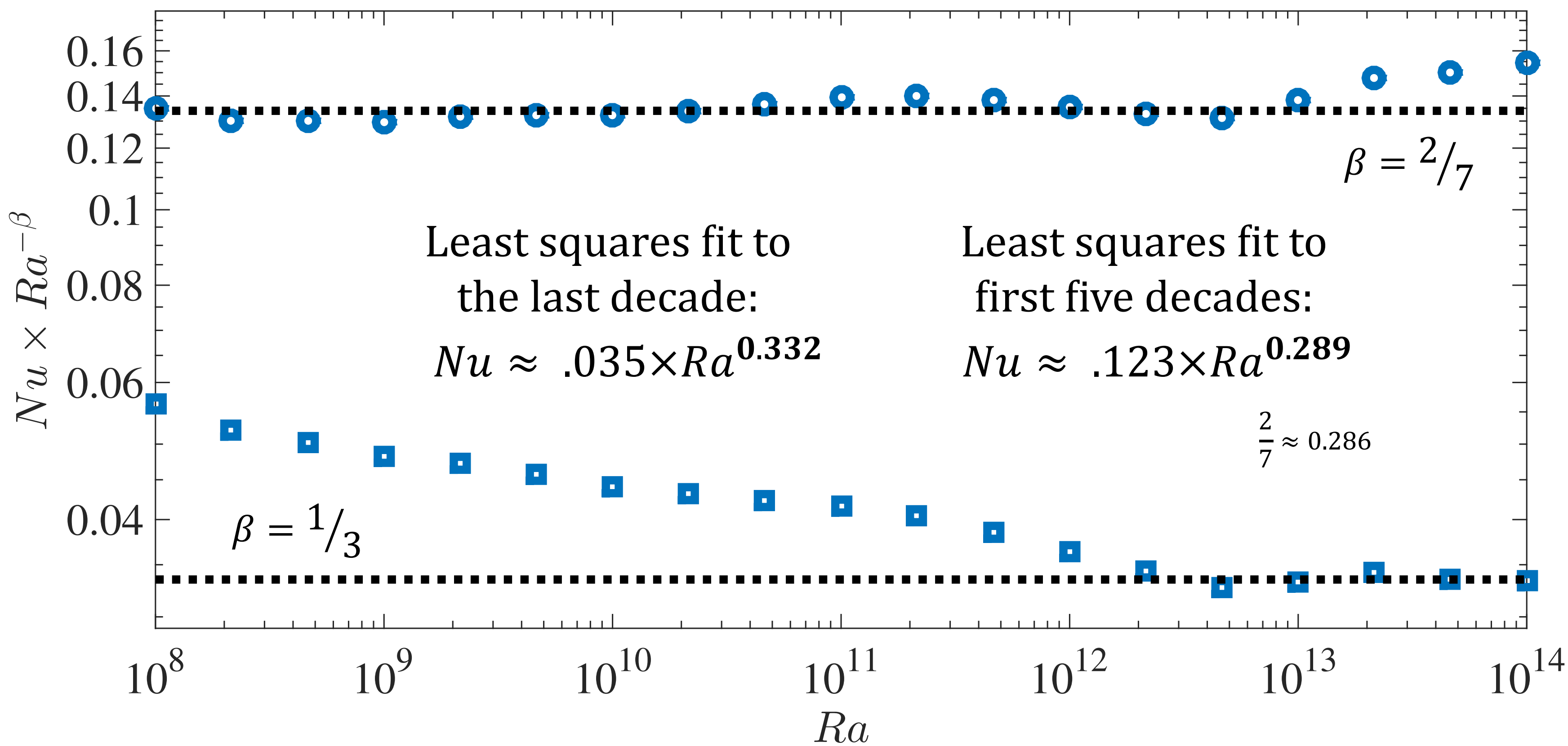
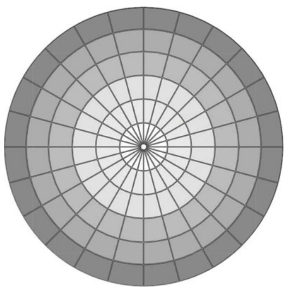


J. Fluid Mech. (2010), vol. 643, pp. 495–507.
doi:10.1017/S0022112009992461

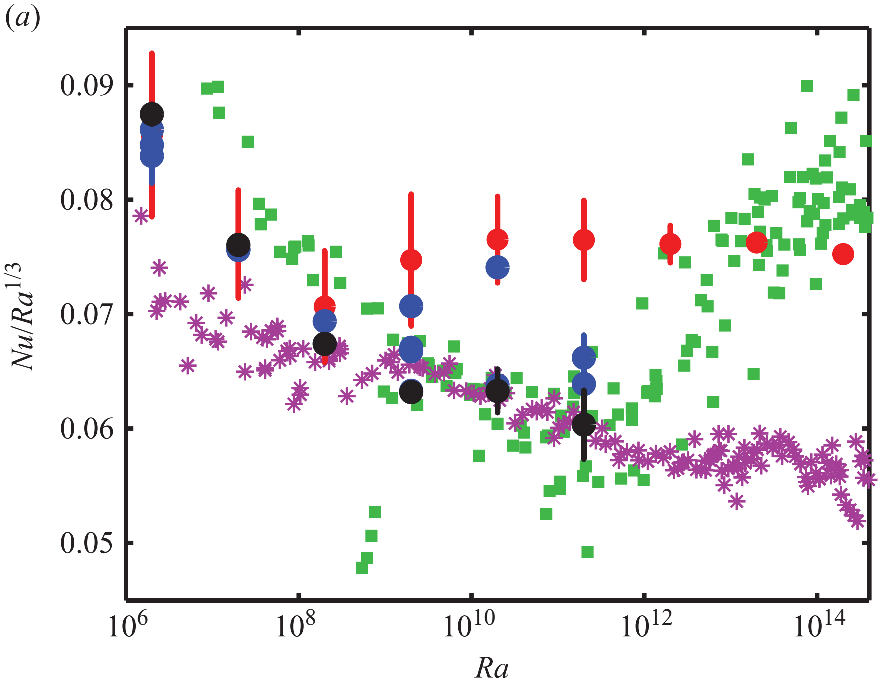
(b)



Radial boundary layer structure and Nusselt number in Rayleigh–Bénard convection

RICHARD J. A. M. STEVENS[1]†, ROBERTO VERZICCO[2] AND DETLEF LOHSE[1]

(a)



Results from direct numerical simulation (DNS) for three-dimensional Rayleigh–Bénard convection in a cylindrical cell of aspect ratio 1/2 and Prandtl number $Pr = 0.7$ are presented. They span five decades of Rayleigh number Ra from 2×10^6 to 2×10^{11}. The results are in good agreement with the experimental data of Niemela *et al.* (*Nature*, vol. 404, 2000, p. 837).

FIGURE 1. (a) Compensated Nusselt number versus the Rayleigh number for $Pr = 0.7$. Purple stars are the experimental data from Niemela *et al.* (2000), and the green squares are the experimental data from Chavanne *et al.* (2001). The DNS results from Verzicco & Camussi (2003) and Amati *et al.* (2005) are indicated in red, and the present DNS results with the highest resolution are indicated by the black dots. When the vertical error bar is not visible the error is smaller than the dot size. The results of the under-resolved simulations of this study are indicated by the blue dots. (b) Sketch of the grid geometry. The cells close to the sidewall are largest, and therefore this region is least resolved.

For what it’s worth ...

There's something happening here
What it is ain't exactly clear
There’s a man with a gun over there
Telling me I got to beware

— *S. Stills* [1]

[1] <https://www.youtube.com/watch?reload=9&v=gp5JCrSXkJY>

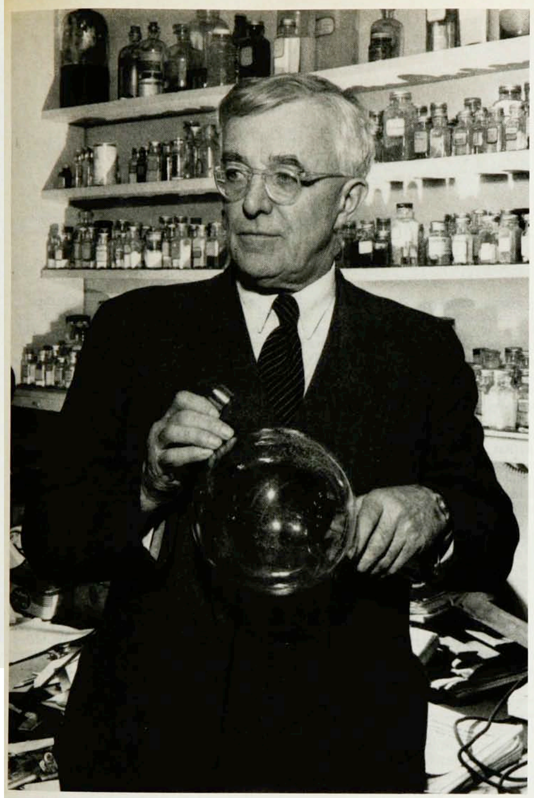
For what it's worth … *Langmuir's lament*

PATHOLOGICAL SCIENCE

Certain symptoms seen in studies of 'N rays' and other elusive phenomena characterize 'the science of things that aren't so.'

Irving Langmuir

Transcribed and edited by Robert N. Hall



Irving Langmuir earned the 1932 Nobel Prize in Chemistry for work dealing with the adsorption of monolayers of molecules on surfaces. He spent his career at the General Electric Company in Schenectady, New York, working there from 1909 until his retirement in 1950. His research included such phenomena as thermionic emission and the properties of liquid surfaces. Over the years, Langmuir also explored the subject of "pathological science," although he never published his investigations on this topic. He died in 1957.

Symptoms of Pathological Science

- ▷The maximum effect that is observed is produced by a causative agent of barely detectable intensity, and the magnitude of the effect is substantially independent of the intensity of the cause.
- ▷The effect is of a magnitude that remains close to the limit of detectability or, many measurements are necessary because of the very low statistical significance of the results.
- ▷There are claims of great accuracy.
- ▷Fantastic theories contrary to experience are suggested.
- ▷Criticisms are met by *ad hoc* excuses thought up on the spur of the moment.
- ▷The ratio of supporters to critics rises up to somewhere near 50% and then falls gradually to oblivion.

Analysis

Theorems

As $Ra \to \infty$,

Are these estimates *sharp*?

$$Nu \le c Ra^{1/2}$$

uniform in Pr, 2D or 3D
no−slip, stress−free and
rough no−slip boundaries

Howard 1963, Busse 1967
Constantin-Doering 1996
Goluskin–Doering 2016

$$Nu \le c' Ra^{1/3} \times (\ln \ln Ra)^{1/3}$$

at $Pr = \infty$ for 2D or 3D
no−slip boundaries

Otto & Seis 2011

J. Fluid Mech. (2014), *vol.* 751, *pp.* 627–662.

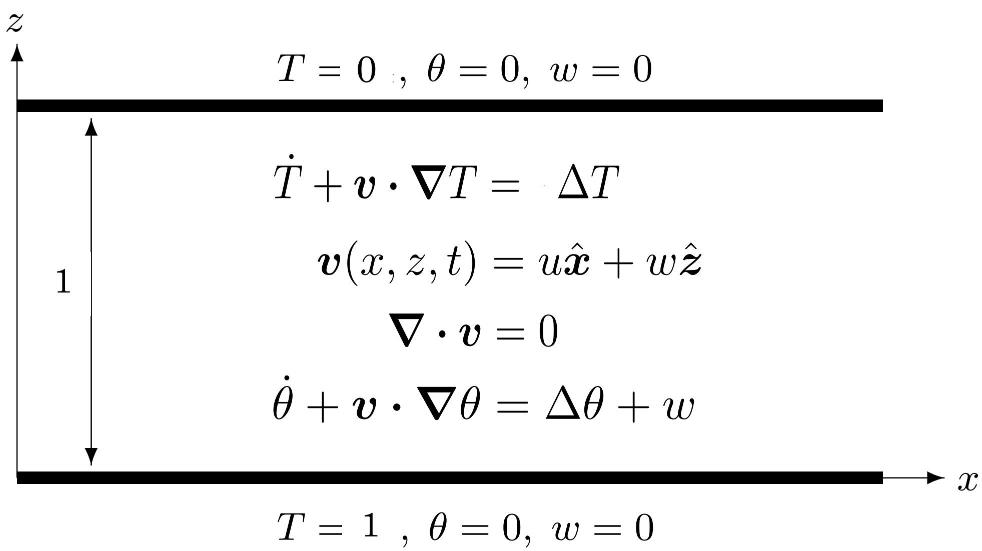
Wall to Wall Optimal Transport

Pedram Hassanzadeh[1], Gregory P. Chini[2], and ~~Charles R.~~ Charlie Doering[3]†

[1]Department of Mechanical Engineering, University of California, Berkeley, CA 94720, USA

[2]Department of Mechanical Engineering, Program in Integrated Applied Mathematics, and Center for Fluid Physics, University of New Hampshire, Durham, NH 03824, USA

[3]Department of Mathematics, Department of Physics, and Center for the Study of Complex Systems, University of Michigan, Ann Arbor, Michigan 48109, USA

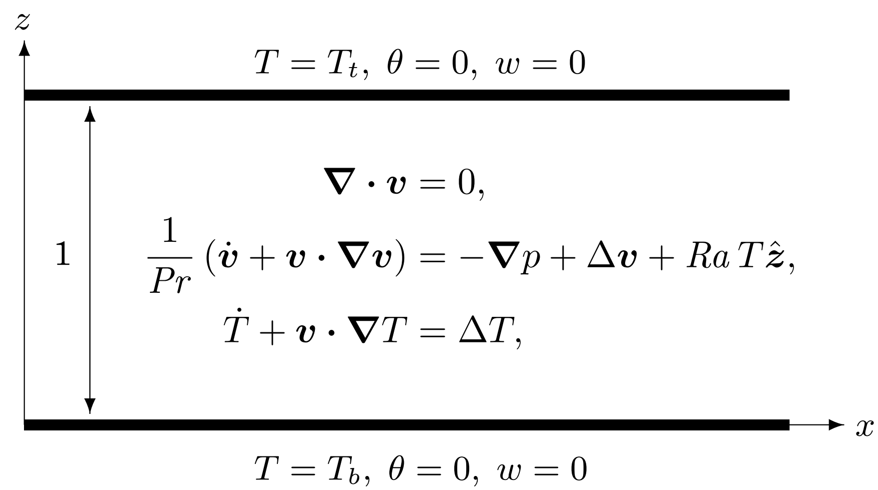


$$Pe^2 = \langle |\boldsymbol{\nabla v}|^2 \rangle$$

$$Nu_{\mathrm{MAX}}(Pe) \equiv \sup_{\Gamma}\{Nu_{\mathrm{max}}(Pe,\Gamma)\}$$

Theorem: $Nu_{MAX} \leq cPe^{2/3}$

Rayleigh-Bénard convection



$$\nabla \cdot \boldsymbol{v} = 0,$$

$$\frac{1}{Pr}\left(\dot{\boldsymbol{v}} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) = -\nabla p + \Delta \boldsymbol{v} + Ra\, T \hat{\boldsymbol{z}}, \qquad \leftarrow \textbf{momentum equation}$$

$$\dot{T} + \boldsymbol{v} \cdot \nabla T = \Delta T,$$

$$\left\langle |\nabla \boldsymbol{v}|^2 \right\rangle = Ra\ \langle wT \rangle$$

$$Pe^2 = Ra\,(Nu - 1)$$

$$\boldsymbol{Nu_{\mathrm{MAX}} \leq c\, Pe^{2/3}} \qquad \Rightarrow \qquad \boldsymbol{Nu_{\mathrm{MAX}} \leq c' Ra^{1/2}}$$

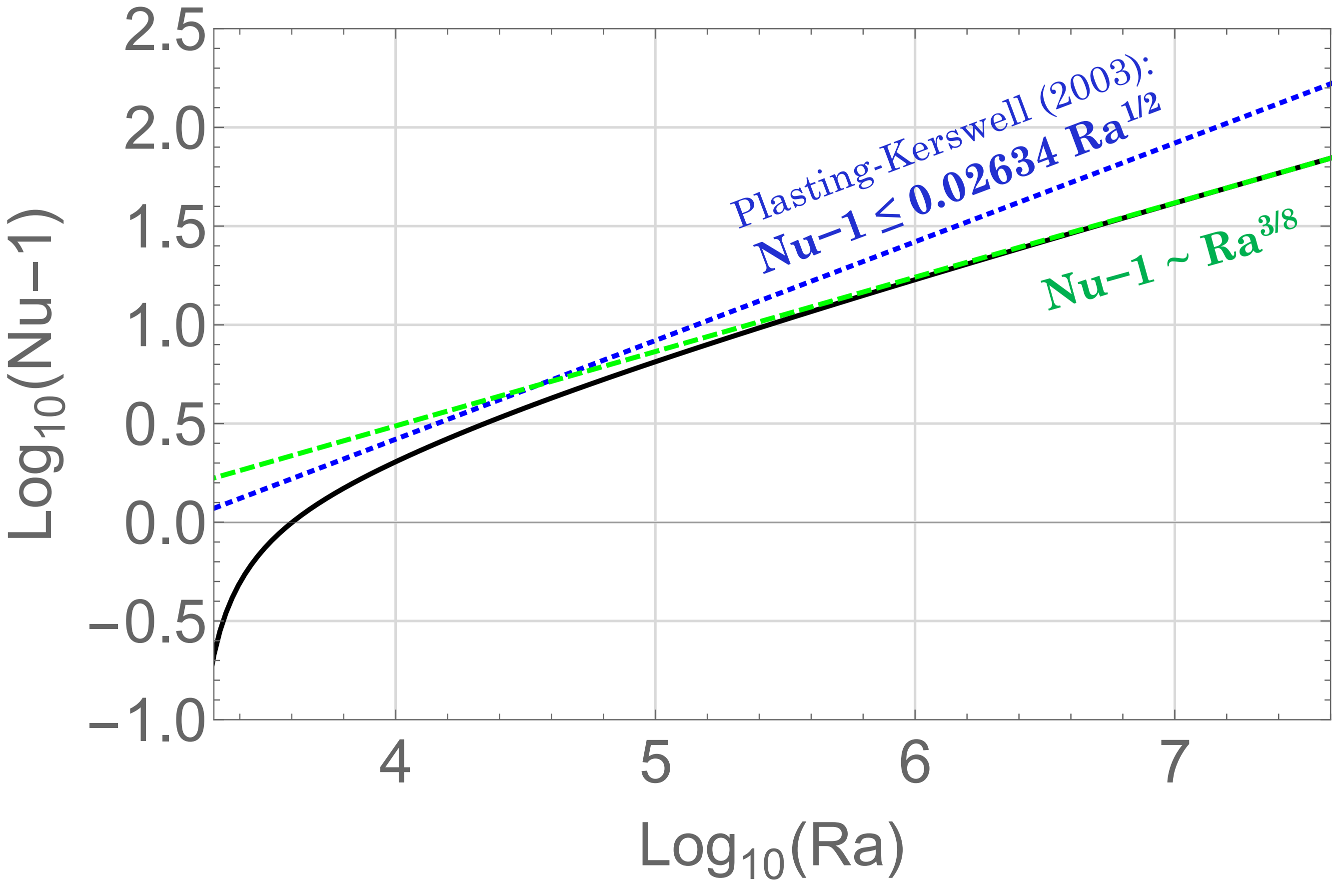
An Optimal Control Approach to Bounding Transport Properties of Thermal Convection

by

Andre N. Souza

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Applied and Interdisciplinary Mathematics)
in the University of Michigan
2016

An Optimal Control Approach to Bounding Transport Properties of Thermal Convection



Optimal Wall-to-Wall Transport by Incompressible Flows

Ian Tobasco[1] and ~~Charles R.~~ Charlie Doering[1,2]
[1]*Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109-1043, USA*
[2]*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1040, USA*
and Center for the Study of Complex Systems, University of Michigan, Ann Arbor, Michigan 48109-1107, USA
(Received 21 December 2016; published 27 June 2017)

Optimal Wall-to-Wall Transport by Incompressible Flows

$$Nu(\mathbf{u}) - 1 = \min_{\eta:\eta|_{\partial\Omega}=0} \left\langle |\nabla\eta|^2 \right\rangle + \left\langle |\nabla\Delta^{-1}(-w + \mathbf{u}\cdot\nabla\eta)|^2 \right\rangle$$

$$= \max_{\xi:\xi|_{\partial\Omega}=0} 2\left\langle w\xi \right\rangle - \left\langle |\nabla\Delta^{-1}\mathbf{u}\cdot\nabla\xi|^2 \right\rangle - \left\langle |\nabla\xi|^2 \right\rangle$$

$$\therefore\ Nu_{MAX} = \max_{\mathbf{u},\xi} \uparrow$$

Optimal Wall-to-Wall Transport by Incompressible Flows

Theorem: there exist *steady incompressible* $2D$ flows $\mathbf{u}_{Pe}$ with

$$\langle |\nabla \mathbf{u}_{Pe}|^2 \rangle \leq Pe^2$$

and

$$Nu(\mathbf{u}_{Pe}) > c\,\frac{Pe^{2/3}}{(\ln Pe)^{4/3}}$$

Recall: $Nu(\mathbf{u}_{Pe}) \leq Nu_{MAX} \leq cPe^{2/3}$

Optimal Wall-to-Wall Transport by Incompressible Flows

Theorem:

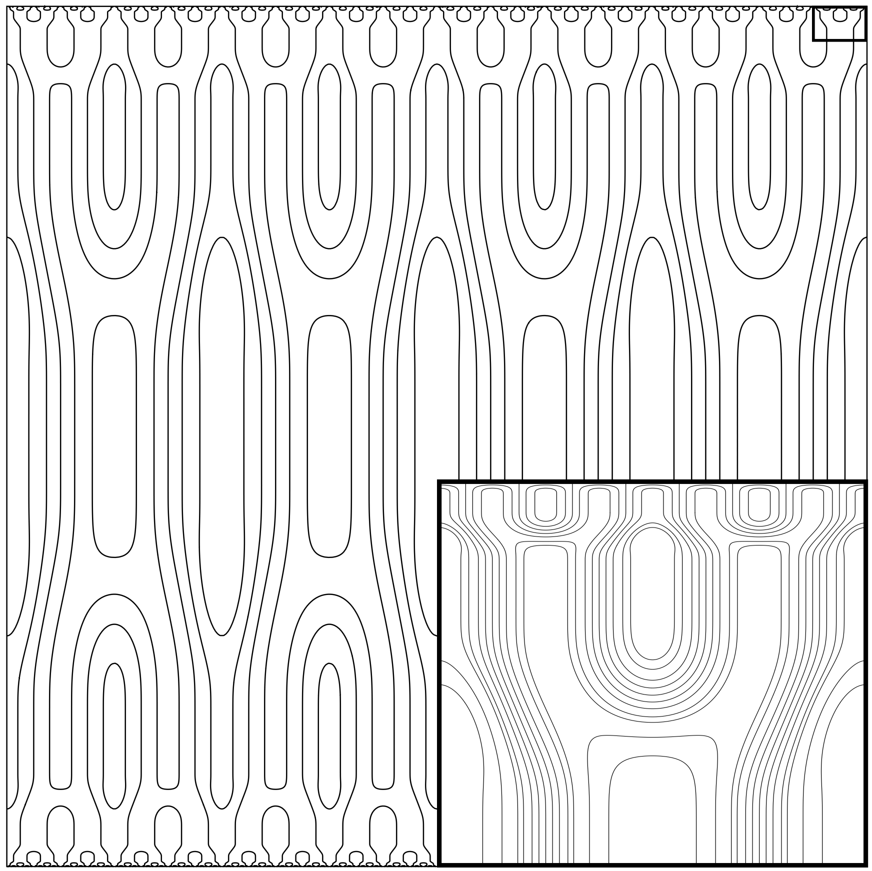
$$c_1 Pe^{2/3}$$

$$\vee$$

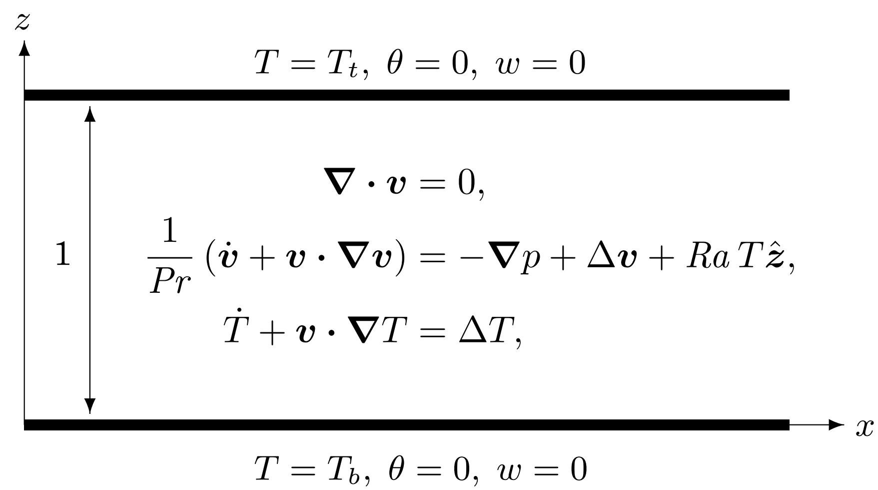
$$\boldsymbol{Nu_{MAX}}$$

$$\vee$$

$$c_2 \frac{Pe^{2/3}}{(\ln Pe)^{4/3}}$$



Rayleigh-Bénard convection

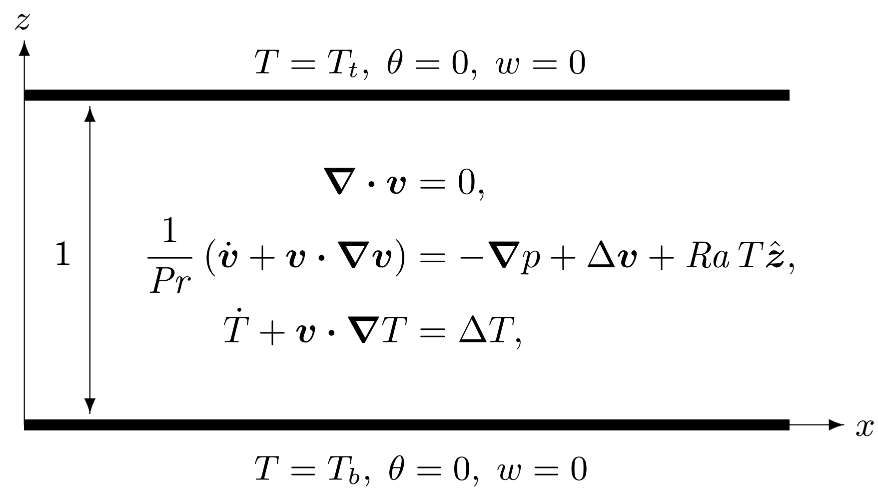


$$\left\langle |\nabla \boldsymbol{v}|^2 \right\rangle = Ra\ \langle wT \rangle$$

$$Pe^2 = Ra\,(Nu - 1)$$

$$c_1 \frac{Pe^{2/3}}{(\log Pe)^{4/3}} \leq \ Nu_{\text{MAX}} \ \leq c_2 Pe^{2/3}$$

Rayleigh-Bénard convection



$$\left\langle |\boldsymbol{\nabla} \boldsymbol{v}|^2 \right\rangle = Ra\ \langle wT \rangle$$

$$Pe^2 = Ra\,(Nu - 1)$$

$$c\,\frac{Ra^{1/2}}{(\log Ra)^2} \le Nu_{\text{MAX}} \le c' Ra^{1/2}$$

Breaking news!

J. Fluid Mech. (2018), *vol.* 851, R4, doi:10.1017/jfm.2018.557

JFM RAPIDS
journals.cambridge.org/rapids

Maximal heat transfer between two parallel plates

Shingo Motoki[1,†], **Genta Kawahara**[1] **and Masaki Shimizu**[1]

[1]Graduate School of Engineering Science, Osaka University, 1-3 Machikaneyama, Toyonaka, Osaka 560–8531, Japan

(Received 18 March 2018; revised 7 July 2018; accepted 9 July 2018)

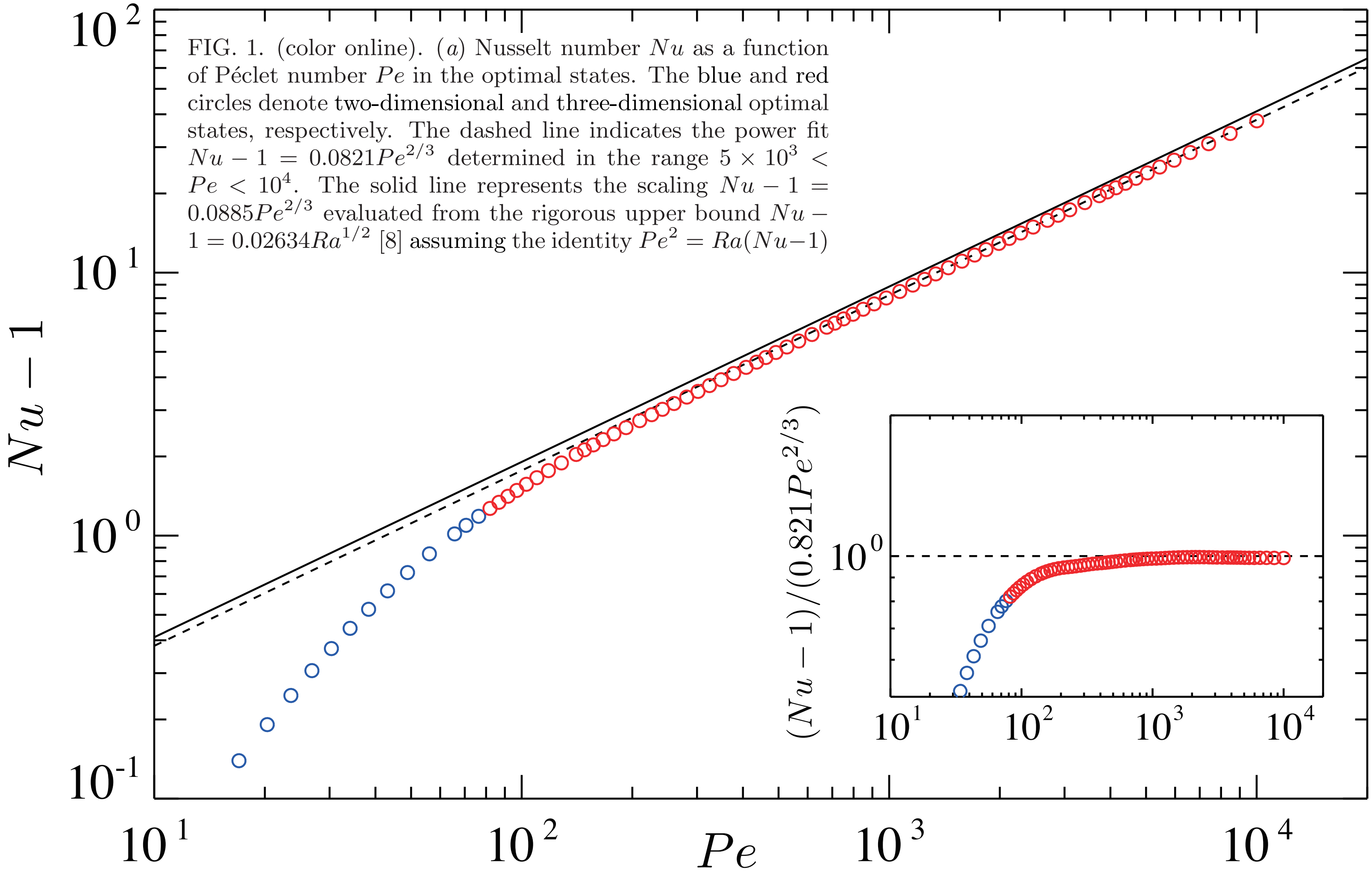


FIG. 1. (color online). (*a*) Nusselt number Nu as a function of Péclet number Pe in the optimal states. The blue and red circles denote two-dimensional and three-dimensional optimal states, respectively. The dashed line indicates the power fit $Nu - 1 = 0.0821Pe^{2/3}$ determined in the range $5 \times 10^3 < Pe < 10^4$. The solid line represents the scaling $Nu - 1 = 0.0885Pe^{2/3}$ evaluated from the rigorous upper bound $Nu - 1 = 0.02634Ra^{1/2}$ [8] assuming the identity $Pe^2 = Ra(Nu-1)$

Maximal heat transfer between two parallel plates

Shingo Motoki, Genta Kawahara and Masaki Shimizu

Graduate School of Engineering Science, Osaka University

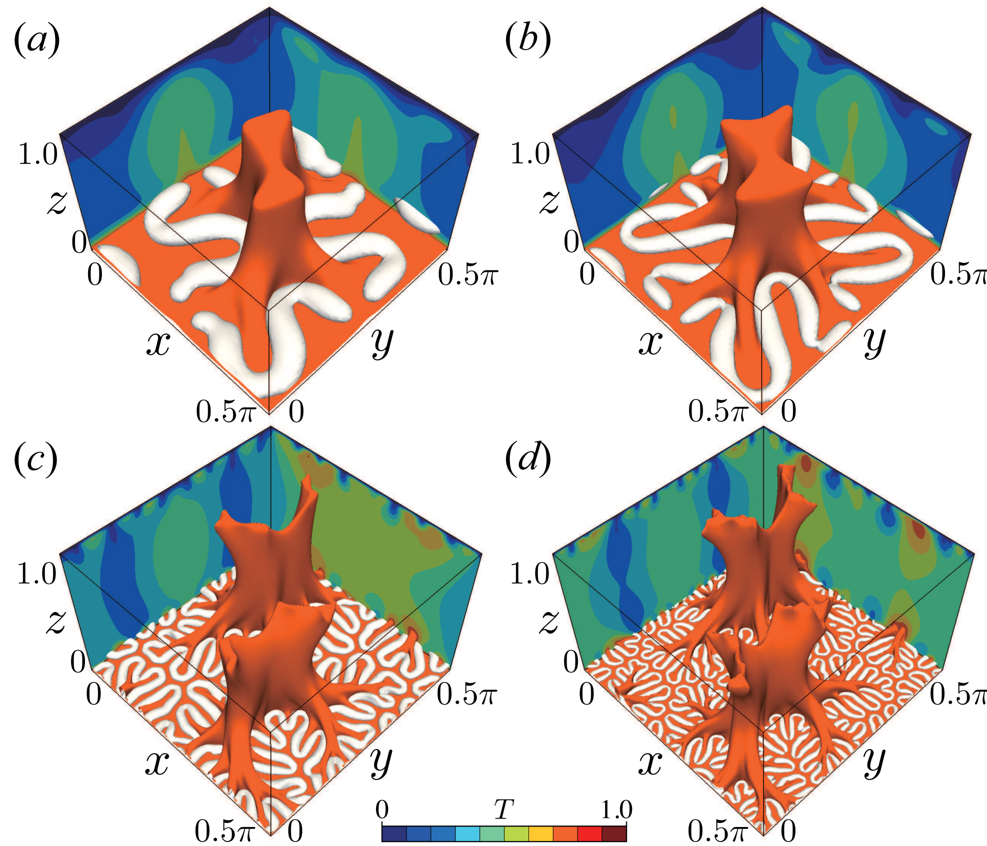
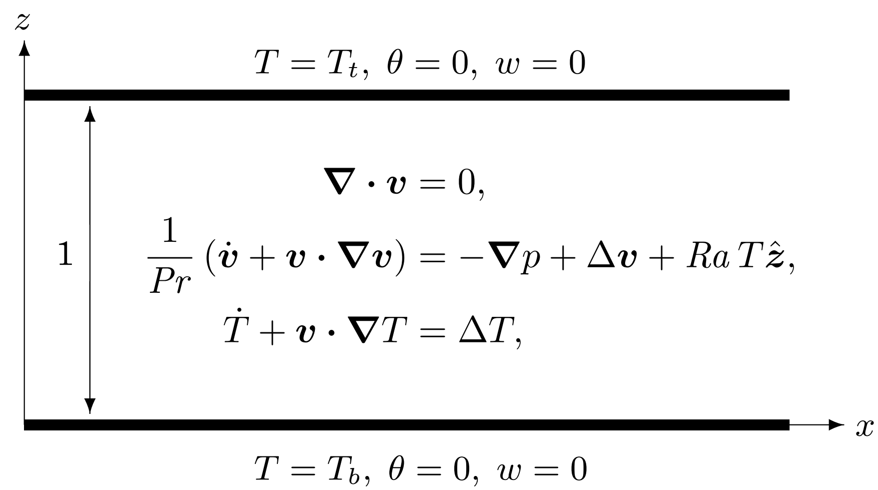


FIG. 3. (color online). (*a*–*d*) Optimal states at Péclet number (*a*) $Pe = 508$, (*b*) $Pe = 1006$, (*c*) $Pe = 5041$ and (*d*) $Pe = 10009$. The orange objects show the isosurfaces of $T = 0.75$. The white tube-like structures are the isosurfaces of (*a*) $Q = 8.0 \times 10^4$, (*b*) $Q = 4.8 \times 10^4$, (*c*) $Q = 1.6 \times 10^7$ and (*d*) $Q = 1.6 \times 10^8$ (note that only those in the lower half of the domain are shown for visualization of the near-wall structures). The contours represent temperature field in the planes $x = 0.5\pi$ and $y = 0$.

Rayleigh-Bénard convection



Can natural buoyancy-driven convection produce $Nu \sim Nu_{MAX} \sim Ra^{1/2}$ (mod logs)?

PRL **106,** 244501 (2011) PHYSICAL REVIEW LETTERS week ending 17 JUNE 2011

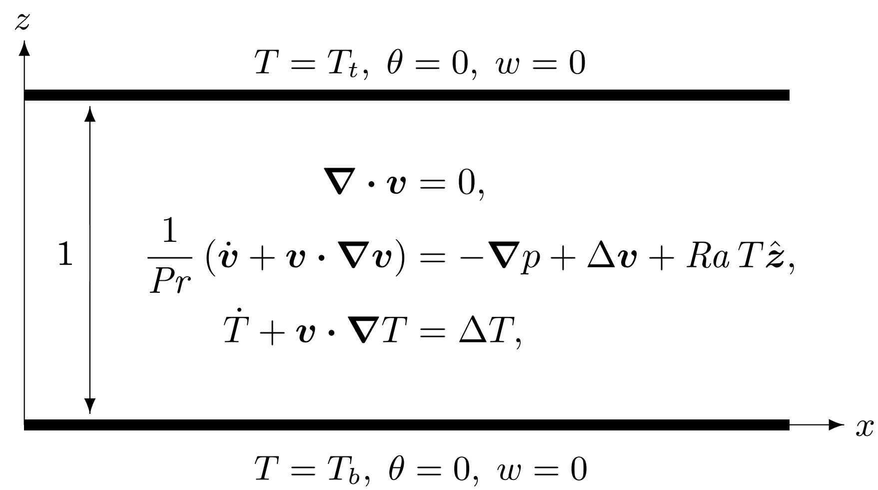
Ultimate State of Two-Dimensional Rayleigh-Bénard Convection between Free-Slip Fixed-Temperature Boundaries

Charlie
Jared P. Whitehead[1,*] and ~~Charles R. Doering~~[1,2,3,†]

[1]*Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109-1034, USA*
[2]*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1040, USA*
[3]*Center for the Study of Complex Systems, University of Michigan, Ann Arbor, Michigan 48109-1107, USA*

Theorem: $$\mathrm{Nu} \le \frac{5^{7/12} \times 3^{3/4}}{2^{13/3}}\,\mathrm{Ra}^{5/12} - \frac{1}{4} \lesssim 0.2891\mathrm{Ra}^{5/12}$$

Rayleigh-Bénard convection



Can natural buoyancy-driven convection produce $Nu \sim Nu_{MAX} \sim Ra^{1/2}$ (mod logs)**?**

$\frac{5}{12} < \frac{6}{12} = \frac{1}{2} \Rightarrow$ **Answer (*FACT*):** ***Not generally!***

For what it's worth …

- Competing theories for the behavior of Nu as $Ra \to \infty$:
- $Nu \sim Ra^{1/3}$

  … with some Pr dependence *?*
- $Nu \sim (Pr \cdot Ra)^{1/2}$

  … perhaps with logarithmic corrections *?*

- Experiments and DNS are inconclusive:

  One experiment (the ***French***) suggests the latter …

  but others (***American***, ***German***) don't, rather supporting the former.

  Both 2D & 3D DNS *and* 3D experiments suggest Rayleigh number transitions to $Ra^{\sim 1/3}$.

- OTOH ***there exist incompressible flows of proper intensity capable of*** $\boldsymbol{Nu \sim Ra^{1/2}}$.

- But it isn't clear if natural buoyancy is capable of sustaining such flows

  … and in at least one case, evidently ***not***.

For what it's worth ... **THANKS FOR YOUR ATTENTION**