GRAPH EDGE COLORING VIZING'S THEOREM AND GOLDBERG'S CONJECTURE

Michael Stiebitz Diego Scheide Bjarne Toft Lene M. Favrholdt



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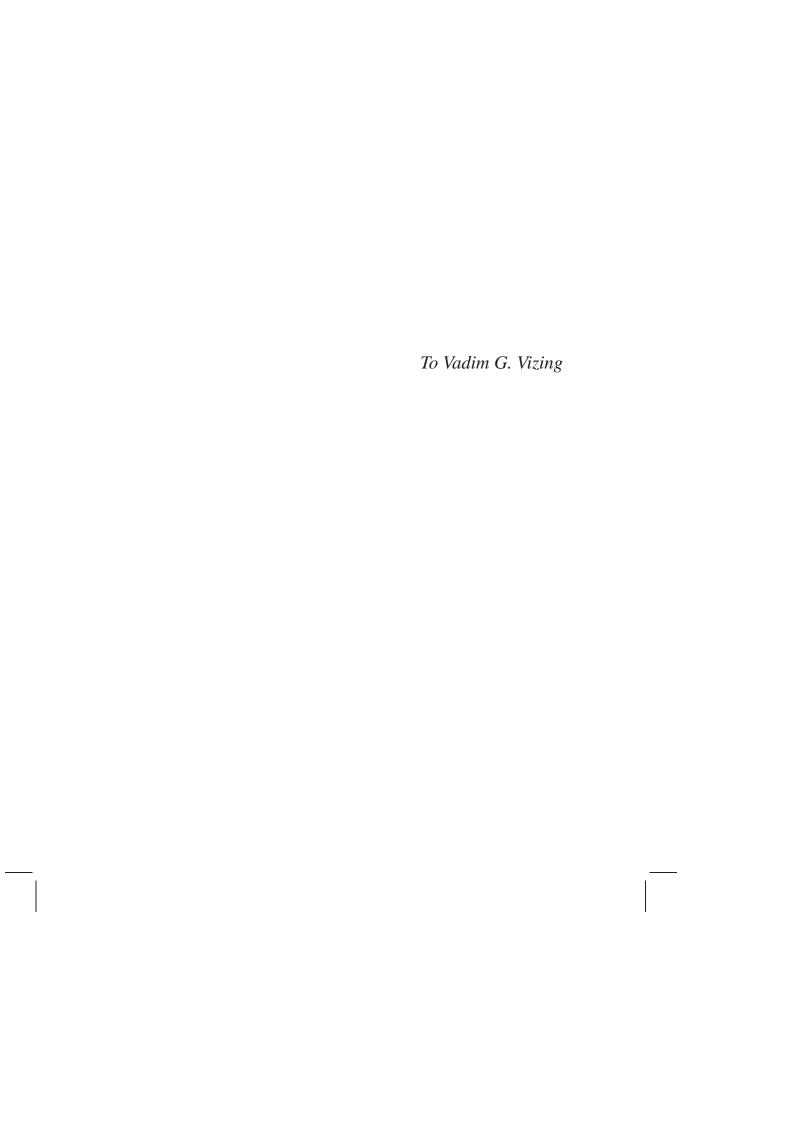
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